Week 2

Defining Probability

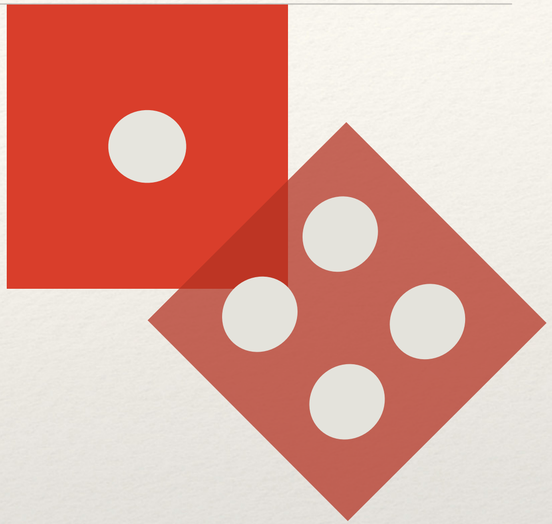
ST 314
Introduction to Statistics for Engineers

Oregon State University

Introducing Ideas in Probability

Suppose we roll two fair six sided dice.

How many possible outcomes are there for rolling the two dice? What are the outcomes?



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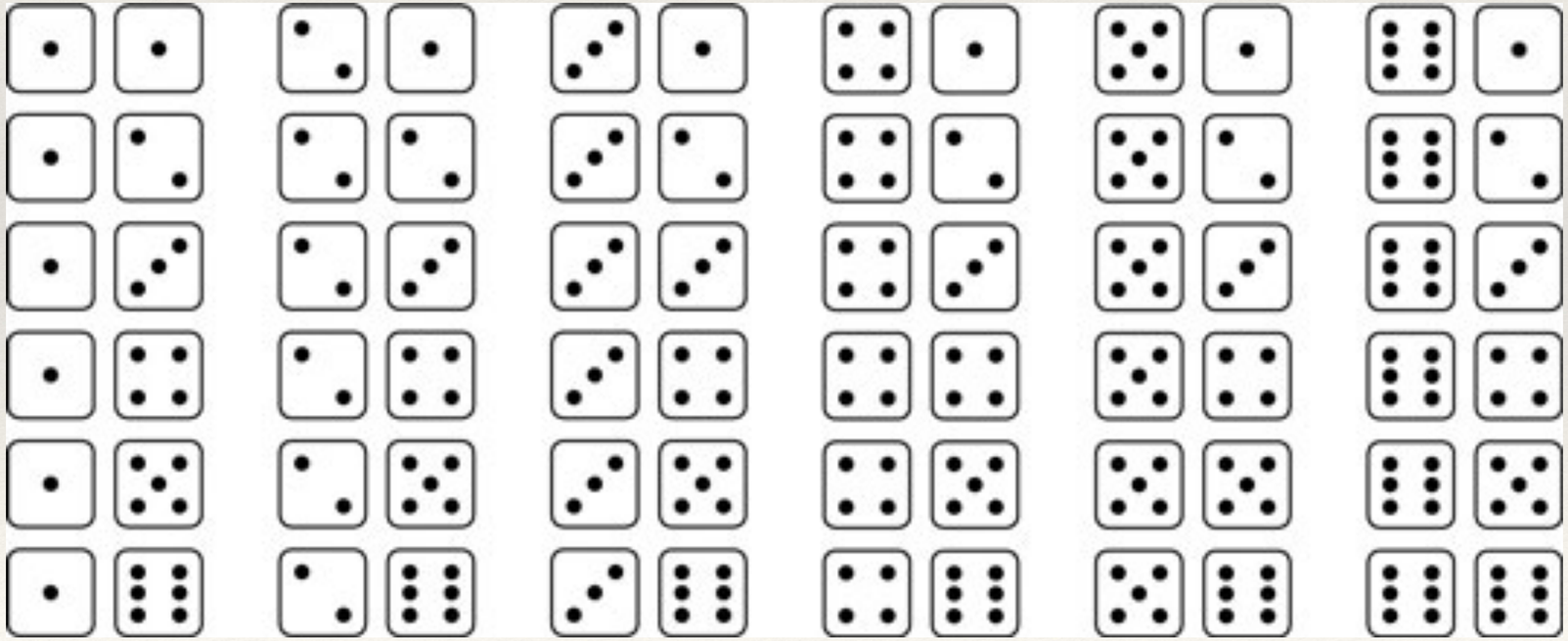
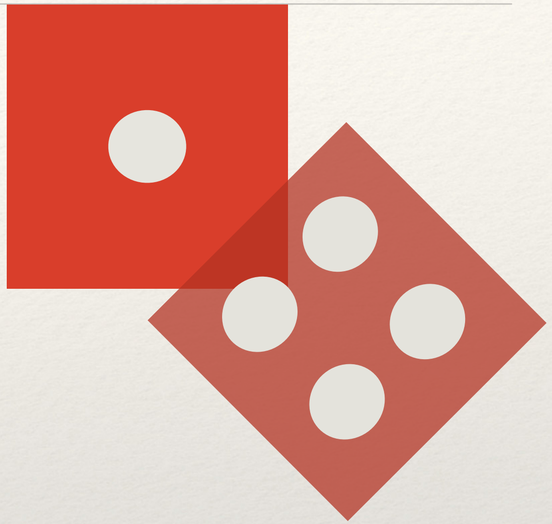


Introducing Ideas in Probability

Suppose we roll two fair six sided dice.

2. How many ways can we roll a double?



$$P(\text{rolling doubles}) = \frac{6}{36} = \frac{1}{6}$$

Introducing Ideas in Probability

Suppose we roll two fair six sided dice.

3. What is the chance that the dice dots sum to seven?

6 ways to roll sum of 7
out of 36 possible outcomes

$$\frac{6}{36} = 0.167$$

Setting: sum when roll 2 dice

Definitions

- Sample space, S set of all possible outcomes in a setting of interest
- Event subset of the sample space for which we are interested in finding the probability
- Probability of event A

$$P(A) = \frac{\text{size } A}{\text{size } S}$$

Basic Probability Models

A table, function, or graph consisting of all events along with probabilities for each outcome is a probability model or distribution.

X	2	3	4	5	6	7	8	9	10	11	12
P(X)	0.028	0.056	0.083	0.111	0.139	0.167	0.139	0.111	0.083	0.056	0.028

outcome for the sum of 2 dice

probabilities

$\frac{6}{36}$

$\frac{3}{36}$

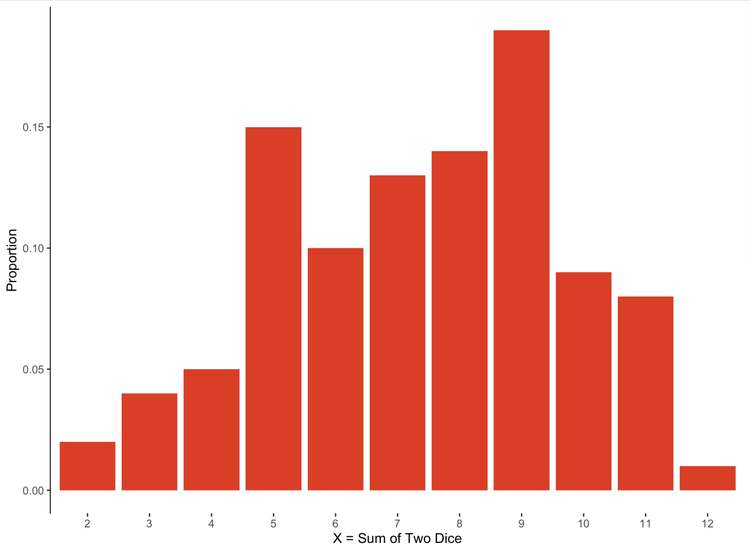
Rules for Probability Distributions

- Rule #1 outcomes must be disjoint
- Rule #2 Each probability must be between 0 and 1
- Rule #3 All probabilities in distribution must sum to 1 $P(S) = 1$

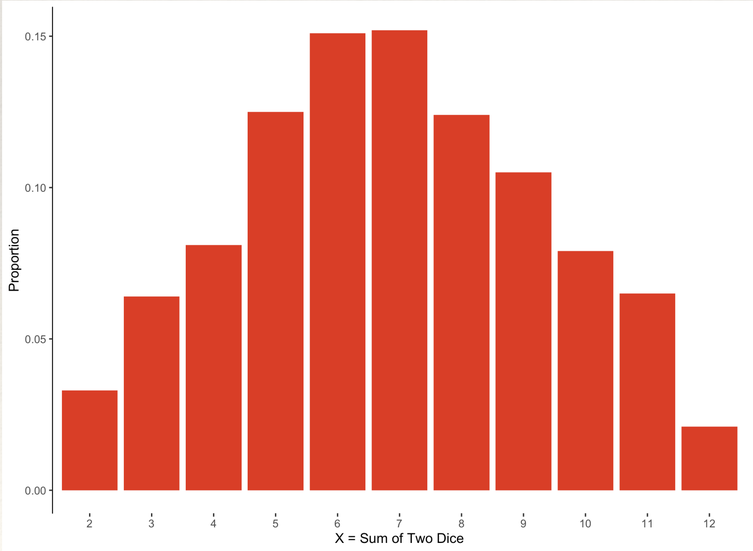
Law of Large Numbers

The **law of large numbers** tells us the more observations that are collected, the proportion of occurrences of an outcome will converge to the true prob. of that outcome

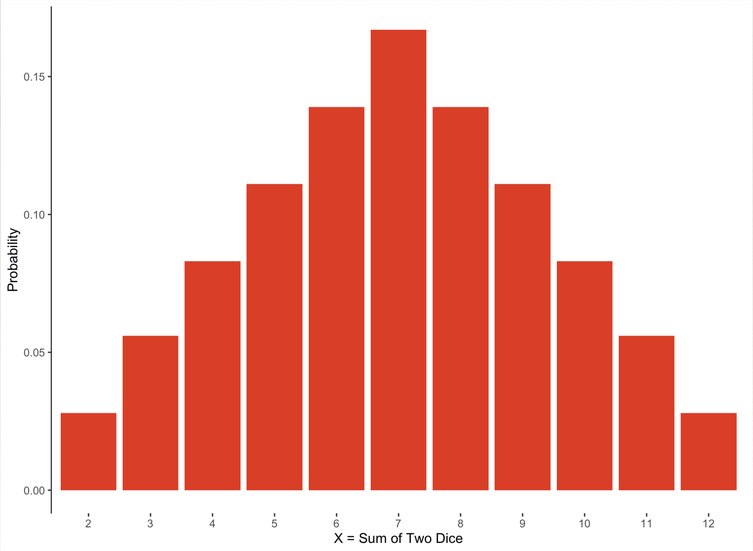
100 Rolls of Two Dice



1000 Rolls of Two Dice



Probability Model: Sum of Two Dice



$P(A) = \frac{\text{size } A}{\text{size } S}$

Example #1

$P(\text{Boy}) = 0.5$

$P(\text{Girl}) = 0.5$

Suppose a couple tells you they have three children. What is the probability that exactly two of them are boys?

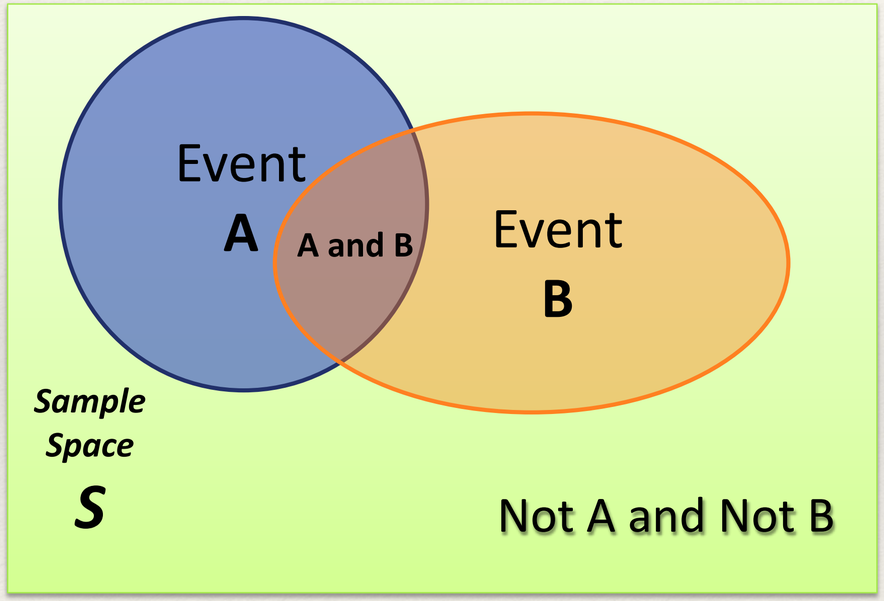
Sample Space

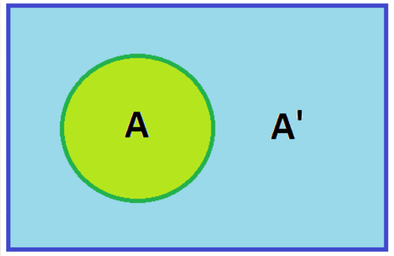
BBB
BBG
BGB
GBB
GGB
GBG
BGG

GGG

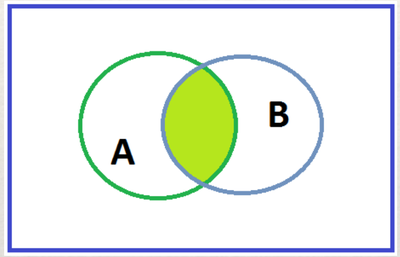
$\frac{3}{8}$

The Venn Diagram

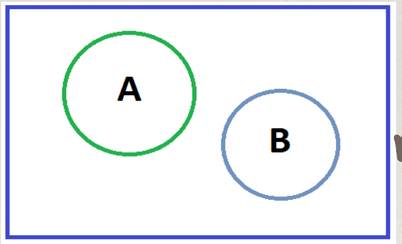




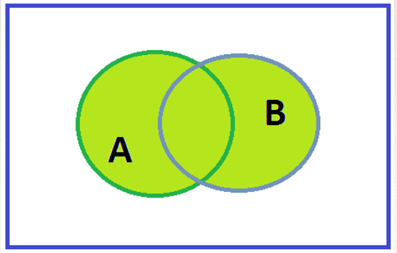
An event is the subset of all the possible outcomes in a random experiment or population called a sample space. The complement of an event A, denoted A′, is the set of all outcomes in the sample space that are not contained in A.



The intersection of events A and B, denoted $A \cap B$ is the event consisting of all outcomes that are in both A and B.



If A and B have no common outcomes, the events are said to be disjoint or mutually exclusive. Therefore, the intersection is an empty set, $A \cap B = \emptyset$.

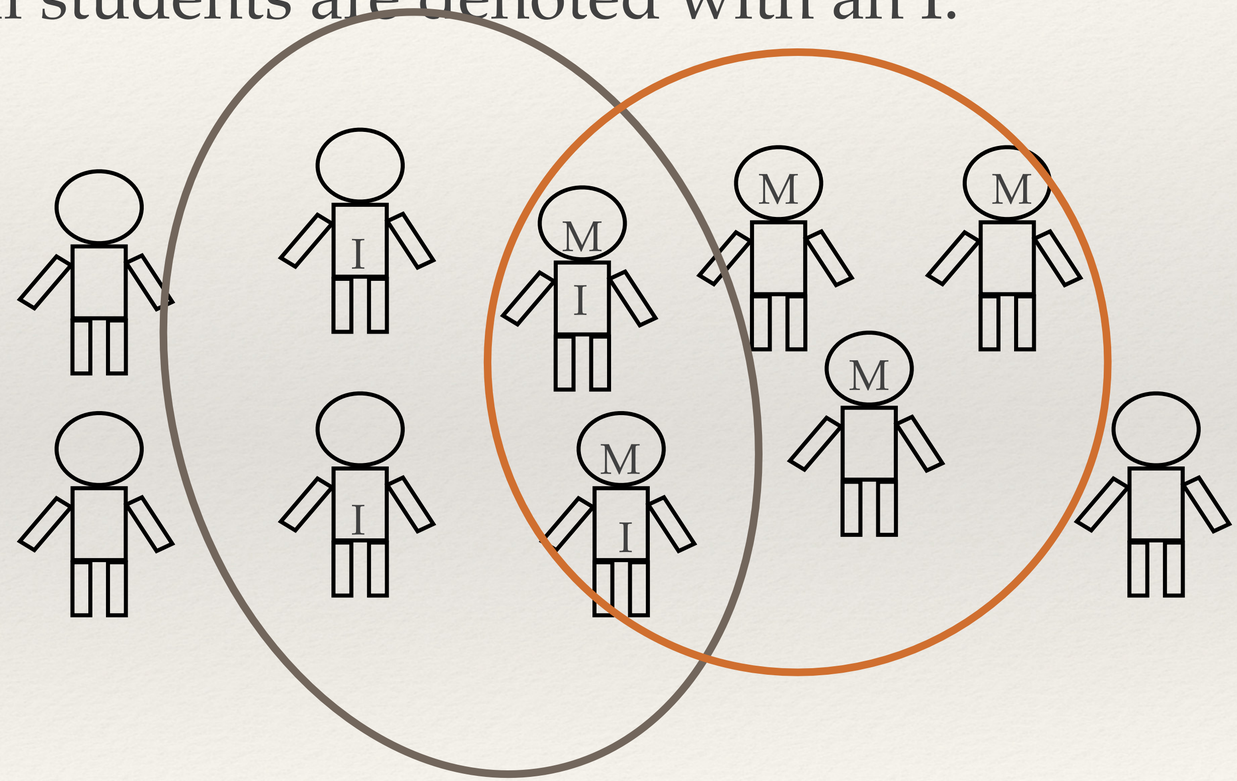


The union of the two events A and B, denoted $A \cup B$ is the event consisting of the elements in A, B, and both A and B.

Set Theory & Diagrams

Consider a club with 10 engineering students. Mechanical engineering students are denoted with an M. International students are denoted with an I.

- $P(M) = \frac{1}{2} = 0.5$
- $P(I) = \frac{4}{10} = \frac{2}{5} = 0.4$
- $P(I \cap M) = \frac{2}{10} = \frac{1}{5} = 0.2$
- $P(I' \cap M') = \frac{3}{10} = 0.3$



The Addition Rule

The **General Addition Rule** calculates the probability of the union between two events A and B, the chance that either A or B or both occur.

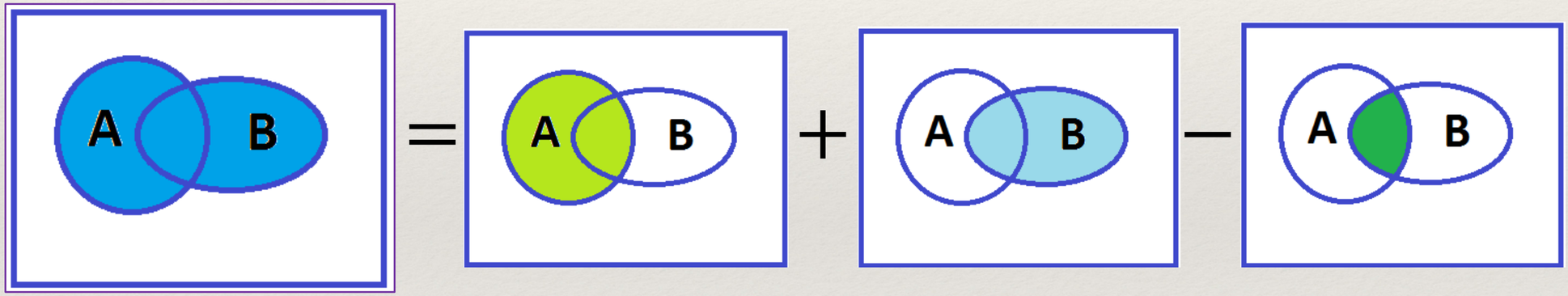
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

If and only if A and B are disjoint, $P(A \cap B) = 0$, the rule simplifies to

$$P(A \cup B) = P(A) + P(B)$$

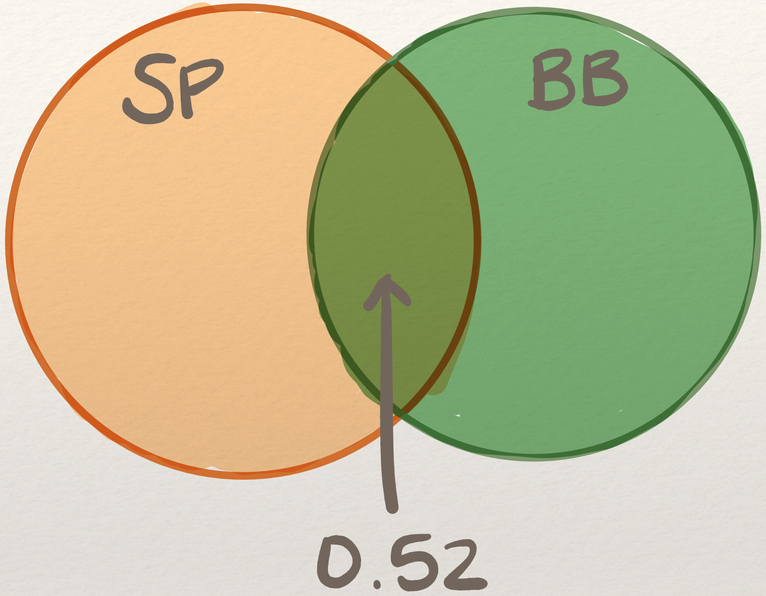
The Addition Rule

$P(A \cup B)$ $\quad$ $P(A)$ $\quad$ $P(B)$ $\quad$ $P(A \cap B)$



Probability Using the Addition Rule

A recent study from Pew Research Center indicates 77% of adults in the United States use a smartphone (SP) to access the internet, while 65% access the internet at home with a broadband (BB) service and 52% access with both a smartphone and broadband.



Calculate $P(SP \cup BB)$.

$$P(SP \cup BB) = P(SP) + P(BB) - P(SP \cap BB)$$

$$= 0.77 + 0.65 - 0.52$$

$$= 0.9$$

Venn Diagram Example Problem

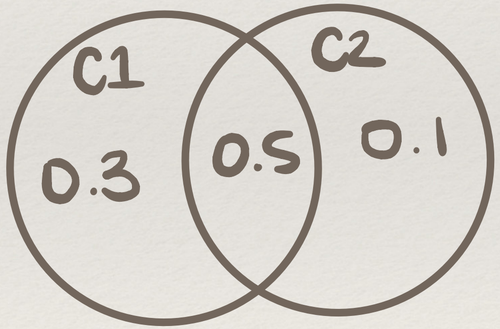
Kai is about to graduate with an industrial engineering degree. After being interviewed at two companies he likes, his probability of getting an offer from company 1 is 0.8 and from company 2 is 0.6. He believes the chance he will get an offer from both companies is 0.5.

Draw a Venn Diagram for the scenario and calculate the probability Kai gets offered at least one of the jobs.

$P(C_1) = 0.8$

$P(C_2) = 0.6$

$P(C_1 \cap C_2) = 0.5$



"At least 1" $= C_1 \cup C_2$

$P(C_1 \cup C_2) = 0.8 + 0.6 - 0.5 = 0.9$