

Single Population Confidence Interval and Hypothesis Test Practice Problems

- Do a majority of US adults believe raising the minimum wage will help the economy, or is there a majority who do not believe this? A Rasmussen Reports survey of 1,000 US adults found that 42% believe it will help the economy. Construct a 95% confidence interval and conduct a hypothesis test at the $\alpha = 0.05$ significance level to help answer the research question.

$$0.42 \pm 1.96 \sqrt{\frac{0.42(1 - 0.42)}{1000}} = (0.389, 0.451)$$

$$H_0 : p = 0.5 \text{ vs. } H_A : p \neq 0.5$$

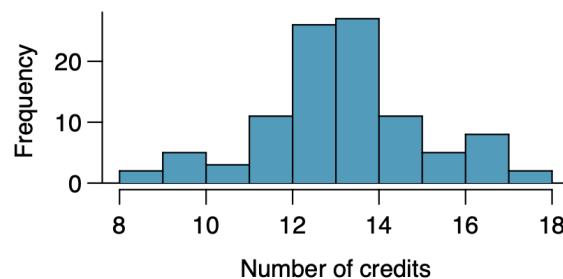
Conditions met!

$$z = \frac{0.42 - 0.5}{\sqrt{\frac{0.5^2}{1000}}} = -5.060$$

$$\text{p-value} = 4.1924 \times 10^{-7}$$

There is convincing evidence that the proportion of US adults that believe raising the minimum wage will help the economy is different from 0.5. The null hypothesis is rejected at the $\alpha = 0.05$ significance level. We are 95% confident that the true proportion of US adults that believe that raising minimum wage will improve the economy is between 0.389 and 0.451 with a point estimate of 0.42.

- A college counselor is interested in estimating how many credits a student typically enrolls in each semester. The counselor decides to randomly sample 100 students by using the registrar's database of students. The histogram below shows the distribution of the number of credits taken by these students. Sample statistics for this distribution are also provided.



Min	8
Q1	13
Median	14
Mean	13.65
SD	1.91
Q3	15
Max	18

- Construct a 90% confidence interval for the average number of credits taken per semester at this college. Interpret the interval.

$$13.65 \pm 1.66 \frac{1.91}{\sqrt{100}} = (13.33, 13.98)$$

We are 90% confident that the average number of credit hours taken per semester by students at this college is between 13.33 and 13.98 with a point estimate of 13.65.

- b. The college expects that students enroll in 13 credit hours per semester on average. Perform a hypothesis test to test if this college's average credit load is more than 13 per semester.

$$H_0 : \mu = 13 \text{ vs. } H_A : \mu > 13$$

$$t = \frac{13.65 - 13}{\frac{1.91}{\sqrt{100}}} = 3.403$$

$$\text{p-value} = 0.0005$$

There is convincing evidence that the average number of credit hours taken per semester by students at this college is greater than 13. We reject the null hypothesis at the 0.1 significance level. (I used the significance level $\alpha = 0.1$ since the confidence level was 90%.)

3. Georgianna claims that in a small city renowned for its music school, the average child takes less than 5 years of piano lessons. We have a random sample of 20 children from the city, with a mean of 4.6 years of piano lessons and a standard deviation of 2.2 years.

- a. Evaluate Georgianna's claim (or that the opposite might be true) using a hypothesis test.

$$H_0 : \mu = 5 \text{ vs. } H_A : \mu < 5$$

$$t = \frac{4.6 - 5}{\frac{2.2}{\sqrt{20}}} = -0.813$$

$$\text{p-value} = 0.213$$

There is no evidence to suggest that the average child takes less than 5 years of piano lessons. We fail to reject the null hypothesis at the 0.01 significance level. (This p-value would be rejected at most commonly used significance levels since it is so large.)

- b. Construct a 99% confidence interval for the number of years students in this city take piano lessons, and interpret it in context of the data.

$$4.6 \pm 2.861 \frac{2.2}{\sqrt{20}} = (3.193, 6.007)$$

We are 99% confident that the average child in this city takes between 3.193 and 6.007 years of piano lessons with a point estimate of 4.6 years.

- c. Assess the conditions for inference required to rely on the results of the confidence interval and hypothesis test.

The sample size is random; however, it is smaller than 30. Since we don't know anything about the population of interest and don't have a visualization of these data, it's hard to say if the distribution of test statistics from this population actually follows the t distribution we expect.