Week 2

Conditional and Independent Probabilities

ST 314
Introduction to Statistics for Engineers

Oregon State University

Example #1

You select a card from a standard deck of 52 cards. It's a queen. You decide to select a second card *without* replacing the first back to the deck.

a. What is the probability the second card is a queen?

$$\frac{3}{51} = 0.059$$

b. Are the events described above disjoint?

NO

c. Are the events described above independent?

No

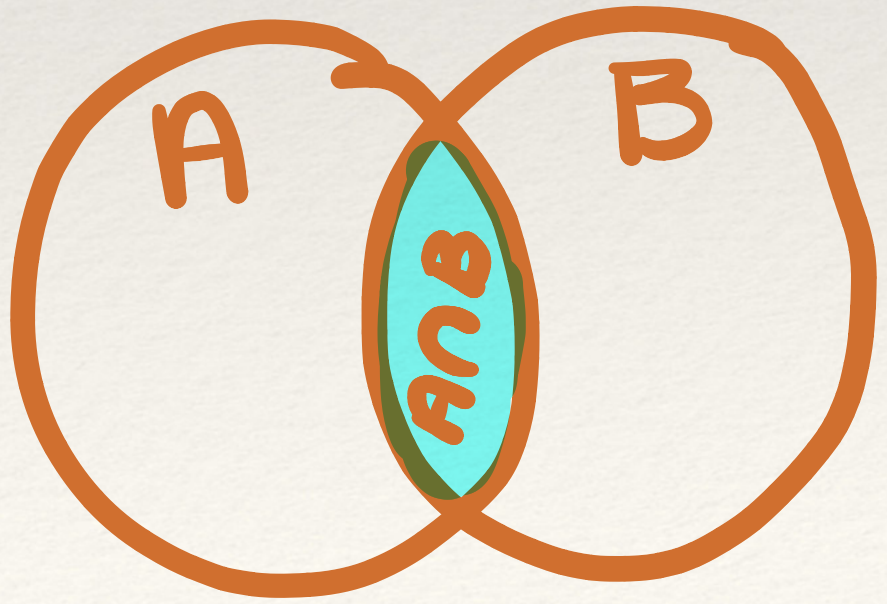
Conditional Probability

For any two events, A and B, with $P(B) > 0$, the conditional probability of A *given* B has occurred is defined by:

Conditional probabilities redefine the sample space to include only events that are in the conditional subset B.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

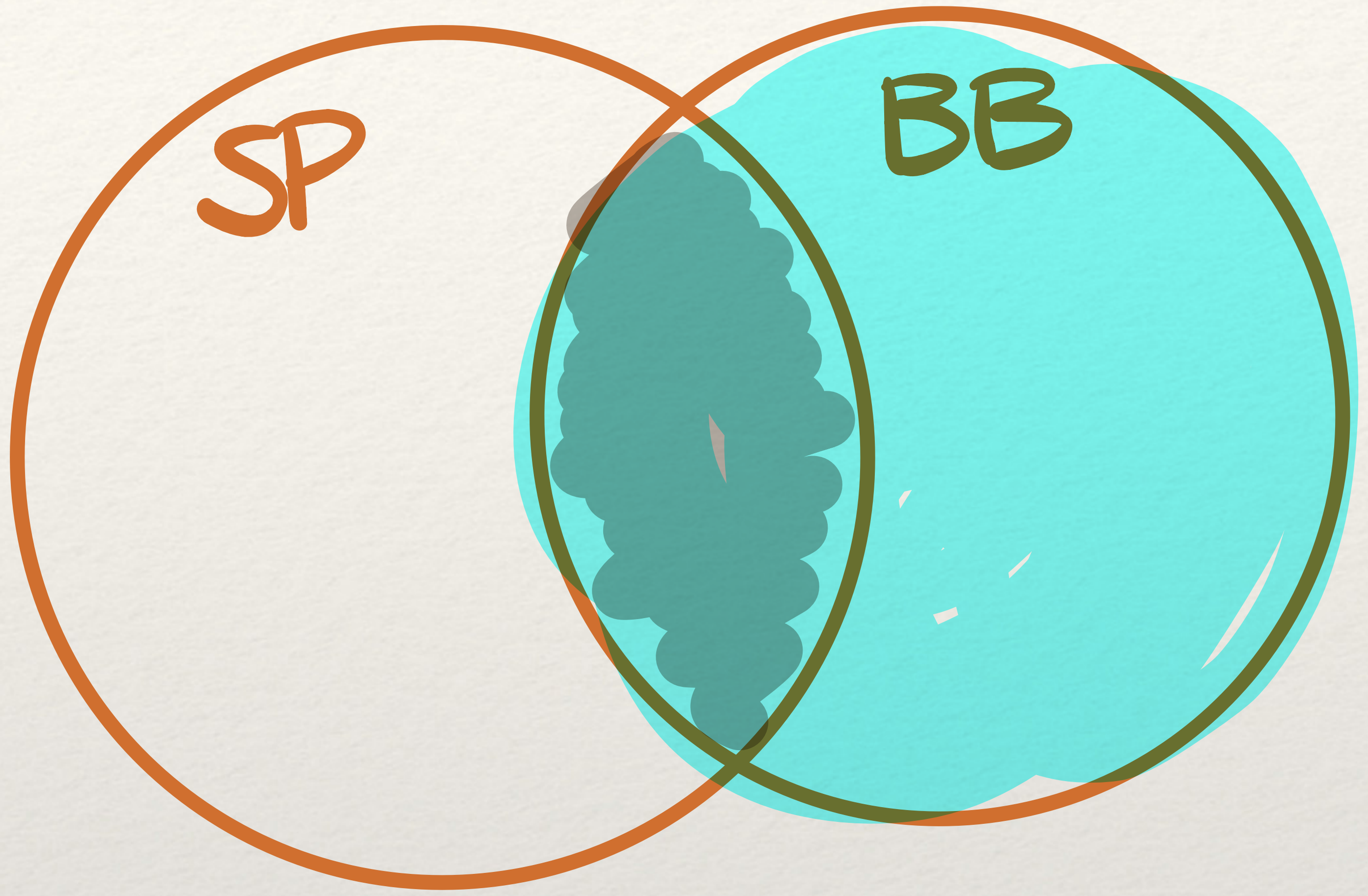
given



Example #2

A recent study from Pew Research Center indicates 77% of adults in the United States use a smartphone (SP) to access the internet, while 65% access the internet at home with a broadband (BB) service and 52% access with both a smartphone and broadband.

Given someone accesses the internet with broadband, what is the chance they also access the internet with a smartphone?



$$P(SP|BB) = \frac{P(SP \cap BB)}{P(BB)}$$

$$= \frac{0.52}{0.65} = 0.8$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$$P(A|B)P(B) = P(A\cap B)$$

Multiplication Rules

- Multiplication rule for independent events:

$P(A\cap B) = P(A)P(B)$ if and only if A and B are independent.

- General multiplication rule:

$$P(A\cap B) = P(A|B)P(B) = P(B|A)P(A)$$

Example #3

a. You select one card from a deck, look at it, then replace it to the deck. You select another card and look at it. What is the probability that both cards selected are queens?

$$\left(\frac{4}{52}\right)\left(\frac{4}{52}\right) = 0.0059$$

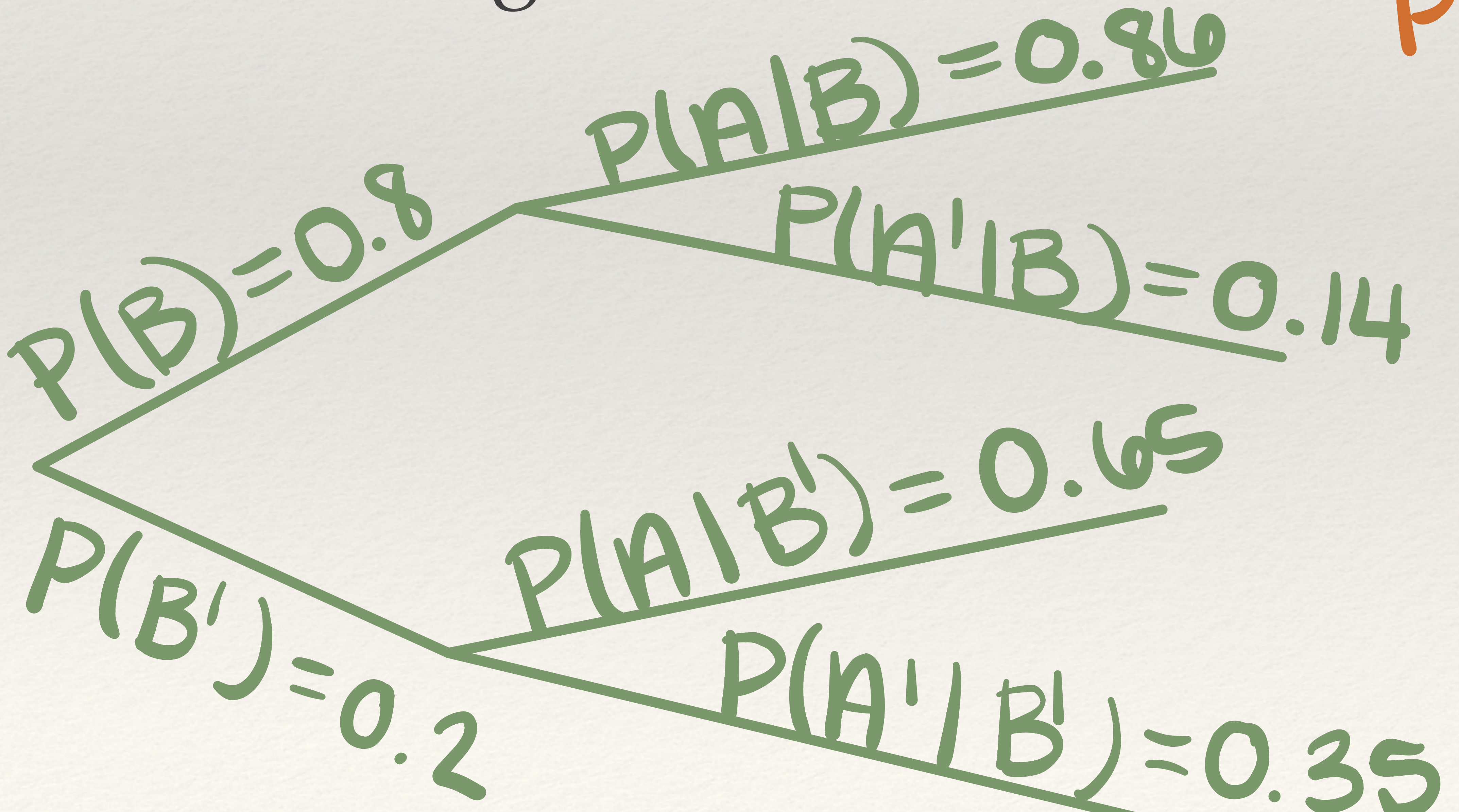
b. You select two cards from the deck at the same time. What is the probability that both cards are queens?

$$\left(\frac{4}{52}\right)\left(\frac{3}{51}\right) = 0.0045$$

Example #4 (Exercise 3.19)

After an introductory statistics course, 80% of students can successfully construct a box plot. Of those can construct box plots, 86% passed, while only 65% of those students who could not construct a box plot passed.

Draw a tree diagram of this scenario. $P(B \cap A) = 0.8(0.86)$

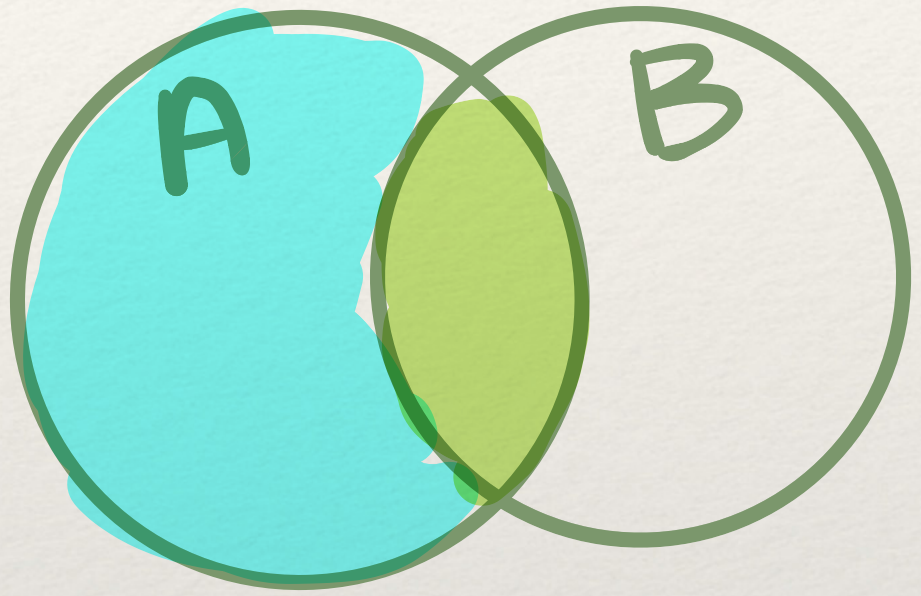


B = can construct a box plot
B′ = cannot construct a box plot
A = passed class
A′ = did not pass class

Law of Total Probability & Bayes Theorem

The Law of total probability finds the probability that event A occurs is the sum of all the mutually exclusive events that contain A:

$$P(A) = P(A \cap B) + P(A \cap B')$$
$$= P(A|B)P(B) + P(A|B')P(B')$$



Bayes' Theorem can be used to find a conditional probability when only other conditional probabilities are known. The formula is a combination of the Law of Total Probability and the Multiplication Rule.

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B')P(B')}$$

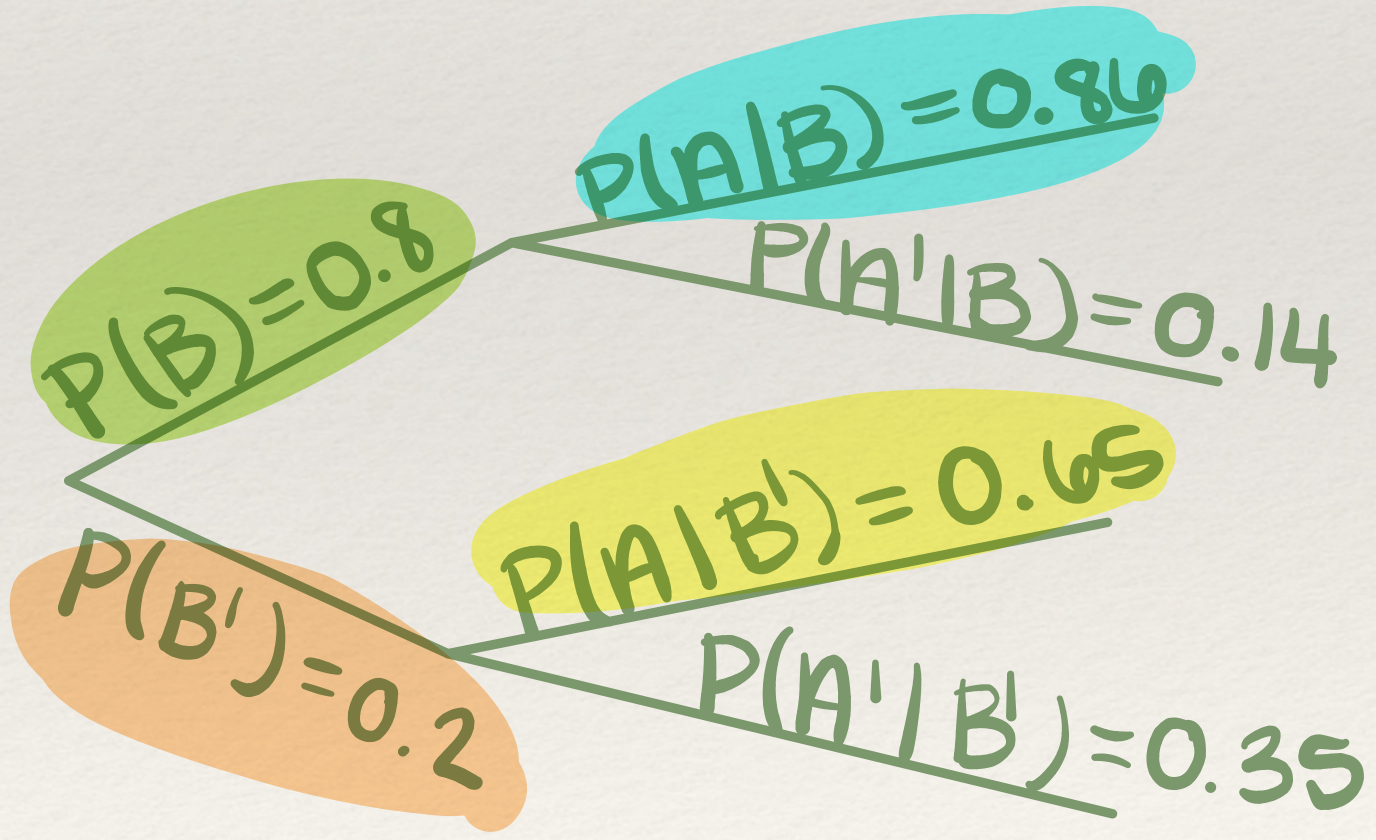
Example #5 (Exercise 3.19)

After an introductory statistics course, 80% of students can successfully construct a box plot. Of those can construct box plots, 86% passed, while only 65% of those students who could not construct a box plot passed.

Calculate the probability that a student is able to construct a box plot if it is known that they passed the class.

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B')P(B')}$$

$$= \frac{0.86(0.8)}{0.86(0.8) + 0.65(0.2)} = 0.841$$



Top Hat Activity

A genetic test is used to determine if people have a predisposition for thrombosis, which is the formation of a blood clot inside a blood vessel that obstructs the flow of blood through the circulatory system. It is believed that 3% of people actually have this predisposition. The genetic test is 99% accurate if a person actually has the predisposition, meaning that the probability of a positive test result when a person actually has the predisposition is 0.99. The test is 98% accurate if a person does not have the predisposition.

Notation:

T = individual has predisposition for thrombosis

T′ = individual does not have predisposition for thrombosis

Pos = tests positive

Neg = tests negative

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