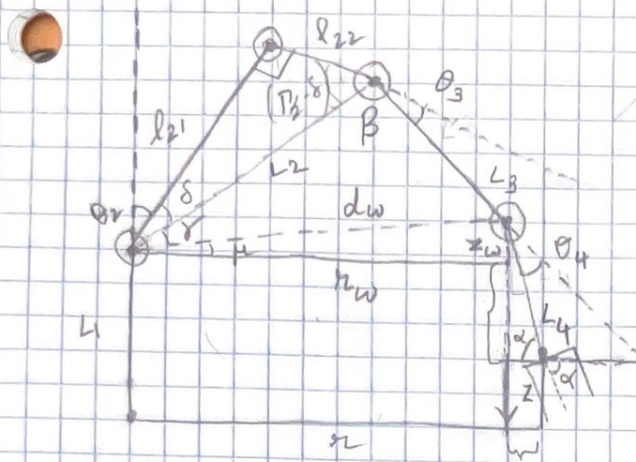
$L_1 = 77\,mm$ $L_2 = 130\,mm$, $L_3 = 124\,mm$, $L_4 = 126\,mm$

$l_{21} = 128\,mm$ $l_{22} = 24\,mm$



We are given the position of the end-effector: (x, y, z, α) → pitch

Must find $\theta_1, \theta_2, \theta_3, \theta_4$

$$\theta_2 = \frac{\pi}{2} - \delta - \gamma - \mu$$

$$\theta_3 = \pi - \beta - (\pi/2 - \delta)$$

α = pitch $= -(\theta_2 + \theta_3 + \theta_4)$

$$\Rightarrow \theta_4 = -\alpha - \theta_2 - \theta_3$$

Side View

$r_\omega = r - L_4 \cos\alpha$, $L_1 = z - (L_4 \sin\alpha - z_\omega) \Rightarrow z_\omega = z + L_4 \sin\alpha - L_1$

$\therefore d_\omega = \sqrt{r_\omega^2 + z_\omega^2}$

$\therefore$ using law of cosines: $d_\omega^2 = L_2^2 + L_3^2 - 2 L_2 L_3 \cos\beta$

$$\Rightarrow \cos\beta = \frac{L_2^2 + L_3^2 - d_\omega^2}{2 L_2 L_3}, \quad \sin\beta = \pm\sqrt{1 - \cos^2\beta}$$

$$\tan\mu = \frac{z_\omega}{r_\omega}$$

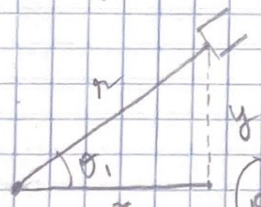
$$\Rightarrow \mu = \operatorname{atan}\left(\frac{z_\omega}{r_\omega}\right) \qquad \Rightarrow \beta = \arctan\left(\frac{\pm\sin\beta}{\cos\beta}\right)$$

for γ using law of cosines: $L_3^2 = L_2^2 + d_\omega^2 + 2 d_\omega L_2 \cos(\gamma)$

$$\Rightarrow \cos\gamma = \frac{L_3^2 - L_2^2 - d_\omega^2}{2 d_\omega L_2}; \quad \sin\gamma = \pm\sqrt{1 - \cos^2\gamma}$$

$$\Rightarrow \therefore \gamma = \arctan\left(\frac{\pm\sin\gamma}{\cos\gamma}\right)$$

Top View



$$\theta_1 = \operatorname{atan}\left(\frac{y}{x}\right)$$