EEE 598

Assignment-3

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WRITTEN PROBLEM.

Problem ① Perceptron

1. Given Dataset,

	x_1	x_2	y
D =	1	1	1
	2	-2	-1
	-1	-1.5	-1
	-2	-1	-1
	-2	1	1
	1.5	-0.5	1

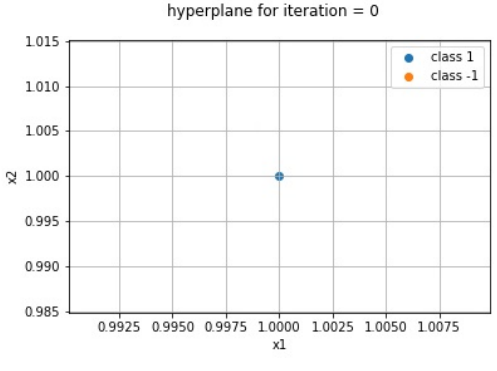
By the perceptron algorithm,

$\hat{y} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$ (w_0: bias)

& by gradient descent, weight updates in the next iteration is given by,

$w_n^{(t)} = w_n^{(t-1)} + y \cdot x_n^{(t)} \cdot \mathbb{1}[y^{(t)} \neq \hat{y}^{(t)}]$, if $\hat{y} \geq 0$, $+1$, if $\hat{y} < 0$, -1.

So, first initialize w_1, w_2 & b to $(0,0,0) \rightarrow (w_0, w_1, w_2)$ (w_0: bias)



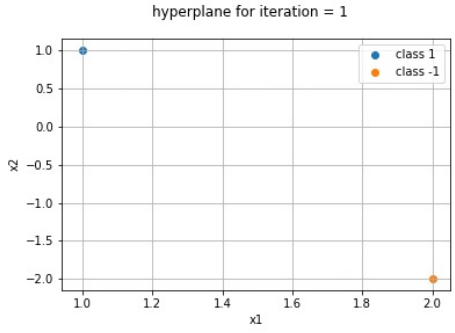
At iteration 0 : $x = (1,1)$, $y = 1$

$w_0 = 0$, $w_1 = 0$, $w_2 = 0$

$\hat{y} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(0 + 0(1) + 0(1)) = +1$

$\Rightarrow$ no misclassification.



At iteration 1 : $x^{(1)} = (2, -2)$, $y^{(1)} = -1$

$w_0 = 0$, $w_1 = 1$, $w_2 = 1$

$\hat{y} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

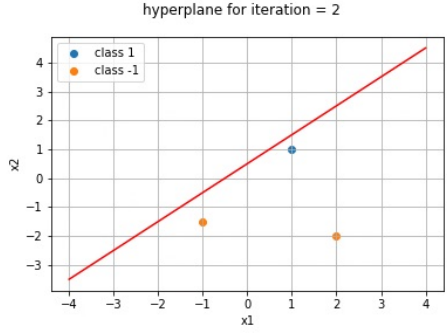
$= \text{Sign}(0 + 0(2) + 0(-2)) = +1$

$\Rightarrow$ missclassification,

update w.

$\Rightarrow w^{(1)} = w^{(0)} + y^{(1)} x^{(1)}$

$\Rightarrow w^{(1)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + -1 \cdot \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$



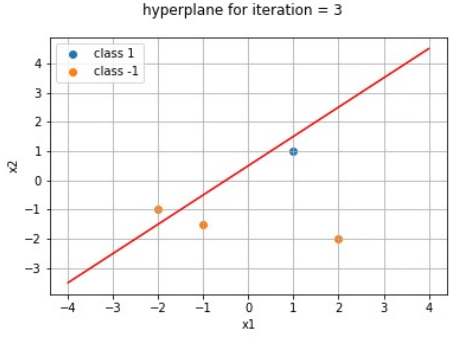
At iteration 2 : $x^{(2)} = (-1, +3)$, $y^{(2)} = -1$

$w_0 = -1$, $w_1 = -2$, $w_2 = 2$

$\hat{y}^{(2)} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(-1 + (-2)(-1) + 2(+3)) = -1$

$\Rightarrow$ no missclassification.



At iteration 3 : $x^{(3)} = (-2, -1)$, $y^{(3)} = -1$

$w_0 = -1$, $w_1 = -2$, $w_2 = 2$

$\hat{y}^{(3)} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

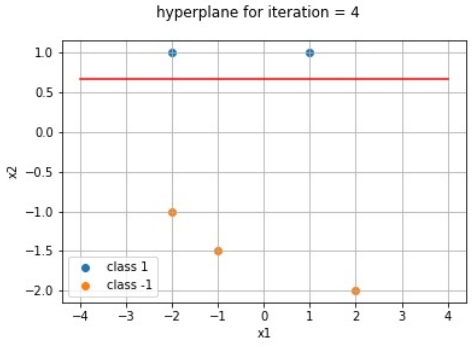
$= \text{Sign}(-1 + (-2)(-2) + (2)(-1)) = +1$

$\hat{y} \neq y \Rightarrow$ missclassification.

update w.

$\Rightarrow w^{(3)} = w^{(2)} + y^{(3)} \cdot x^{(3)}$

$\Rightarrow w^{(3)} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix} + (-1) \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}$



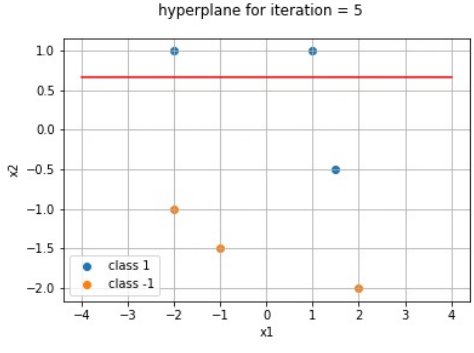
At iteration 4 : $x^{(4)} = (-2, 1)$, $y^{(4)} = 1$

$w_0 = -2$, $w_1 = 0$, $w_2 = 3$

$\hat{y}^{(4)} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(-2 + (0)(-2) + 3(1)) = 1$

$\Rightarrow$ no missclassification.



At iteration 5 : $x^{(5)} = (1.5, 0.5)$, $y^{(5)} = +1$

$w_0 = -2$, $w_1 = 0$, $w_2 = 3$

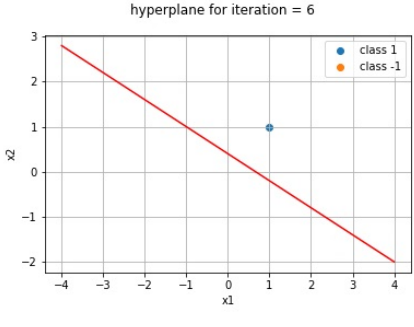
$\hat{y}^{(5)} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(-2 + (0)(1.5) + (3)(0.5)) = -1$

$\Rightarrow$ missclassification, update W.

$W^{(5)} = W^{(4)} + y^{(5)} \cdot x^{(5)}$

$\Rightarrow W^{(5)} = \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 1.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} -1 \\ 1.5 \\ 2.5 \end{bmatrix}$



Start second epoch for all points:

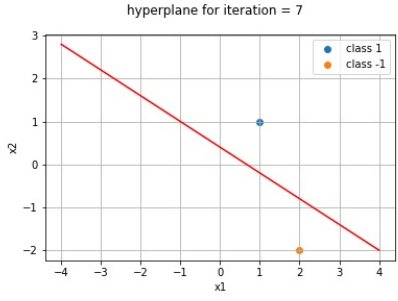
At iteration 6 : $x^{(6)} = (1, 1)$, $y^{(6)} = 1$

$w_0 = -1$, $w_1 = 1.5$, $w_2 = 2.5$

$\hat{y} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(-1 + 1.5(1) + 2.5(1)) = +1$

$\Rightarrow$ no missclassification.



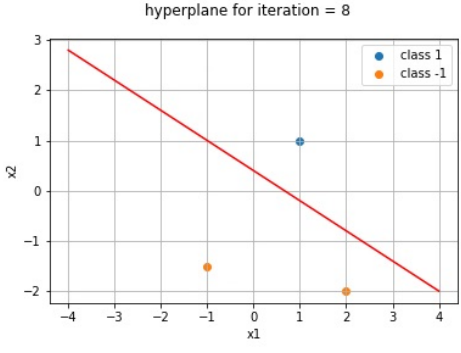
At iteration 7 : $x^{(7)} = (2, -2)$, $y^{(7)} = -1$

$w_0 = -1$, $w_1 = 1.5$, $w_2 = 2.5$

$\hat{y}^{(7)} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(-1 + 1.5(2) + 2.5(-2)) = -1$

$\Rightarrow$ no misclassification,



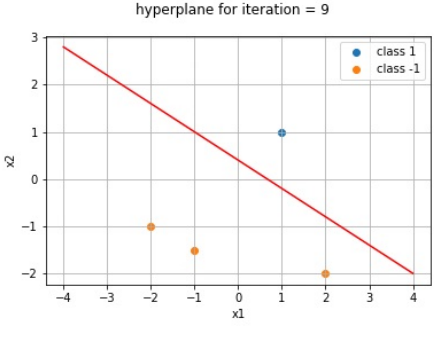
At iteration 8 : $x^{(8)} = (-1, -1.5)$, $y^{(8)} = -1$

$w_0 = -1$, $w_1 = 1.5$, $w_2 = 2.5$

$\hat{y}^{(8)} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(-1 + 1.5(-1) + 2.5(-1.5)) = -1$

$\Rightarrow$ no misclassification,



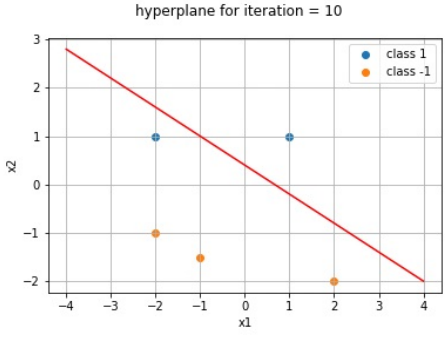
At iteration 9 : $x^{(9)} = (-2, -1)$, $y^{(9)} = -1$

$w_0 = -1$, $w_1 = 1.5$, $w_2 = 2.5$

$\hat{y}^{(9)} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(-1 + 1.5(-2) + 2.5(-1)) = -1$

$\Rightarrow$ no misclassification,



At iteration 10: $x^{(10)} = (2, 1)$, $y = +1$

$w_0 = -1$, $w_1 = 1.5$, $w_2 = 2.5$

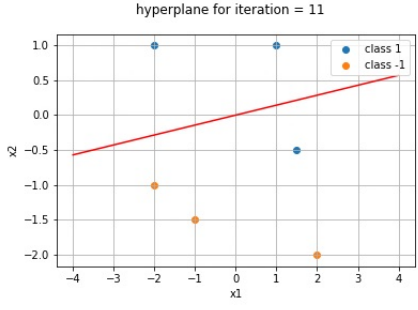
$\hat{y}^{(10)} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(-1 + 1.5(2) + 2.5(1)) = -1$

$\Rightarrow$ misclassification, update W

$W^{(10)} = W^{(9)} + y^{(10)} \cdot x^{(10)}$

$\Rightarrow W^{(10)} = \begin{bmatrix} -1 \\ 1.5 \\ 2.5 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -0.5 \\ 3.5 \end{bmatrix}$



At iteration 11: $x^{(11)} = (1.5, 0.5)$, $y^{(11)} = +1$

$w_0 = 0$, $w_1 = -0.5$, $w_2 = 3.5$

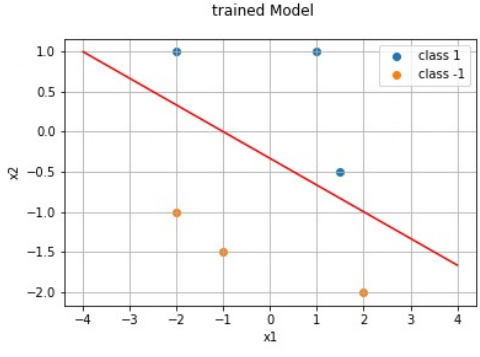
$\hat{y}^{(11)} = \text{Sign}(w_0(1) + w_1 x_1 + w_2 x_2)$

$= \text{Sign}(0 + (0.5)(1.5) + (3.5)(0.5)) = -1$

$\Rightarrow$ misclassification, update W

$W^{(11)} = W^{(10)} + y^{(11)} \cdot x^{(11)}$

$\Rightarrow W^{(10)} = \begin{bmatrix} 0 \\ -0.5 \\ 3.5 \end{bmatrix} + 1 \begin{bmatrix} 1 \\ 1.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$



As it can be seen that all the points get correctly classified after the weight updates, there will be no further updates

$\Rightarrow$ The final weights for the perceptron algorithm: $w_1 = 1$, $w_2 = 3$, bias $= 1$

Decision boundary: $x_1 + 3x_2 + 1 = 0$