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The use of vehicle acceleration measurements to estimate road roughness

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The use of vehicle acceleration measurements to estimate road roughness

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Road roughness is a broad term that incorporates everything from potholes and cracks to the random deviations that exist in a profile. To build a roughness index, road irregularities need to be measured first. Existing methods of gauging the roughness are based either on visual inspections or using one of a limited number of instrumented vehicles that can take physical measurements of the road irregularities. This paper proposes the collection of data from accelerometers fixed in a specific vehicle type and the use of this data to estimate the road condition. Although the estimate is approximate, accelerometers are being increasingly used by car manufacturers to improve suspension performance and the proposed method is relatively inexpensive to implement and provide road managers with constantly updated measurements of roughness. This approach is possible due to the relationship between the power spectral densities of road surface and vehicle accelerations via a transfer function. This paper shows how road profiles can be accurately classified using axle and body accelerations from a range of simulated vehicle–road dynamic scenarios.

Keywords: roughness; classification; dynamics; accelerometer; ISO

2000 Mathematics Subject Classifications: 34A30; 62M15

1. Introduction

The analysis and maintenance of a road surface is a difficult problem that pavement engineers have been facing for many years. Detection of the condition of a road profile is important for many reasons, such as safety and economic savings, but profiling methods using direct measurements of the road itself are usually expensive [1–6]. The aim of this paper is to develop a method of estimating the general condition of a road by utilising technology that increasingly exists in everyday vehicles. It is becoming common in the luxury end of the car industry that accelerometers are being fitted in the suspensions in order to improve suspension performance and increase ride comfort [7,8]. These accelerometers are effectively measuring the response of the vehicle to the

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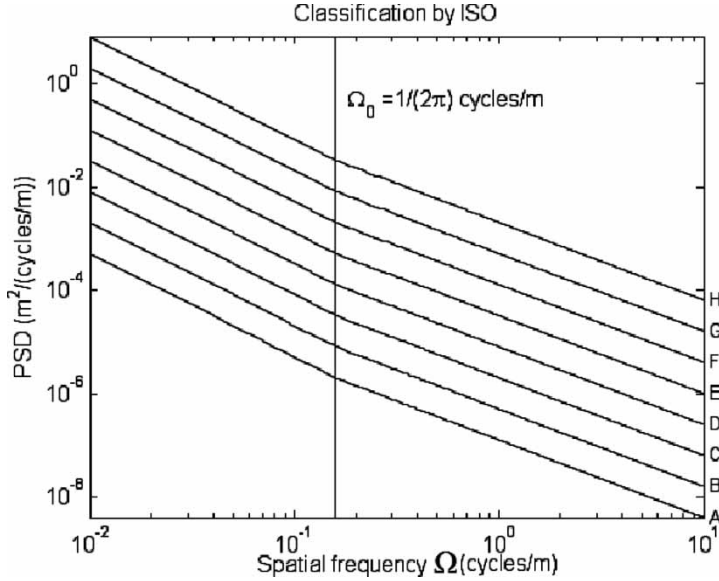


Figure 1. Classification of road roughness by ISO.

road surface. It is proposed herein to collect and store these acceleration measurements to estimate the average road condition. Other researchers developed methods for the collection of such data by road network management authorities [9]. The roughness estimate can be used to identify sections of road where the profile roughness has increased and hence to prioritise sections and optimise the use of profilometers for more detailed investigations.

Two popular methods to classify the roughness of a profile are the International Roughness Index [10] and the International Standards Organisation (ISO) classification [11]. In this paper, the road condition is classified according to ISO, which uses Fourier analysis to calculate the power spectral density (PSD) function of the surface. This PSD is compared with some boundaries that depend on the value of a geometric spatial mean, a , as described by Equation (1).

$$\text{PSD}(\Omega) = \begin{cases} a(2\pi\Omega)^{-2} & \Omega \leq \frac{1}{2\pi} \\ a(2\pi\Omega)^{-1.5} & \Omega > \frac{1}{2\pi} \end{cases} \quad (1)$$

where Ω is the spatial frequency in cycles/m. It classifies the profile into ‘A’ (very good), ‘B’ (good), ‘C’ (average), ‘D’ (poor) and ‘E’ (very poor) roughness indices, as shown in Figure 1 [11].

2. Vehicle and road profile models

A two-axle vehicle is modelled as a 4 degree-of-freedom suspension system. This model, known as a half-car, is illustrated in Figure 2.

The sprung body mass of the vehicle, m_3 , has vertical body displacement $y_3(t)$ and rotation $\varphi(t)$. The body mass moment of inertia is I_3 . The two unsprung masses corresponding to the front and rear axles, m_1 and m_2 , have vertical axle displacements $y_1(t)$ and $y_2(t)$, respectively. The tyre stiffness is modelled as a linear spring of constant K_t and the suspension system as a linear spring K_s in parallel with a damper C_s . The horizontal distance from the centroid of the

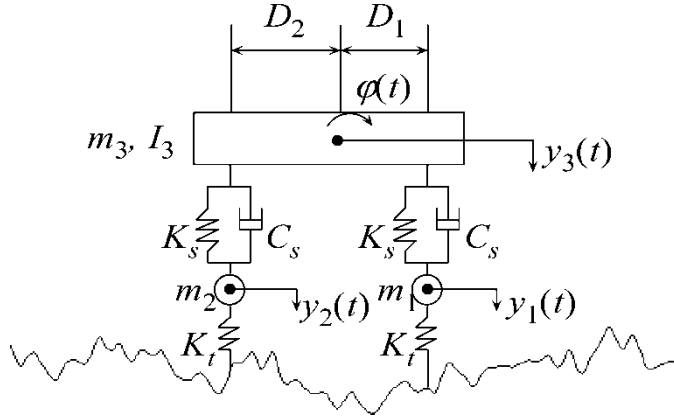


Figure 2. Half-car model.

vehicle to the front axle is described as D_1 , while the distance from the centroid to the rear axle is D_2 . The motion controlling the vehicle response when travelling at uniform speed is defined by the ordinary differential Equations (2)–(4) [12]:

$$-I_3 \frac{d^2 \phi(t)}{dt^2} - D_1 [Z_1(t) + Z_{b1}(t)] + D_2 [Z_2(t) + Z_{b2}(t)] = 0 \quad (2)$$

$$-m_3 \frac{d^2 y_3(t)}{dt^2} - [Z_1(t) + Z_{b1}(t) + Z_2(t) + Z_{b2}(t)] = 0 \quad (3)$$

$$(m_i + m_{3i})g - m_i \frac{d^2 y_i(t)}{dt^2} + Z_i(t) + Z_{bi}(t) - R_i(t) = 0 \quad i = 1, 2 \quad (4)$$

where $Z_i(t)$ and $Z_{bi}(t)$ are the spring and damping forces between the i th axle and the vehicle body, respectively; m_i the sprung mass corresponding to the i th axle and $y_{3i}(t)$ the displacement of the contact point between the i th axle and the vehicle body.

$$Z_i(t) = K_s [y_{3i}(t) - y_i(t)] \quad i = 1, 2 \quad (5)$$

$$Z_{bi}(t) = C_s \left[\frac{dy_{3i}(t)}{dt} - \frac{dy_i(t)}{dt} \right] \quad i = 1, 2 \quad (6)$$

$$m_{31} = m_3 \frac{D_2}{D_1 + D_2} \quad (7)$$

$$m_{32} = m_3 \frac{D_1}{D_1 + D_2} \quad (8)$$

$$y_{3i}(t) = y_3(t) - (-1)^i D_i \phi(t) \quad i = 1, 2 \quad (9)$$

The tyre force imparted to the road, $R_i(t)$, is given by Equation (10).

$$R_i(t) = K_t [y_i(t) - r(x_i)] \geq 0 \quad i = 1, 2 \quad (10)$$

where $r(x_i)$ is the surface roughness corresponding to the location of the i th axle at time t . This system of equations can be solved using the Runge–Kutta–Nyström method [13].

Based on the $\text{PSD}(\Omega_i)$ given by Equation (1), the road surface irregularities can be generated using the discrete form of Equation (11) [14].

$$r(x) = \sum_{i=1}^N \sqrt{4\text{PSD}(\Omega_i)\Delta\Omega} \cos(2\pi\Omega_i x + \theta_i) \quad (11)$$

where x is the distance measured from the left end of the road, N the total number of waves used to construct the road surface, $\Delta\Omega$ the frequency interval ($\Delta\Omega = (\Omega_{\max} - \Omega_{\min})/N$) and θ_i the random phase angle uniformly distributed within $[0, 2\pi]$, which corresponds to the spatial frequency Ω_i , is defined by Equation (12).

$$\Omega_i = \Omega_{\min} + (i - 0.5)\Delta\Omega \quad i = 1, 2, 3, \dots, N \quad (12)$$

3. Relationship between measured acceleration and road roughness

As a result of unsprung mass motion being strongly influenced by the road profile, a measured time history of the vehicle accelerations, $\ddot{y}(t)$, can be used to describe the overall condition of the road profile, $r(x)$. Vibrations and road profile can be related in a linear system through a transform function defined by Equation (13) [15,16,17].

$$H(\Omega) = \frac{\text{PSD}_{\text{acc}}(\Omega)}{\text{PSD}_{\text{road}}(\Omega)} \quad (13)$$

where $\text{PSD}_{\text{acc}}(\Omega)$ and $\text{PSD}_{\text{road}}(\Omega)$ are the PSD for a frequency Ω due to the vehicle accelerations and road profile, respectively. Once PSD_{road} is obtained, the road can be classified according to Figure 1.

3.1. Calibration

In practice, these transform functions will be unknown, but they can be experimentally obtained using Equation (13) and driving the vehicle over a measured profile. Once the transform function is calibrated, it can be used to classify any other profile, although periodic calibrations of $H(\Omega)$ are recommended. For the purpose of illustrating this procedure, a theoretical known road profile is generated. Then, a ‘typical’ two-axle vehicle is excited with this profile using Equations (2)–(10). Finally, the transform function of the vehicle is obtained from the PSDs of road profile and simulated vehicle accelerations via Equation (13). All calculations are carried out using MATLAB software [18].

Figure 3 shows the generated random road profile, classified by ISO as ‘D’ (poor). It is made up of 8001 spatial frequencies between 0.01 and 4 cycles/m.

Table 1 shows the values used for the parameters of the ‘typical’ half-car model employed in the simulation [16,19].

Axle and body mass accelerations, $\ddot{y}_1(t)$, $\ddot{y}_2(t)$ and $\ddot{y}_3(t)$, are illustrated in the Figure 4a–c, respectively, for a speed of 50 km/h.

Then, the PSD of axle accelerations are calculated. However, because fast fourier transform (FFT) uses a constant bandwidth, it over-exaggerates the fluctuations at high frequencies. This problem is overcome applying a smoothing procedure defined in ref. [11]. The resulting smoothed PSD is illustrated in log–log scale in Figure 5 for the road profile and in Figure 6 for vehicle accelerations.

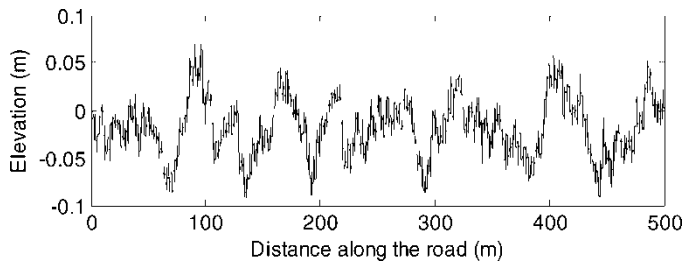


Figure 3. Class 'D' road profile.

Table 1. Parameters of 'typical' half-car model.

| Spacings (m) | | masses (kg) | | | | Stiffness (kN/m) | | |
|--------------|-------|-------------|-------|-------|----------------------------|------------------|-------|---------------|
| D_1 | D_2 | m_1 | m_2 | m_3 | I_3 (kg/m ²) | K_t | K_s | C_s (kNs/m) |
| 1.5 | 1.5 | 181 | 181 | 1814 | 1965 | 704 | 88 | 10.884 |

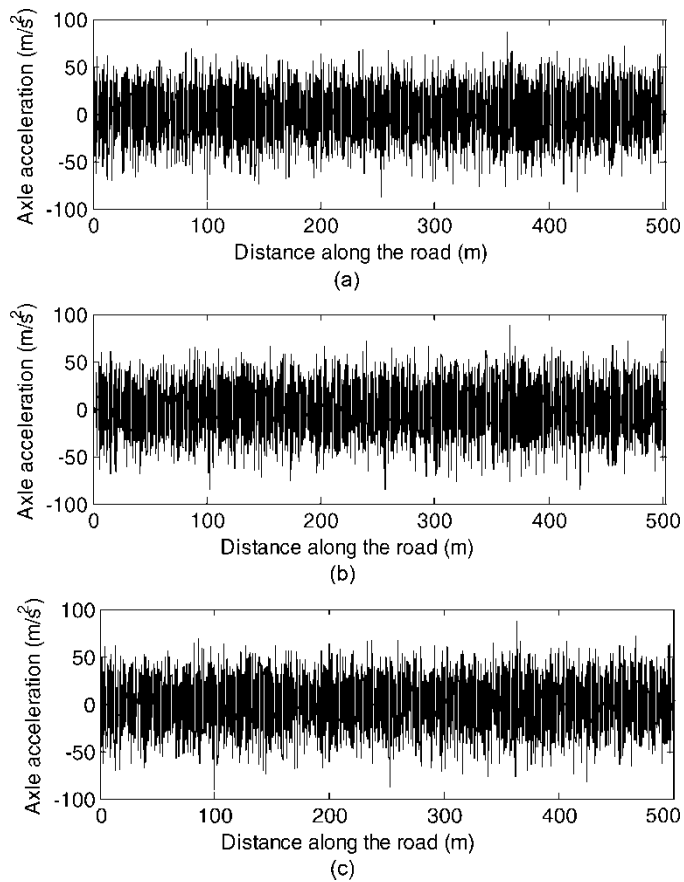


Figure 4. Vehicle accelerations: (a) front axle $\ddot{y}_1(t)$, (b) rear axle $\ddot{y}_2(t)$ and (c) body $\ddot{y}_3(t)$.

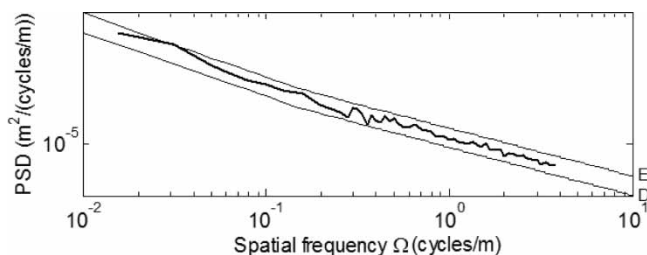


Figure 5. Smoothed PSD of the road profile.

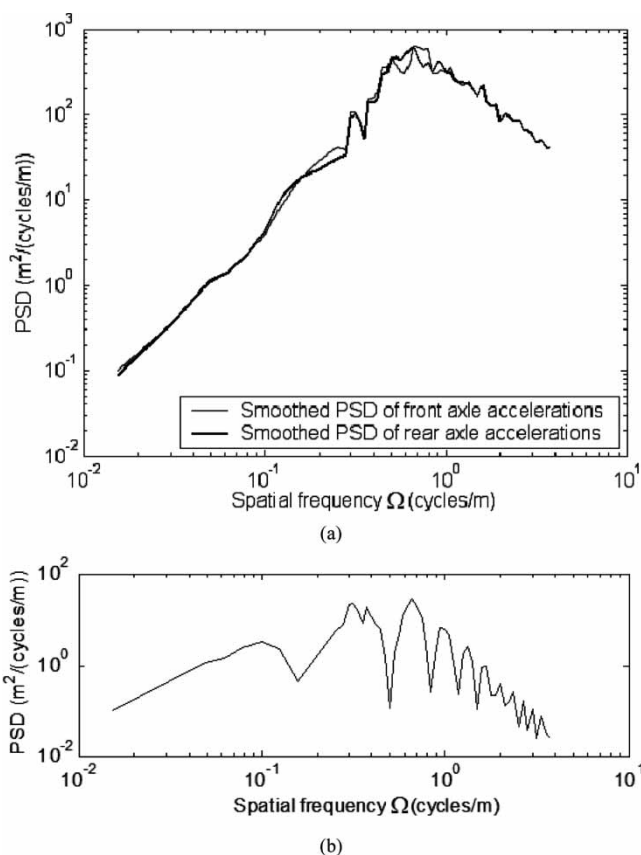


Figure 6. Smoothed PSD for: (a) front- and rear-axle accelerations and (b) body accelerations.

Figure 7 shows the calibrated transform functions for each vertical acceleration variable in this half-car model, derived from applying Equation (13) to the Figures 5 and 6.

This function can be used to transform the PSD for axle or body accelerations to the PSD for the road profile and vice versa. For unknown road profiles, it can be used to find the PSD of the profile from axle or body acceleration measurements.

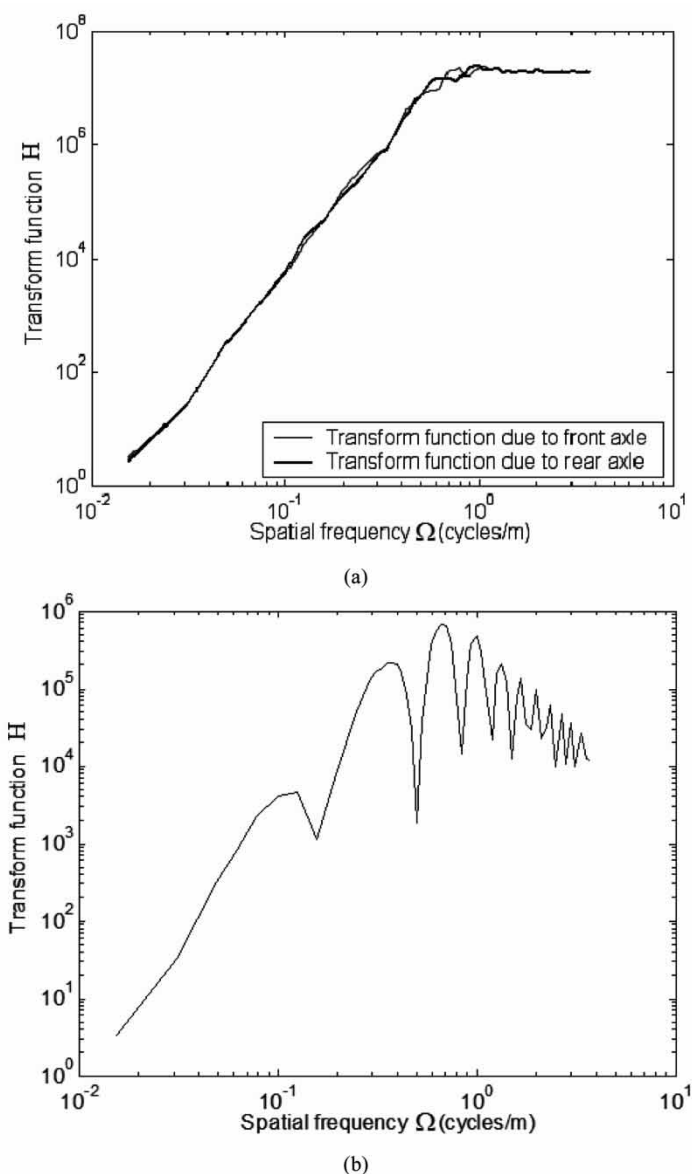


Figure 7. Transform function ($H(\Omega)$) for: (a) front and rear axle accelerations and (b) body accelerations.

4. Numerical testing

4.1. Validation for different road profiles

Two new theoretical profiles are generated with the same spatial frequency range (0.01–4 cycles/m) but different geometric spatial means, falling into classes ‘C’ and ‘D’. The axle and body accelerations due to the crossing of the ‘typical’ half-car model (as defined in Table 1) are obtained for a speed of 50 km/h. The PSD of the road profile is estimated from the PSD of axle and body accelerations using the transform functions of Figure 7a and b, respectively. Figures 8 and 9 compare the ‘true’ PSD, directly obtained from the road profile, to the estimated

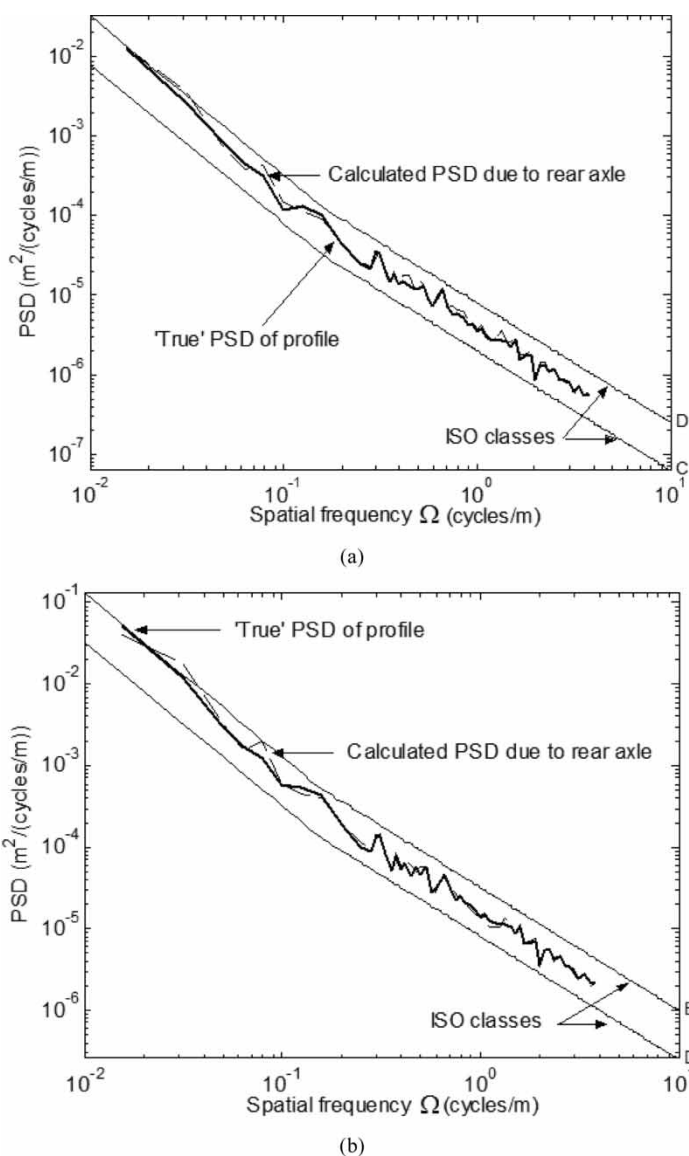


Figure 8. Comparison of 'true' and estimated $\text{PSD}(\Omega)$ of road profile using front-axle accelerations: (a) class 'C' profile and (b) class 'D' profile.

PSD for both road profiles when using front- and rear-axle accelerations, respectively. It can be seen that both axle acceleration measurements can be used for an estimate of the PSD and accurately classify the road without the need for profiling equipment. A small deviation appears between 0.07 and 0.08 cycles/m (~ 1 Hz, close to the body modes of vibration) of the PSDs in these two figures.

Figure 10 does the same comparison when using body accelerations. Sprung mass accelerations seem to provide a road spectra as accurate as the unsprung masses, except for small deviations at high frequencies and a significant peak around 0.5 cycles/m, ~ 7 Hz, related to axle-hop interference. The road class can be reasonably predicted once the spectra is not falling very close to the divisory line between two road classes.

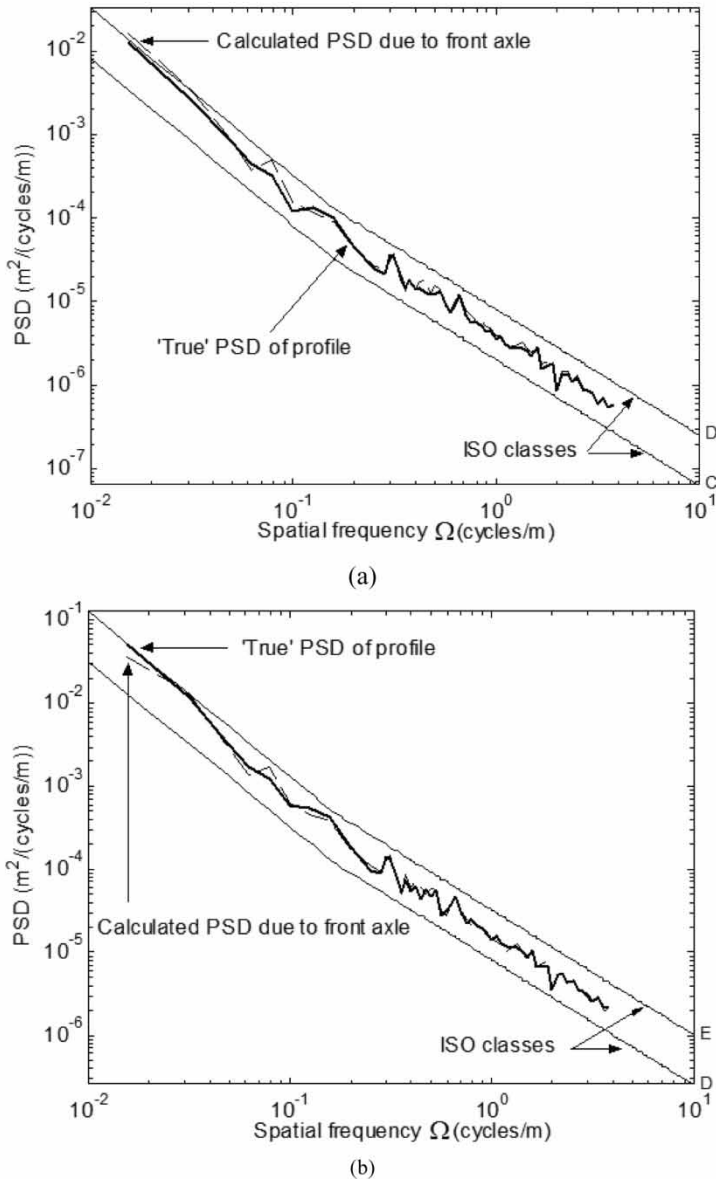


Figure 9. Comparison of 'true' and estimated $\text{PSD}(\Omega)$ of road profile using rear-axle accelerations: (a) class 'C' profile and (b) class 'D' profile.

4.2. Validation for different speeds

When measuring axle accelerations, the vehicle may drive at different speeds depending on traffic conditions. It is therefore important to evaluate the sensitivity of the transform function to speed. Figure 11 shows the 'true' and estimated PSD when driving the same vehicle at different speeds if using only one transform function (corresponding to 50 km/h). It is found that the closer the driving speed to the calibration speed, the more accurate the results become.

Hence, accuracy can be improved by using a set of transform functions for a range of speeds and choosing that transform function appropriate to the speed closest to that for which axle

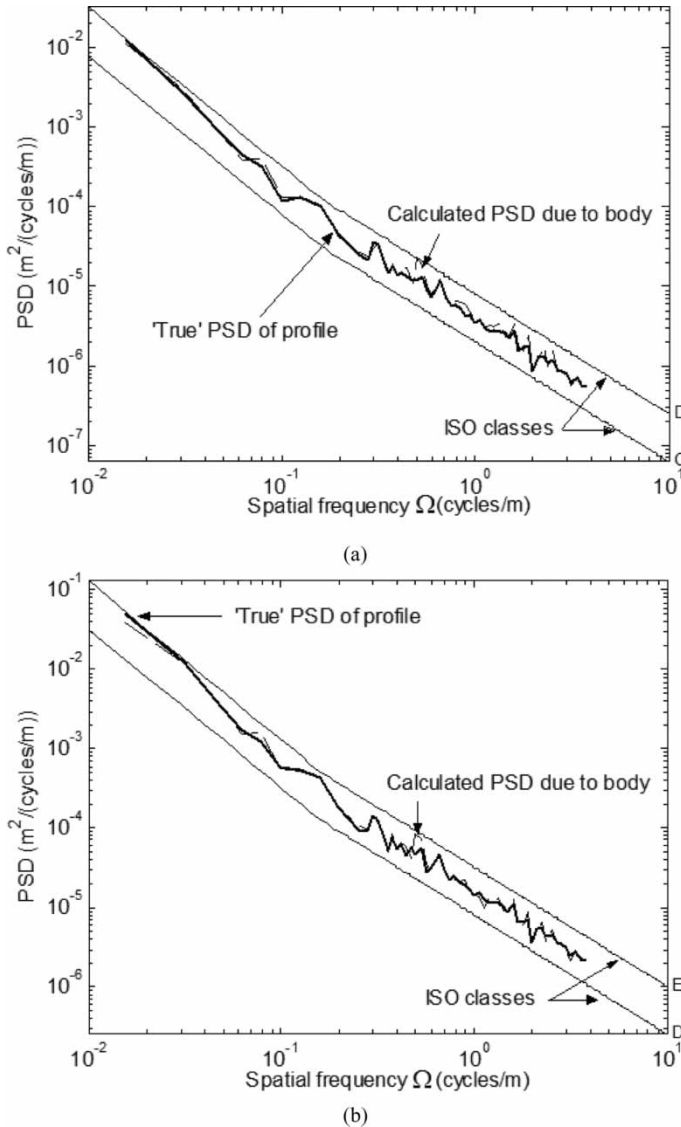


Figure 10. Comparison of 'true' and estimated $\text{PSD}(\Omega)$ of road profile using body accelerations: (a) class 'C' profile, and (b) class 'D' profile.

accelerations are taken. Transform functions are illustrated in Figure 12 for 40, 45, 50, 55 and 60 km/h speeds.

Calibration of the transform function every 5 km/h is found to provide sufficient accuracy for a correct ISO classification. For example, PSD results of Figure 13 are obtained using a transform function calibrated at 40 km/h for axle measurements taken at 42 km/h.

4.3. Validation for a vehicle fleet

The PSD of front-axle accelerations in the temporal frequency domain (cycles/s), $\text{PSD}(f)$, can be obtained by dividing the PSD of axle accelerations in the spatial frequency domain, $\text{PSD}(\Omega)$ (Figure 6a), by speed, and the result is illustrated in Figure 14.

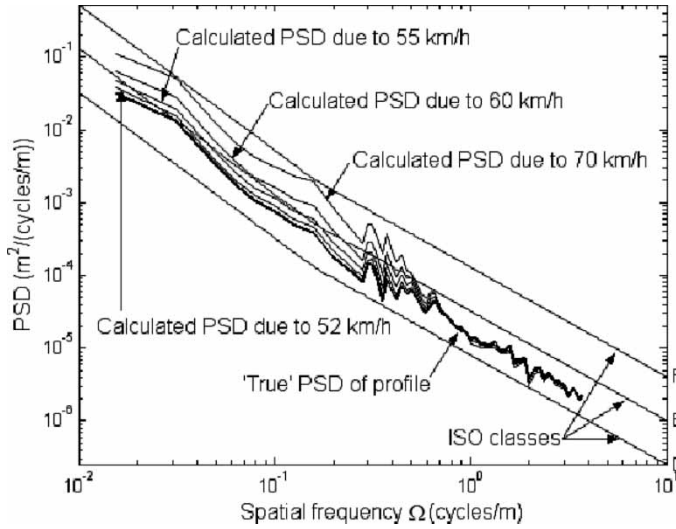


Figure 11. Comparison of 'true' and estimated $\text{PSD}(\Omega)$ of profiles for a range of speeds.

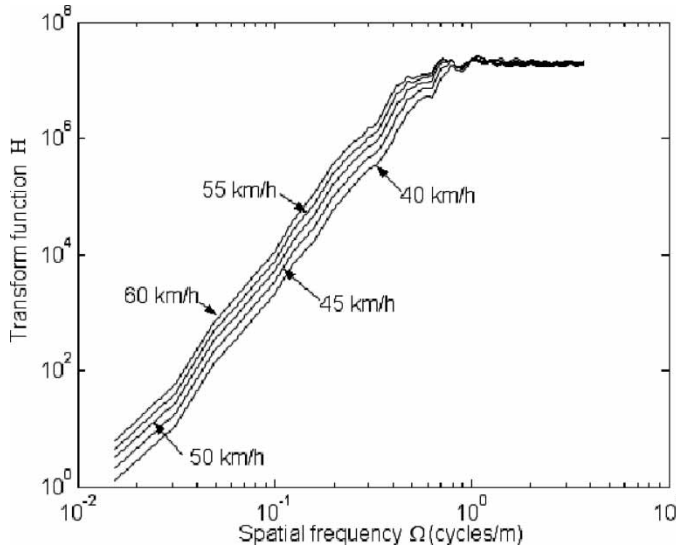


Figure 12. Transform function ($H(\Omega)$) for a range of speeds.

It can be seen that the critical frequency, where the peak values of $\text{PSD}(f)$ occur, is 10.56 cycles/s, which represents axle-hop frequency. It can be assumed that the PSD due to axle acceleration measurements will be similar if the axle-hop natural frequency is similar and other parameters vary within a reasonable range. In other words, if the transform function calculated for a particular vehicle was applied to the axle accelerations from other vehicles with similar properties, a reasonable degree of accuracy can be expected.

In order to test this assumption, the accelerations of six vehicles of the same type (*e.g.*, a fleet of public service vehicles) are simulated using the parameters specified in Table 2.

Only the transform functions of the 'typical' half-car or calibration vehicle (Figure 7) are employed to determine the PSD of the road profile from the axle accelerations of each vehicle.

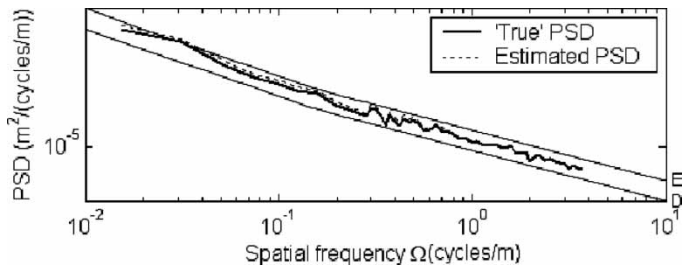


Figure 13. Comparison of 'true' and estimated PSD(Ω) of profiles at 42 km/h.

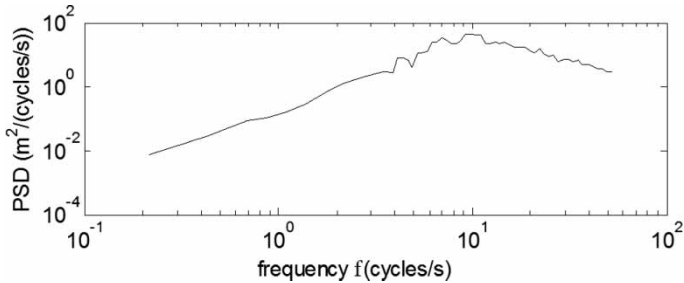


Figure 14. PSD(f) of the front-axle accelerations versus temporal frequency (f in cycles/s).

Table 2. Parameters of test vehicles.

| Symbols | Case | | | | | |
|---------------------|--------|--------|--------|--------|--------|-------|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| D_1 (m) | 2.5 | 2.0 | 1.5 | 1.5 | 1.5 | 1.5 |
| D_2 (m) | 2.5 | 1.0 | 1.5 | 1.5 | 1.5 | 1.5 |
| m_1 (kg) | 181 | 181 | 271.5 | 181 | 271.5 | 181 |
| m_2 (kg) | 181 | 181 | 271.5 | 181 | 271.5 | 181 |
| m_3 (kg) | 1814 | 1814 | 2721 | 1814 | 2721 | 1814 |
| I_3 (kg/m²) | 4384 | 1965 | 2948 | 1965 | 2948 | 1965 |
| K_t (kN/m) | 704 | 704 | 704 | 1056 | 1056 | 704 |
| K_s (kN/m) | 88 | 88 | 88 | 132 | 132 | 88 |
| C_s (kNs/m) | 10.884 | 10.884 | 10.884 | 10.884 | 10.884 | 15 |
| c (km/h) | 41 | 48 | 54 | 59 | 60 | 51 |
| Front axle hop (Hz) | 10.56 | 10.56 | 8.62 | 12.93 | 10.55 | 10.56 |
| Body bounce (Hz) | 2.12 | 2.31 | 1.73 | 2.6 | 2.12 | 2.12 |
| Body pitch (Hz) | 1.48 | 1.32 | 1.2 | 1.81 | 1.48 | 1.48 |
| Road class | D | C | D | C | D | C |

Figure 15 shows the PSD results for each of the six scenarios. It is found that the closer the axle-hop frequency of the test vehicle is to the calibration vehicle, the more accurate is the estimate. It can be concluded that the transform function of the calibrated vehicle can be used to estimate road roughness accurately once the vehicle has similar axle-hop frequency and vehicle parameters.

4.4. Validation for noise-corrupted acceleration data

The results discussed so far has been somewhat idealised because they have ignored the effects of noise. For high-quality sensors, the average power of the measurement noise will be small in

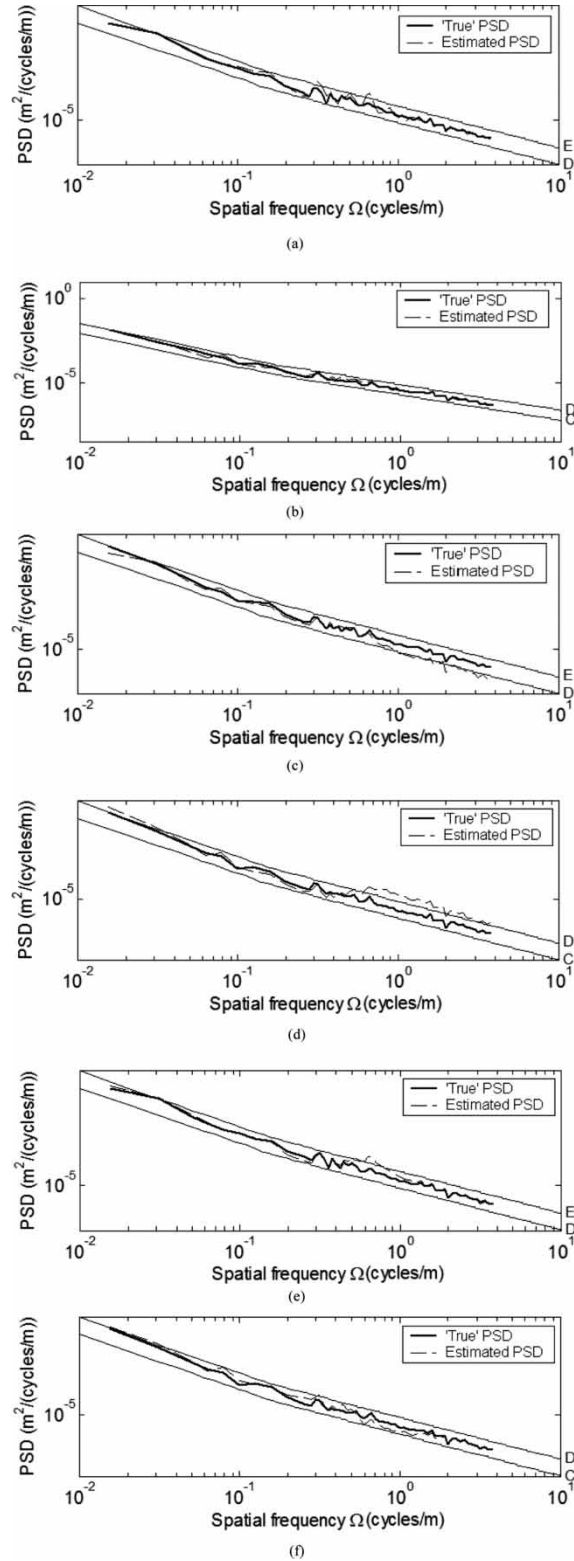


Figure 15. Comparison of 'true' and estimated $\text{PSD}(\Omega)$ of profiles for a range of vehicle parameters.

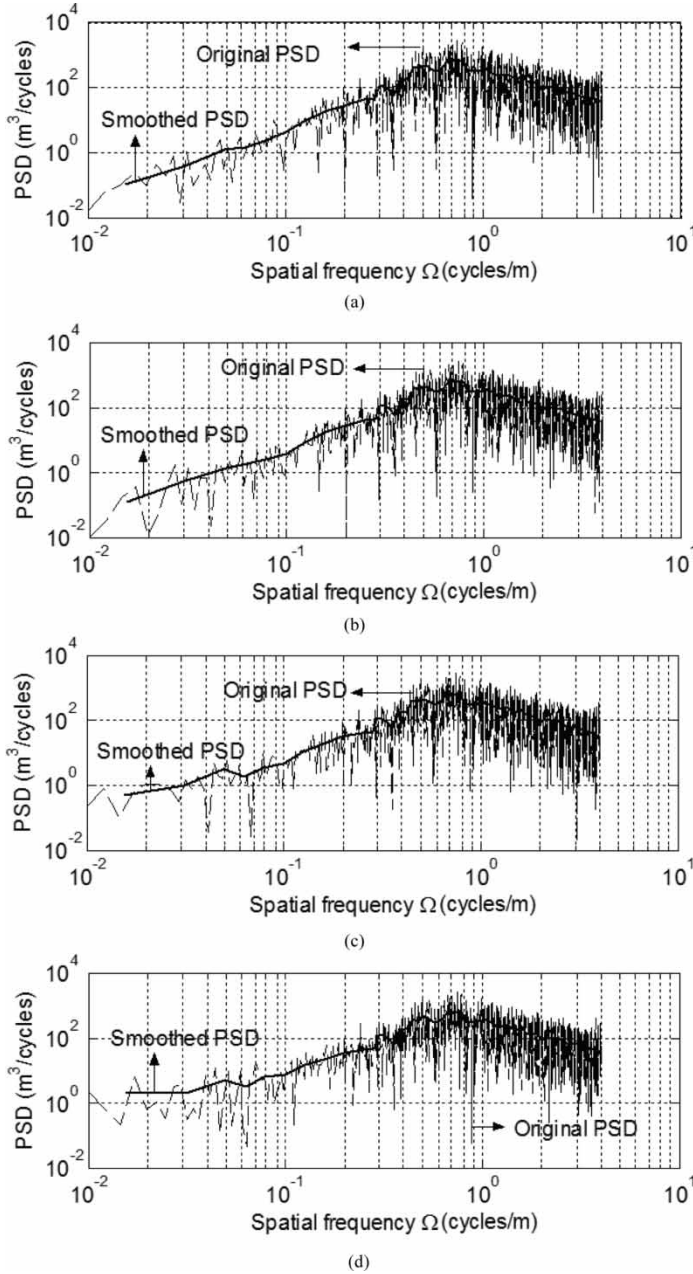
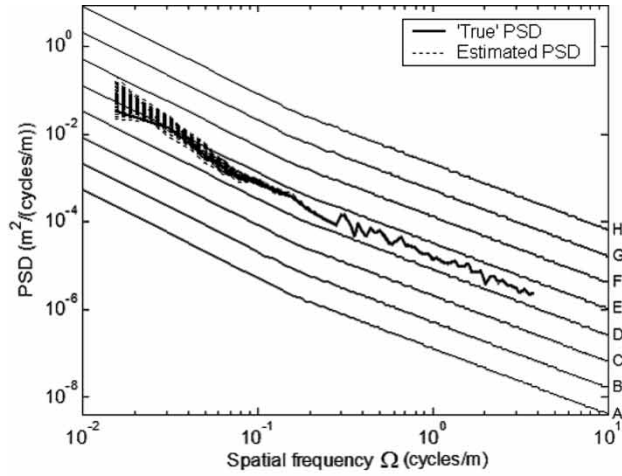


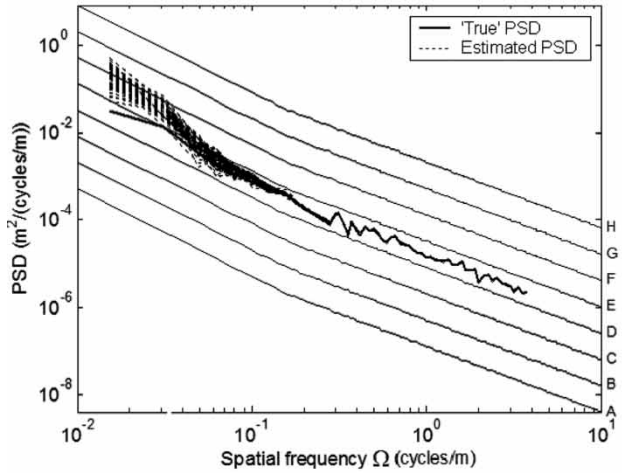
Figure 16. Original and smoothed PSD(Ω) of front-axle accelerations for noise levels of: (a) noise-free, (b) $S/N = 20$, (c) $S/N = 10$ and (d) $S/N = 5$.

comparison with the average power of the underlying accelerations. However, the noise power is typically spread out over a range of frequencies, and therein lies the problem. An additive noise model is used here to describe the noisy data that invariably is part of the measurements. So, the simulated acceleration, $a(t)$, is calculated as the sum of the true acceleration, $\ddot{y}(t)$, and the noise, $n(t)$:

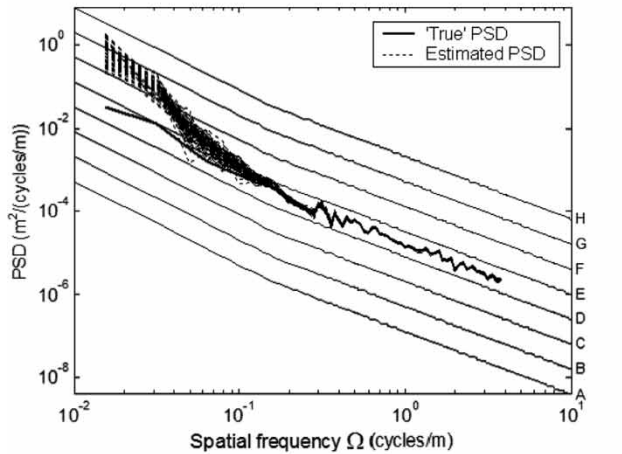
$$a(t) = \ddot{y}(t) + n(t) \quad \text{for any instant } t \quad (14)$$



(a)



(b)



(c)

Figure 17. Comparison of 'true' and estimated PSDs(Ω) of road profile from 100 randomly generated noise-corrupted axle accelerations with: (a) $S/N = 20$, (b) $S/N = 10$ and (c) $S/N = 5$.

The noise, $n(t)$, is zero-mean and described by its variance σ_n^2 . The impact of the noise on the signal is described by the signal-to-noise ratio (S/N), which is given by:

$$\frac{S}{N} = \frac{\sigma_y}{\sigma_n} = \sqrt{\frac{\sigma_a^2}{\sigma_n^2} - 1} \quad (15)$$

where σ_y^2 and σ_a^2 are the variances of the true and the recorded accelerations, respectively.

The algorithm is tested for the accelerations of the front axle shown in Figure 4a, which are corrupted with different levels of noise. Following smoothing procedure described in Section 3.1, Figure 16 shows the original and smoothed [PSDs of the noise-free ($\ddot{y}(t)$)] and noise-corrupted accelerations [$a(t)$] for S/N of 20 (relative error in measurement of 5%), 10 (relative error of 10%) and 5 (relative error of 20%). It can be seen how the influence of noise on the PSD is very relevant for low frequencies, but it hardly has any effect on the high frequencies, even for low S/N.

One hundred different noise signals [$n(t)$] are randomly generated varying the initial seed for each of the three S/N, which results into a total of 300 noise-corrupted acceleration samples being tested. The 100 estimated PSDs of road profiles are shown and compared with the ‘true’ PSD for each S/N in Figure 17. If taking into account only the low frequency components, the deviations introduced by noise classify the road into a poorer class than in reality. The higher S/N, the poorer prediction for the low frequencies class. For the simulations under study, S/N of 20 will accurately classify the road class as ‘D’ except for very low spatial frequencies (Figure 17a). S/N of 10 will classify the road class as ‘E’ for spatial frequencies below 0.04 cycles/m (Figure 17b), while S/N of 5 will classify the road as ‘F’ for spatial frequencies below 0.04 cycles and as ‘E’ for spatial frequencies between 0.04 and 0.1 cycles/m (Figure 17c). However, the influence of S/N on the high spatial frequency components of the spectra is not so important, and once frequencies are above 0.1 cycles/m, the road class is generally accurately predicted regardless the noise level.

5. Conclusions

Current methods of estimating the road condition generally involve taking direct measurements of the profile using lasers or other measuring devices. Although there have been major improvements in recent years in the quality of road-profiling equipment, these devices remain expensive to purchase, inefficient in terms of time and specialised in terms of operation. With maintenance and construction costs demanding a large portion of road management authorities budgets, this paper has presented a convenient, fast and economical method of estimating the road condition, using data harnessed from vehicle accelerometers. Then, the PSD of a road profile can be estimated from the PSD of the axle or body accelerations measured over the road profile. Such an approach requires prior knowledge of the vehicle transform function. This transform function depends only on vehicle parameters and it can be obtained by dividing the PSD of accelerations measured over a known profile by the PSD of that profile.

Computer simulations of half-car models have been carried out to analyse the sensitivity of the proposed road classification method to road roughness, speed, vehicle parameters and noise. Although it is hard to predict the exact interaction between a vehicle and a road in a simulation and the vehicles that are used are somewhat simplified, the findings give definite indications of the good levels of accuracy for the PSD estimate. The method is shown to be robust enough for a number of road profiles, vehicle speeds and dynamic scenarios. The possibility of using a fleet of vehicles to calculate profiles simultaneously is also successfully tested. This would be far more time-efficient than using a single vehicle and opens up the possibility of using data from existing public sector vehicle fleets. In the presence of a significant random noise component, the low

spatial frequency components of the road are classified into unrealistic poorer classes, but high spatial frequency components are generally classified within the correct road class. It has been shown how data from accelerometers fixed to the sprung mass can give a road spectra as accurate as axle acceleration data. Nevertheless, axle accelerations seem to provide slightly better estimates of the PSD than body accelerations for high spatial frequencies, which have been shown to be less sensitive to noise. Once the PSD of the road is identified, it is possible to generally classify the road as being very good/good/poor, etc., thereby prioritising the roads that need further attention. Should a particular road be classified as poor for example, a more detailed investigation of the profile could then be carried out using specialised equipment such as profilometers.

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