4CBLA30 Energy Storage and Transport

Measurement Practical

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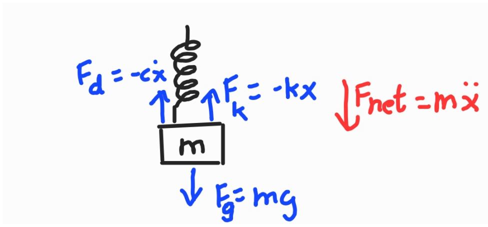
Modelling Objective

To create a model that can be used to understand the motion and eigenfrequency of a mass-spring system, so it can be used to optimise frequency and resonance duration.

Conceptual Model and Mathematical Elaboration

The conceptual model can be described as a mass spring system suspended freely in its equilibrium state. It is initially subjected to a small displacement of 2 cm after which the displacement and acceleration of the system is observed undisturbed. This model is used to describe the motion and find the eigenfrequency of the system.



The motion of the mass-spring system can be simply described by the following equation of motion

$$\begin{aligned} m\ddot{x} &= mg - c\dot{x} - kx \\ \ddot{x} &= g - 2\zeta\omega_n\dot{x} - \omega_n^2 x \end{aligned} \tag{1}$$

It is assumed that the system is lightly damped (due to air resistance and friction), and the damping ratio is assumed to be **0.016**.

Model Parameters and Simulink Implementation

Parameter	Value
Mass (m)	173.2 g
Initial Displacement (x0)	0.05 m
Acceleration due to Gravity (g)	9.8 m/s^2
Damping Ratio (ζ)	0.016
Spring Constant (k)	42.73 N/m
Natural Frequency (ω_n)	15.71 rad/s

Table 1: Model Parameters

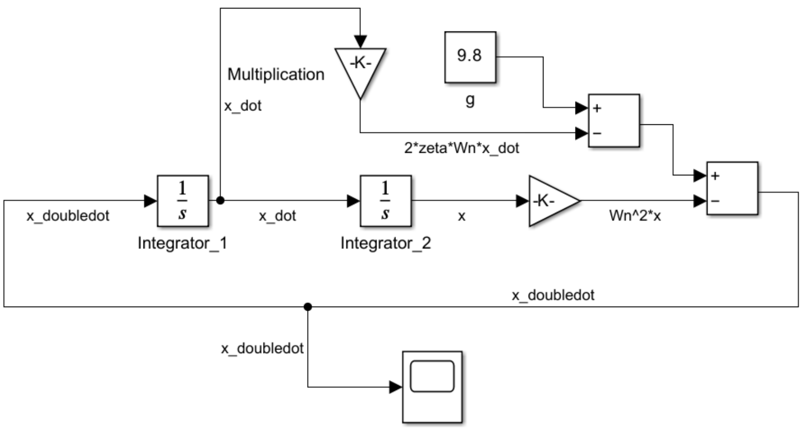


Figure 1: Simulink Model

Following eqn (2), the signals have been labelled accordingly. The initial displacement (relative to unstretched state, x0) is set to 0.05 m in the integrator. Different step sizes were compared to reduce discrepancies between the model, and exact solution (solution and comparison fabricated in Matlab). This can be seen in fig. 2

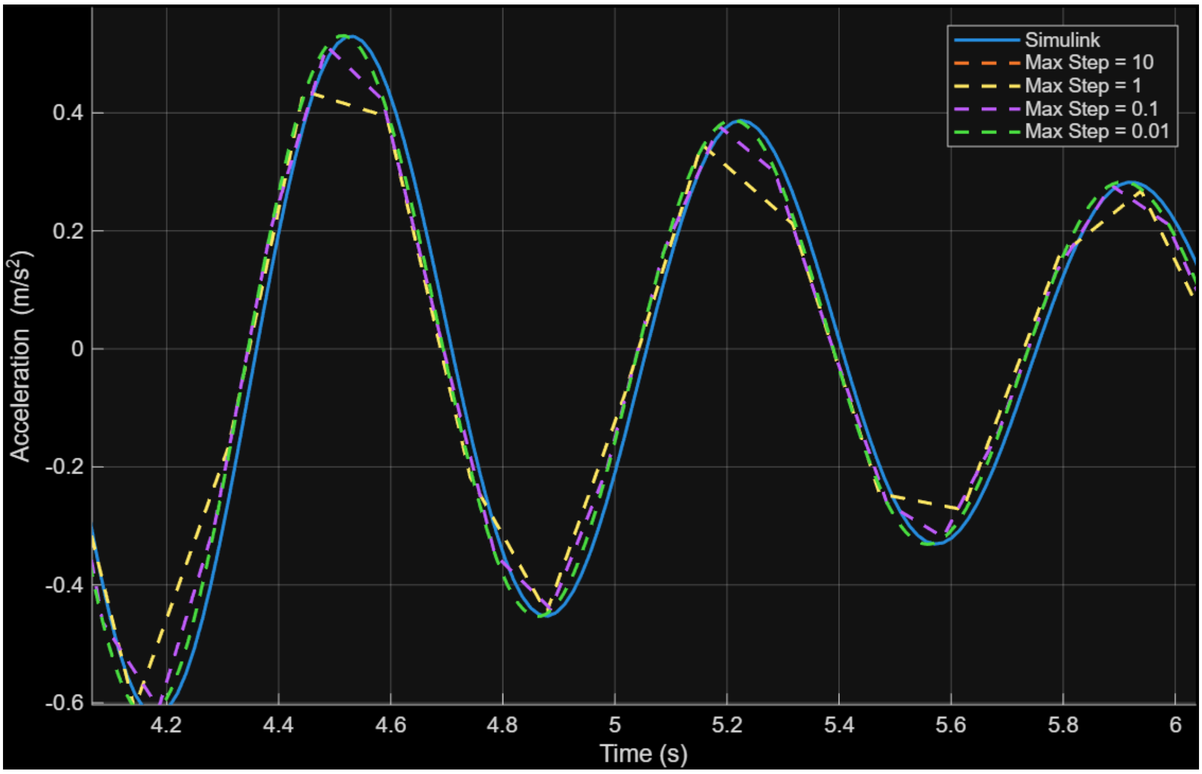


Figure 2: Comparison between different step sizes

Since no further accuracy was obtained from a smaller timestep, from this comparison a maximum timestep setting of **0.01** was deemed ideal. There is a very small phase difference, however this can be caused due to non-uniform time steps in the ode solver of Simulink.

Experimental Data

The data from carrying out three experiments (sample rate of 100 Hz) is displayed in Fig. 3. The mean Eigenfrequency ($\bar{f_n}$) is found to be **2.17 Hz** (Using Fourier transform) with a standard deviation of **+/- 0.06**, indicating only a small measurement uncertainty.

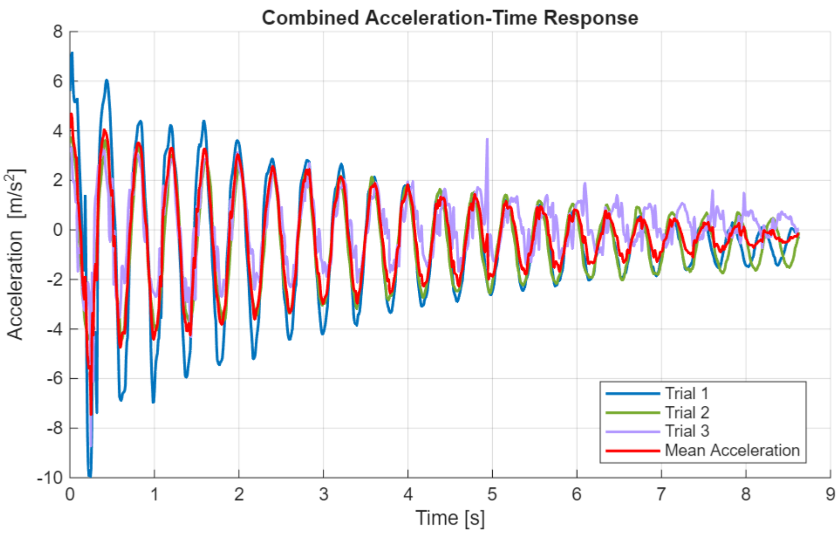


Figure 3: Experimental Data

Validation of Data

The model parameters defined in table 1 were defined to emulate the experiment environment, hence the same parameters have been used to draw comparisons to the experiment. The spring constant was measured by measuring the displacement of the spring from the equillibrium position when freely suspending a mass of **173.2 g**. Further, the natural frequency was determined using the spring constant and mass (173.2 g). The initial displacement was kept to **5 cm** to create predictable behavior from the experiment.

Fig. 4 shows the graphical interpretation. The predicted Eigenfrequency was **2.5 Hz**, which is a factor of **1.15** larger than the experimental result of **2.17 Hz**

This is not a significant difference, and is possible due to inconsistencies in sensor measurement or the assumption that the spring-mass-damper system is ideal.

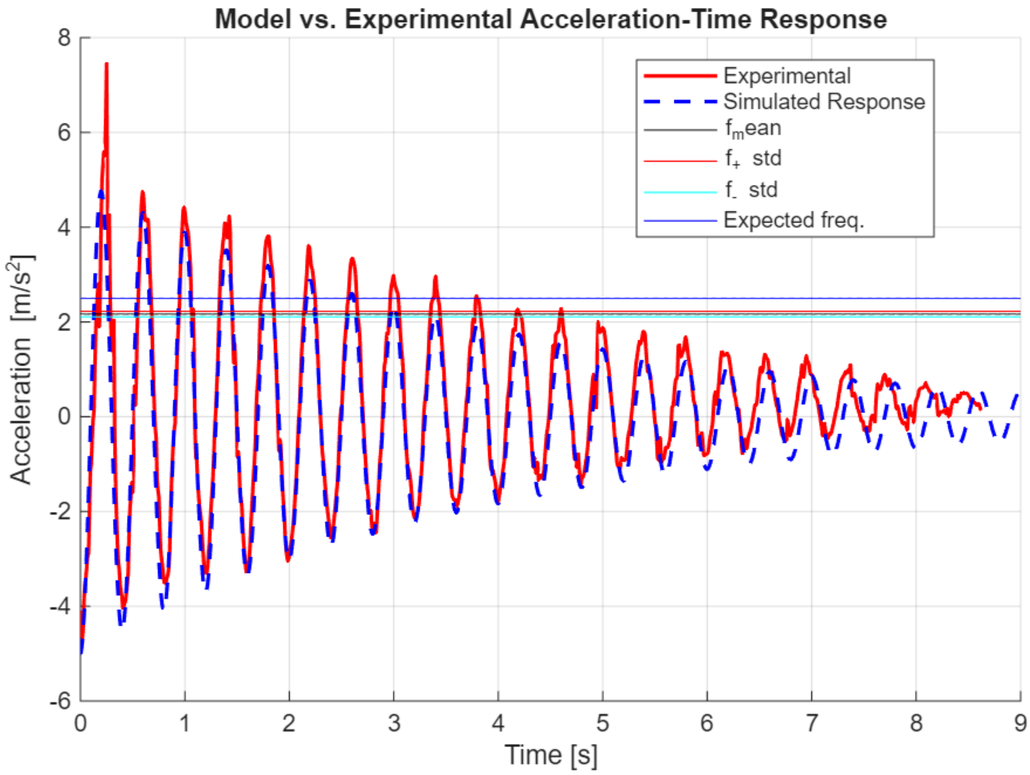


Figure 4: Validation of model indicating clear frequency difference

Reflection

Upon the validation of the theoretical model, it is found that the amplitudes and shapes of the two graphs match to a great extent. The model can be improved by varying zeta values to better match the experimental data even further (as shown in fig. 5). This comparison concludes that a damping ratio of **0.016** is most appropriate based on the clear superpositional similarity of the experimental and modeled graph.

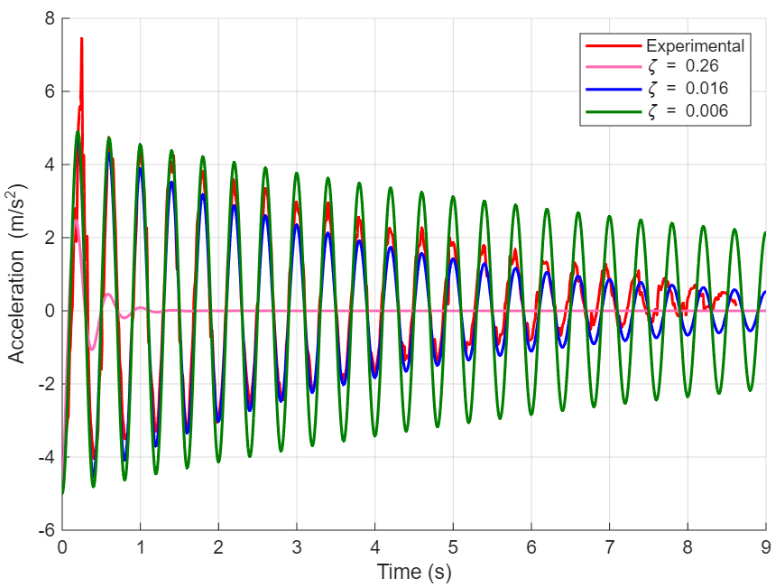


Figure 5: Zeta Comparison