4CBLB00 Solar Heat System

Self-Study Assignment
Group 16

SSA No.	Description
5	Modelling Sand
SSA Owner	
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Introduction

After discussions from the last meeting, it was deemed necessary to model the sand layer (independently) to study its thermal behavior under the artificial sun.

Goal

- To model the temperature gradient in the sand
- To model the power losses that occur by radiation

Conclusion

- After verfiying a model of sand temperature against time (see fig. 1 and verification below it) when placed under an artificial sun, different d_{values} were compared (see fig. 4 and 5) to conclude that a sand height of 10 mm is ideal (see fig. 6) since the temperature drop between the top and bottom layer of sand is only around 1 °C.

Recommendations

- A sand height of 10 mm is used for the sand solar mat design

1 Elaboration

1.1 Theory Behind The Model

Based on the design details of the system, it is given that:

- The sand solar mat is designed to be right up to the glass of the artificial sun, ie; there is no air-pocket between the glass (of the artificial sun) and the sand of the solar mat
- The surface area of the sand bed (A_{sand}) will be 590 x 640 mm
- The height of the sand bed (d_{sand}) has been kept at 50 mm (preliminary)
- The ambient temperature is set to 20 °C
- The conductive coeffecient of heat transfer of sand (k_{sand}) is 0.15 W/m K
- The heat capacity of sand (c_{sand}) is 840 J/kg K
- The absorpivity of sand (α) is assumed to be 0.35 (it is used to understand the $\dot{Q}_{absorbed}/\dot{Q}_{available-from-sun}$ ratio, it is a thermal radiative property of a surface and can be assumed to be constant for the duration of the experiment)

The equations which govern the heating of the sand are the following:

$$
\begin{aligned}
\dot{Q}_{sand} &= m_{sand} * c_{sand} * \frac{T_{final} - T_{initial}}{dt} \\
\dot{Q}_{radiation-absorbed} &= I * A_{sand} * \alpha \\
\dot{Q}_{radiation-loss} &= \sigma * e * A * (T_{sand}^4 - T_{ambient}^4) \\
\dot{Q}_{conduction-through-sand} &= \frac{k_{sand} * A_{sand} * (T_{sand-top} - T_{sand-bottom})}{d_{sand}} \\
m_{sand} * c_{sand} * \frac{dT}{dt} &= \dot{Q}_{radiation-absorbed} - \dot{Q}_{radiation-loss} - \dot{Q}_{conduction-through-sand}
\end{aligned}
\tag{1}
$$

Conceptually, heat enters the sand system through the mechanism of radiation, and the losses that occur are due to the poor conductivity of sand and heat lost due to radiation. No convection is assumed to occur since it is assumed above that there is no air medium between the sand layer and sun (glass), and since the sand is stationary and tightly packed, no convective flows can occur inside the sand bed.

The constants that need to be defined are the following:

- Stefan-Boltzmann Constant (σ): 5.6704 * 10^{-8} W/m^2 K^4
- Irradiance of the Artificial Sun (I): 1000 W/m^2

From this model, the following results are desired

- The temperature at the bottom of the sand layer ($T_{sand-bottom}$)
- The height of the sand layer (d_{sand})
- The mass of the sand required to create this height (m_{sand})

First, a model of the preliminary situation (d = 50 mm), which will be followed by iterations to optimize the sand height.

1.2 Modelling Preliminary Sand Layer (d=50 mm)

The best way to model this simple system is with an ode45 solver. Based on the last eqn (in the above subsection), T needs to be solved for. $T_{initial}$, $T_{ambient}$ and $T_{sand-bottom}$ are all equal to 20 °C (initially), and only $T_{sand-bottom}$ rises with time. The ode is expressed as follows:

$$
\begin{aligned}
m_{sand} * c_{sand} * \frac{dT}{dt} &= \dot{Q}_{radiation-absorbed} - \dot{Q}_{radiation-loss} - \dot{Q}_{conduction-through-sand} \\
\frac{dT}{dt} &= \frac{1}{m_{sand} * c_{sand}} * \dot{Q}_{radiation-absorbed} - \dot{Q}_{radiation-loss} - \dot{Q}_{conduction-through-sand} \\
\frac{dT}{dt} &= \frac{1}{m_{sand} * c_{sand}} * [(I * A_{sand} * \alpha) - (\sigma * e * A * (T^4 - T^4_{ambient})) \\
&\quad - \frac{k_{sand} * A_{sand} * (T - T_{ambient})}{d_{sand}}]
\end{aligned} \tag{2}
$$

After defining all the neccesary constants and fixed values, the above equation has been replicated in the matlab ode solver as follows:

```
odefun = @(t,T) (1/(m*cp))*( Qin - (T - T_ambient)/Rcond - e*sigma*A*(T.^4 - T_ambient^4) );
```

The constants (and fixed values) are defined as follows:

```
rho = 1210;            % density of sand in kg/m^3
A   = 0.59*0.65;       % Cross-section of sand bed in m^2
d   = 50e-3;           % Depth of sand layer in m
m   = rho*A*d;         % Mass of sand in kg

I     = 1000;          % Irradiance of artificial sun in W/m^2
alpha = 0.35;          % absorptivity of sand (dimensionless)
k     = 0.25;          % Thermal conductivity of sand in W/(m K)
cp    = 800;           % Heat capacity of sand in J/(kg K)
e     = 0.65;          % Emissivity of sand (dimensionless)
sigma = 5.67e-8;       % Stefan-Boltzmann Constant
T_ambient   = 293.15;  % Ambient temperature in K

Rcond = d/(k*A);       % Conductive thermal resistance (K/W)

Qin = I*A*alpha;       % Power going into the system (W)
```

To check if the model is functioning properly, first a simulation with $d_{sand} = 50\ mm$ has been run with a timespan of 60*10^3 s (a randomly chosen large timespan to check behaviour of model). This is done using the following simulation settings:

```
tspan = [0 60000]; % Time span of 30 minutes (in seconds)
T0 = T_ambient;    % Explicitly defining initial temperature condition (= 20 deg C)
[t,T] = ode45(odefun, tspan, T0);
```

To visualize the results, temperature (T) has been plot against time (t) by the following piece of code after the simulation:

```
figure;
plot(t/3600, T-273.15); %Converting time to hours and temperature from Kelvin to Degrees C
xlabel('Time (hours)'); ylabel('Sand temperature (C)');
title('Sand heating under I = 1000 W/m^2');
grid on;
```

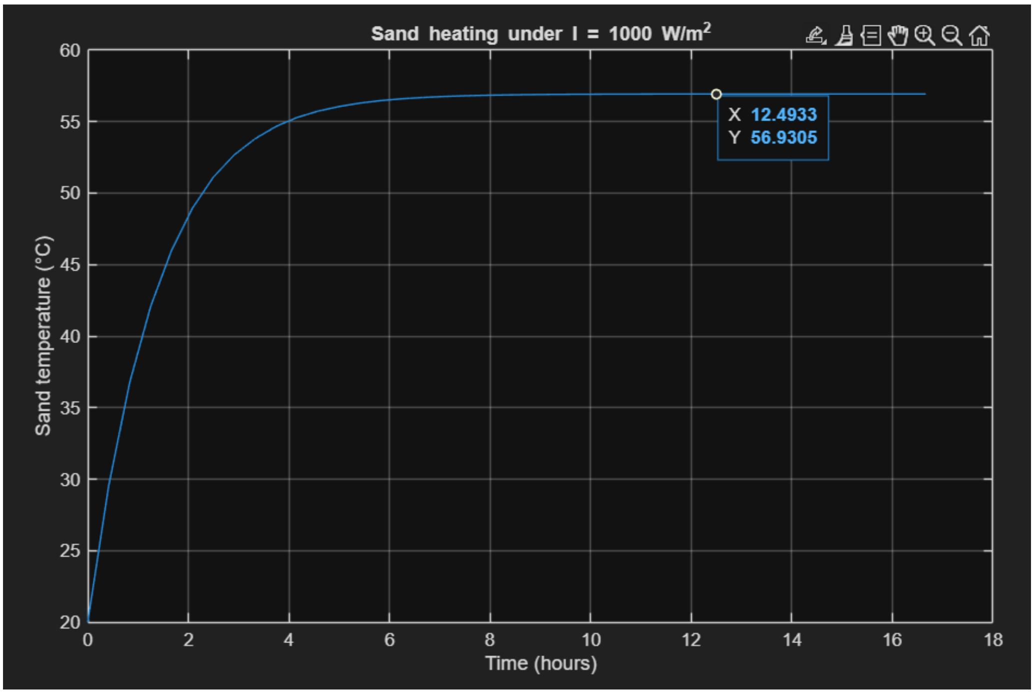

Figure 1: Sand Model Sanity Check

A few things to note about this version of the model are as follows:

- It treats the entire sand bed as one heating entity, and doesn't give the temperature of the top or bottom surface, it gives the average temperature of the sand bed against time
- The steady state value corresponds with the solution of the ode, hence the model has been verified succesfully

To verify the model, quite simply the model was simulated for a long time-period so that it achieves steady state (ie; when dT/dt = 0). If the temp. at steady state matches the mathematical solution (for temperature) of the ode at steady state, the model can be considered verified. Hence, if the model is accurate it should give a temperature of:

$$\frac{dT}{dt} = 0 = \frac{1}{m_{sand} * c_{sand}} * [(I * A_{sand} * \alpha) - (\sigma * e * A * (T^4 - T^4_{ambient})) \\ - \frac{k_{sand} * A_{sand} * (T - T_{ambient})}{d_{sand}}] \tag{3}$$

$$\sigma * e * A * (T^4 - T^4_{ambient}) + \frac{k_{sand} * A_{sand} * (T - T_{ambient})}{d_{sand}} = (I * A_{sand} * \alpha)$$

$$5.67 * 10^{-8} * 0.65 * (0.59 * 0.65) * (T^4 - 293^4) + \frac{0.25 * (0.59 * 0.65) * (T - 293)}{50 * 10^{-3}} = (1000 * (0.59 * 0.65) * 0.35)$$

$$1.4134 * (10^{-8}) * T^4 + 1.9175 * T = 799.0935 \tag{4}$$

This quartic equation was solved using an online quartic equation solver ([1]), the four solutions are displayed below:

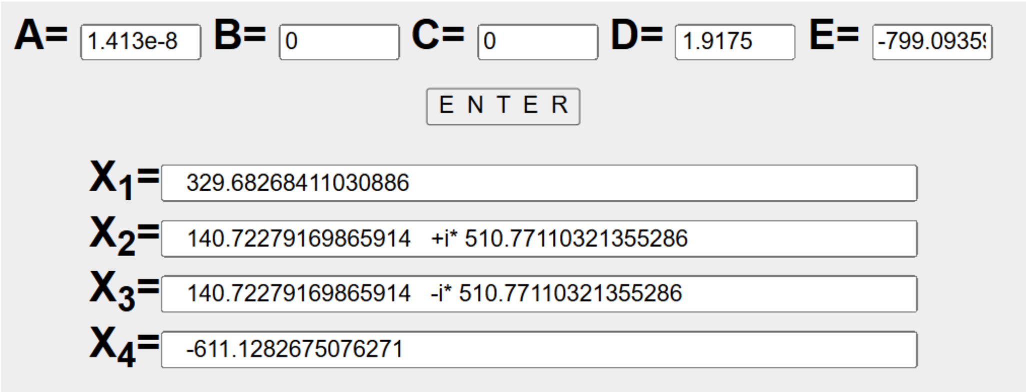

Figure 2: Quartic Equation Solutions (x1,x2,x3 and x4)

Only x1 is considered as a valid solution out of the four, since temperature cannot be in the imaginary domain or negative (since this is a heating process). This solution is in Kelvin, when converted to degree celcius it gives a temperature of **56.53 °C**, which is roughly equal to the **56.93 °C** predicted by the model. Hence, it is considered that the model has been verified successfully.

1.3 Finding Temperature at The Bottom of The Sand Bed

Between the sand layers itself, the primary form of heat transfer can be considered to be conduction, hence after the first layer of sand (say 5 mm of sand), the layers that follow can be described by the following ode:

$$\frac{dT_{nth-layer}}{dt} = \frac{1}{m_{sand} * c_{sand}} * \dot{Q}_{conduction-absorbed} - \dot{Q}_{conduction-loss}$$

$$\frac{dT_{nth-layer}}{dt} = \frac{1}{m_{sand} * c_{sand}} * \left[\frac{k_{sand} * A_{sand} * (T_{above} - T_{nth-layer})}{d_{sand}} - \frac{k_{sand} * A_{sand} * (T_{nth-layer} - T_{(n+1)-layer})}{d_{sand}}\right] \tag{5}$$

Here, T_{above} is the time varying temperature of the layer above the nth layer and $T_{(n+1)-layer}$ is the time-varying temperature of the layer below the nth layer. The last layer is assumed to be perfectly insulated against heat losses, hence it only conducts heat from the layer above it, this is a reasonable assumption since this effect is achieved by the insulation of the design. This ode depends on the time-varying temperature outputs of the layer above and below it, hence this system can be replicated in MATLAB as follows:

This ode can be replicated in Matlab using the following function:

```
odefun = @(t,Y) [
(1/(m*cp))*(Qin - (Y(1) - Y(2))/Rcond - e*sigma*A*(Y(1).^4 - T_ambient^4) );
(1/(m*cp))*(((Y(1) - Y(2))/Rcond)-(Y(2) - Y(3))/Rcond);
(1/(m*cp))*(((Y(2) - Y(3))/Rcond)-(Y(3) - Y(4))/Rcond)
(1/(m*cp))*(((Y(3) - Y(4))/Rcond)-(Y(4) - Y(5))/Rcond)
(1/(m*cp))*(((Y(4) - Y(5))/Rcond)-(Y(5) - Y(6))/Rcond)
(1/(m*cp))*(((Y(5) - Y(6))/Rcond)-(Y(6) - Y(7))/Rcond)
(1/(m*cp))*(((Y(6) - Y(7))/Rcond)-(Y(7) - Y(8))/Rcond)
(1/(m*cp))*(((Y(7) - Y(8))/Rcond)-(Y(8) - Y(9))/Rcond)
(1/(m*cp))*(((Y(8) - Y(9))/Rcond)-(Y(9) - Y(10))/Rcond)
(1/(m*cp))*(((Y(9) - Y(10))/Rcond))
];
```

Here, 10 layers have been used, the height of each layer depends on variable 'd' when simulated, and the number of layers has been saved in a variable called 'N'.

Following this, to run a simulation for a total $d_{sand} = 50\ mm$, the height of the individual 10 layers needs to be:

$$\begin{aligned} d &= \frac{d_{sand}}{n} \\ &= \frac{50 * 10^{-3}}{10} \\ &= 50 * 10^{-4} \end{aligned} \tag{6}$$

To run this simulation, the ode function is called as shown below:

```
tspan = [0 1800]; % Time span of 30 minutes (in seconds)

Y0=T_ambient*ones(N,1);
[t, Y] = ode45(odefun, tspan, Y0);

T_top = Y(:,1);
T_lower = Y(:,N);
```

A timespan of 30 minutes is used to gain an understanding of the sand heating system within the experimental time-frame.

The results for a d_{sand} of 50 mm have been displayed in fig. 3

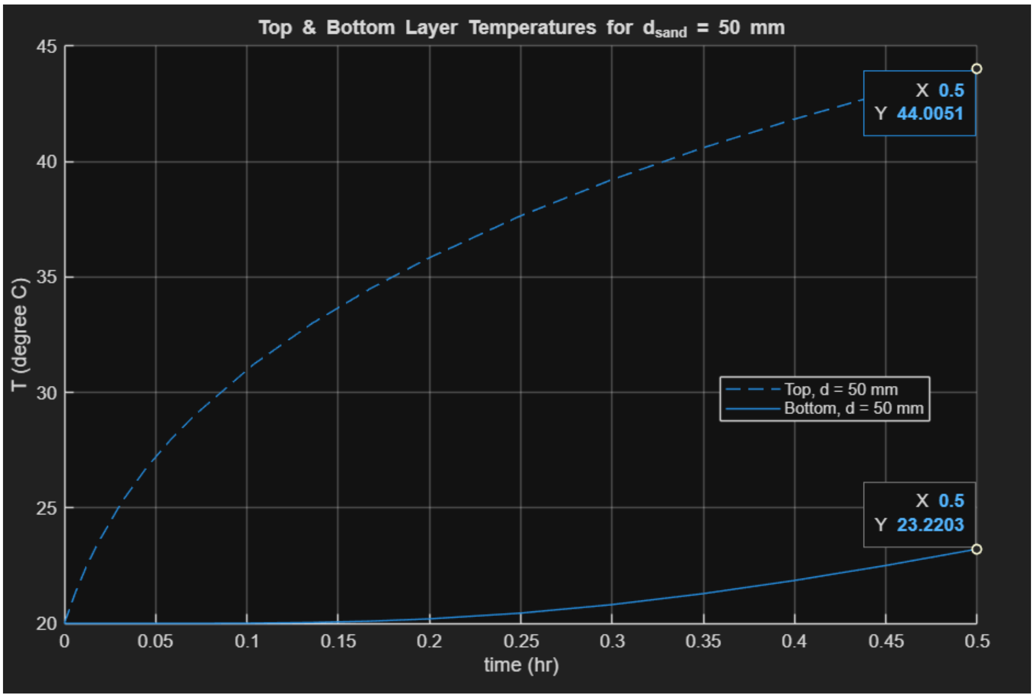

Figure 3: Temperature of the bottom-most layer when sand bed depth (d_{sand}) is 50 mm

The model suggests that the bottom-most layer would not achieve steady state within 30 minutes and the temperature reaches a maximum of to **23.22 °C** approximately. This is undesirable, since this is barely higher than the room temperature of 20 °C. Along with this, the graph clearly shows that a lot of energy is lost (due to conduction) between the top and bottom-most layer.

1.4 Finding an Optimum Sand Height

Figure 3 clearly suggests that a sand depth of 50 mm is undesirable. To find a desirable height of sand, the following steps are followed

- A series of depths will be simulated (both above and below the 50 mm mark)
- These simulations will be graphed together to obtain a conclusion on optimum d_{sand} and notice any pattern/relationship with depth of sand bed

To make things easier, a function was setup to adapt for different no.s of layers (N) and different depths (d_values). This function is shown below:

```
function dYdt = sand_layers_ode(¬,Y,N,m_node,cp,Rcond,Qin,e,sigma,A,T_amb)
    dYdt = zeros(N,1);

    % Top node
    dYdt(1) = (1/(m_node*cp))*(Qin - (Y(1)-Y(2))/Rcond - e*sigma*A*(Y(1)^4 - T_amb^4));

    % Interior nodes
    for i = 2:N-1
        dYdt(i) = (1/(m_node*cp))*( (Y(i-1)-Y(i))/Rcond - (Y(i)-Y(i+1))/Rcond );
    end

    % Bottom node
    dYdt(N) = (1/(m_node*cp))*( (Y(N-1)-Y(N))/Rcond);
end
```

Here node is used as a synonym for layer. The logic used to make this function follows from the equations and theory discussed in the previous subsection.

For a range of the following depths:

```
d_values = [10,20,30,40,50,60,70,80,90,100]; % Sand bed depths in mm
```

With the help of the loop shown below:

```
figure; hold on;
for j = 1:length(d_values)
    colors = lines(length(d_values));   % Used to make different colors for each plot

    d = d_values(j) / 1000;  % Converts sand depth from mm -> m
    N= d_values(j)/5;        % No. of 5 mm layers
    m = A*d*rho;             % mass of entire sand bed in kg


    dz = d / N;             % Height of one layer
    m_node = rho*A*dz;      % Mass of one layer
    Rcond = dz / (k*A);     % Conductive thermal resistance of one layer (K/W)

    %Defining the ODE
    odefun = @(t,Y) sand_layers_ode(t,Y,N,m_node,cp,Rcond,Qin,e,sigma,A,T_ambient);


    % Solving the  ODE
    Y0 = T_ambient*ones(N,1);
    [t, Y] = ode15s(odefun, tspan, Y0);

    % Extracting temperatures from solution
    T_top   = Y(:,1) - 273.15;
    T_lower = Y(:,end) - 273.15;

    % Plotting results

    plot(t/3600, T_top, '--', 'Color', colors(j,:), ...
        'DisplayName', sprintf('Top, d = %d mm', d_values(j)));

    plot(t/3600, T_lower, '-', 'Color', colors(j,:), ...
        'DisplayName', sprintf('Bottom, d = %d mm', d_values(j)));
end

xlabel('time (hr)')
ylabel('T (degree C)');
legend show;
legend('Location','best');
grid on;
title('Top & Bottom Layer Temperatures for Different Thicknesses');
```

A graph was made to visualize the effect of d_sand on the temperature of the bottom-most layer. This is seen in fig. 4

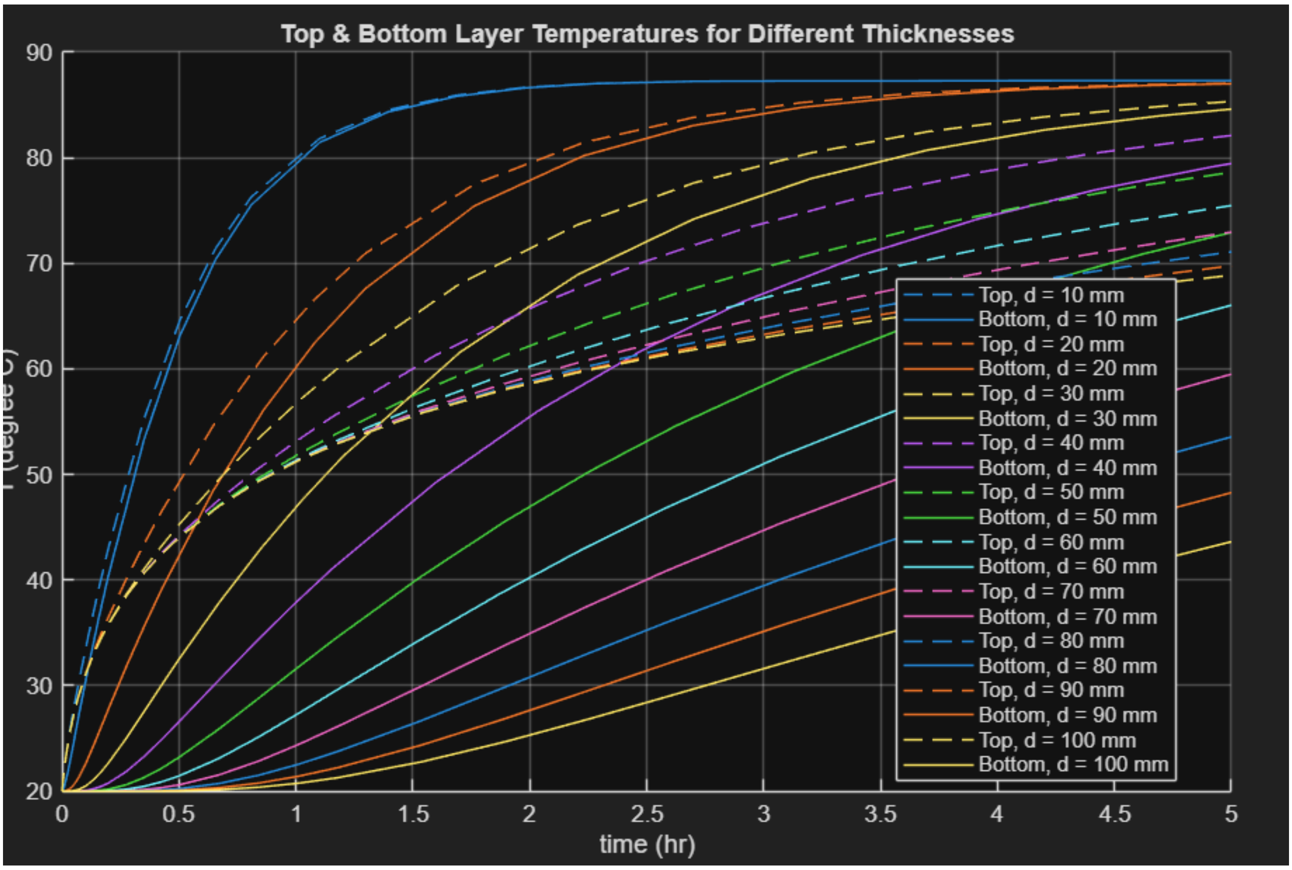

Figure 4: Depth Comparison

The graph is quite chaotic to comprehend at first, however, just looking at the graphs of 10 mm, 20 mm and 30 mm is enough to understand the pattern, this is shown below in fig. 5

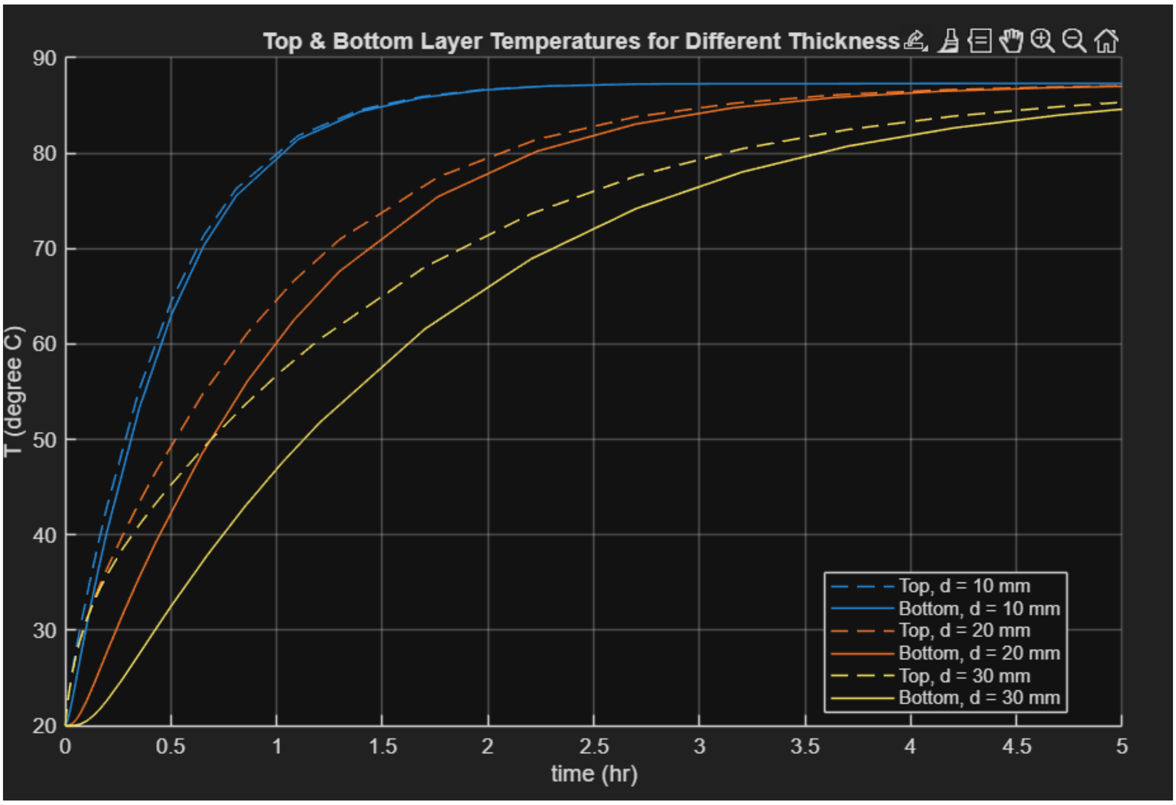

Figure 5: Depth comparison for d_sand=10,20 and 30 mm

As the sand layer reduces in height, the temperature difference between the top and bottom layer tends to 0. With a layer of 10 mm the temperature drop is negligible (hardly 1 °C), hence it is recommended that a **10 mm** sand layer is used. The figure 5 shows results for a time period of 5 hours (it was done to see if the model was working correctly), when simulated for a period of 30 minutes, the results of a 10 mm high sand layer are shown below in fig. 6:

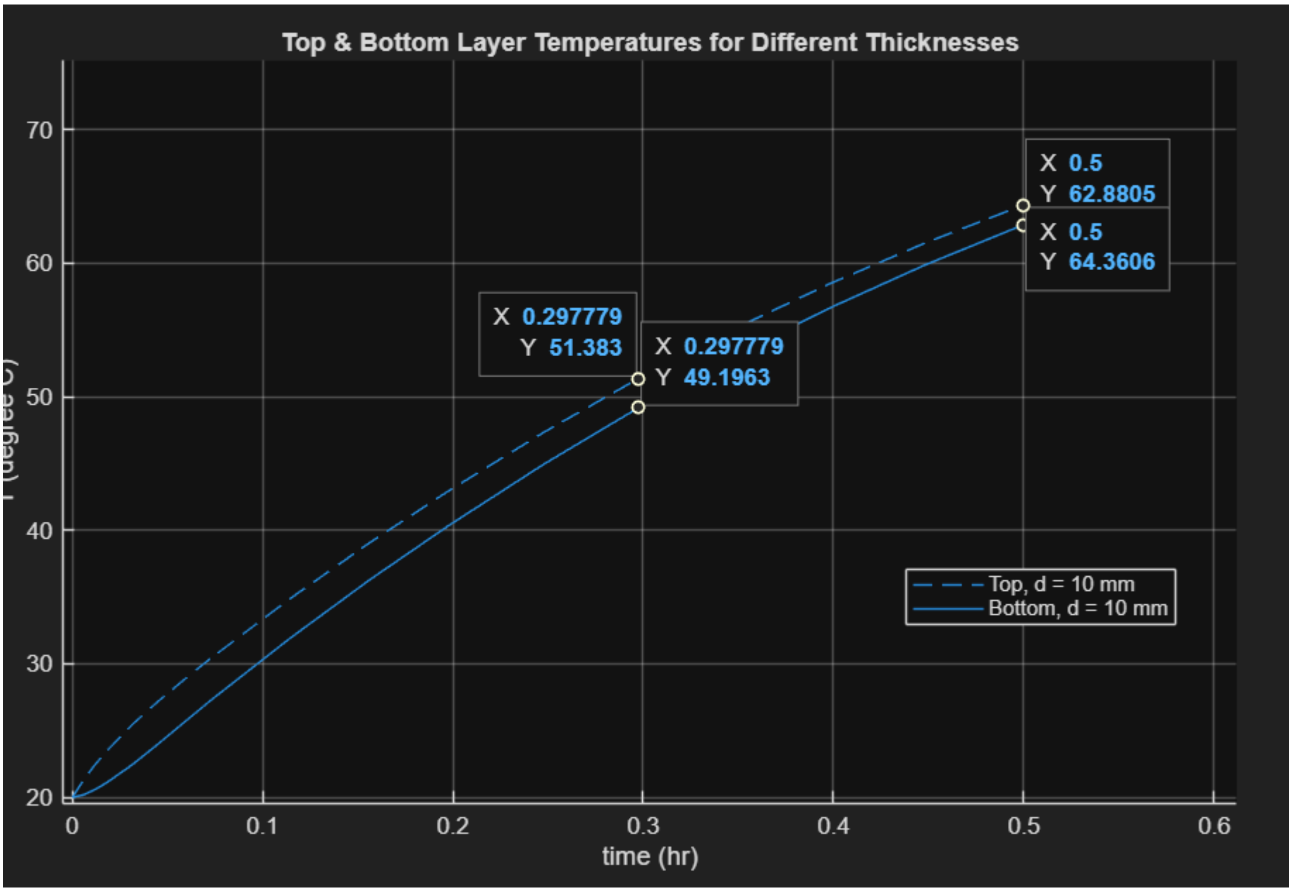

Figure 6: Temperature against time for $d_{sand} = 10$ mm

Overleaf Link to this SSA

https://www.overleaf.com/read/djzcvfzxshnb#7f20c0

References

[1] *Quartic Equation Calculator*. URL: https://www.1728.org/quartic.htm. (accessed: 20.09.2025).