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(1) (1) $rref(A) = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

basis of $\ker(T) = \left\{ \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$

Pivots are columns 1, 2, 3.

basis of $image(T) = \left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 10 \\ -8 \\ 2 \end{bmatrix} \right\}$

(2) Yes. a_4 is reduntant

$$\begin{bmatrix} 2 \\ 2 \\ -4 \\ 2 \end{bmatrix} = a \begin{bmatrix} 1 \\ 2 \\ -1 \\ 1 \end{bmatrix} + b \begin{bmatrix} 3 \\ 4 \\ -1 \\ 3 \end{bmatrix} + c \begin{bmatrix} 6 \\ 10 \\ -8 \\ 2 \end{bmatrix}$$

From rref, we have

$a = -4$, $b = 0$, $c = 1$

(3)

No. Because, $\vec{a}_4 = -4\vec{a}_1 + \vec{a}_3$

$\Rightarrow$ They are linearly dependent

Hence, $\vec{a}_2$ cannot be formed from them.

(4)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -1 \\ 1 \end{bmatrix} \times s \quad , \quad s \in \mathbb{R}$$

(2)

(a) No. $\vec{a_1} = x_1 \vec{a_2} \Rightarrow x_1 = 0$

Hence, Lineerly independent

(b)

$$\begin{bmatrix} -1 & -4 & 1 \\ -1 & -4 & 1 \\ 4 & 17 & -3 \end{bmatrix}$$

$$rref = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

dim $= 2 < 3 \Rightarrow$ lineerly dependent

(c) $$rref = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow$ linerly independent

(d) $$rref = \begin{bmatrix} 1 & 0 & 0 & -\frac{31}{5} \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & \frac{24}{5} \end{bmatrix}$$

dim $= 3 < 4 \Rightarrow$ lineerly dependent

1 free variable.

(2)

(c) forms a basis of R^3

(3)

(1) $$\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

(2) Yes. It can have no solution.

Suppose $\vec{r}_4 = \vec{r}_1 + \vec{r}_2 + \vec{r}_3$ (r_i are rows of A)

and $c_4 = c_1 + c_2 + c_3 + 1$ (where $\vec{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$)

We have no zero in last column

(3) NO. It can have atmost 1 solution.

Because $\text{rank}(A) = 3 =$ number of columns

④

(1) $M^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 2 & 4 & 7 \end{bmatrix}$

(2) $\vec{x} = M^{-1} \vec{b} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 2 & 4 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\vec{x} = \begin{bmatrix} 6 \\ 14 \\ 13 \end{bmatrix}$

(3) Yes. Because M is inVertible

we Can multiply both sides by

M^{-1} 5 times

and get

$\vec{x} = (M^{-1})^5 \vec{b}$

(5)

(1) $\vec{0}$ is present in S_n — [i]

let $A, B \in S_n$

$$(A+B)^T = A^T + B^T = A+B$$

$\Rightarrow (A+B) \in S_n$ — [ii]

let $A \in S_n$ and $r \in \mathbb{R}$

$$(rA)^T = r\,A^T = rA$$

$\Rightarrow (rA) \in S_n$ — [iii]

$\Rightarrow$ From [i], [ii] & [iii]

S_n is a Vector Space

(2) Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$A^T = A$

$$\begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$\therefore a_{12} = a_{21}$

$$\Rightarrow A = a_{11} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + a_{12} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} + a_{22} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \text{basis} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

as $c_1 b_1 + c_2 b_2 + c_3 b_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\Rightarrow c_1 = c_2 = c_3 = 0$

$\therefore$ Dimension $= 3$

(6)

$$I_n = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$E_{ij} = E_{ji} \Rightarrow (E_{ij})^T = E_{ij}$$

$$E_i(r) = E_i[r]$$

(7)

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) - 2\,T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 3 \\ 5 \\ 3 \end{bmatrix} - \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \; ; \; T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$T(\vec{x}) = \begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 3 & 3 \end{bmatrix} \vec{x}$$

(8) $\vec{u} + k\vec{v},\ \vec{u} + \vec{w},\ \vec{u} + \vec{w}$

$$\begin{vmatrix} 1 & k & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = 1(0-1) - k(1-0) = 0$$

$$\Rightarrow \boxed{k = -1}$$

It can be seen that, $\boxed{\vec{u} - \vec{v} = (\vec{v} + \vec{w}) - (\vec{u} + \vec{w})}$

(9)

(1) False

(2) True

(3) True

(4) True

(5) False

(6) False

(7) True

(8) False

(10) M = [

let, a_1, a_2, a_3 are numbers (non-zero]
(or 1x1 matrices)

M is also 1x1.

rank(M) = 1 = n

nullity = 0.