Math 5231 - Fall 2019
Problem Set 6

Systems of Linear Equations with Complex Coefficients Solve the following systems of differential equations. Draw a trajectory chart consisting of the behavior at the eigenvectors, and showing the long term behavior in each sector.

1)
$$x_1' = x_1 + 2x_2$$
$$x_2' = -2x_1 + x_2$$

2)
$$x_1' = -3x_1 + 2x_2$$
$$x_2' = -4x_1 + x_2$$

3)
$$x_1' = -x_1 + x_2$$
$$x_2' = -5x_1 + x_2$$

4)
$$x_1' = -x_1 + x_2 - 2x_3$$
$$x_2' = -10x_1 + 7x_2 + 4x_3$$
$$x_3' = 3x_2 + 3x_3$$

In 4, it is given that one of the eigenvalues is $3i$.

Describe the solutions to the following systems of equations. State whether the solutions is a **node**, **focus** or **saddle**. Describe it's stability, either as **stable** or **unstable** for a node and a focus, or its directions of stability for a **a saddle**. Finally, if it is a focus state the direction of rotation as **clockwise** or **counter clockwise**.

5)
$$x_1' = x_1 + x_2$$
$$x_2' = -x_1 + x_2$$

6)
$$x_1' = x_1 + -x_2$$
$$x_2' = x_1 + x_2$$

7)
$$x_1' = 5x_1 - 3x_2$$
$$x_2' = 9x_1 - 7x_2$$

8)
$$x_1' = 2x_1 - 8x_2$$
$$x_2' = 5x_1 - 2x_2$$

Answers:

1) $$\vec{x} = Ce^{(1+2i)t} \begin{bmatrix} -i \\ 1 \end{bmatrix} + \bar{C}e^{(1-2i)t} \begin{bmatrix} i \\ 1 \end{bmatrix}.$$

2) $$\vec{x} = Ce^{(-1+2i)t} \begin{bmatrix} 1 \\ 1-i \end{bmatrix} + \bar{C}e^{(-1-2i)t} \begin{bmatrix} 1 \\ 1+i \end{bmatrix}.$$

3) $$\vec{x} = Ce^{-2it} \begin{bmatrix} 1 \\ 1-2i \end{bmatrix} + \bar{C}e^{2it} \begin{bmatrix} 1 \\ 1+2i \end{bmatrix}.$$

4) $$\vec{x} = C_1e^{-3it} \begin{bmatrix} 1 \\ 1-i \\ i \end{bmatrix} + \bar{C}_1e^{3it} \begin{bmatrix} 1 \\ 1+i \\ -i \end{bmatrix} + C_2e^{9t} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

(5) Unstable focus, Clockwise. (6) Unstable focus, Counterclockwise. (7) Saddle, unstable direction $[1, 1]$, stable direction $[1, 3]$. (8) Stable focus, Counterclockwise.