MATH 5110

Hints for Lab 2a - Areas and Determinants

$\boxed{\textbf{Task 4:}}$ Consider a circle C of radius r in the xy-plane, centered at (a,b): its equation is

$$(x-a)^2 + (y-b)^2 = r^2$$

Let L be an invertible linear map from $\mathbb{R}^2$ to itself, represented by the 2×2 matrix A. Then the image of a point (x,y) under the mapping L is the point (u,v), where

$$\begin{pmatrix} u \\ v \end{pmatrix} = A \begin{pmatrix} x \\ y \end{pmatrix}$$

Since L is invertible this is equivalent to

$$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1} \begin{pmatrix} u \\ v \end{pmatrix}$$

Therefore there are scalars d, e, f, g such that

$$x = du + ev, \quad y = fu + gv$$

Suppose now that (x,y) lies on the circle C, then the point (u,v) must lie on the curve $L(C)$ described by the equation

$$(du + ev - a)^2 + (fu + gv - b)^2 = r^2$$

This is a second order equation in the coordinates (u,v) with positive coefficients in highest order, and therefore describes an ellipse. You can show more generally that the image under L of any ellipse is another ellipse.