

14-12. Suppose you are interested in investigating the factors that affect the prevalence of tuberculosis among intravenous drug users. In a group of 97 individuals who admit to sharing needles, 24.7% had a positive tuberculin skin test result; among 161 drug users who deny sharing needles, 17.4% had a positive test result [15].

(a) Assuming that the population proportions of positive skin test results are in fact equal, estimate their common p-value.

Tuberculin skin test result				
Sharing needle s		Positive	Negative	Total
	Admit	$97 \times 24.7\% = 23.959$	$97 \times 75.3\% = 76.041$	97
	Deny	$161 \times 17.4\% = 28.014$	$161 \times 82.6\% = 132.986$	161
	Total	51.973	209.027	258

The common p-value $\hat{p} = \frac{51.973}{258} \cong 0.20$

(b) Test the null hypothesis that the proportions of intravenous drug users who have a positive tuberculin skin test result are identical for those who share needles and those who do not.

$$H_0: p_1 = p_2; H_1: p_1 \neq p_2$$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\frac{pq}{n_1} + \frac{pq}{n_2}}} = \frac{(0.247 - 0.174) - 0}{\sqrt{(0.20)(0.80)\left(\frac{1}{97} + \frac{1}{161}\right)}} = \frac{0.073}{0.0514} = 1.4202 < 1.96$$

The p-value of the test is approximately 0.078, so we cannot reject the null hypothesis.

(c) What do you conclude?

The samples we collected in this particular study do not provide evidence that the proportions of people having positive tuberculin skin test result differ between those who had admitted to share needles and those who had denied.

(d) Construct a 95% confidence interval for the true difference in proportions.

$$(\hat{p}_1 - \hat{p}_2) \pm 1.96 \sqrt{\frac{\hat{p}_1 \hat{q}_1}{n_1} + \frac{\hat{p}_2 \hat{q}_2}{n_2}} = (0.247 - 0.174) \pm 1.96 \sqrt{\frac{(0.247)(0.753)}{97} + \frac{(0.174)(0.826)}{161}} = (-0.0309, 0.3789)$$

This 95% confidence interval contains the value 0, indicating that there is no difference between these two rates.