Math 5231 - Fall 2019
Problem Set 8: 2D Linear Systems Practice

Systems of Linear Equations with Complex Coefficients Solve the following systems of differential equations. Draw a trajectory chart consisting of the behavior at the eigenvectors, and showing the long term behavior in each sector. State whether the solutions is a **node**, **focus** or **saddle**. Describe it's stability, either as **stable** or **unstable** for a node and a focus, or its directions of stability for a **a saddle**. Finally, if it is a focus state the direction of rotation as **clockwise** or **counter clockwise**.

1) $x_1' = 3x_1 - 2x_2$ $\quad x_1(0) = 1$
$x_2' = 9x_1 - 3x_2$ $\quad x_2(0) = 3$

2) $x_1' = -x_1 - x_2$ $\quad x_1(0) = 2$
$x_2' = 4x_1 - x_2$ $\quad x_2(0) = -4$

3) $x_1' = 2x_1 - x_2$ $\quad x_1(0) = -1$
$x_2' = x_1 + 2x_2$ $\quad x_2(0) = 2$

4) $x_1' = 4x_1 + 3x_2$ $\quad x_1(0) = -2$
$x_2' = 4x_1$ $\quad x_2(0) = 4$

5) $x_1' = -3x_1 + 3x_2$ $\quad x_1(0) = 0$
$x_2' = x_1 - 5x_2$ $\quad x_2(0) = 4$

6) $x_1' = 6x_1 - 12x_2$ $\quad x_1(0) = 1$
$x_2' = 4x_1 - 8x_2$ $\quad x_2(0) = 2$

7) $x_1' = -2x_1 + \frac{3}{2}x_2$ $\quad x_1(0) = 2$
$x_2' = -5x_1 + \frac{7}{2}x_2$ $\quad x_2(0) = -2$

8) $x_1' = x_1 + 2x_2$ $\quad x_1(0) = -4$
$x_2' = -\frac{9}{2}x_1 + x_2$ $\quad x_2(0) = 6$

9) $x_1' = x_1 + 2x_2$ $\quad x_1(0) = 0$
$x_2' = \frac{9}{2}x_1 + x_2$ $\quad x_2(0) = 1$

Answers: Your answer may look different, check to see that they are the same.

1) $$\vec{x} = .5e^{3it}\begin{bmatrix} i+1 \\ 3 \end{bmatrix} + .5e^{-3it}\begin{bmatrix} 1-i \\ 3 \end{bmatrix}.$$

2) $$\vec{x} = -(1-i)e^{(2i-1)t}\begin{bmatrix} i \\ 2 \end{bmatrix} + (i-1)e^{(-2i-1)t}\begin{bmatrix} -i \\ 2 \end{bmatrix}.$$

3) $$\vec{x} = \frac{1}{4}(3-i)e^{(2+i)t}\begin{bmatrix} i-1 \\ i+1 \end{bmatrix} + \frac{1}{4}(3+i)e^{(2-i)t}\begin{bmatrix} -i-1 \\ -i+1 \end{bmatrix}.$$

4) $$\vec{x} = 0 \times e^{6t}\begin{bmatrix} 3 \\ 2 \end{bmatrix} + 2e^{-2t}\begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

5) $$\vec{x} = e^{-2t}\begin{bmatrix} 3 \\ 1 \end{bmatrix} + 3e^{-6t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

6) $$\vec{x} = 3e^{-2t}\begin{bmatrix} 3 \\ 2 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

7) $$\vec{x} = -16e^{t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + 6e^{t/2}\begin{bmatrix} 3 \\ 5 \end{bmatrix}.$$

8) $$\vec{x} = (1+i)e^{(1-3i)t}\begin{bmatrix} 2i \\ 3 \end{bmatrix} + (1-i)e^{(1+3i)t}\begin{bmatrix} -2i \\ 3 \end{bmatrix}.$$

9) $$\vec{x} = \frac{1}{3}e^{4t}\begin{bmatrix} 2 \\ 3 \end{bmatrix} + \frac{1}{3}e^{-2t}\begin{bmatrix} -2 \\ 3 \end{bmatrix}.$$