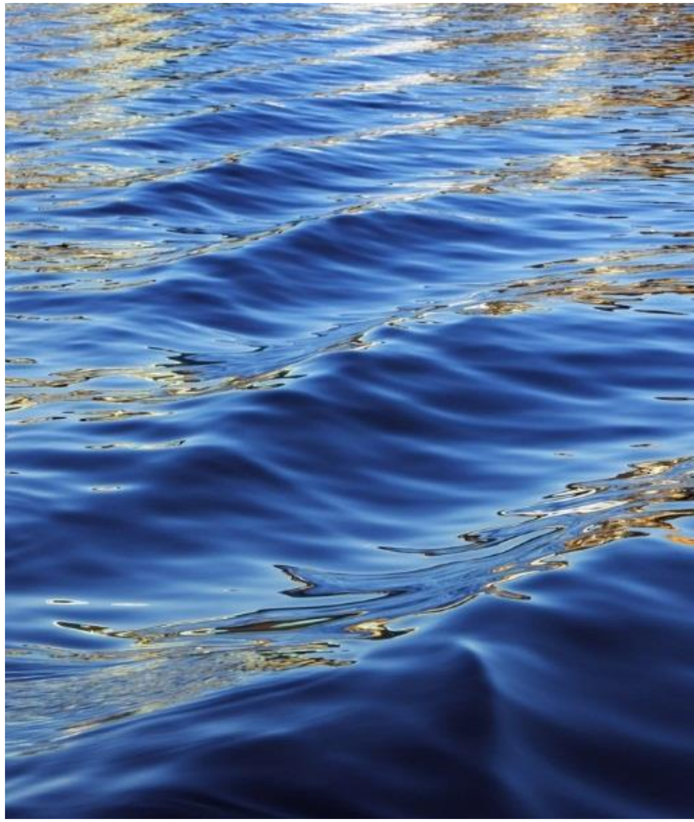
Predator-Prey Model with Weak Allee Effect and Pesticide usage

MATH 5131 – Final Project



Introduction

- The predator-prey interactions in animals are governed the following set of non-linear first order differential equations (proposed by Lotka-Volterra),

$$\frac{dX}{dt} = aX - C_1XY$$

$$\frac{dY}{dt} = C_2XY - bY$$

Extension from Malthusian to Logistic growth for Prey

- The assumption of growth rate for Prey as exponential, under the absence of Predator is impractical. Hence, an improvisation to this model can be using the logistic growth which has a carrying capacity K.

$$\frac{dX}{dt} = aX\left(1 - \frac{X}{K}\right) - C_1XY$$

$$\frac{dY}{dt} = C_2XY - bY$$

Extension from Logistic to Allee Effect Growth for Prey

- Logistic growth rate says that per-capita growth is maximum when the prey population is minimum. This may work for species like bacteria and other parasites, but it does not work in species where it is difficult to find a mate. Hence, an improvisation to logistic growth model is applying the Allee effect with an Allee effect constant A, where $0 < A < K$.

$$\frac{dX}{dt} = aX\left(\frac{X}{A} - 1\right)\left(1 - \frac{X}{K}\right) - C_1XY$$

$$\frac{dY}{dt} = C_2XY - bY$$

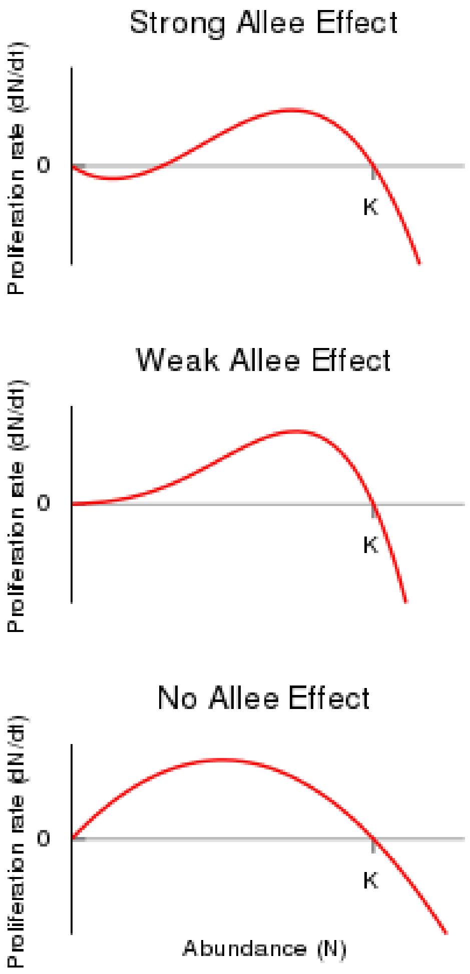
Variations in Allee Effect Growth for Prey

- When the density is below the critical threshold, the population affected by the strong Allee effect will have a negative average growth rate. Under deterministic dynamics, we find that populations that do not exceed this threshold will be extinct. This is called as Strong Allee Effect.
- The opposite is called Weak Allee effect which can be modelled powerfully using Holling's Type II function. So, the growth rate is not negative when population is small, but it is almost equal to zero.

$$\frac{dX}{dt} = aX\left(\frac{X}{X+A}\right)\left(1-\frac{X}{K}\right) - C_1XY$$

$$\frac{dY}{dt} = C_2XY - bY$$

Plots



Source: https://en.wikipedia.org/wiki/Allee_effect

Weak Allee Effect in presence of a Pesticide

- Most research only considers Strong Allee Effect. This research considers Weak Allee Effect.
- In addition to this, we also take into consideration of effect of usage of a Pesticide on both prey and predator. The model looks like the following:

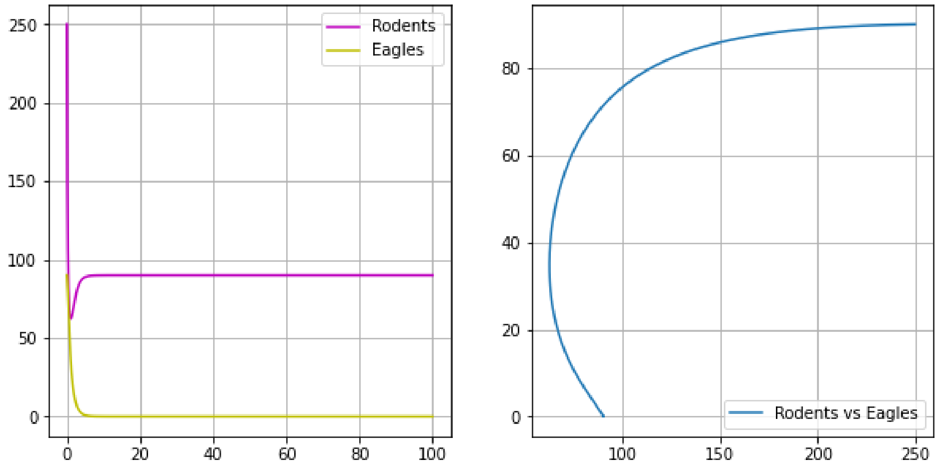
$$\frac{dX}{dt} = aX\left(\frac{X}{X+A}\right)\left(1-\frac{X}{K}\right) - C_1XY - P_1X$$

$$\frac{dY}{dt} = C_2XY - bY - P_2Y$$

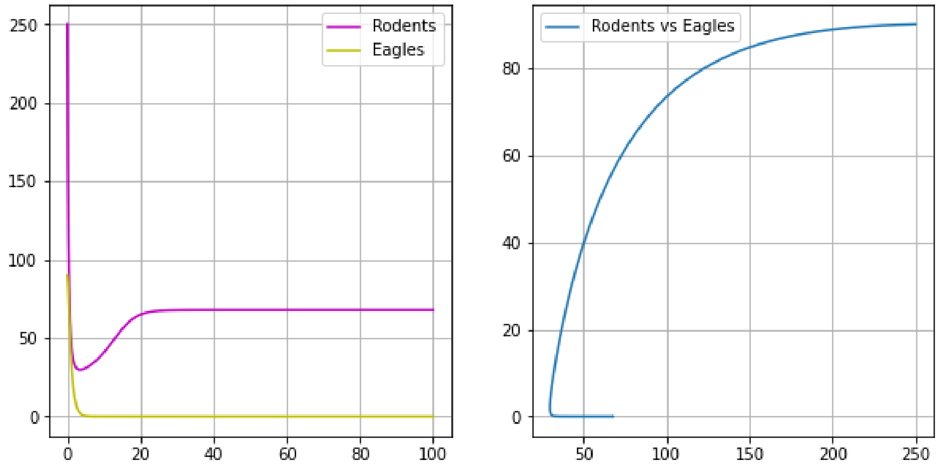
- To Simplify things we assumed $P_1 = P_2$, i.e., the effect of pesticide is the same on both the prey and the predator.

Plots

(P1, P2, K, A) = (0.25, 0.25, 100, 0.01)

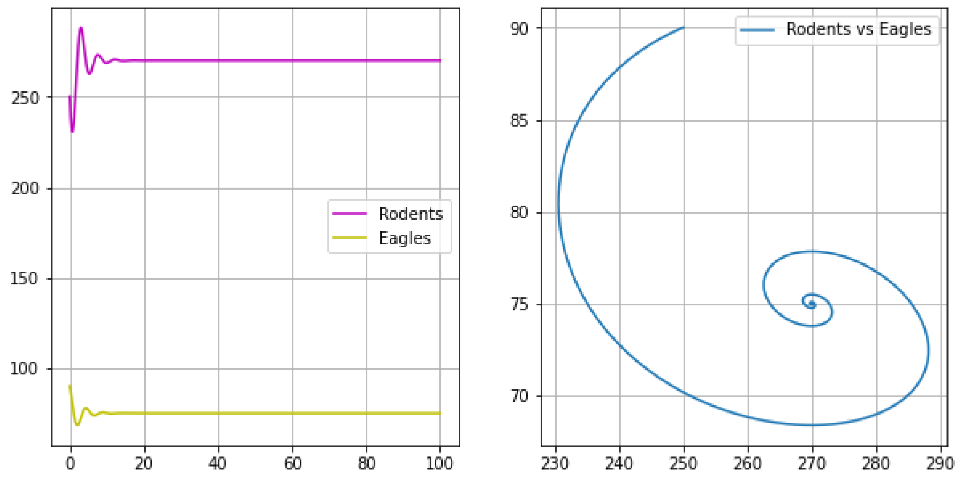


(P1, P2, K, A) = (0.25, 0.25, 100, 150)

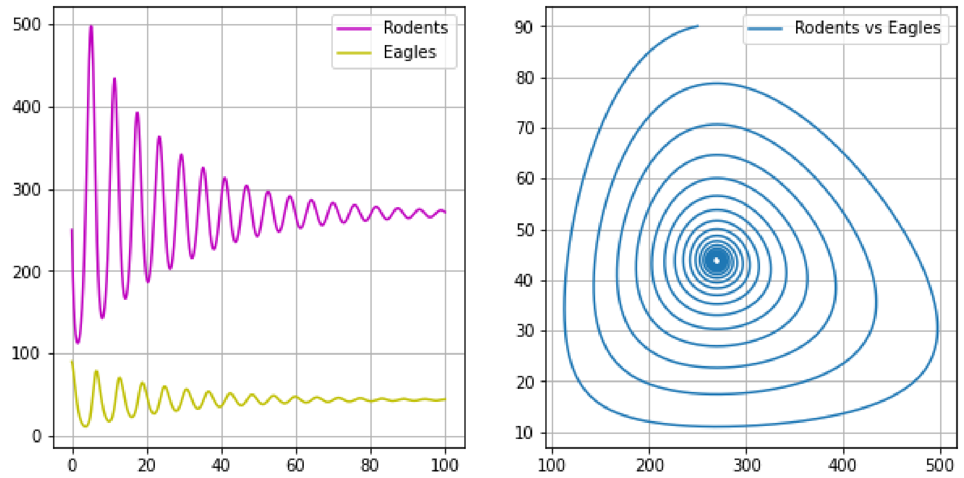


Plots (Contd.)

(P1, P2, K, A) = (0.25, 0.25, 900, 0.01)

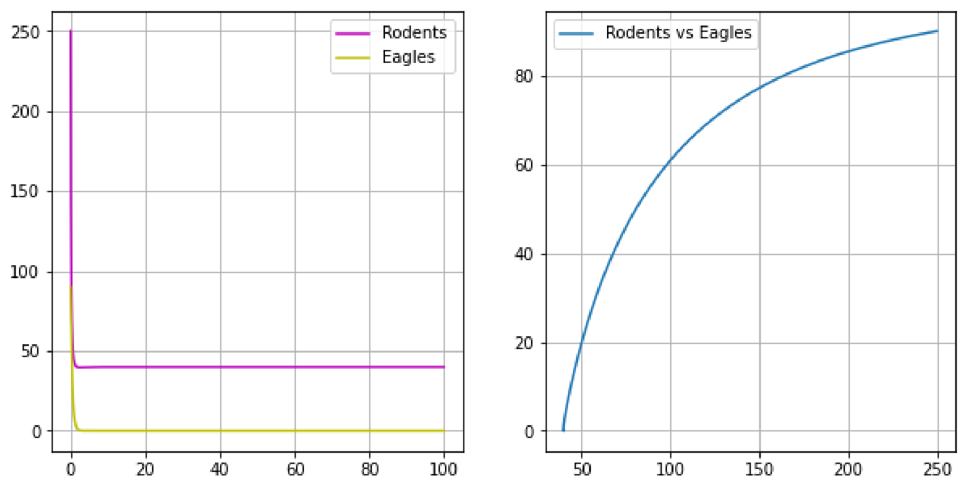


(P1, P2, K, A) = (0.25, 0.25, 900, 150)

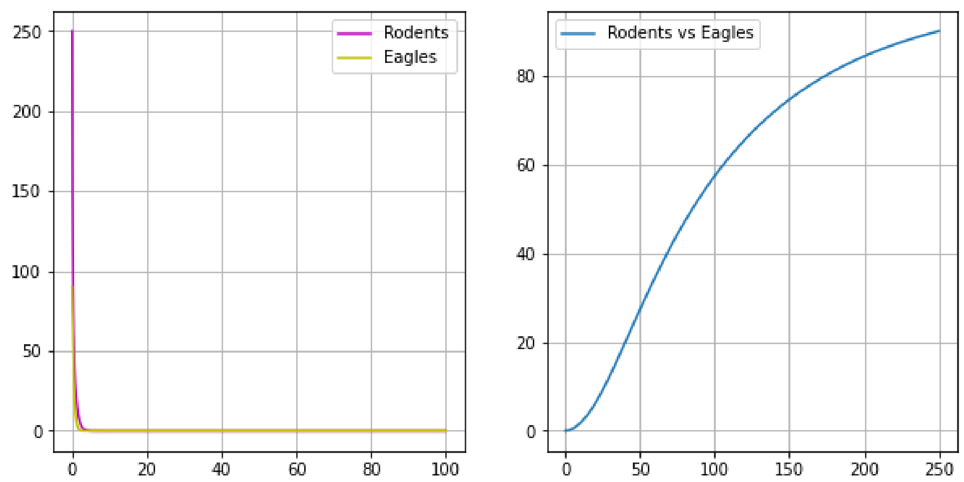


Plots (Contd.)

(P1, P2, K, A) = (1.5, 1.5, 100, 0.01)

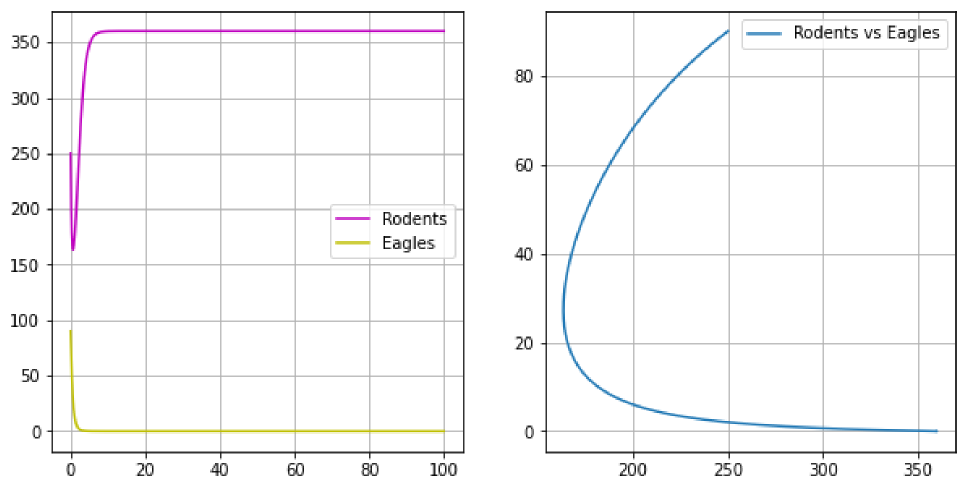


(P1, P2, K, A) = (1.5, 1.5, 100, 150)

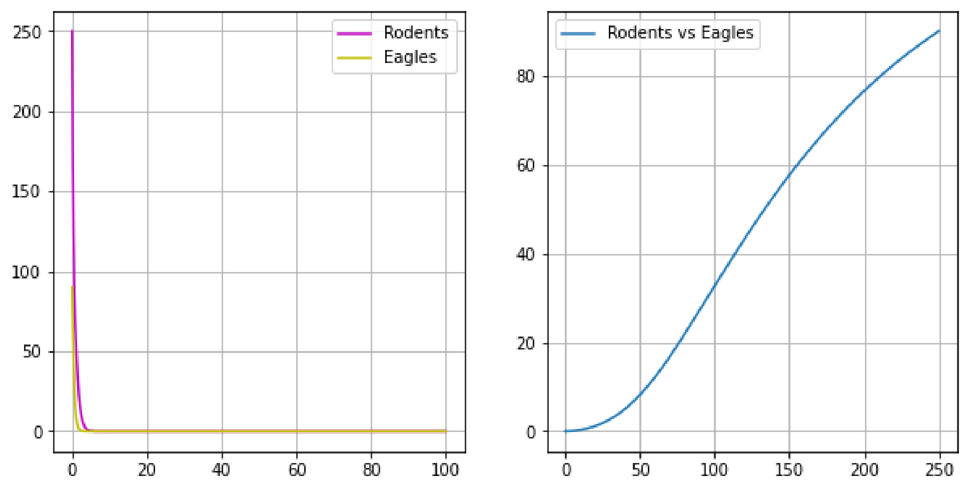


Plots (Contd.)

(P1, P2, K, A) = (1.5, 1.5, 900, 0.01)

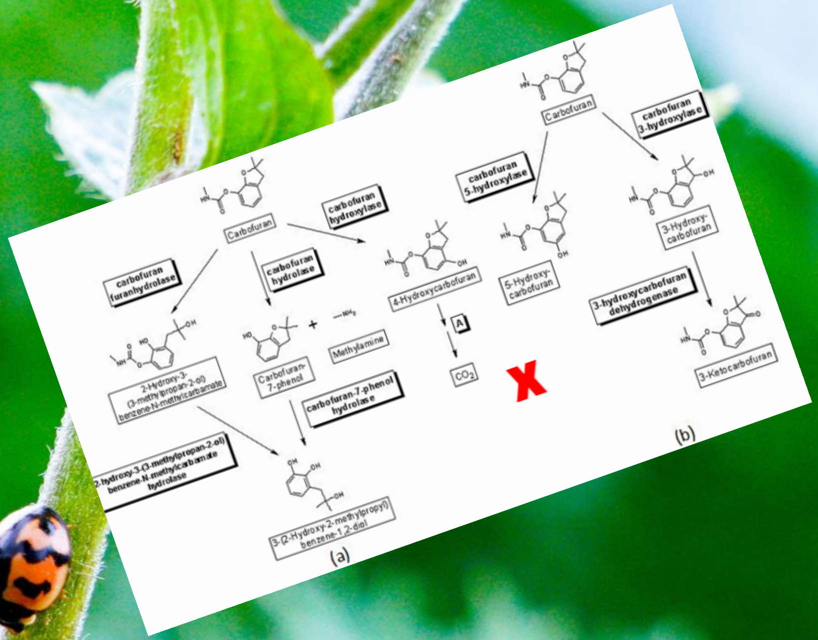


(P1, P2, K, A) = (1.5, 1.5, 900, 150)



Extension

- We would like to extend this research further by estimating the best values for the parameters involved and comment on the results for the model above using a cost function with both **least-squares** and **cubic-spline techniques** and compare their results.


Carbofuran
carbofuran hydroxylase
carbofuran furanhydrolase
carbofuran hydrolase
4-Hydroxycarbofuran
A
CO_2
2-Hydroxy-3-(3-methylpropan-2-ol) benzene-N-methylcarbamate
Carbofuran-7-phenol
Methylamine
carbofuran-7-phenol hydrolase
2-hydroxy-3-(3-methylpropan-2-ol) benzene-N-methylcarbamate hydrolase
3-(2-Hydroxy-2-methylpropyl) benzene-1,2-diol
(a)
Carbofuran
carbofuran 5-hydroxylase
carbofuran 3-hydroxylase
5-Hydroxy-carbofuran
3-Hydroxy-carbofuran
3-hydroxycarbofuran dehydrogenase
3-Ketocarbofuran
(b)

Questions?