Homework #2

1. (10 points) The blood type among Americans is approximately distributed as: 37% type A, 13% type B, 44% type O, and 6% type AB. Suppose the blood types are distributed the same in both male and female populations. And assume that the blood types are independent of marriage.
(A) An individual with type B blood can safely receive transfusions only from persons with type B or type O blood. What is the probability of a husband has type B or type O blood? If a woman has type B blood, what is the probability that her husband is an acceptable donor for her?
(B) What is the probability that in a randomly chosen couple the husband has type A blood and the wife has type B blood?

2. (10 points) To overcome the difficulty in getting truthful answers on sensitive issues, "randomized response" is proposed. As an example, we ask the students whether they have ever cheated on exams. However, they are asked to toss a coin first in private (which has a 50% chance landing on heads and 50% chance landing on tails). The coin toss result is known only to the students themselves. If a "head" is tossed, they are to answer "Yes" regardless of whether they cheated or not. If a "tail" is tossed, they are to answer truthfully. This way, the surveyor has no way of telling whether a particular individual cheated or not.
Suppose that in fact 40% of the students have cheated on exams before. What is the probability of a student answering "No" (that is, he/she has never cheated AND tossed a "tail")? What is the probability that a student never cheated and answered "yes" (because a "head" toss)? What is the conditional probability of a student never cheated given that he/she answered "yes"?

3. (10 points each) Exercises 12, 16, 19 in section 7.6.

4. (5 points)
(a) Compute P($987 < Z < 1032.6$) for the random variable Z following a Chi-squared distribution with degrees of freedom 1000.
(b) Use a normal approximation to calculate this probability.
(c) Include in the submission those R commands calculating probabilities in (a) and (b).

5. (10 points)
20 employees of a company drive to work. During the time period of 11am to 11:10am, each employee has a 2% chance of driving in front their office building, and they do this independent of each other. Other cars driving by during the same time period follows an independent Poisson distribution with mean 2. Let X and Y denote, respectively, the number of employee cars and other cars during this time period. W=X+Y denote the total number of cars driving by.
   (a) Find the mean and variance of W.
   (b) Find the probability that exactly W=1 car drove by.
   (c) Use the normal approximation to calculate probability in part (b) instead.
   (d) Submit R codes calculating the probabilities in parts (b) and (c).

R commands for computing probabilities

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> ##### Example program to produce a table of CDF
> ##Set x values where to calculate the probability: 0,1,2,...,20
> probTable <- data.frame(x=(0:20))
> ## Calculate CDF using exact Binomial, then Poisson and Norm
al approximation.
> probTable$binprob = pbinom(probTable$x, size=1000, p=0.01)
> probTable$posprob = ppois(probTable$x, lambda=10)
> probTable$normprob = pnorm((probTable$x+0.5-10)/sqrt(9.9))
> ## Display results, round to 5 digits for better display
> round(probTable, digits=5)
```

	x	binprob	posprob	normprob
1	0	0.00004	0.00005	0.00127
2	1	0.00048	0.00050	0.00345
3	2	0.00268	0.00277	0.00857
4	3	0.01007	0.01034	0.01942
5	4	0.02869	0.02925	0.04023
6	5	0.06614	0.06709	0.07633
7	6	0.12888	0.13014	0.13299
8	7	0.21886	0.22022	0.21344
9	8	0.33169	0.33282	0.31678
10	9	0.45730	0.45793	0.43687
11	10	0.58304	0.58304	0.56313
12	11	0.69735	0.69678	0.68322
13	12	0.79251	0.79156	0.78656
14	13	0.86556	0.86446	0.86701
15	14	0.91759	0.91654	0.92367
16	15	0.95213	0.95126	0.95977
17	16	0.97361	0.97296	0.98058
18	17	0.98617	0.98572	0.99143
19	18	0.99310	0.99281	0.99655
20	19	0.99671	0.99655	0.99873
21	20	0.99850	0.99841	0.9995