

**Separation of Variables** Use separation of variables to find a general solution to the following differential equations. What is the domain of each solution?

1)  $\frac{dy}{dx} = \frac{\cos x}{y^2}$

2)  $\frac{dy}{dx} = x e^y$

3)  $\frac{dy}{dt} = y^2$

4)  $y' - ty^2 = t$

Use separation of variables to solve the initial value problems

5)  $\frac{dy}{dx} = \frac{1}{3}y^{\frac{1}{3}}, y(1) = 1$

6)  $\frac{dy}{dt} = \frac{y}{t+1}, y(0) = 10$

7)  $\frac{dy}{dx} = e^{2x+y}, y(0) = -\log(2)$

8)  $y' = y^2 + 1, y(\pi) = 1$

Answers:

$$1) \quad y = (3 \sin x + C)^{\frac{1}{3}}, \quad D = \mathbb{R}$$

$$2) \quad y = -\log\left(-\frac{1}{2}x^2 + C\right), \quad D = \{x \in [-\sqrt{2C}, \sqrt{2C}], C > 0\}$$

$$3) \quad y = -\frac{1}{t+C}, \quad D = \{x \neq C\}$$

$$4) \quad y = \tan\left(\frac{t^2}{2} + C\right), \quad D = \left\{\frac{t^2}{2} + C \neq n\pi + \frac{1}{2}\pi, \forall n \in \mathbb{Z}\right\}$$

$$5) \quad y = (2x - 1)^{\frac{3}{2}}$$

$$6) \quad y = 10(t + 1)$$

$$7) \quad y = -\log\left(\frac{5}{2} - \frac{1}{2}e^{2x}\right)$$

$$8) \quad y = \tan\left(t - \frac{3\pi}{4}\right)$$