Q1)

A fair coin is flipped 4 times. What is the probability of exactly two heads being flipped?

Please show all work and explain your methods

Sol:

It's a binomial problem with n=4, p=1/2, x = 2

P(x=2) = 4C2*(1/2)^2(1/2)

P(x=2) = 6(1/4)(1/2)

P(x=2) = 6/8 = 3/4

Q2)

> A few days ago, parts of Italy (and, for example, of England too) saw the wettest January since records began. Prof. Leighton decides to investigate whether these "rare" events are indeed so rare. To simplify her calculations, she makes the following two approximations: first, months are equally long; second, the occurrence on a given month of a "rare" event – namely, the month is the wettest on record – is independent from the occurrence of a "rare" event on previous and following months. By reading old newspapers, she discovers that the probability *p* of occurrence of a "rare" event is 1/80.

(a) How large is the probability of observing at least one "rare" event in a year?

(b) How large is the probability of observing no "rare" events in a decade?

Sol:

as we can see there are independent rom the occurrence of a "rare" event on previous and following months

so we have the probability of occurrence of a rate event is 1/80

a)

Probability at least one rare event in a year

we can use a poisson distribution to find this probability

with landa = 1/80 but this is for a month and we need for a year

in a year we have 12 months

so landha (for years) = 1/80 * 12 = 12/80

$$P(x \geq 1) = 1 - P(x < 1)$$
$$P(x < 1) = P(x = 0)$$
$$P(x = 0) = \frac{e^{-12/80} * (12/80)^0}{0!} = 0.8607$$
$$P(x \geq 1) = 1 - 0.8607 = 0.1393$$

$b)$

$$\lambda = 1/80 * 12 * 10 = 120/80$$
$$P(x = 0) = \frac{e^{-120/80} * (120/80)^0}{0!} = 0.2231$$

Q3)

Suppose that you are interested in determining whether the advice given by a physician during a routin physical examination is effective in encouraging patients to stop smoking. In a study of current smokers, one group of patients was given a brief talk about the hazards of smoking and was encouraged to quit. A second group received no advice pertaining to smoking. All patients were given a follow-up exam. In the sample of 114 patients who had received the advice, 11 reported that they had quit smoking; in the sample of 96 patients who had not, 7 had quit smoking.

A) Estimate the true difference in population proportions $p_1 - p_2$.
B) Construct a 95% CI for this difference.

C) At the 0.05 level of significance, test the null hypothesis that the proportions of patients who quit smoking are identical for those who received advice and those who did not.

D) Do you believe that the advice given by physicians is effective? Why or why not?

Sol:

Advised group:
Number of people in the group, $n_1 = 114$
Number of people who quit smoking, $X_1 = 11$
The sample proportion of people who quit smoking, $\hat{p}_1 = \frac{X_1}{n_1} = \frac{11}{114} = 0.0965$

No advise group:
Number of people in the group, $n_2 = 94$
Number of people who quit smoking, $X_2 = 7$
The sample proportion of people who quit smoking, $\hat{p}_2 = \frac{X_2}{n_2} = \frac{7}{94} = 0.0745$

a)The estimate for the difference of proportion, $(\hat{p}_1 - \hat{p}_2) = 0.0965 - 0.0745 = 0.022$

b)The 95% confidence interval for the difference,

$$(\hat{p}_1 - \hat{p}_2) \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$
$$= (0.0965 - 0.0745) \pm 1.96\sqrt{\frac{0.0965(1-0.0965)}{114} + \frac{0.0745(1-0.0745)}{94}}$$
$$= 0.022 \pm 0.0759$$
$$= (-0.0539, 0.0979)$$

C)
The $H_0: p_1 = p_2$ against $H_1: p_1 \neq p_2$

The test statistic,

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_c(1-\hat{p}_c)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \text{ where } \hat{p}_c = \frac{X_1 + X_2}{n_1 + n_2} = \frac{11+7}{114+94} = 0.0865$$
$$= \frac{0.0965 - 0.0745}{\sqrt{0.0865(1-0.0865)\left(\frac{1}{114} + \frac{1}{94}\right)}}$$
$$= 0.56$$

The P-value $= 2P(Z > 0.56) = 2[1 - P(Z \leq 0.56)] = 2[1 - 0.71226] = 0.5755$

D)
Since the P-value is higher than the significance level, do not reject the null hypothesis.

Hence, conclude that the proportions of patients who quit smoking are identical and the advice given by the physicians is not effective.

Q4)

Aparrently 16.6% of police officers surveyed have more than 24 months of experience in other jobs , and 6.68% of police officer have less than 9 month of experience in other jobs. If this data assumes normal distribution, what is the value of the mean and standard deviation of this distribution?

Sol:

well we have 2 values to find in our table of normal distribution

$P(Z > z) = 0.166$

$z = 0.97$

$0.97 = \dfrac{24-\mu}{\sigma_x}$

$P(Z < z) = 0.0668$

$z = -1.50$

$-1.50 = \dfrac{9-\mu}{\sigma_x}$

sustitute in one equation

we will find the values

$\mu = 18.11$

$\sigma_x = 6.07$

Q5)

In New York City, a study was conducted to evaluate whether any information that is available at the time of birth can be used to identify children with special educational needs. In a random sample of 45 third-graders enrolled in the special education program of the public school system, 4 have mothers who have had more than 12 years of schooling.

A) Construct a 90% CI for the population proportion of children with special educational needs whose mothers have had more than 12 years of schooling.

B) In 1980, 22% of all third-graders enrolled in the NYC public school system had mothers who had more than 12 years of schooling. Suppose you wish to know whether this proportion is the same for children in the special education program. What are the null and alternative hypotheses of the appropriate test?

C) Conduct the test at the 0.05 level of significance.

Sol:

a)
Confidence Interval For Proportion
CI = p ± Z a/2 Sqrt(p*(1-p)/n)))
x = Mean
n = Sample Size
a = 1 - (Confidence Level/100)
Za/2 = Z-table value
CI = Confidence Interval
Mean(x)=4
Sample Size(n)=45
Sample proportion = x/n =0.0889
Confidence Interval = [0.0889 ±Z a/2 (Sqrt (0.0889*0.9111) /45)]
= [0.0889 - 1.64* Sqrt(0.0018) , 0.0889 + 1.64* Sqrt(0.0018)]
= [0.0193,0.1585]

b)
Set Up Hypothesis
Null, H0:P=0.22
Alternate, H1: P!=0.22
c)
Test Statistic
No. Of Success chances Observed (x)=4
Number of objects in a sample provided(n)=45
No. Of Success Rate (P)= x/n = 0.0889
Success Probability (Po)=0.22
Failure Probability (Qo) = 0.78
we use Test Statistic (Z) for Single Proportion = P-Po/Sqrt(PoQo/n)
Zo=0.08889-0.22/(Sqrt(0.1716)/45)
Zo =-2.1232
| Zo | =2.1232
Critical Value
The Value of |Z α| at LOS 0.05% is 1.96
We got |Zo| =2.123 & | Z α | =1.96
Make Decision
Hence Value of | Zo | > | Z α| and Here we Reject Ho
P-Value: Two Tailed (double the one tail) - Ha : (P != -2.12318) = 0.03374
Hence Value of P0.05 > 0.0337,Here we Reject Ho

Q6)

Suppose that you select a random sample of 40 children from the population of newborn infants in Mexico. The probability that a child in this population weights at most 2,500 grams is 0.15.

A) For the sample of size 40, what is the probability that four or fewer of the infants weigh at most 2,500 grams? Compute the exact binomial probability.

B) Using the normal approximation to the binomial distribution, estimate the probability that four or fewer of the children weigh at most 2,500 grams.

Sol:

Binomial Distribution

PMF of B.D is = f (k) = (n k) p^k * (1- p) ^ n-k
Where
k = number of successes in trials
n = is the number of independent trials
p = probability of success on each trial

a)
P(X < = 4) = P(X=4) + P(X=3) + P(X=2) + P(X=1) + P(X=0)
= (40 4) * 0.15^4 * (1- 0.15) ^36 + (40 3) * 0.15^3 * (1- 0.15) ^37 + (40 2) * 0.15^2 * (1- 0.15) ^38 + (40 1) * 0.15^1 * (1- 0.15) ^39 + (40 0) * 0.15^0 * (1- 0.15) ^40
= 0.263

b)
Normal Approximation to Binomial Distribution
Mean (np) =40 * 0.15 = 6
Standard Deviation (√npq)= √40*0.15*0.85 = 2.2583
Normal Distribution = Z= X- u / sd
P(X > 4) = (4-6)/2.2583
= -2/2.2583 = -0.8856
= P (Z >-0.886) From Standard Normal Table
= 0.8121
P(X<=4) = 1 - 0.8121 = 0.1879

Q7)

For each case, state the null and alternative hypothesis as done in class. For example:
H0: $\mu \leq 23$
Ha: $\mu > 23$
a) The average time workers spent commuting to work in Verona five years ago was 38.2 minutes. The Verona Chamber of Commerce asserts that the average is less now.
b) The mean salary for all men in a certain profession is $58,291. A special interest group thinks that the mean salary for women in the same profession is different.

c) The accepted figure for the caffeine content of an 8-ounce cup of coffee is 133 mg. A dietitian believes that the average for coffee served in a local restaurants is higher.
d) The average yield per acre for all types of corn in a recent year was 161.9 bushels. An economist believes that the average yield per acre is different this year.
e) An industry association asserts that the average age of all self-described fly fishermen is 42.8 years. A sociologist suspects that it is higher.

Sol:

a) Ho : μ = 38.2 v/s H_1: μ<38.2
b) Ho : $\mu_1 - \mu_2 = 0$ v/s H_1: $\mu_1 - \mu_2 \neq 0$.
c) Ho : $\mu = 133$ v/s H_1 : $\mu > 133$
d) Ho : $\mu = 161.9$ v/s H_1 : $\mu \neq 161.9$
e) Ho : $\mu = 42.8$ v/s H_1 : $\mu > 42.8$

Q8)

Compute the range and standard deviation for strength of the concrete (in psi)

3940, 4080, 3300, 3200, 2910, 3810, 4080, 4020

The range is __ psi

s=__ psi

Sol:

a)
Range = Max - Min = 1170

b)
Mean= 3940 + 4080 + 3300 + 3200 + 2910 + 3810 + 4080 + 4020/8 = 3667.5

Step 1:Add them up:
3940 + 4080 + 3300 + 3200 + 2910 + 3810 + 4080 + 4020 = 29340

Step 2: Square your answer:
29340 × 29340 = 860835600
…and divide by the number of items. We have 8 items in our example so:
860835600 / 8 = 107604450
Set this number aside for a moment.

Step 3: Take your set of original numbers from Step 1+and square them individually this time:
3940^2 + 4080^2 + 3300^2 + 3200^2 + 2910^2 + 3810^2 + 4080^2 + 4020^2 = 109091000

Step 4: Subtract the amount in Step 2 from the amount in Step 3.
109091000 – 107604450 = 1486550
Set this number aside for a moment.

Step 5: Subtract 1 from the number of items in your data set. For our example:
8 – 1 = 7

Step 6: Divide the number in Step 4 by the number in Step 5. This gives you the variance:
1486550/ 7 = 212364.28
How to find the sample variance and standard deviation: Standard Deviation

Step 7: Take the square root of your answer from Step 6. This gives you the standard deviation:

$\sqrt{212364.28}$= 460.829

Q9)

Solve for the mean and standard deviation of the following binomial distributions. a. n = 30 and p = 0.50 b. n = 70 and p = 0.45 c. n = 110 and p = 0.50

Sol:

a)
Mean (np) =30 * 0.5 = 15
Standard Deviation ($\sqrt{npq}$)= $\sqrt{30*0.5*0.5}$ = 2.7386
b)
Mean (np) =70 * 0.45 = 31.5
Standard Deviation ($\sqrt{npq}$)= $\sqrt{70*0.45*0.55}$ = 4.1623
c)
Mean (np) =110 * 0.5 = 55
Standard Deviation ($\sqrt{npq}$)= $\sqrt{110*0.5*0.5}$ = 5.244

Q10)

listed below are the measured radiation emissions (in W/kg) corresponding to certain cell phones. Construct a 90% confidence interval estimate of the population mean radiation emissions.
.38 .55 1.54 1.55 .5 .6 .92 1.0 .86 1.46
give the point estimate:
distribution chosen:
reason for choosing distribution:
symbolic form of confidence interval:
explanatory sentence:

Sol:

CI = x ± t a/2 * (sd/ Sqrt(n))
Where,
x = Mean
sd = Standard Deviation
a = 1 - (Confidence Level/100)
ta/2 = t-table value
CI = Confidence Interval
Point of estimate = Mean(x)=0.936
Standard deviation(sd)=0.446
Sample Size(n)=10
Confidence Interval = [0.936 ± t a/2 (0.446/ Sqrt (10))]
= [0.936 - 1.833 * (0.141) , 0.936 + 1.833 * (0.141)]
= [0.677,1.195]
Interpretations:
1) We are 90% sure that the interval [-7.8646 , -3.9353] contains the
true population mean
2) If a large number of samples are collected, and a confidence interval is created
for each sample, 95% of these intervals will contain the true
population mean