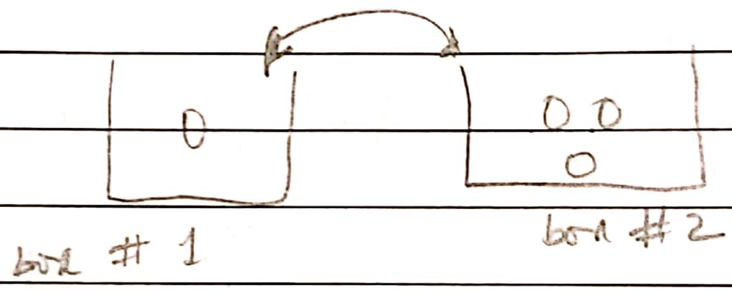
(1)

$$P\left(\begin{matrix}6 \text{ by atleast}\\ \text{one person}\end{matrix}\right) = 1 - P(6 \text{ by none})$$

$$= 1 - \left(\frac{5}{6}\right)^5$$

(2)



$$p(H) = p \; ; \; p(T) = 1 - p$$

$$P_{00} = p \times (1) + (1-p)\, 0 = p$$

$$P_{01} = p \times (0) + (1-p)\, 1 = (1-p)$$

$$
P = \begin{array}{c c}
\begin{pmatrix}
p & (1-p) & 0 & 0 & 0 \\
p & 0 & (1-p) & 0 & 0 \\
0 & p & 0 & (1-p) & 0 \\
0 & 0 & p & 0 & (1-p) \\
0 & 0 & 0 & p & (1-p)
\end{pmatrix} & \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} \\
\begin{matrix} 0 \quad & 1 \quad & 2 \quad & 3 \quad & 4 \end{matrix} &
\end{array}
$$

(3) $N_{11} = 1\left(1 + N_{21}\right)$

$$N_{21} = \frac{1}{2}\left(1+N_{11}\right) + \frac{1}{2}\left(1+N_{31}\right)$$

$$N_{31} = \frac{1}{2}\left(1+N_{21}\right) + \frac{1}{2}\left(1+N_{41}\right)$$

$$N_{41} = \frac{1}{2}\left(1+N_{31}\right) + \frac{1}{2}\left(1+N_{51}\right)$$

$$N_{51} = 1\left(1+N_{41}\right)$$

$$N_{11} - N_{21} = 1$$

$$N_{21} - \frac{1}{2} N_{11} - \frac{N_{31}}{2} = 1$$

$\vdots$

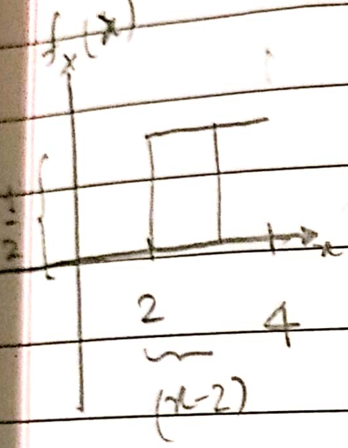
$$\left(\begin{array}{ccccc|c}
1 & -1 & 0 & 0 & 0 & 1 \\
-\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 0 & 1 \\
0 & -\frac{1}{2} & 1 & -\frac{1}{2} & 0 & 1 \\
0 & 0 & -\frac{1}{2} & 1 & -1/2 & 1 \\
0 & 0 & 0 & -1 & 1 & 1
\end{array}\right)$$

The equations are linearly dependet

$\therefore$ No solutions exist.

Looks like $N_{11} = 0$

(4) $$P(X \le Y+2) = \int_{-\infty}^{\infty} P(X \le y+2 \mid Y=y)\, f_Y(y)\, dy$$



$$= \int_{0}^{1} P(X \le y+2) \times f_Y(y)\, dy$$

$$= \int_{0}^{1} \frac{1}{2} \times y \times 1 \times dy$$

$$= \frac{1}{2} \times \frac{y^2}{2} \Big]_0^1 = \frac{1}{4}$$

(5) 3^{rd} Computer is independent of 1^{st} and 2^{nd}.

$$\therefore P(T_3 > 4) = e^{-(1 \times 4)}$$

$$= e^{-4}$$

$$\approx 1.83\%$$

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(6) $\sum_{n=1}^{n} P(A_n) \geq \sqrt{n+4}$

$\therefore \lim_{n\to\infty} \sum_{n=1}^{n} P(A_n) = \infty$

As A_i are independent,

$\therefore$ According to B.C. Lemma 2,

$$P(A_n \text{ i.o.}) = 1$$

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(7) $$\lim_{n\to\infty} \sum_{k=1}^{n} P_{ii}(k) = \lim_{n\to\infty} \left(3 - \frac{9}{\sqrt{n+8}}\right)$$

$$= 3 < \infty$$

$\therefore$ state "i" is transient

(8) (a) $E[N_4 | N_3 = k]$

$$= E[N_4 | N_3 = k, H_{k+1}] \cdot (p)$$

$$+ E[N_4 | N_3 = k, T_{k+1}] \cdot (1-p)$$

$$= (k+1+E[N_4])p + (k+1)(1-p)$$

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$$= (k+1) + E(N_4)\,p$$

$$\therefore E(N_4 \mid N_3 = k) = (k+1) + E(N_4)\,p$$

(b) $$E(N_4) = \sum_{k=3}^{\infty} \left[(k+1) + E(N_4) \cdot p\right] \times p(N_3 = k)$$

$$E(N_4) = E[N_3] + 1 + \sum_{k=3}^{\infty} E(N_4)\,p \cdot P(N_3 = k)$$

$$= \frac{1}{p} + \frac{1}{p^2(1-p)} + \sum_{k=3}^{\infty} E(N_4)\,p\,p(N=k)$$

$$E(N_4)\left[1-p\right] = \frac{1}{p} + \frac{1}{p^2(1-p)}$$

$$E[N_4] = \frac{1}{p(1-p)} + \frac{1}{p^2(1-p)^2}$$