

# MATH 7241 Fall 2020: Problem Set #6

Due date: Tuesday November 3

**Reading:** relevant background material for these problems can be found on Canvas ‘Notes 4: Finite Markov Chains’. Also Grinstead and Snell Chapter 11.

**Grinstead and Snell:** see pages 442-443 and pages 444, 467-468 on Canvas. The text is available online (free!) at

<http://www.dartmouth.edu/~chance/>

Click on the link “A GNU book”.

**Exercise 1** A box contains  $N$  balls, some red and some blue. At each step, a coin is flipped with probability  $p$  of coming up Heads, and probability  $1 - p$  of coming up Tails. If the coin comes up Heads, a ball is chosen at random from the box and is replaced by a red ball; if the coin comes up Tails then a ball is chosen randomly from the box and replaced by a blue ball. Let  $X_n$  denote the number of red balls in the box after  $n$  steps. Find the transition matrix for the chain  $\{X_n\}$ , and find the stationary distribution. [Hint: is the chain reversible?] Compute  $\lim_{n \rightarrow \infty} E[X_n]$ , and explain why you could have guessed your answer without doing the calculation.

**Exercise 2** A knight moves randomly on a standard  $8 \times 8$  chessboard. At each step it chooses at random one of the possible legal moves available. Given that the knight starts in a corner of the chessboard, find the expected number of steps until its first return to its initial position. [Hint: model the knight’s position using a Markov chain, and try to show that the chain is reversible]