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1) a) $E[x] = E[x|A] \times P[A] + E[x|B] \times P[B]$

$+ E[x|C] \times P[C]$

$= 2.6 \times \frac{1}{3} + 3 \times \frac{1}{3} + 3.4 \times \frac{1}{3}$

$= \boxed{3}$

For poisson distribution,

$E[x^2] = \lambda^2 + \lambda$

$\therefore E[x^2|A] = 2.6^2 + 2.6$

$E[x^2|B] = 3^2 + 3$

$E[x^2|C] = 3.4^2 + 3.4$

$$E[x^2] = \frac{\left(E[x^2|A] + E[x^2|B] + E[x^2|C]\right)}{3}$$

$\text{var}[x] = E[x^2] - (E[x])^2$

$$= \frac{2.6^2 + (2.6) + 3^2 + 3 + 3.4^2 + 3.4}{3} - 9$$

$= \boxed{3.1067}$

b)

$$E[X] = E[X_A] + E[X_B] + E[X_C]$$

$$= 2.6 + 3 + 3.4$$

$$= 9$$

$$Var[X] = \underline{Var[X_A] + Var[X_B] + Var[X_C]}$$

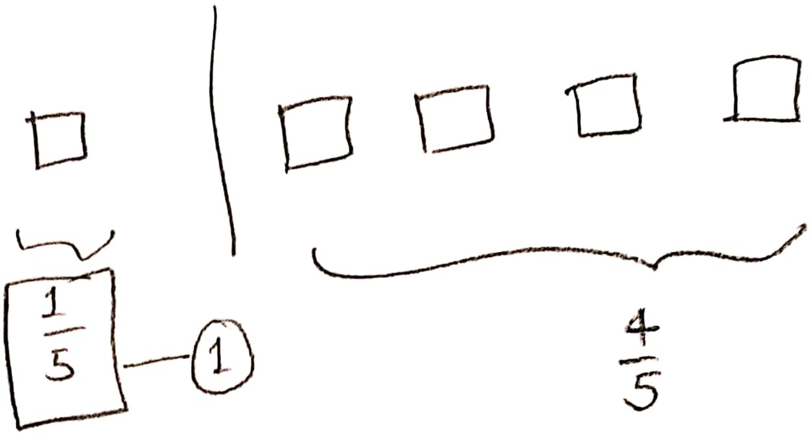
$$= 2.6 + 3 + 3.4$$

$$= 9$$

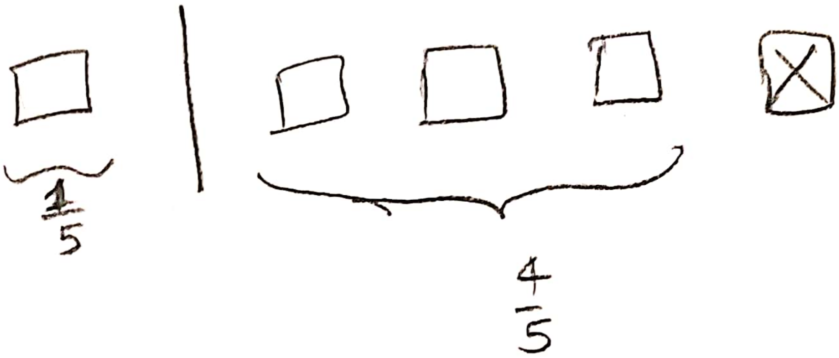
Because A, B, C are independent the Covariance is zero

(2)

Initial:



Later:



$\therefore$ Required probability $= \frac{4}{5} \times$ Probability of choosing 1 out of 3 doors

$$= \frac{4}{5} \times \frac{1}{3} = \boxed{\frac{4}{15}}$$ (2)

Because, $\boxed{\frac{4}{15}} > \boxed{\frac{1}{5}}$,

switching gives more chance to win

(4) $\lim_{n \to \infty} P\left(\sum_{i=1}^{n} X_i \geq \sqrt{n} \right)$

For uniform distribution over $[-1, 1]$,

$$\mu = \frac{-1+1}{2} = 0$$

$$Var(\sigma^2) = \frac{(1-(-1))^2}{12} = \frac{4}{12} = \frac{1}{3}.$$

$$\therefore \lim_{n \to \infty} P\left(\sum_{i=1}^{n} X_i \geq \sqrt{n} \right)$$

$$= P\left(\frac{\sum_{i=1}^{n} X_i - \mu}{\sqrt{n \times \sigma^2}} \geq \frac{\sqrt{n} - \mu}{\sqrt{n \times \sigma^2}} \right)$$

$$= P\left(Z \geq \frac{\sqrt{n} - 0}{\sqrt{n \times \frac{1}{3}}} \right)$$

$$= P\left(Z \geq \sqrt{3} \right)$$

Comparing, we get

$$\boxed{a = \sqrt{3}}$$

⑤ Randomly distribute "r" balls over "n" boxes

$$\frac{r}{n} \to c \Rightarrow r \to n \times c$$

$$\therefore E[N_n] = E[X_1] + E[X_2] + \cdots E[X_n]$$

$$= n \times E[X_1]$$

$$\Rightarrow \frac{E[N_n]}{n} = E[X_1]$$

$$= \left(\frac{n-1}{n}\right)^r = \left(1 - \frac{1}{n}\right)^{nc}$$

$$\therefore \lim_{n\to\infty} \frac{E[N_n]}{n} = \lim_{n\to\infty} \left[\left(1 - \frac{1}{n}\right)^n\right]^c$$

$$= \boxed{e^{-c}}$$

⑥

By memoryless property, the distribution of

$g(X) \mid X > x$ is the same as that of

$g(x)$, but shifted to the right by "x".

$$\therefore \; E[X^2 \mid X > 1] = E[X^2] + 1$$

$\downarrow$

Proof :-

$$\int_{-\infty}^{\infty} t \, f_{X^2 \mid X > 1}(t)\, dt$$

$$= \int_{-\infty}^{\infty} t \, f_{X^2}(t-1)\, dt$$

$$= \int_{-\infty}^{\infty} (u+1)\, f(u)\, du$$

$$= \int_{-\infty}^{\infty} u f(u)\, du + \int_{-\infty}^{\infty} f(u)\, du$$

$$= E[X^2] + 1$$