Math 5110- Applied Linear Algebra-Fall 2020

Instructor: He Wang

Test 3.

Submission deadline: Dec. 3 (Thursday) at 8 pm Boston time.

Student Name: ____________________ __________**/50**

Rules and Instructions for Exams:

1. Unless otherwise specified, to receive full credits you must show **all** necessary work. The grading is based on your work shown.

2. You need to finish the exam yourself. Any discussions with the other people will be considered as **academic dishonesty**. **Cheating, Unauthorized Collaboration, and Facilitating Academic Dishonesty are not allowed.** You can read a description of each here http://www.northeastern.edu/osccr/academic-integrity-policy/

3. You are allowed to look at notes or textbook. However, you are **not** allowed to asked help using any online platform.

4. Scan your solutions, merge into **one .pdf**, and upload. This is plenty of time to use a scanner or scanning app and clearly scan every page.

1. (10 points) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 0 \\ 1 & 2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 2 \end{bmatrix}$.

(1) Find the orthogonal protection of $\vec{b}$ onto $\mathrm{im}(A)$.

(2) Find the shortest distance from the vector $\vec{b}$ (or the point $(2, 4, 0, 2)$) to the space $\mathrm{im}(A)$.

2. (10 points) Suppose $\vec{u}$ and $\vec{v}$ are two non-zero vectors in an real inner product space V.

Prove that $\vec{u}$ and $\vec{v}$ are orthogonal if and only if $||\vec{u}|| \leq ||\vec{u} + a\vec{v}||$ for any $a \in \mathbb{R}$.

3. (10 points) Suppose that the rule

$$\langle A, B\rangle := \text{trace}(A^T B)$$

defines an inner product on the vector space of all $m \times n$ matrices. (We already verified this in homework.)

Find the angle between $A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$.

4. (10 points) **True or False** Questions: Prove your the true statement and provide a counter example for the false statement.

(1) Suppose that A is any matrix in $\mathbb{R}^{n\times n}$. Then $AA^T = A^T A$?

(2) Suppose that A is an orthogonal matrix in $\mathbb{R}^{n\times n}$. Then the rows of A must also form an orthonormal basis for $\mathbb{R}^n$.

5. (10 points) Let A be a 2×2 matrix of the form $A = \begin{bmatrix} a & b \\ 1-a & 1-b \end{bmatrix}$ for $0 \leq a, b \leq 1$.

Find $\lim_{n \to \infty} A^n$.