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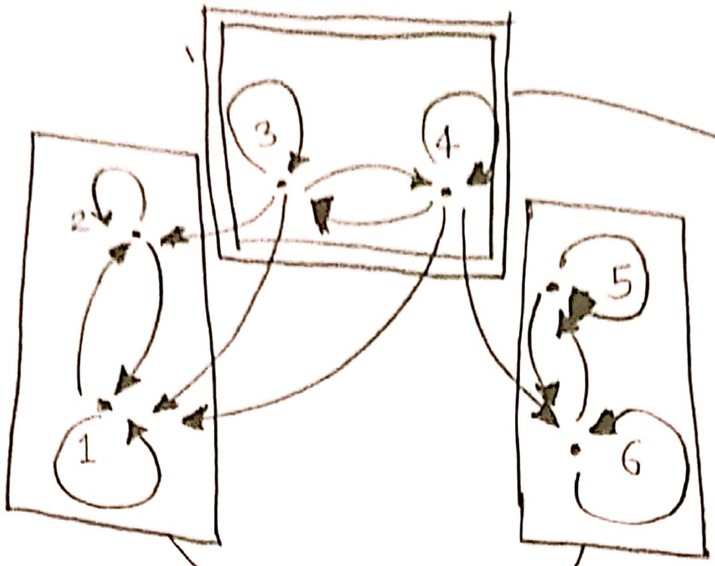


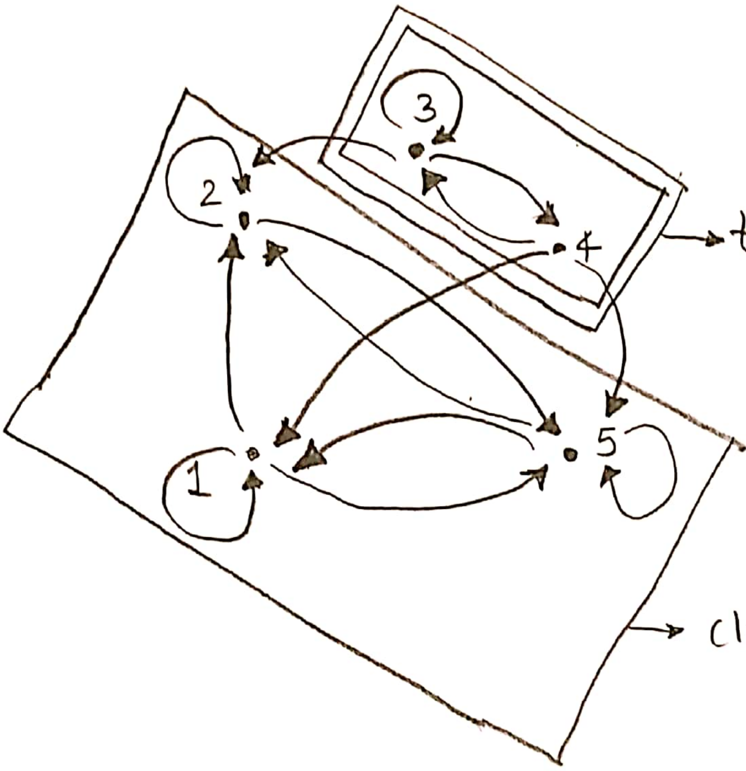
transient states

(Because there are no incoming edges to this group. The probability that it will eventually remain in these states = 0)

Closed irreducible classes

2

a)



transient states

closed irreducible class

b)

Let, $(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5)$ be the stationary distribution

$\pi P = P$ and $\sum_{i=1}^{5} \pi_i = 1$

including 5 varibles and 6 equations (including normalization),

we get,

$$\left[\begin{array}{ccccc|c} -0.5 & 0 & 0 & 0.3 & 0.5 & 0 \\ 0.3 & -0.5 & 0.4 & 0 & 0.2 & 0 \\ 0 & 0 & -0.6 & 0.2 & 0 & 0 \\ 0 & 0 & 0.2 & -1 & 0 & 0 \\ 0.2 & 0.5 & 0 & 0.5 & -0.7 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

Solving for rref,

we get,

$$\begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 0 \\ 0 \\ 1/3 \end{bmatrix}$$

Yes, the solution is unique

(3) Let,

$$X_n = \begin{cases} 0, & \text{failure} \\ 1, & \text{Success} \end{cases}$$

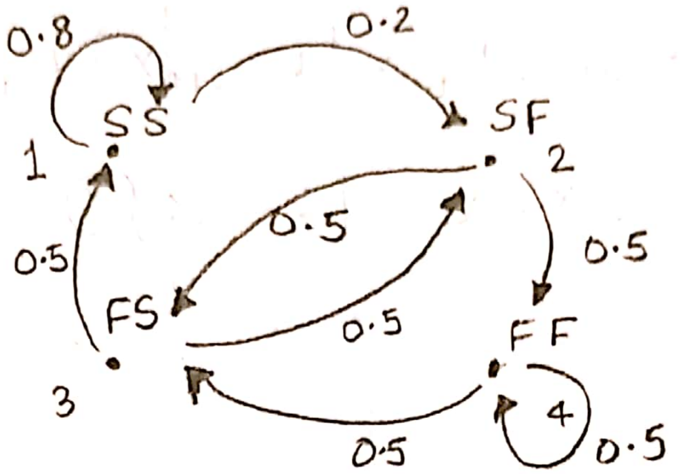
To make a Markov chain, let us Combine, X_n and X_{n-1}.

Let us define $\{Y_n\}$ by $Y_n = (X_n, X_{n-1})$

Then Y_n is a Markov Chain

S → Success

F → failure



$$\therefore \quad P = \begin{bmatrix} 0.8 & 0.2 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 \end{bmatrix}$$

For stationary distribution,

$\boxed{\pi = \pi P}$ Solving,

$$\left[\begin{array}{cccc|c} -0.2 & 0 & 0.5 & 0 & 0 \\ 0.2 & -1 & 0.5 & 0 & 0 \\ 0 & 0.5 & -1 & 0.5 & 0 \\ 0 & 0.5 & 0 & -0.5 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

$$\pi = \left(\frac{5}{11}, \frac{2}{11}, \frac{2}{11}, \frac{2}{11}\right)$$

$$\lim_{n\to\infty} P(X_n = S) = \lim_{n\to\infty} \left(P(Y_n = SS) + P(Y_n = SF)\right)$$

$$= \pi_{SS} + \pi_{SF}$$

$$= \frac{7}{11}$$

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(4)

$$P = \begin{pmatrix} 1/2 & 1/3 & 1/6 \\ 3/4 & 0 & 1/4 \\ 0 & 1 & 0 \end{pmatrix}$$

a) A Simple matlab program with a loop yields,

$$P^3 = \frac{1}{48}\begin{pmatrix} 24 & 16 & 8 \\ 27 & 12 & 9 \\ 18 & 24 & 6 \end{pmatrix}$$

$\therefore$ P is regular as P^n, contains all non-zero entries, with $n=3$.

b) $P(X_2 = 3 \mid X_0 = 1) = (P^2)_{13} = \frac{1}{6}$

c)

$$w_1 = \frac{1}{2} w_1 + \frac{3}{4} w_2$$

$$w_2 = \frac{1}{3} w_1 + w_3$$

$$w_3 = \frac{1}{6} w_1 + \frac{1}{4} w_2$$

$$w_1 + w_2 + w_3 = 1$$

$$W = \left[\begin{array}{ccc|c} -1/2 & 3/4 & 0 & 0 \\ 1/3 & -1 & 1 & 0 \\ 1/6 & 1/4 & -1 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right]$$

Solving this we get,

$$\left.\begin{array}{l}\text{limiting probability vector } (w) \\ \text{(stationary distribution)}\end{array}\right\} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/6 \end{bmatrix}$$