Modified SIR Modeling on the Spread of Ideas

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December 2nd 2019

***Abstract*—The power of sharing is stronger than we imagined. People can share viral videos among their friends or plan a flash mob across the Internet, but what exactly does it take to keep an idea flowing around without becoming a forgotten thought? In this paper, we introduce and construct a system of differential equations using a combination of SIR-like and population growth models to simulate the behavior of the breadth of a shared idea. We also investigate the possibility of a cyclical pattern within the model. We analyze quantitatively in cases when the flow rates of the users are identical and when it differs. In addition, we provide probable explanations for the behaviors and results. Finally, we attempt to simulate a more realistic model through the addition of a capacity limit within a community and observe the behavior after the total population reaches the capacity.**

I. Introduction

Long before the evolution of homo sapiens, ancestors have been spreading ideas with one another. From the discovery of fire million years ago to influencers spreading memes with their social media followers nowadays, we can see how idea-sharing has engraved in our lives, specifically after the introduction of the Internet. The Internet helps us communicate instantaneously with people all around the globe through different platforms. Websites like Google and Wikipedia put information out in the open, accessible to everyone. On the other hand, social media sites help you limit sharing your personal information and ideas with just your friends. To make the communication even more private, applications like Whatsapp and Messenger help you create groups where content can be shared in private chats, within a small group of people. Sharing information is like a branching process, as the information passes, new layers are added which then exponentially pass on the information. American business writer Tom Peter once said, “Innovation comes only from readily and seamlessly sharing information rather than hoarding it.” After realizing the importance of the phenomenon, we decided to take it up as our project topic.

To examine the spread of ideas between people, we will like to create models specifically as the spread of an inside joke within a social media space where people exit or enter constantly. In the initial model, we will look at the interactions between the informed population and uninformed population given that the entering and exiting rates are identical. This keeps the population flowing while maintaining the total population constant. Using this model, we will like to investigate whether the piece of information or the inside joke, will die out eventually. Furthermore, we will like to see whether the model is possible to become a cyclical pattern provided with specific constants. For the second model, we will remove the constraint of the constant population and set the entering rate different from the exiting rate to see the behavior of each population. In addition, we will also examine how each group behaves when the social space has an upper limit or cap in the total population.

II. Approach

We start by setting the total population as H and then splitting it into three groups. People who are unaware of the joke (Ignorant (I)); People who enjoy the joke and share it with others (Sharers (S)); and people who find the joke dull and will not share with anyone (Bored (B)). Sharers will share the joke to a portion of people regardless of their status. When a person from the ignorant population hears the joke, the person either finds it interesting and starts sharing to others or uninteresting and will not share with others afterward. However, when a person from the bored population hears the joke again from one of the sharers, there is a possibility that the person gets interested in it again. At every unit of time, a proportion of the shared population grows tired of the joke and stops sharing. We assume the model to be a community where it has an initial total population and also an incoming rate and an exciting rate (N). People can only enter as ignorants but a proportion of each population groups will leave every unit of time.

The following are the variables and constants we introduced in both of our models:

Variable	Description
I	Ignorant
S	Shared
B	Bored
H	Total Population ($I + S + B$)
N	Rate of Ignorant Entering
y	Rate of Sharing from Sharers **(Proportion of H)**
c	Chance that friends they shared to is an Ignorant and the person saw the shared post and the person is a sharer
h	Chance that sharer got bored by it
d	Chance that friends they shared to is already bored by it and the person saw the shared post and gets interested again
r	Rate of Ignorant Exiting
w	Additional Exiting Rate for Bored Population

We assume the value of the initial total population (H_0) as 1000 and we set the other constants through studies and surveys conducted on the relevant subjects. According to Fox News, an average American has around 9 friends (Gervis, 2019). This helped us determine the y constant. Through a survey conducted by Olapic, a visual content solutions provider, we set the base value of the frequency and willingness of sharing (c) as 40%, the rate of getting bored (h) as 35% and the rate of becoming interested again (d) as 2.5% (Elieen Brown, 2017)

III. Modeling

A. *Constant Total population*

1) Equations: To set up the desired system of differential equations for the initial model, we express the factors that affect the change in each population as word equations.

$$\frac{dI}{dt} = \left\{\begin{matrix}\text{Rate of}\\ \text{entering room}\end{matrix}\right\} - \left\{\begin{matrix}\text{Ignorant}\\ \text{become Sharer}\end{matrix}\right\} - \left\{\begin{matrix}\text{Ignorant}\\ \text{become Bored}\end{matrix}\right\} - \left\{\begin{matrix}\text{Rate of}\\ \text{exiting room}\end{matrix}\right\}$$

$$\frac{dS}{dt} = \left\{\begin{matrix}\text{Ignorant}\\ \text{become Sharer}\end{matrix}\right\} - \left\{\begin{matrix}\text{Sharer}\\ \text{become Bored}\end{matrix}\right\} + \left\{\begin{matrix}\text{Bored}\\ \text{become Shared}\end{matrix}\right\} - \left\{\begin{matrix}\text{Rate of}\\ \text{exiting room}\end{matrix}\right\}$$

$$\frac{dB}{dt} = \left\{\begin{matrix}\text{Sharer}\\ \text{become Bored}\end{matrix}\right\} - \left\{\begin{matrix}\text{Ignorant}\\ \text{become Bored}\end{matrix}\right\} + \left\{\begin{matrix}\text{Sharer}\\ \text{become Bored}\end{matrix}\right\} - \left\{\begin{matrix}\text{Rate of}\\ \text{exiting room}\end{matrix}\right\}$$

We can convert them to the following numerical expression

$$\frac{dI}{dt} = N - ySI - \frac{NI}{H} \tag{1}$$

$$\frac{dS}{dt} = ycSI - hS + ydSB - \frac{NS}{H} \tag{2}$$

$$\frac{dB}{dt} = hS - ydSB + y(1-c)SI - \frac{NB}{H} \tag{3}$$

2) Equilibrium Points: From the expression above, we learned that this is an autonomous system of differential equations. Therefore, we first find the nullclines for each equation.

$$\frac{dI}{dt} = \frac{dS}{dt} = \frac{dB}{dt} = 0$$

Nullcline for I:

$$I = \frac{HN}{HyS + N} \tag{4}$$

Nullclines for S:

$$S = 0 \text{ or } I = \frac{-HydB + N + hH}{ycH} \tag{5}$$

Nullcline for B:

$$I = \frac{NB - hHS + dyHSB}{yHS(1-c)} \tag{6}$$

We then find the equilibrium points of the four nullclines. Notice when $S = 0$, Equation (6) will not hold due to a total of 0 in the denominator. Therefore, we first assume $S \neq 0$ and try to find the equilibrium points from the rest of the nullclines. The results are in terms of S due to the complexity of the solution.

Equilibrium Points

$$I = \frac{HN}{HyS + N} \tag{7}$$

$$B = \left(\frac{N + hH}{ycH} - \frac{HN}{HyS + N}\right)\frac{c}{d} \tag{8}$$

$$S = \frac{-\beta \pm \sqrt{\beta^2 - 4\alpha\Delta}}{2\alpha} \tag{9}$$

$$\text{where } \alpha = NHy, \quad \beta = \left(1 + \frac{1}{d}\right)N^2 + \frac{NHh}{d} - NH^2y$$

$$\Delta = \frac{N^3}{Hyd} + \frac{N^2h}{yd} - \frac{N^2Hc}{d}$$

We then assume when $S = 0$. This simplifies the system equation to the following

$$\frac{dI}{dt} = N - \frac{NI}{H} \tag{10}$$

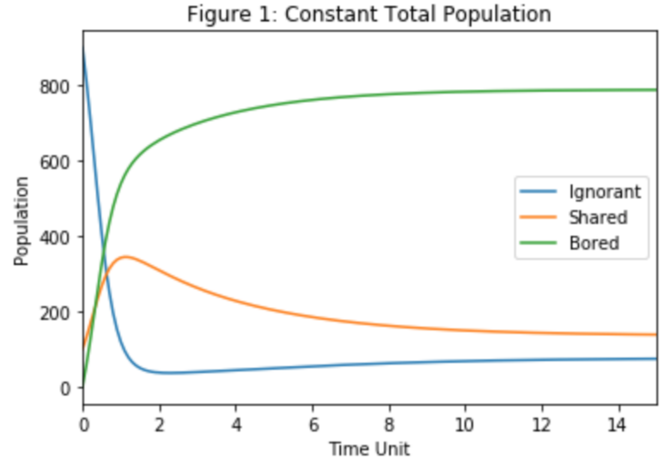
$$\frac{dB}{dt} = -\frac{NB}{H} \tag{11}$$

By setting both equations to 0, we can easily get the equilibrium point to be $(H, 0, 0)$

Implementing the constants declared in *Section II*, we get the following behavior given

$$N = 100, I_0 = 900, S_0 = 100, B_0 = 0$$

Figure 1: Constant Total Population



Based on *Figure 1*, we can see that the constants eventually caused the three population groups to reach an equilibrium at some unit of time. This agrees with the equilibrium we calculated earlier. To investigate further how each constant affects the equilibrium, we fixed all but one constant and observed the changes in the equilibrium of each group compared to the base-value equilibrium. The behaviors are shown below

Note: $+$ indicates the value is greater than that of the base-value equilibrium

$-$ indicates the value is lesser than that of the base-value equilibrium

	+10%			−10%		
Variables	I	S	B	I	S	B
H	+	−	−	−	+	+
N	−	−	+	+	+	−
y	+	−	−	−	+	+
c	+	−	+	−	+	−
h	−	+	−	+	−	+
d	+	−	+	−	+	−

By observing the equilibrium points, we also noticed that the initial population of each group will not affect the outcome with the exception of when $S = 0$. This provided us the answer to our proposed question; Given the condition of our model, the idea will never die out when the conversion of ignorant to sharer is positive, the sharing rate is positive, and at least one person is sharing the joke initially.

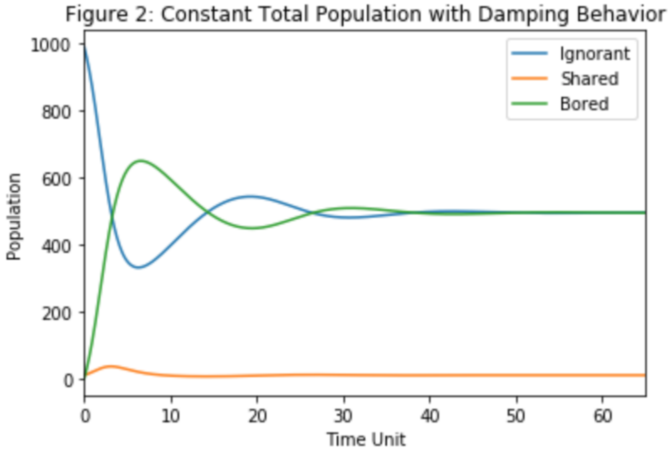
3) Cyclical Pattern/Damping Behavior: To answer the question of whether there is a cyclical pattern exist in this model, we look at the Jacobian matrix at the equilibrium points and check whether imaginary eigenvalues exist or not. After trial and error, we learned that the model will produce imaginary eigenvalues with specific constants.

Consider when

$$y = \frac{101}{9900}, c = \frac{1897}{10100}, d = 0.03, N = 100$$

We learned that given these constants, the Jacobian Matrix outputs imaginary eigenvalues when $h \geq 0.5466$. To best represent the cyclical behavior, we set $h = 1$.

Figure 2: Constant Total Population with Damping Behavior



As we can see in *Figure 2*, both the ignorant and bored population shows a behavior similar to a damped sine wave and eventually reaches equilibrium ($I = 495, S = 10, B = 495$). We can therefore conclude that the model is possible to become a cyclical pattern.

B. *Inconsistent Total Population*

1) Equation: We now investigate the case when the entering rate is not equal to the exiting rate. In this model, we set the entering and exiting rate to be dependent on respective population groups. The rate of entering depends on the magnitude of sharing at the specific time unit while the exiting rate is dependent on the bored and the ignorant population with constants r and w, with the assumption that the bored population will leave with a higher rate. We also assumed that the shared population will not be leaving due to the enthusiasm of the interaction and the desire of sharing. This resulted in the following equations

$$\frac{dI}{dt} = yS - ySI - rI \tag{12}$$

$$\frac{dS}{dt} = ycSI - hS + ydSB \tag{13}$$

$$\frac{dB}{dt} = hS - ydSB + y(1-c)SI - (r+w)B \tag{14}$$

2) Equilibrium: Instead of following the steps as in *Section III-A2*, we calculate the equilibrium points through extrema approximation for constant r and w. We chose to apply approximations from the two extremes to help us obtain a range for our equilibrium point. The following are the approximation of the two extrema for r and w

Variable	Minimum	Maximum
r	0.001	0.0025
w	0.002	0.005

We will also assume $d = 0.05$ due to the constant defined in *Section II* was not able to properly show the various behaviors of the model.

$$I_0 = 990, S_0 = 10, B_0 = 0$$

Figure 3: Inconsistent Total Population - Minimum

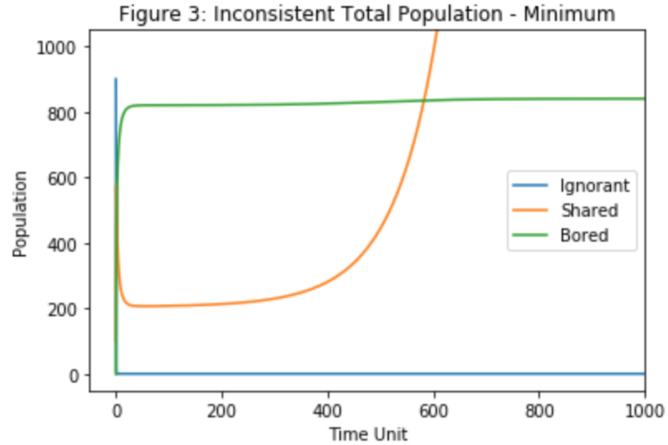
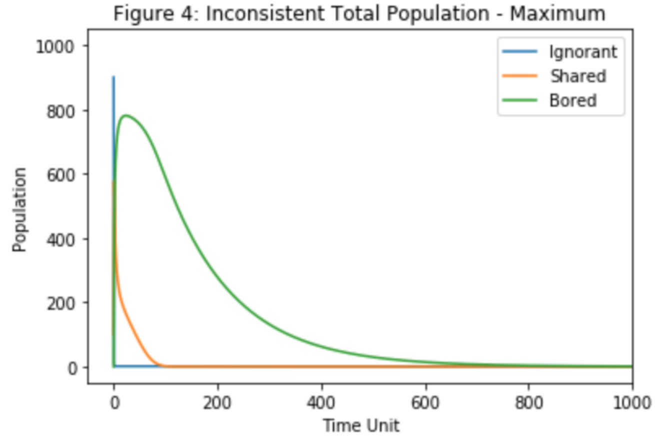


Figure 4: Inconsistent Total Population - Maximum



In *Figure 3*, the shared population grows exponentially and increases to infinity as time goes on. This is due to the fact that the incoming rate is dependent on the magnitude of shares and the only source of outflow of the shared population is fixed constant (h). On the other hand, the ignorants and bored population reached an equilibrium regardless of the behavior of S.

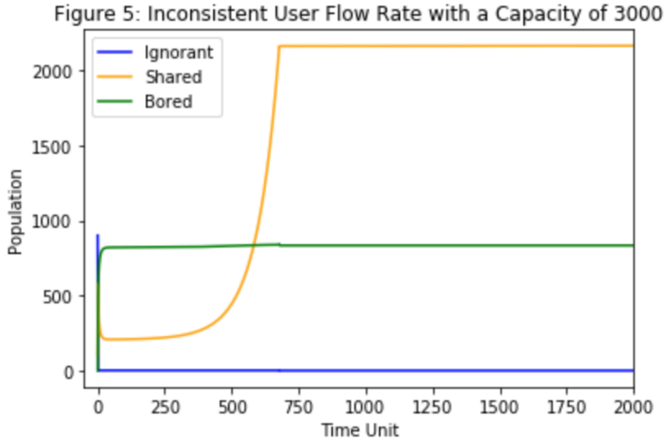
In *Figure 4*, we can see that all three population groups decrease to 0 after some point of time. The drop of the bored population declines at a slow rate due to the setup of the equation where the source of outflow of the bored population is $r + w$ of the group at every time unit. The result indicates that one of the equilibrium points for the model is $(0, 0, 0)$.

Overall, we can conclude that depending on the constants along with the magnitude of the entering rate and exiting rate, the total population will either increase to infinity and becomes a renowned joke, or the joke dies off and becomes a forgotten idea.

C. Inconsistent User Flow Rate with Capacity

1) Modifications: Although the conclusion we established in the previous section is logical, it is not realistic. To strive for realism, we will modify the differential equations to account for the space capacity and observe the behaviors between the three population groups. Numerically, we were unable to set up the proper system of equations to reflect the capacity trigger. However, we were able to plot the behaviors of them by combining the previous two models. In this model, assume the same condition as the inconsistent total population model, but once it reaches the capacity (C), it switches to the consistent total population model where the incoming rate is proportional to the ignorant and bored population. We set the capacity to 3000, and the result is shown below

Figure 5: Inconsistent User Flow Rate with a Capacity of 3000

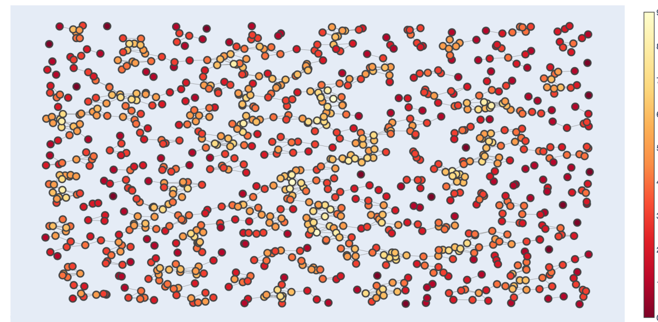


2) Observations: As we can see in *Figure 5*, the shared population spiked up quickly after a certain point in time. However, once the total population reached its capacity, the shared population stays leveled and reaches an equilibrium point along with the rest of the population groups. We believe that this can be interpreted as that every point of S is the equilibrium point for S when $C = S + I + B$.

IV. Simulation

Through a connection map, we were able to capture what the nodes will look like at a particular instant. We did attempt to simulate them overtime but weren't able to do so due to lack of time. Within the graph, the gradient describes the number of shares per node for H nodes. 10 being the most, colored with yellow, and 0 being the least, colored in dark red. The reason why we wanted to represent the process through a graph is to visualize how the group would like at an active state and eventually plot the process with the selected constants.

Figure 6: Attempted Simulation (Connection map)



V. Conclusion

By modifying the SIR model, we were able to model the spread of ideas. Specifically, we modeled a combination of the SIR-like model and the population growth model. We were able to get a general view on how the three population groups interact with each other and discovered that the model will create cyclical patterns when the birth (enter) rate and death (exit) rate is identical. We also discovered that when the birth and death rate is different, the idea either spreads exponentially or disappeared after a certain point of time. In conclusion, in order to prevent an idea disappear within a community, sharers must exist and the idea needs to be share-worthy enough for the ignorants to be willing to pass on to other ignorants. That way, we can maintain to keep the good ideas flowing, aside from documenting the ideas in writing.

VI. Future Work

Although we would like to analyze further on these models, certain topics have been left for the future due to lack of time. If time permits, these are the possible topics that we investigate. We would like to introduce functions within the constants and see how the population groups interact with each other. For example, constant h could be a function that grows increasingly faster as time goes on to reflect that that more people get bored from the joke as time goes on. We could try to determine the general solution in getting the cyclical pattern within the constant total population model. We would also like to investigate the timing of each population group reaching its equilibrium, especially with the inconsistent total population model. It is interesting to see that all population groups start out steady, but the Shared population spiked up afterward. Another possible topic we would like to work on is to find the general solution in determining the shared population going to infinity or dropping to 0.

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