Math 5231 - Fall 2019
Problem Set 5

Systems of Linear Equations Solve the following systems of differential equations. Draw a trajectory chart consisting of the behavior at the eigenvectors, and showing the long term behavior in each sector.

1) $$\begin{aligned} x_1' &= x_1 - 5x_2 \\ x_2' &= -5x_1 + x_2 \end{aligned}$$

2) $$\begin{aligned} x_1' &= -x_1 - 3x_2 \\ x_2' &= -4x_2 \end{aligned}$$

3) $$\begin{aligned} x_1' &= 5x_1 - 2x_2 \\ x_2' &= 3x_1 \end{aligned}$$

4) $$\begin{aligned} x_1' &= -4x_1 + 3x_2 - 3x_3 \\ x_2' &= -x_1 + 2x_2 - x_3 \\ x_3' &= 5x_1 - 3x_2 + 4x_3 \end{aligned}$$

Solve the following IVPs. Use the information provided to make an approximate drawing of the solution.

5) $$\begin{aligned} x_1' &= 2x_2 & x_1(0) &= 3 \\ x_2' &= x_1 + x_2 & x_2(0) &= -1 \end{aligned}$$

6) $$\begin{aligned} x_1' &= -4x_1 + x_2 & x_1(0) &= -1 \\ x_2' &= 2x_1 - 5x_2 & x_2(0) &= -1 \end{aligned}$$

7) $$\begin{aligned} x_1' &= 7x_1 - x_2 & x_1(0) &= 1 \\ x_2' &= -2x_1 + 8x_2 & x_2(0) &= 0 \end{aligned}$$

8) $$\begin{aligned} x_1' &= 5x_1 - 4x_2 & x_1(0) &= 2 \\ x_2' &= 8x_1 - 7x_2 & x_2(0) &= 3 \end{aligned}$$

Answers:

1) $\vec{x} = C_1 e^{6t} \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{-4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

2) $\vec{x} = C_1 e^{-t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + C_2 e^{-4t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

3) $\vec{x} = C_1 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

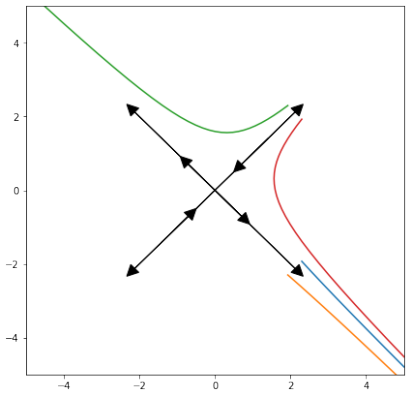
4) $\vec{x} = C_1 e^{-t} \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} + C_2 e^{2t} \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + C_3 e^{t} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.

5) $\vec{x} = -\frac{4}{3} e^{-t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + \frac{1}{3} e^{2t} \begin{bmatrix} -1 \\ -1 \end{bmatrix}$.

6) $\vec{x} = -e^{-3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

7) $\vec{x} = \frac{2}{3} e^{6t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{3} e^{9t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$.

8) $\vec{x} = e^{t} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^{-3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.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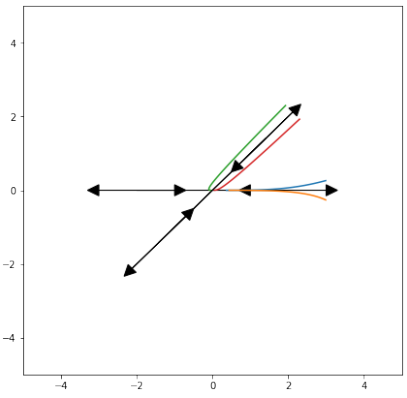
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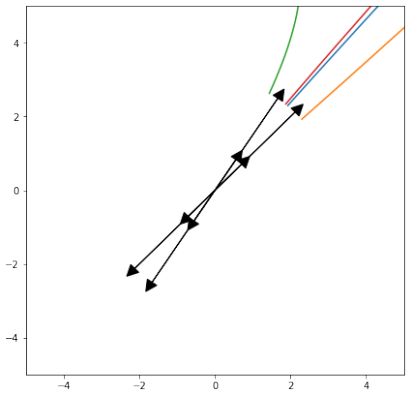
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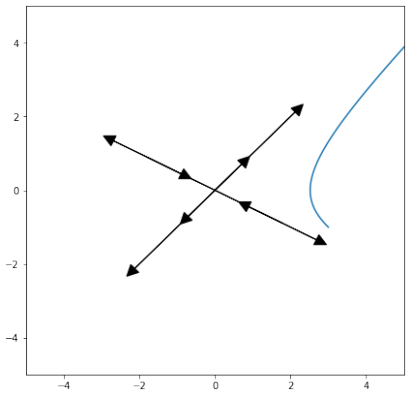
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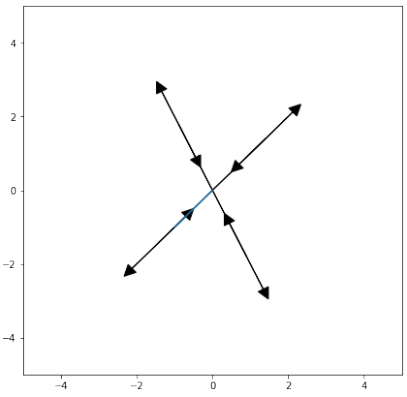
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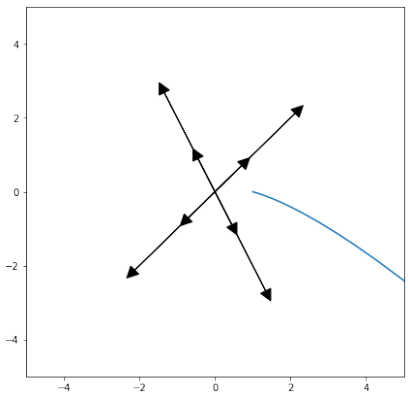
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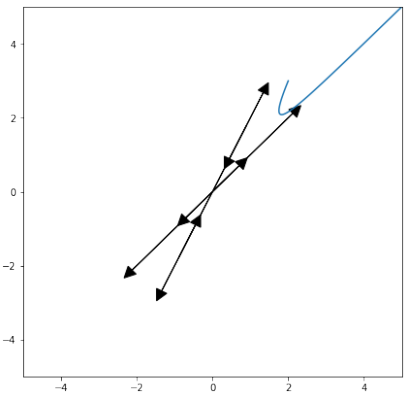
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