

$$1) \quad a) \quad E[x] = E[x|A] \cdot P[A] + E[x|B] \cdot P[B] + E[x|C] \cdot P[C]$$

$$= 2.6 \times \frac{1}{3} + 3 \times \frac{1}{3} + 3.4 \times \frac{1}{3}$$

$$= \boxed{3}$$

For Poisson distribution,

$$E[x^2] = \lambda^2 + \lambda$$

$$\therefore E[x^2|A] = 2.6^2 + 2.6$$

$$E[x^2|B] = 3^2 + 3$$

$$E[x^2|C] = 3.4^2 + 3.4$$

$$E[x^2] = \frac{E[x^2|A] + E[x^2|B] + E[x^2|C]}{3}$$

$$\text{Var}[x] = E[x^2] - (E[x])^2$$

$$= \frac{2.6^2 + (2.6) + 3^2 + 3 + 3.4^2 + 3.4}{3} - 9$$

$$= \boxed{3.1067}$$

b)

$$E[X] = E[X_A] + E[X_B] + E[X_C]$$

$$= 2 \cdot 6 + 3 + 3 \cdot 4$$

$$= 9$$

$$\text{Var}[X] = \text{Var}[X_A] + \text{Var}[X_B] + \text{Var}[X_C]$$

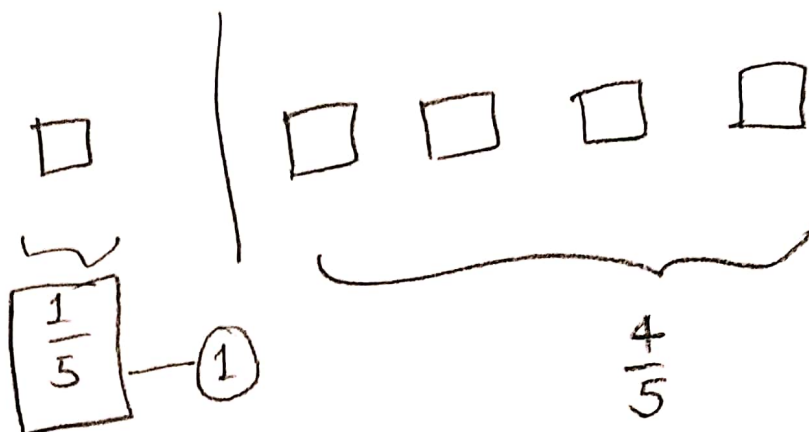
$$= 2 \cdot 6 + 3 + 3 \cdot 4$$

$$= 9$$

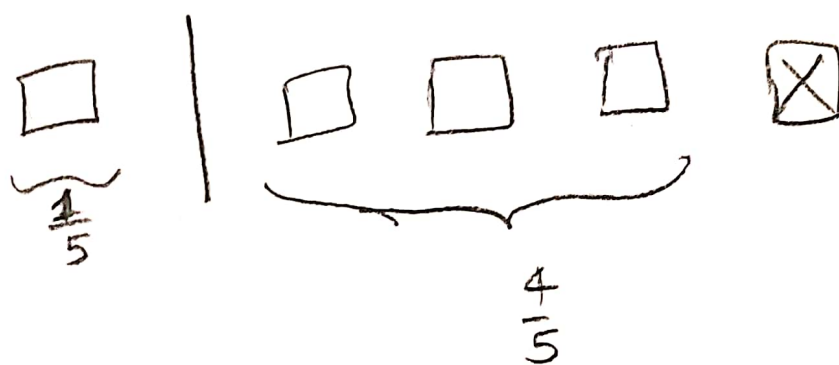
↓
Because A, B, C
are independent
the covariance
is zero

②

Initial:



Later:



\therefore Required probability } = $\frac{4}{5} \times$ Probability of choosing 1 out of 3 doors

$$= \frac{4}{5} \times \frac{1}{3} = \boxed{\frac{4}{15}} - \textcircled{2}$$

Because, $\boxed{\frac{4}{15}} > \boxed{\frac{1}{5}}$, we have

switching gives more chance to win

$$\textcircled{4} \quad \lim_{n \rightarrow \infty} P\left(\sum_{i=1}^n X_i \geq \sqrt{n}\right)$$

For uniform distribution over $[-1, 1]$,

$$\mu = \frac{-1+1}{2} = 0$$

$$\text{Var}(\sigma^2) = \frac{(1 - (-1))^2}{12} = \frac{4}{12} = \frac{1}{3}$$

$$\therefore \lim_{n \rightarrow \infty} P\left(\sum_{i=1}^n X_i \geq \sqrt{n}\right)$$

$$= P\left(\frac{\sum_{i=1}^n X_i - \mu}{\sqrt{n \times \sigma^2}} \geq \frac{\sqrt{n} - \mu}{\sqrt{n \times \sigma^2}}\right)$$

$$= P\left(Z \geq \frac{\sqrt{n} - 0}{\sqrt{n \times \frac{1}{3}}}\right)$$

$$= P(Z \geq \sqrt{3})$$

Comparing, we get

$$\boxed{a = \sqrt{3}}$$

⑤

Randomly distribute "r" balls over "n" boxes

$$\frac{r}{n} \rightarrow c \Rightarrow r \rightarrow n \cdot c$$

$$\therefore E[N_n] = E[X_1] + E[X_2] + \dots + E[X_n]$$

$$= n \times E[X_1]$$

$$\Rightarrow \frac{E[N_n]}{n} = E[X_1]$$

$$= \left(\frac{n-1}{n} \right)^r = \left(1 - \frac{1}{n} \right)^{nc}$$

$$\therefore \lim_{n \rightarrow \infty} \frac{E[N_n]}{n} = \lim_{n \rightarrow \infty} \left[\left(1 - \frac{1}{n} \right)^n \right]^c$$

$$= \boxed{e^{-c}}$$

⑥

By memoryless property, the distribution of $g(X) | X > x$ is the same as that of $g(x)$, but shifted to the right by " x ".

$$\therefore E[X^2 | X > 1] = E[X^2] + 1$$

Proof:

$$\downarrow$$
$$\int_{-\infty}^{\infty} t f_{X^2 | X > 1}(t) dt$$

$$= \int_{-\infty}^{\infty} t f_{X^2}(t-1) dt$$

$$= \int_{-\infty}^{\infty} (u+1) f(u) du$$

$$= \int_{-\infty}^{\infty} u f(u) du + \int_{-\infty}^{\infty} f(u) du$$

$$= E[X^2] + 1$$