1a

2.1

(a)

We have a non-zero element in rref and this indicates there is no solution for Ax=b. MATLAB’s approximation (A\b) yields the following solution.

```
Warning: Matrix is singular to working precision.
> In Solution_2_1 (line 4)

   NaN
   Inf
   Inf
```

(b)

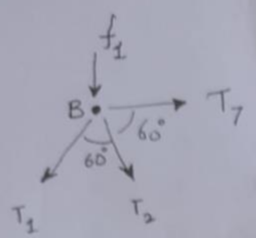
```
    0.5000
    0.5000
    0.5000
```

(c)

```
         0
   -0.3333
    0.3333
```

3.1

1. R4 is missing in the question diagram. Included it and solved.



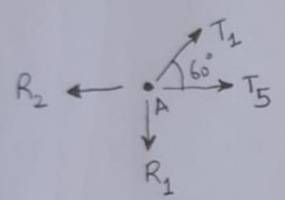
Vertical :-

$$-f_1 - T_1\cos 30^\circ - T_2\cos 30^\circ = 0$$

$$-T_1\cos 30^\circ - T_2\cos 30^\circ = f_1$$

Horizontal :-

$$+T_7 + T_2\cos 60^\circ - T_1\cos 60^\circ = 0$$

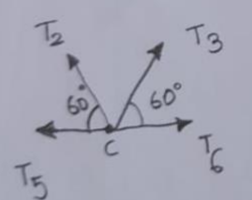


vertical :-

$$T_1\cos 30^\circ - R_1 = 0$$

Horizontal :-

$$T_1\cos 60^\circ + T_5 - R_2 = 0$$

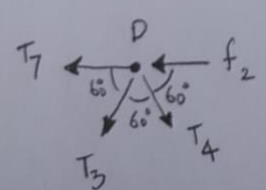


vertical :-

$$T_3\cos 30^\circ + T_2\cos 30^\circ = 0$$

Horizontal :-

$$T_3\cos 60^\circ + T_6 - T_2\cos 60^\circ - T_5 = 0$$

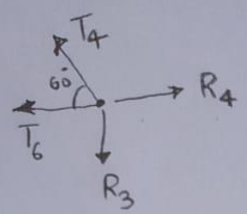


vertical:

$$T_3\cos 30^\circ + T_4\cos 30^\circ = 0$$

Horizontal:

$$T_4\cos 60^\circ - T_3\cos 60^\circ - T_7 = f_2$$



Vertical:

$$T_4 \cos 30° - R_3 = 0$$

Horizontal:

$$R_4 - T_4 \cos 60° + T_6 = 0$$

Whole system

vertical:

$$f_1 = -R_1 - R_3$$

Horizontal:

$$-R_2 - f_2 + R_4 = 0$$

$$R_4 - R_2 = f_2$$

$$\begin{array}{ccccccccccc} T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & R_1 & R_2 & R_3 & R_4 \end{array}$$

$$\begin{bmatrix}
-\cos 30° & -\cos 30° & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
-\cos 60° & \cos 60° & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \\
\cos 30° & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \\
\cos 60° & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & \\
0 & \cos 30° & \cos 30° & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & -\cos 60° & \cos 60° & 0 & -1 & 1 & 0 & 0 & 0 & 0 & \\
0 & 0 & \cos 30° & \cos 30° & 0 & 0 & 0 & 0 & 0 & 0 & \\
0 & 0 & -\cos 60° & \cos 60° & 0 & 0 & -1 & 0 & 0 & 0 & \\
0 & 0 & 0 & \cos 30° & 0 & 0 & 0 & 0 & 0 & -1 & \\
0 & 0 & 0 & \cos 60° & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & +1
\end{bmatrix}
\begin{bmatrix} f_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ f_2 \\ 0 \\ 0 \\ f_1 \\ f_2 \end{bmatrix}$$

$$\begin{pmatrix} T1 \\ T2 \\ T3 \\ T4 \\ T5 \\ T6 \\ T7 \\ R1 \\ R2 \\ R3 \\ R4 \end{pmatrix} = \begin{pmatrix} -430.9401 \\ 315.4701 \\ -315.4701 \\ 315.4701 \\ -473.2051 \\ -157.7350 \\ -684.5299 \\ -373.2051 \\ -688.6751 \\ 273.2051 \\ 311.3249 \end{pmatrix}$$

2.

$$\begin{pmatrix} T1 \\ T2 \\ T3 \\ T4 \\ T5 \\ T6 \\ T7 \\ R1 \\ R2 \\ R3 \\ R4 \end{pmatrix} = \begin{pmatrix} (2*3\text{^}(1/2)*\mathrm{f1})/(3*(3\text{^}(1/2)-2)) \\ -(2*(3*\mathrm{f1}-3\text{^}(1/2)*\mathrm{f1}))/(3*(3\text{^}(1/2)-2)) \\ -2*\mathrm{f1}-(2*3\text{^}(1/2)*\mathrm{f1})/3 \\ (2*3\text{^}(1/2)*(\mathrm{f1}-3\text{^}(1/2)*\mathrm{f1}))/(3*(3\text{^}(1/2)-2)) \\ -(3\text{^}(1/2)*(\mathrm{f1}-3\text{^}(1/2)*\mathrm{f1}))/(3\text{^}(1/2)-2) \\ -(3\text{^}(1/2)*(\mathrm{f1}-3\text{^}(1/2)*\mathrm{f1}))/(3*(3\text{^}(1/2)-2)) \\ -(6*\mathrm{f1}-6*\mathrm{f2}-2*3\text{^}(1/2)*\mathrm{f1}+3*3\text{^}(1/2)*\mathrm{f2})/(3*(3\text{^}(1/2)-2)) \\ \mathrm{f1}/(3\text{^}(1/2)-2) \\ (9*\mathrm{f1}-2*3\text{^}(1/2)*\mathrm{f1})/(3*(3\text{^}(1/2)-2)) \\ -(3\text{^}(1/2)*(3*\mathrm{f1}-3\text{^}(1/2)*\mathrm{f1}))/(3*(3\text{^}(1/2)-2)) \\ (9*\mathrm{f1}-6*\mathrm{f2}-2*3\text{^}(1/2)*\mathrm{f1}+3*3\text{^}(1/2)*\mathrm{f2})/(3*(3\text{^}(1/2)-2)) \end{pmatrix}$$

It can be clearly seen that **T3** is independent of **f2.**
So, no matter how big or small **f2** is, **T3** cannot be changed.

3.

If T3 = 1000 then,

$$-2*\mathrm{f1}-\frac{2*3^{\frac{1}{2}}*\mathrm{f1}}{3}=1000$$

⇨ f1 ≈ -316.9873

f2 can be anything.

1b

2.1

Refer to “*1/1b/Solution_2_1.m*” for code.

```
   2.4455   -0.4558   -0.2476   -0.0588
  10.6574   -1.4300   -1.2112   -0.2585
 235.8636  -35.3400  -28.4804   -5.7000
  81.1064  -17.0496   -6.5352   -1.6280
```

2.2

```
   5.1036
  14.8974
  51.1443
-147.6006
```

1c

1.3.1 - 1.3.5

1.3.1

$$\begin{bmatrix} 0.67 & 0.14 & 0.21 \\ 0.33 & 0.08 & 0.71 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = 0$$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = t \begin{bmatrix} 413/37 \\ -2032/37 \\ 1 \end{bmatrix}, \quad t \in \mathbb{R}$$

$\therefore$ basis of null space is

$$\left\{ \begin{bmatrix} 413/37 \\ -2032/37 \\ 1 \end{bmatrix} \right\}$$

$$S = u + v + w = \left(\frac{413}{37} - \frac{2032}{37} + 1 \right) t$$

$$= \boxed{\frac{-1582}{37} t}$$

1.3.2

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0.67 & 0.14 \\ 0.33 & 0.08 \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}$$

$$A = \begin{bmatrix} 0.67 & 0.14 \\ 0.33 & 0.08 \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} 10.8108 & -18.9189 \\ -44.5946 & 90.5405 \end{bmatrix}$$

1.3.3

$$(r-c)R + (b-d)B + gG = 0$$

From 1.3.1

$$\begin{bmatrix} r-c \\ b-d \\ g \end{bmatrix} = \alpha \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\begin{bmatrix} r \\ b \\ g \end{bmatrix} = \begin{bmatrix} \alpha u + c \\ \alpha v + d \\ \alpha w \end{bmatrix}$$

$$r+b+g = 1 \Rightarrow c+d+\alpha(u+v+w) = 1$$

$$\Rightarrow \boxed{\alpha = \frac{1-(c+d)}{(u+v+w)}}$$

$$\therefore \begin{bmatrix} r \\ b \\ g \end{bmatrix} = \begin{bmatrix} \left[\frac{1-(c+d)}{s}\right] u + c \\ \left[\frac{1-(c+d)}{s}\right] v + d \\ \left[\frac{1-(c+d)}{s}\right] w \end{bmatrix}$$

1.3.4

$$\begin{bmatrix} 0.3 \\ 0.5 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.67 & 0.14 & 0.21 \\ 0.33 & 0.08 & 0.71 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} r \\ b \\ g \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} r \\ b \\ g \end{bmatrix} = \begin{bmatrix} 0.2257 \\ 0.1972 \\ 0.5771 \end{bmatrix}$$

1.3.5

$$x' = \begin{bmatrix} 0.3800 \\ 0.2467 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 0.3800 \\ 0.2467 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.67 & 0.14 & 0.21 \\ 0.33 & 0.08 & 0.71 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} r \\ b \\ g \end{bmatrix}$$

$$\begin{bmatrix} r \\ b \\ g \end{bmatrix} = \begin{bmatrix} 0.4416 \\ 0.4694 \\ 0.0895 \end{bmatrix}$$

1d

1.4.1 - 1.4.3

1.4.1

$$\vec{P} = 117.67 \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} / 600.0016 = \begin{bmatrix} 0.7845 \\ 0.1961 \\ 0.5883 \end{bmatrix}$$

$$\vec{W} = \frac{1}{3} \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$$

$$\vec{Q} = \vec{W} \times \vec{P}$$

$$\vec{Q} = \begin{bmatrix} -0.5230 \\ 0.7191 \\ 0.4576 \end{bmatrix}$$

1.4.2

$$[Id]\_EU = [P \ Q \ W]$$

$$= \begin{bmatrix} 0.7845 & -0.5230 & -0.3333 \\ 0.1961 & 0.7191 & -0.6667 \\ 0.5883 & 0.4576 & 0.6667 \end{bmatrix}$$

$$[Id]\_UE = \begin{bmatrix} 0.7845 & 0.1961 & 0.5883 \\ -0.5230 & 0.7191 & 0.4576 \\ -0.3333 & -0.6667 & 0.6667 \end{bmatrix}$$

1.4.3

$$T\_UU = \begin{bmatrix} P & Q & W \end{bmatrix}$$

$$T\_EE = [P\ Q\ W]\ T\_UU\ [P\ Q\ W]^{-1}$$

$$\Rightarrow T\_EE = [P\ Q\ W]$$

$$|\hat{R}_3|_{EE} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\theta = \omega t = 3.91 \times 0.5 = 1.955$$

$$\hat{R}_3(0.5) = \begin{bmatrix} \cos(1.955) & -\sin(1.955) & 0 \\ \sin(1.955) & \cos(1.955) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{x}(0.5) = \hat{R}_3(0.5) \times [P\ Q\ W]\ \vec{x}(0)$$

$$= \begin{bmatrix} -17.0669 \\ 198.0656 \\ 566.1101 \end{bmatrix}$$

||'y

$$\vec{x}(1) = \begin{bmatrix} -177.2290 \\ -90.0619 \\ 566.1101 \end{bmatrix}$$

$$\vec{x}(1.5) = \begin{bmatrix} 149.9253 \\ -130.5515 \\ 566.1101 \end{bmatrix}$$