Math 5231 - Fall 2018
Problem Set 4

Autonomous Differential Equations Solve the following autonomous differential equations for an explicit real solution. The solution may be implicitly defined.

1) $\dfrac{dy}{dx} = y(y-1)$

2) $\dfrac{dy}{dx} = (y+4)^2$

3) $\dfrac{dy}{dx} = \dfrac{y^2+2}{y}$

4) $\dfrac{dy}{dt} = \dfrac{1}{2y-3}\,,\ y(0) = 10$

5) $y' = y - y^3\,,\ y(1) = 2\,,$ also $y(1) = 0$

Find the equalibria of the following differential equations and classify them. You need not solve the equations.

6) $\dfrac{dR}{dx} = -7R(R+3)$

7) $\dfrac{dy}{dx} = y^2 + y + 2$

8) $\dfrac{dy}{dt} = y^2$

9) $\dfrac{dy}{dx} = e^y - 1$

10) $y' = \cot(y)a$

Challenge: Solve the following:

11) $\dfrac{dy}{dx} = y^2 + y + 2$

Answers:

1) $y = \dfrac{1}{e^{C+x}+1}$

2) $y = -\dfrac{4x+4C+1}{C+x}$

3) $y = \pm\left(e^{2x+C}-2\right)^{\frac{1}{2}}$

4) $y^2 - y = 10t + 90$

5) $\dfrac{y}{y^2-1} = \dfrac{2}{3}e^{t-1}\,,\ \ y = 0$

6) $R = 0\,,\text{Stable}\,,R = -3\,,\text{Unstable}$

7) None

8) $y = 0\,,$ Unstable from above, but stable from below

9) $y = 0\,,$ Unstable

10) $y = \pi\dfrac{n+1}{2}\,,$ All points are stable