Math 5231 - Fall 2019
Problem Set 9: Higher Order Differential Equations with Constant Coefficients

Solve the following homogeneous initial value problems:

1) $x'' - 6x' + 8x = 0\,, \qquad x(0) = 1\,,\ x'(0) = 0$

2) $x' + 3x = 0\,, \qquad x(0) = 5$

3) $x''' = 0\,, \qquad x(0) = 1\,,\ x'(1) = 0,\ x''(0) = 2$

4) $x'' + 16x = 0\,, \qquad x(0) = 1\,,\ x'(0) = 6$

Solve the following inhomogeneous problems for a general solution

5) $x'' + 2x' - 3x = 3t - 2\,,$

6) $x' - 2x + 4t^2 + 3t - 1 = 0\,,$

7) $x'' + x = -t^2\,,$

8) $x'' - 4x = 1\,,$

Challenge: To find a particular solution we can find the most general form of the sum of all derivatives of the inhomogeneous part. Use this principle to find

9) $x' - x = 2\sin(4t) - \cos(4t)\,,$

10) $x'' + 2x - 8 = \sin(t)\,,$

11) $x' - 5x = \sin(3t) + 3\cos(t)\,,$

12) $2x' + 4x = e^{-4t}\,,$

Answers: Your answer may look different, check to see that they are the same.

1) $$x(t) = 2e^{2t} - e^{4t}$$

2) $$x(t) = 5e^{-3t}$$

3) $$x(t) = t^2 - 2t + 1$$

4) $$x(t) = \frac{3}{2}\sin(4t) + \cos(4t)$$

5) $$x(t) = -t + c_1e^{-3t} + c_2e^{t}$$

6) $$x(t) = 2t^2 + \frac{7}{2}t + \frac{5}{4} + ce^{2t}$$

7) $$x(t) = -t^2 + 2 + ce^{it} + \bar{c}e^{-it}$$

8) $$x(t) = \frac{1}{4} + c_1e^{2t} + c_2e^{-2t}$$