Boundary Value Problems (BVP).

$$\frac{d^2y}{dx^2} = -4y \qquad y = e^{\alpha x}$$

$$y(x_L) = \alpha \qquad \alpha^2 + 4 = 0 \qquad \alpha = \pm 2i$$

$$y(x_R) = \beta \qquad y(x) = c_1' e^{2ix} + c_2'' e^{-2ix}$$

$$y(x) = c_1 \cos 2x + c_2 \sin 2x$$

\# $y(0) = -2$ $\quad y(0) = c_1 = -2$ $\quad y = -2\cos 2x + 10 \sin 2x$

$y(\pi/4) = 10$ $\quad y(\pi/4) = c_2 = 10$

\# $y(0) = -2$ $\quad y(0) = c_1 = -2$ $\quad y = -2\cos 2x + c_2 \sin 2x$

$y(2\pi) = -2$ $\quad y(2\pi) = c_1 = -2$

infinitely many solns

\# $y(0) = -2$ $\quad y(0) = c_1 = -2$

$y(2\pi) = 3$ $\quad y(2\pi) = c_1 = 3$ $\quad$ no solution !!

Shooting Method

$\boxed{\text{2pt BVP}} \longrightarrow \boxed{\text{IVP}}$

$y'' = f(x, y, y')$

$y(a) = \alpha$

$y(b) = \beta$

$x \in [a, b]$

$y'' = f(x, y, y')$

$y(a) = \alpha$

$y'(a) = z$ [guess]

Solve by RK4 get approx soln. y_z

if $y_z(b) \neq \beta$ try another z

Find the correct z by root finding $\Rightarrow$ Secant Method.

Let $\phi(z) = y_z(b) - \beta$

$z_1 \rightarrow y_{z_1} \longrightarrow \phi(z_1) = y_{z_1}(b) - \beta$

$z_2 \rightarrow y_{z_2} \longrightarrow \phi(z_2) = y_{z_2}(b) - \beta$

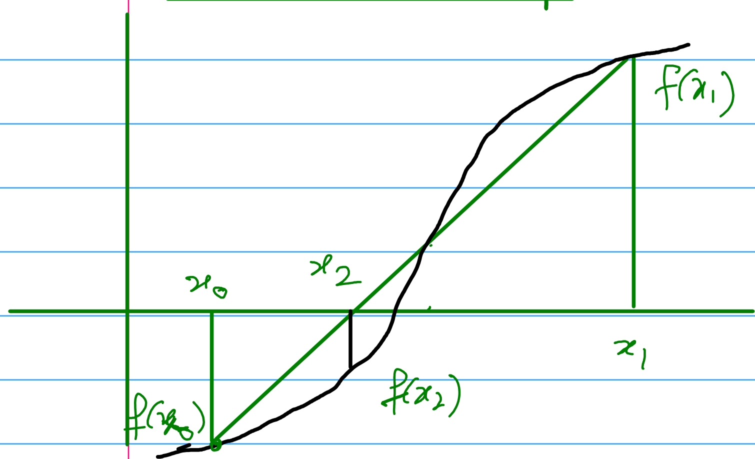
$$z_3 = z_2 - \phi(z_2)\frac{z_2 - z_1}{\phi(z_2) - \phi(z_1)}$$

$\vdots$

$$z_{k+1} = z_k - \phi(z_k)\frac{z_k - z_{k-1}}{\phi(z_k) - \phi(z_{k-1})}$$

Stop if $\phi(z_k) < \epsilon$

Secand Method Recap



$$y = \boxed{\frac{f(x_1) - f(x_0)}{x_1 - x_0}}(x - x_1) + f(x_1)$$

slope intercept

$y = 0 \Rightarrow$

$$x_2 = x_1 - f(x_1)\frac{x_0 - x_1}{f(x_0) - f(x_1)}$$

$\vdots$

x_k stop such that $f(x_k) \sim \epsilon$