

Worksheet 5

Problem 1 Consider a random walker on a 1-d lattice hopping left or right with equal probability ($p = q = 1/2$) starting from origin (0). Store the final position after N steps ($N \gg 1$). Repeat the exercise for an ensemble of M random walkers ($M \gg 1$). Plot the distribution of final positions (normalized). Calculate the mean position and the variance. Vary N and show that the variance $\propto N$.

Problem 2 Numerically solve the 1-d Diffusion equation:

$$\frac{\partial \rho(x, t)}{\partial t} = D \frac{\partial^2 \rho(x, t)}{\partial x^2} \quad (1)$$

where $D = \frac{(\Delta x)^2}{2\Delta t}$ and set the initial condition at $t = 1$ as

$$\rho(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad (2)$$

with $\sigma^2 = 2Dt$. Integrate over time by (a) Euler (b) RK4 and plot $\rho(x, t)$ at time $\{t_1, t_2, t_3\}$. Discretize space $x \in [-2 : 2]$ with $\Delta x = 0.01$.