

Boundary Value Problems (BVP)

$$\left. \begin{array}{l} \frac{d^2 y}{dx^2} = -4y \\ y(x_L) = \alpha \\ y(x_R) = \beta \end{array} \right\} \quad \begin{array}{l} y = e^{\alpha x} \\ \alpha^2 + 4 = 0 \quad \alpha = \pm 2i \\ y(x) = C_1' e^{2ix} + C_2' e^{-2ix} \end{array}$$

$$y(x) = C_1 \cos 2x + C_2 \sin 2x$$

$$\begin{array}{lll} \# & \begin{array}{l} y(0) = -2 \\ y(\pi/4) = 10 \end{array} & \begin{array}{l} y(0) = C_1 = -2 \\ y(\pi/4) = C_2 = 10 \end{array} & y = -2 \cos 2x + 10 \sin 2x \end{array}$$

$$\begin{array}{lll} \# & \begin{array}{l} y(0) = -2 \\ y(2\pi) = -2 \end{array} & \begin{array}{l} y(0) = C_1 = -2 \\ y(2\pi) = C_1 = -2 \end{array} & \begin{array}{l} y = -2 \cos 2x + C_2 \sin 2x \\ \text{infinitely many solutions} \end{array} \end{array}$$

$$\begin{array}{lll} \# & \begin{array}{l} y(0) = -2 \\ y(2\pi) = 3 \end{array} & \begin{array}{l} y(0) = C_1 = -2 \\ y(2\pi) = C_1 = 3 \end{array} & \begin{array}{l} \text{no solution} \\ \text{"} \end{array} \end{array}$$

Shooting Method

2pt BVP

IVP

$$y'' = f(x, y, y')$$

$$y(a) = \alpha$$

$$y(b) = \beta$$

$$x \in [a, b]$$

$$y'' = f(x, y, y')$$

$$y(a) = \alpha$$

$$y'(a) = \gamma \quad [\text{guess}]$$

Solve by RK4 get approx soln, y_z
if $y_z(b) \neq \beta$ try another γ

Find the correct γ by root finding \Rightarrow Secant Method.

$$\text{let } \phi(\gamma) = y_z(b) - \beta$$

$$z_1 \rightarrow y_{z_1} \rightarrow \phi(z_1) = y_{z_1}(b) - \beta$$

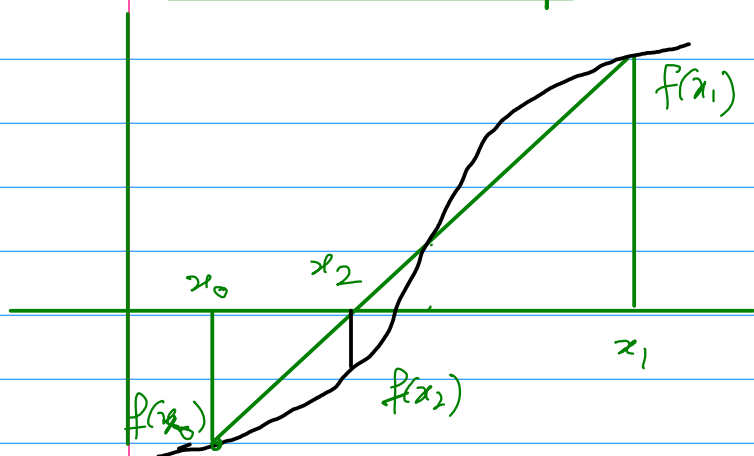
$$z_2 \rightarrow y_{z_2} \rightarrow \phi(z_2) = y_{z_2}(b) - \beta$$

$$z_3 = z_2 - \phi(z_2) \frac{z_2 - z_1}{\phi(z_2) - \phi(z_1)}$$

\vdots

$$z_{k+1} = z_k - \phi(z_k) \frac{z_k - z_{k-1}}{\phi(z_k) - \phi(z_{k-1})} \quad \text{stop if } \phi(z_k) \leq \epsilon$$

Secant Method Recap



$$y = \underbrace{\frac{f(x_1) - f(x_0)}{x_1 - x_0}}_{\text{slope}} (x - x_1) + \underbrace{f(x_1)}_{\text{intercept}}$$

$$y=0 \Rightarrow$$

$$x_2 = x_1 - f(x_1) \frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

\vdots

x_k stop such that $f(x_k) \approx 0$