Worksheet 7

Problem 1 Numerically solve the boundary value problem

$$y''(x) + 4y(x) = 0$$

for $x \in [0, \pi/4]$ by *shooting method* with boundary conditions

$$\{y(0) = -2; y(\pi/4) = 10\} \ .$$

Exact solution: $y(x) = -2cos(2x) + 10(sin2x)$

Plot exact and numerical solutions.

Show error as a function of stepsize Δx.

Problem 2 Numerically solve the boundary value problem

$$y''(x) = -y(x) + \frac{2(y'(x))^2}{y(x)}$$

for $x \in [-1, 1]$ by *shooting method* with boundary conditions

$$y(1) = y(-1) = \frac{1}{e+e^{-1}} \qquad = y_0(say).$$

(a) shoot from one end $\{y(-1) = y_0; y'(-1) = \epsilon\}$ and match $y(x = 1)$

(b) shoot from both ends $\{y(\pm 1) = y_0; y'(\pm 1) = \epsilon\}$ and match $y(x = 0)$.

Exact solution: $y(x) = \frac{1}{e^x + e^{-x}}$

Plot exact and numerical solutions.