# Worksheet 6

**Problem 1** Integrate Black-Scholes equation $dx = \mu x dt + \sigma x dW(t)$ using Euler-Maruyama method, with noise property $dW\ dW \propto dt$, i.e. $\Delta W_i = z_i\sqrt{dt}$ where $z_i \in \mathcal{N}(0,1)$.
(a) Plot the numerically integrated trajectory and the corresponding exact solution $x(t) = x_o exp(\mu t - \sigma^2 t/2 + \sigma\eta(t))$ where $\eta(t) = \int_0^t dW(t)$.
(b)Compute the rms error and plot with varying $dt$.

**Problem 2** Integrate stochastic Birth-Death model $dn = k_1 dt - k_2 n dt + dW(t)$ using Euler-Maruyama method, with noise property $dW\ dW \propto (k_1 + k_2 n)dt$, i.e. $\Delta W_i = z_i\sqrt{dt(k_1 + k_2 n)}$ where $z_i \in \mathcal{N}(0,1)$.
(a) Compute the mean trajectory $n(t)$, averaged over N realizations and plot with the deterministic solution $n(t) = (k_1/k_2)(1 - exp(-k_2 t))$.
(b) From the simulated data find the steady state distribution $P(n)$ and fit to a Poisson distribution and determine the mean.