Q) what is probability & explain diff. types of probability.

Probability: is the measure of quantity of uncertainity

(or)

it is the branch of mathematics concerning the occurance of a random event, and four main types of probability exist:

Types of probability: 1. Theoritical
2. Experimental
3. Axiomatic.

1. Theoritical probability: which comes from throughout experiment is called theoritical / clasical probability.

* It can be found without doing the experiment.

$$P(E) = \frac{\#\text{ sample points in } E}{\#\text{ of sample points in } S.}$$

Eg:- A coin is tossed one time then $P(H) = \frac{1}{2}$, $P(T) = \frac{1}{2}$.

2. Experimental probability: probability which comes from Practical experiment is called experimental/ emperical probability

* It is found by repeating the experiment & obseeving the outcome. $P(E) = \frac{\#\text{ desired outcomes}}{\text{Trial.}}$

Eg:- A coin is tossed one time a head is recorded one time & a tail is recorded '0' times

$$P(H) = \frac{1}{1} \qquad P(\text{Tail}) = \frac{0}{1}$$

3. Subjective probability: probability which comes from an educated way is called subjective probability.

* It is found by observing/ analysing the past data.

Ex:- As long as you pay the bill, the power won't cut off.

(1) Experiment: an experiment is an activity with observable result.

(2) Sample space: The Sample space is set of all outcomes of an experiment.

(3) outcome :- The outcome (sample point) is the result of an experiment.

(4) Event :- The subset of possible outcomes of an experiment.

(5) In loan defaulters older people make up only 1.4%. Now the probability that someone defaults on a loan is 0.184, find the prob. of default on loan knowing that he is an old person, old people make only 0.8%.

Sol:- Given $P(\text{old} | \text{defaulters}) = 1.4\% \Rightarrow 0.014$.

$P(\text{defaults}) = 0.184$.

$P(\text{default} | \text{old}) = ?$

$P(\text{old}) = 0.8\% = 0.008$.

$$P(\text{default} | \text{old}) = \frac{P(\text{default} \cap \text{old})}{P(\text{old})}$$

$$= P(\text{default}) \frac{P(\text{old} | \text{default}) \cdot P(\text{default})}{P(\text{old})}$$

$$= \frac{0.014 * 0.184}{0.008}$$

$$= 0.322.$$

(6) Define Bayes theorem & write the formula.

Bayes theorem can be derived for events & random variables separately using defination of conditional probability & density.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \rightarrow (1)$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \longrightarrow (2)$$

from eqn(1) & eqn(2);

$$P(A|B).P(B) = P(A \cap B)$$
$$P(B|A).P(A) = P(A \cap B)$$
$$P(A|B).P(B) = P(B|A).P(A)$$
$$P(A|B) = \frac{P(B|A).P(A)}{P(B)}$$

5) Spam assassin works by having users train the system. It looks for patterns in the words in email marked as spam by the user. For eg; it may have learned that the word "free" appears in 30% of the mails marked as spam, i.e P(free/spam) = 0.30. Assuming 1% of non-spam mail includes the word "free" & 50% of all mails received by the user are spam. find the prob. that a mail is spam if word "free" appears in it

Sol:- Given that
$P(\text{free}|\text{spam}) = 30\% = 0.3 \longrightarrow (1)$
$P(\text{free}|\text{non-spam}) = 1\% = 0.01 \longrightarrow (2)$
$P(\text{spam}) = 50\% = 0.5$
$P(\text{spam}|\text{free}) = ?$

$$P(\text{non spam}) = 1 - P(\text{spam})$$
$$= 1 - 0.5$$
$$= 0.5$$

$$P(\text{spam}|\text{free}) = \frac{P(\text{spam} \cap \text{free})}{P(\text{free})}$$

$P(\text{free}) = ?$

from (1); (2) eqns $P(\text{free}) = P(\text{free}|\text{spam}) * P(\text{spam}) + P(\text{free}|\text{non spam}) * P(\text{non spam})$

$$P(\text{free}) = 0.3 * 0.5 + 0.01 * 0.5$$
$$= 0.165$$

$$\therefore P(\text{spam} \mid \text{free}) = \frac{P(\text{free} \mid \text{spam})\, p(\text{spam})}{P(\text{free})}$$

$$= \frac{0.3 * 0.5}{0.165}$$

$$= 0.909.$$