Catalog

MA 225 — Mid-semester Examination
Probability Theory and Random Processes — 9:00–9:45 IST
IIT GUWAHATI — September 21, 2022

PART A

Instructions:

1. ***Answers must be given exclusively on this sheet: answers given on the other sheets will be ignored. USE Rough space at the End of Part B for Rough work.***
2. ***Be very careful while bubbling the Roll No. and answers. Once bubbled, it cannot be changed. Bubble properly, otherwise computer will not be able to detect it.***
3. ***Use black or blue ball pen only for bubbling.***
4. ***Answer all the questions. Total marks in Part A is 10.***

Name and Roll Number

Please bubble your Roll No:

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◯2 ◯2 ◯2 ◯2 ◯2 ◯2 ◯2 ◯2 ◯2
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Signature of Invigilator:_____

Name:

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Roll No.:

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Signature of Student:_____

Questions and Responses

Question [q1]**:** (2 points) Let $(\Omega, \mathcal{F}, P)$ be a probability space. Let A and B are two events such that $0 < P(A) < 1$ and $0 < P(B) < 1$. Then which of the following statements is/are **TRUE**?

- ● If $P(A) = P(A|B)$, then A and B are independent.
- ◯ If $A \cap B = \phi$, then it is possible that A and B are independent.
- ● If $P(A^c) = P(A^c|B^c)$, then A and B are independent.
- ◯ If A and B are exhaustive, then it is possible that A and B are independent.

Question [q2]**:** (2 points) Let F and G are cumulative distribution functions (CDFs). Then which of the following statements is/are **TRUE**?

- ● For any $0 \leq \lambda \leq 1$, $H(x) = \lambda F(x) + (1 - \lambda)G(x)$ is a CDF.
- ● $H(x) = \sqrt{F(x)}$ is a CDF.
- ● $H(x) = (F(x))^2$ is a CDF.
- ● $H(x) = F(x)G(x)$ is a CDF.

Question [q3]: (2 points) Let f be a probability density function (PDF). Then which of the following statements is/are **TRUE**?

- ◯ $\lim_{x\to\infty} f(x) = 0$
- ◯ $0 \le f(x) \le 1$ for all $x \in \mathbb{R}$
- ● $\lim_{n\to\infty} \int_{-\infty}^{x+\frac{x}{n^2}} f(t)dt = \int_{-\infty}^{x} f(t)dt$ for all $x \in \mathbb{R}$
- ● $\lim_{n\to\infty} \int_{-\infty}^{x-\frac{1}{n}} f(t)dt = \lim_{n\to\infty} \int_{-\infty}^{x+\frac{x}{n}} f(t)dt$ for all $x \in \mathbb{R}$

Question [q4]: (2 points) Let X be a random variable with cumulative distribution function (CDF),

$$F(x) = \begin{cases} 1 - (\frac{1}{2})^{[x]+1}, \text{ if } x \ge 0 \\ 0 \text{ otherwise,} \end{cases}$$

where $[x]$ denotes the largest integer not exceeding x. Then which of the following statements is/are **TRUE**?

- ◯ X is a continuous random variable.
- ◯ $P(1 \le X < 4) = \frac{15}{32}$
- ● $P(X > n + m | X \ge n) = P(X > m)$ for positive integers n and m.
- ● $P(1.5 \le X \le 2.5 | 1 \le X < 4) = \frac{2}{7}$

Question [q5]: (2 points) Let $U \sim U(0,\, 1)$ and $Y = \{-\ln(1-U)\}^{\frac{1}{2}}$. Then which of the following statements is/are **TRUE**?

- ● Y^2 and Z have same distribution, where $Z \sim Exp(1)$.
- ◯ $P\left(Y^2 = Z\right) = 1$, where $Z \sim Exp(1)$.
- ● $P\left(-\sqrt{\pi} \le Y < \sqrt{\pi}\right) = 1 - e^{-\pi}$.
- ● The moment generating function of $X = \ln Y$ is

$$M_X(t) = \Gamma\left(\frac{t}{2} + 1\right) \quad \text{for } t > -2.$$