# Probability Theory and Random Processes (MA225)

## LECTURE SLIDES
## Lecture 17

Indian Institute of Technology Guwahati

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# Computing Probability by Conditioning

We have seen that $P\left(X \in A\right) = E\left(I_A(X)\right)$, where $I_A$ is the indicator function of the set $A$. Also, note that $E\left(I_A(X)|Y = y\right) = P\left(X \in A|Y = y\right)$. Therefore, we can write

$$P(A) = P\left(X \in A\right) = E\left(I_A(X)\right) = EE\left(I_A(X)|Y\right)$$
$$= \begin{cases} \sum_{y \in S_Y} P(A|Y = y)P(Y = y) & \text{for } Y \text{ discrete} \\ \int_{-\infty}^{\infty} P(A|Y = y) f_Y(y) dy & \text{for } Y \text{ continuous.} \end{cases}$$

Example 1: Let $X$ and $Y$ be independent CRVs having PDFs $f_X$ and $f_Y$, respectively. Compute $P(X < Y)$.

Example 2: Let $X$ and $Y$ be i.i.d. CRVs having common PDF $f$ and CDF $F(\cdot)$. Then $P(X < Y) = P(X > Y) = 0.5$. And $P(X = Y) = 0$.

Example 3: Suppose $X$ and $Y$ are two independent RVs, either discrete or continuous. What is the distribution of $X + Y$ ?

Def: Let $(X, Y)$ be a random vector. Then

$$E\left(h(X,Y)|(X,Y)\in A\right) = \frac{E(h(X,Y)I_A(X,Y))}{P\left((X,Y)\in A\right)}.$$

Example 4: $X \sim Exp(1)$. Find $E\left(X|X \geq 2\right)$.

Example 5: $(X, Y)$ is uniform on unit square. Find $E\left(X|X+Y>1\right)$.