Probability Theory and Random Processes (MA225)

Lecture Slides

Lecture 16

Indian Institute of Technology Guwahati

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Let (X, Y) be a random vector.

Def: $Var(X|Y) = h(Y)$ where $h(y) = E((X - E(X|Y))^2|Y = y)$
$= E(X^2|Y = y) - (E(X|Y = y))^2$.

Theorem: $Var(X) = E(Var(X|Y)) + Var(E(X|Y))$.

Example 1: Let $X_0, X_1, X_2, \ldots, X_n$ be a sequence of i.i.d. RVs with mean μ and variance σ^2. Let $N \sim Bin(n, p)$, independent of $\{X_i\}$. Define $S = \sum_{i=0}^{N} X_i$. Find $E(S)$ and $Var(S)$.