

# Probability Theory and Random Processes (MA225)

LECTURE SLIDES  
Lecture 28



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# Example

Example 1:

$$P = \begin{bmatrix} 0 & 0 & 1/2 & 1/2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Example 2:

$$P = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 0 & 0 & 1/2 \end{bmatrix}$$

# Example

**Example 3:** Consider a simple random walk:  $S = \{0, \pm 1, \pm 2, \dots\}$ ,  
 $p_{i,i-1} = p = 1 - p_{i,i+1}$ .

- 1 The chain is irreducible.
- 2 If  $p \neq 1/2$ , the state 0 is transient.
- 3 If  $p = 1/2$ , the state 0 is recurrent.

$$[n! \sim n^{n+1/2} e^{-n} \sqrt{2\pi}]$$

# Period

**Def:** The period of a state  $i$  is defined by the greatest common divisor of all integers  $n \geq 1$  for which  $p_{ii}^{(n)} > 0$ , *i.e.*,

$$d(i) = \begin{cases} \gcd \{n \geq 1 : p_{ii}^{(n)} > 0\} & \text{if } \{n \geq 1 : p_{ii}^{(n)} > 0\} \neq \phi \\ 0 & \text{if } \{n \geq 1 : p_{ii}^{(n)} > 0\} = \phi. \end{cases}$$

**Example 4:**  $S = \{0, \pm 1, \pm 2, \dots\}$ .  $p_{i,i+1} = a$ ,  $p_{i,i-1} = b$ ,  $p_{ii} = c$ , where  $a + b + c = 1$ ,  $a > 0$ ,  $b > 0$ ,  $c \geq 0$ .

**Theorem:** If  $i \leftrightarrow j$ , then  $d(i) = d(j)$ .