Probability Theory and Random Processes (MA225)

LECTURE SLIDES
Lecture 30

Indian Institute of Technology Guwahati

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Theorem: Let $\{X_n\}$ be a irreducible MC. Then the following are equivalent.

1. All states are positive recurrent.
2. One state is positive recurrent.
3. $\{X_n\}$ has a stationary distribution.

If any of the above holds, then

1. the stationary distribution is unique and is given by $\pi_i = \frac{1}{E(T_i|X_0=i)}$.
2. for any initial distribution μ and for any $j \in S$, $L_n(j) \to \pi_j$ with porbability 1.
3. for any initial distribution μ and for any $j \in S$, $\frac{1}{n+1}\sum_{k=0}^{n} p_{jj}^{(n)} \to \pi_j$.
4. if $\{X_n\}$ is aperiodic (i.e. $d(i) = 1$), then for any initial distribution μ and for any $j \in S$, $\lim_{n\to\infty} P_\mu\left(X_n = j\right) = \pi_j$.

Some Problems

Example 1: A problem of interest to sociologists is to determine the proportion of society that belongs to upper class, middle class and lower class (in terms of wealth). One possible model would be to assume that transitions between economic classes of successive generations in a family happens according to a MC, i.e., we assume that economic condition of the child depends only on his or her parents economic condition. If such a model is true and the TPM is given by

$$\begin{bmatrix} 0.45 & 0.48 & 0.07 \\ 0.05 & 0.70 & 0.25 \\ 0.01 & 0.50 & 0.49 \end{bmatrix}$$

then in the long run what proportion of the society will be in each class?

$$\pi_0 = 0.07,\ \pi_1 = 0.62,\ \pi_2 = 0.31.$$

Example 2: [A Gambling Model] Consider a gambler who at each play of the game either wins Re. 1 with probability p or losses Re. 1 with probability $1 - p$. Suppose that the gambler quits play either when he goes broke or he attains a fortune of Rs. N. What is the probability that starting with i units of wealth the gambler will go home a winner.