1. Let (X, Y) be a continuous random vector with JPDF $f(\cdot, \cdot)$. Show that X and Y are independent if and only if $f(x, y) = g(x)h(y)$ for all $(x, y) \in \mathbb{R}^2$.   conditional JPDF

2. Let (X, Y) be uniform over the interior of the triangle with vertices $(0, 0), (2, 0)$ and $(1, 2)$. Find $P(X \leq 1, Y \leq 1)$.

3. If X_1 and X_2 are independent random variables each having PDF $2xe^{-x^2} (0 < x < \infty)$, then find the PDF of the random variable $\sqrt{X_1^2 + X_2^2}$.

4. Two numbers are independently chosen at random between 0 and 1. What is the probability that their product is less than a constant $k (0 < k < 1)$?

5. A vertical board is ruled with horizontal parallel lines at constant distance b apart. A needle of length $a (< b)$ is thrown at random on the board. Find the probability that it will intersect one of the lines.

6. Let X_1, X_2, X_3 have the joint PDF

$$f(x_1, x_2, x_3) = \begin{cases} 48x_1x_2x_3 & \text{if } 0 < x_1 < x_2 < x_3 < 1 \\ 0 & \text{otherwise.} \end{cases}$$

Find the marginal distributions of $Y_1 = \frac{X_1}{X_2}$, $Y_2 = \frac{X_2}{X_3}$, and $Y_3 = X_3$.

7. Let X_1, X_2, X_3 be i.i.d. $Exp(1)$ random variables. Find the joint PDF of $Y_1 = \frac{X_1}{X_1+X_2+X_3}$, $Y_2 = \frac{X_2}{X_1+X_2+X_3}$, and $Y_3 = X_1 + X_2 + X_3$. Also find the marginal PDF of Y_1, Y_2, and Y_3.

8. Let X_1, X_2, X_3 be i.i.d. $Exp(1)$ random variables. Find the joint PDF of $Y_1 = \frac{X_1}{X_1+X_2+X_3}$, $Y_2 = \frac{X_1+X_2}{X_1+X_2+X_3}$, and $Y_3 = X_1 + X_2 + X_3$. Also find the marginal PDF of Y_1, Y_2, and Y_3.

9. Let X and Y be two independent random variables having $Gamma(\alpha_1, \beta)$ and $Gamma(\alpha_2, \beta)$ distributions, respectively. Find the PDF of $\frac{X}{X+Y}$.

10. Let X be a random variable of continuous type. The integral part, Y, of X has a $P(\lambda)$ distribution and the fractional part, Z, has a $U(0, 1)$ distribution. Find the CDF of X, assuming that Y and Z are independent. Using the CDF find the PDF of X.

11. Let $X_1, X_2, \ldots, X_n$ be i.i.d. $U(0, 1)$ random variables. Define $X_{(n)} = \max\{X_1, \ldots, X_n\}$ and $X_{(1)} = \min\{X_1, \ldots, X_n\}$. Find the joint and marginal distributions of $X_{(1)}$ and $X_{(n)}$.

12. Let X_1 and X_2 be i.i.d. $P(\lambda)$ random variables. Find the PMF of $X_{(2)} = \max\{X_1, X_2\}$.