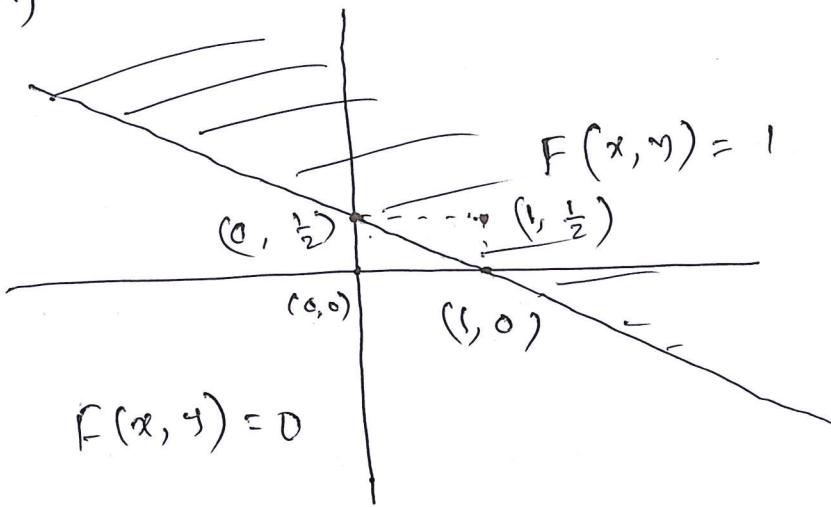


1a)

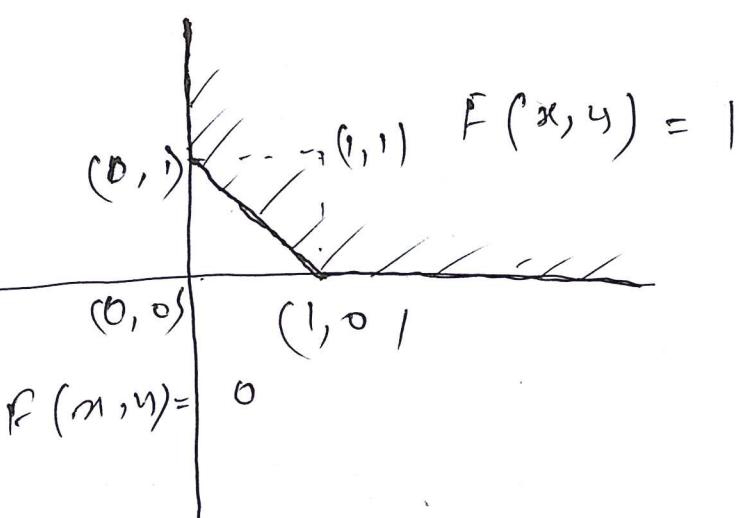


$$\text{Let } a_1 = 0, b_1 = 1, \cancel{a_2 = 0}, b_2 = \frac{1}{2}$$

$$\begin{aligned} \text{Now } F(b_1, b_2) - F(a_1, b_2) - F(b_1, a_2) + F(a_1, a_2) \\ = F(1, \frac{1}{2}) - F(0, \frac{1}{2}) - F(1, 0) + F(0, 0) \\ = 1 - 1 - 1 + 0 = -1 \neq 0. \end{aligned}$$

Thus  $F(x, y)$  is not a distribution function.

b)



$$a_1 = 0, b_1 = 1, a_2 = 0, b_2 = 1$$

$$\begin{aligned} \text{Now } F(b_1, b_2) - F(a_1, b_2) - F(b_1, a_2) + F(a_1, a_2) \\ = F(1, 1) - F(0, 1) - F(1, 0) + F(0, 0) = -1 \neq 0. \end{aligned}$$

Thus  $F(x, y)$  is not a distribution function.

Qa)

$$\begin{aligned}
 F(x, y) &= P(X \leq x, Y \leq y) \leq P(X \leq x) \\
 &\leq P(Y \leq y) \\
 &\leq \min\{P(X \leq x), P(Y \leq y)\} \\
 &= \min\{F_1(x), F_2(y)\} \\
 &= U(x, y)
 \end{aligned}$$

Now  $F(x, y) \geq 0$ .

$$\begin{aligned}
 \text{Also } F(x, y) &= P(X \leq x, Y \leq y) \\
 &= P(X \leq x) + P(Y \leq y) - P(X \leq x \text{ or } Y \leq y) \\
 &\geq F_1(x) + F_2(y) - 1
 \end{aligned}$$

$$\begin{aligned}
 \text{Thus } F(x, y) &\geq \max\{F_1(x) + F_2(y) - 1, 0\} \\
 &= L(x, y).
 \end{aligned}$$

$$\begin{aligned}
 b) \lim_{x \rightarrow \infty} \lim_{y \rightarrow \infty} U(x, y) &= \lim_{x \rightarrow \infty} \lim_{y \rightarrow \infty} \min\{F_1(x), F_2(y)\} \\
 &= \lim_{x \rightarrow \infty} \min\{F_1(x), 1\} \\
 &= \lim_{x \rightarrow \infty} F_1(x) = 1
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow -\infty} U(x, y) &= \lim_{x \rightarrow -\infty} \min\{F_1(x), F_2(y)\} \\
 &= \min\{0, F_2(y)\} = 0
 \end{aligned}$$

$$\text{Sim., } \lim_{y \rightarrow -\infty} U(x, y) = 0.$$

$$\lim_{h \downarrow 0} L(x+h, y) = \lim_{h \downarrow 0} \min \{ F_1(x+h), F_2(y) \}$$

$$= \min \{ F_1(x), F_2(y) \} = L(x, y).$$

Sim. right cont. in  $y$ .

$$\text{Suppose } F_1(a_1) \leq F_1(b_1) \leq F_2(a_2) \leq F_2(b_2) \quad \begin{cases} a_1 < b_1 \\ a_2 < b_2 \end{cases}$$

$$\begin{aligned} \text{Then } U(b_1, b_2) - U(a_1, b_2) - U(b_1, a_2) + U(a_1, a_2) \\ &= \min \{ F_1(b_1), F_2(b_2) \} - \min \{ F_1(a_1), F_2(b_2) \} \\ &\quad - \min \{ F_1(b_1), F_2(a_2) \} + \min \{ F_1(a_1), F_2(a_2) \} \\ &= F_2(b_2) - F_1(a_1) - F_1(b_1) + F_1(a_1) \\ &= 0 \end{aligned}$$

$$\text{Suppose } F_1(a_1) \leq F_2(a_2) \leq F_1(b_1) \leq F_2(b_2)$$

$$\begin{aligned} \text{Then we get, } U(b_1, b_2) - U(a_1, b_2) - U(b_1, a_2) + U(a_1, a_2) \\ &= F_1(b_1) - \cancel{F_1(a_1)} - F_2(a_2) + \cancel{F_1(a_1)} \\ &\geq 0. \end{aligned}$$

Similarly for other cases. (Remember  $F_1(a_1) \leq F_1(b_1)$   
 $\& F_2(a_2) \leq F_2(b_2)$   
by prop. of dist. function).

$$\begin{aligned} \text{Now } \lim_{x \rightarrow \infty} \lim_{y \rightarrow \infty} L(x, y) &= \lim_{x \rightarrow \infty} \lim_{y \rightarrow \infty} \max \{ F_1(x) + F_2(y) - 1, 0 \} \\ &= \lim_{x \rightarrow \infty} \max \{ F_1(x), 0 \} = \lim_{x \rightarrow \infty} F_1(x) = 1. \end{aligned}$$

$$\lim_{y \rightarrow \infty} L(x, y) = \lim_{y \rightarrow -\infty} \max \{ F_1(x) + F_2(y) - 1, 0 \}$$

$$= \cancel{\max} \{ F_1(x) - 1, 0 \} = 0.$$

Sim.  $\lim_{x \rightarrow -\infty} L(x, y) = 0$ .

$$\lim_{h \rightarrow 0} L(x, y+h) = \lim_{h \rightarrow 0} \max \{ F_1(x) + F_2(y+h) - 1, 0 \}$$

$$= \max \{ F_1(x) + F_2(y) - 1, 0 \}$$

$$= L(x, y).$$

Sim. right cont. in  $x$ .

If  $F_1(b_1) + F_2(b_2) < 1$ , trivial  $\begin{cases} a_1 < b_1 \\ a_2 < b_2 \end{cases}$

If  $F_1(a_1) + F_2(a_2) \geq 1$ , trivial.

Suppose  $F_1(b_1) + F_2(b_2) \geq 1$  and

$$F_1(a_1) + F_2(a_2) < 1.$$

Suppose  $F_1(a_1) + F_2(b_2) < 1$

$$F_1(\phi_1) + F_2(a_2) \geq 1.$$

Then  $L(b_1, b_2) - L(a_1, b_2) - L(a_2, b_1) + L(a_1, a_2)$   
 $= \cancel{F_1(b_1)} + F_2(b_2) - \cancel{F_1(a_2)} - \cancel{F_2(a_1)} + \cancel{F_1(a_2)} + F_2(a_1) + 0 + 0$   
 ~~$\cancel{F_1(b_1)} + F_2(b_2) - F_1(a_2) - F_2(a_1) + F_1(a_2) + F_2(a_1) + 0 + 0$~~   
 $\geq 0.$  Sim. consider other cases.

$$c) \lim_{x \rightarrow \infty} L(x, y) = \lim_{x \rightarrow \infty} \max \{ F_1(x) + F_2(y) - 1, 0 \}$$

$$= \max \{ F_1(x), 0 \} = F_1(x).$$

$$\lim_{y \rightarrow \infty} L(x, y) = \lim_{y \rightarrow \infty} \max \{ F_1(x) + F_2(y) - 1, 0 \}$$

$$= \max \{ F_2(y), 0 \} = F_2(y).$$

$$\lim_{x \rightarrow \infty} U(x, y) = \lim_{x \rightarrow \infty} \min \{ F_1(x), F_2(y) \}$$

$$= \min \{ 1, F_2(y) \} = F_2(y).$$

$$\text{Sim. } \lim_{y \rightarrow \infty} U(x, y) = F_1(x)$$

③ Suppose  $F_1(x) \leq F_2(y)$

$$\Rightarrow P(X \leq x) \leq P(X \leq g^{-1}(y))$$

$$\Rightarrow x \leq g^{-1}(y)$$

$$\text{Now } P(X \leq x, Y \leq y) = P(X \leq x, X \leq g^{-1}(y))$$

$$= P(X \leq x) = F_1(x)$$

Suppose  $F_1(x) > F_2(y)$

$$\Rightarrow P(X \leq x) > P(X \leq g^{-1}(y))$$

$$\Rightarrow x > g^{-1}(y). \text{ Thus}$$

$$P(X \leq x, Y \leq y) = P(X \leq g^{-1}(y)) = P(g(X) \leq y)$$

$$= F_2(y).$$

$$b) \text{ If } F_1(x) + F_2(y) < 1 \Rightarrow P(X \leq x) + P(g(x) \leq y) < 1$$

$$\Rightarrow P(X \leq x) + P(X \geq g^{-1}(y)) < 1 \Rightarrow x < g^{-1}(y)$$

$$\text{Then } F_1(x) + F_2(y) \geq 1 \Rightarrow x \geq g^{-1}(y).$$

$$\begin{aligned} \text{Now } P(X \leq x, g(x) \leq y) &= P(X \leq x, X \geq g^{-1}(y)) \\ &= P(g^{-1}(y) \leq X \leq x) = 0 \quad \text{if } F_1(x) + F_2(y) < 1 \\ &= F_1(x) - (1 - P(X \geq g^{-1}(y))) \\ &= F_1(x) + F_2(y) - 1 \quad \text{if } F_1(x) + F_2(y) \geq 1 \end{aligned}$$

$$\begin{aligned} 4a) \quad P(X+Y < 8) &= P(X=4, Y=1) + P(X=4, Y=2) \\ &\quad + P(X=4, Y=3) + P(X=5, Y=1) + P(X=5, Y=2) \\ &\quad + P(X=6, Y=1) \\ &= 0.08 + 0.11 + 0.09 + 0.04 + 0.12 + 0.09 \\ &= 0.53 \end{aligned}$$

$$\begin{aligned} P(X+Y > 7) &= \cancel{P(X+Y \leq 7)} \quad 1 - P(X+Y \leq 8) \\ &= 1 - 0.53 = 0.47 \end{aligned}$$

$$\begin{aligned} P(X+Y \leq 14) &= \cancel{P(X+Y > 14)} \quad 1 - P(X \geq 15) \\ &= 1 - P(X=4, Y=4) - P(X=5, Y=3) \\ &\quad - P(X=5, Y=4) - P(X=6, Y=3) - P(X=6, Y=4) \\ &= 1 - 0.03 - 0.21 - 0.05 - 0.08 - 0.04 \\ &= 0.59 \end{aligned}$$

6)

~~Exponential distribution + Poisson~~

b) Do the calculations.

$$\begin{aligned}
 5. \quad P(X=x) &= \frac{(1-\theta_1-\theta_2)^K \theta_1^x \theta_2^{x+k} \sum_{y=0}^{\infty} \frac{(x+k-1+y)!}{y! (x+k-1)!} \theta_2^y (1-\theta_2)^{y+k}}{x! (k-1)! (1-\theta_2)^{x+k}} \\
 &= \frac{(x+k-1)!}{x! (k-1)!} \left( \frac{\theta_1}{1-\theta_2} \right)^x \left( 1 - \frac{\theta_1}{1-\theta_2} \right)^K
 \end{aligned}$$

for  $x = 0, 1, 2, \dots$

Thus  $X \sim \text{NB}(K, 1 - \frac{\theta_1}{1-\theta_2})$

$$\begin{aligned}
 P(Y=y) &= \frac{(1-\theta_1-\theta_2)^K \theta_2^y (y+k-1)!}{y! (k-1)! (1-\theta_2)^{y+k}} \sum_{x=0}^{\infty} \frac{(y+k-1+x)!}{x! (y+k-1)!} \theta_2^x (1-\theta_2)^{y+1} \\
 &= \binom{y+k-1}{k-1} \left( \frac{\theta_2}{1-\theta_2} \right)^y \left( 1 - \frac{\theta_2}{1-\theta_2} \right)^K
 \end{aligned}$$

for  $y = 0, 1, 2, \dots$

Thus  $Y \sim \text{NB}(K, 1 - \frac{\theta_2}{1-\theta_2})$

6. a) The possible configurations are

$$3, 0, 0 \quad N=1 \quad \checkmark$$

$$2, 1, 0 \quad N=2$$

$$1, 1, 1 \quad N=3$$

$$0, 3, 0 \quad N=1 \quad \checkmark$$

$$0, 2, 1 \quad N=2 \quad \checkmark$$

$$1, 2, 0 \quad N=2$$

$$0, 0, 3 \quad N=1 \quad \checkmark$$

$$0, 1, 2 \quad N=2 \quad \checkmark$$

$$1, 0, 2 \quad N=2$$

$$2, 0, 1 \quad N=2$$

a)  ~~$P(X_1=0, X_2=0)$~~   $P(N=1, X_1=0) = \frac{1}{5}$

$$P(N=1, X_1=3) = \frac{1}{10}, \quad P(N=2, X_1=2) = \frac{1}{5}$$

$$P(N=2, X_1=0) = \frac{1}{5} \quad P(N=3, X_1=1) = \frac{1}{10}$$

$$P(N=2, X_1=1) = \frac{1}{5}.$$

b)  $P(X_1=0, X_2=0) = \frac{1}{10} \quad P(X_1=1, X_2=2) = \frac{1}{10}$

$P(X_1=0, X_2=1) = \frac{1}{10} \quad P(X_1=2, X_2=0) = \frac{1}{10}$

$P(X_1=0, X_2=2) = \frac{1}{10} \quad P(X_1=2, X_2=1) = \frac{1}{10}$

$P(X_1=0, X_2=3) = \frac{1}{10} \quad P(X_1=3, X_2=0) = \frac{1}{10}$

$P(X_1=1, X_2=0) = \frac{1}{10}$

$P(X_1=1, X_2=1) = \frac{1}{10}$

$$\begin{array}{ll}
 \text{c)} & P(N=1) = \frac{3}{10} \quad P(X_2=0) = \frac{4}{10} \\
 & P(N=2) = \frac{6}{10} \quad P(X_2=1) = \frac{3}{10} \\
 & P(N=3) = \frac{1}{10} \quad P(X_2=2) = \frac{2}{10} \\
 & \qquad \qquad \qquad P(X_2=3) = \frac{1}{10} .
 \end{array}$$

$$\begin{array}{l}
 \text{d)} \quad P(X_1=0) = \sum_{k=0}^3 P(X_1=0, X_2=k) = \frac{4}{10} \\
 P(X_1=1) = \sum_{k=0}^2 P(X_1=0, X_2=k) = \frac{3}{10} \\
 P(X_1=2) = \sum_{k=0}^1 P(X_1=0, X_2=k) = \frac{2}{10} \\
 P(X_1=3) = P(X_1=3, X_2=0) = \frac{1}{10} .
 \end{array}$$

Q7

$$E(\bar{x}) = \frac{1}{n} \sum_{i=1}^n E(x_i) = \mu.$$

$$\text{Var}(\bar{x}) = \frac{1}{n^2} \text{Var}\left(\sum_{i=1}^n x_i\right)$$

$$= \frac{1}{n^2} \left[ \sum_{i=1}^n \text{Var}(x_i) + \sum_{i=1}^n \sum_{j=1, j \neq i}^n \text{Cov}(x_i, x_j) \right]$$

$$= \frac{1}{n^2} \sum_{i=1}^n \sigma^2$$

$$= \sigma^2/n.$$

$$\text{Cov}(\bar{x}, x_i - \bar{x}) = \text{Cov}(\bar{x}, x_i) - \text{Var}(\bar{x})$$

$$= \frac{1}{n} \text{Cov}(x_i, x_i) - \text{Var}(\bar{x})$$

$$= \frac{\sigma^2}{n} - \frac{\sigma^2}{n}$$

$$= 0.$$

□

Q8  $P(x=x) = P(x=x|x_1=1)P(x_1=1) + P(x=x|x_1=0)P(x_1=0).$

Now for  $x=1, 2, \dots, n,$

$$P(x=x|x_1=1) = \binom{n-1}{x-1} p^{x-1} (1-p)^{n-x}$$

$$P(x=x|x_1=0) = 0$$

for  $x = 1, 2, \dots, n-1,$

$$P(x=x|x_1=0) = 1.$$

$$P(x=x) = \binom{n-1}{x-1} p^x (1-p)^{n-x}$$

Hence for  $x = 1, 2, \dots, n-1,$

$$P(x=x) = p^m + 1-p.$$

As  $\sum_{x=1}^n P(x=x) = 1,$  the PMF of  $x$  is

$$f_x(x) = \begin{cases} \binom{n-1}{x-1} p^x (1-p)^{n-x} & \text{if } x = 1, 2, \dots, n-1 \\ p^n + 1 - p & \text{if } x = n \\ 0 & \text{o.w.} \end{cases}$$

[Q9] The marginal PDF of  $X$  is

$$\begin{aligned} f_x(x) &= \int_{-\infty}^{+\infty} f_{x,y}(x,y) dy \\ &= \begin{cases} \frac{\Gamma(\theta_1 + \theta_2 + \theta_3)}{\Gamma(\theta_1)\Gamma(\theta_2)\Gamma(\theta_3)} x^{\theta_1-1} (1-x)^{\theta_3-1} \int_0^{1-x} y^{\theta_2-1} \left(1 - \frac{y}{1-x}\right)^{\theta_3-1} dy & \text{if } 0 < x < 1 \\ 0 & \text{o.w.} \end{cases} \\ &= \frac{\Gamma(\theta_1 + \theta_2 + \theta_3)}{\Gamma(\theta_1)\Gamma(\theta_2)\Gamma(\theta_3)} x^{\theta_1-1} (1-x)^{\theta_3-1 + \theta_2-1} \int_0^1 y^{\theta_2-1} (1-y)^{\theta_3-1} dy \\ &\quad \text{if } 0 < x < 1. \\ &= \frac{\Gamma(\theta_1 + \theta_2 + \theta_3)}{\Gamma(\theta_1)\Gamma(\theta_2)\Gamma(\theta_3)} x^{\theta_1-1} (1-x)^{\theta_2 + \theta_3 - 1} \frac{\Gamma(\theta_2)\Gamma(\theta_3)}{\Gamma(\theta_2 + \theta_3)} \\ &\quad \text{if } 0 < x < 1 \\ &= \begin{cases} \frac{\Gamma(\theta_1 + \theta_2 + \theta_3)}{\Gamma(\theta_1)\Gamma(\theta_2 + \theta_3)} x^{\theta_1-1} (1-x)^{\theta_2 + \theta_3 - 1} & \text{if } 0 < x < 1 \\ 0 & \text{o.w.} \end{cases} \end{aligned}$$

Similarly, the marginal PDF of  $X$  is

$$f_x(x) = \begin{cases} \frac{\Gamma(\theta_1 + \theta_2 + \theta_3)}{\Gamma(\theta_2)\Gamma(\theta_1 + \theta_3)} y^{\theta_2-1} (1-y)^{\theta_1+\theta_3-1} & \text{if } 0 < y < 1 \\ 0 & \text{o.w.} \end{cases} \quad \square$$

Q10(a) The marginal PDF of  $X$  is

$$\begin{aligned} f_x(x) &= \int_0^x y dy && \text{if } 0 < x < 1 \\ &= \begin{cases} 2x & \text{if } 0 < x < 1 \\ 0 & \text{o.w.} \end{cases} \end{aligned}$$

The marginal PDF of  $Y$  is

$$f_y(y) = \begin{cases} 2y & \text{if } 0 < y < 1 \\ 0 & \text{o.w.} \end{cases} \quad \square$$

Q10(b) As  $f_{x,y}(x,y) = f_x(x)f_y(y)$  for all  $(x,y) \in \mathbb{R}^2$ ,  $x$  &  $y$  are independent.  $\square$

Q10(c)  $P(0 < X < 0.5, 0.25 < Y < 1)$

$$\begin{aligned} &= P(0 < X < 0.5) P(0.25 < Y < 1) \\ &= \left(\frac{1}{2}\right)^2 \times \left\{1 - \left(\frac{1}{4}\right)^2\right\} = \frac{15}{64}. \end{aligned} \quad \square$$

$$\begin{aligned}
 P(X+Y \leq 1) &= \iint_{x+y \leq 1} f_{x,y}(x,y) dx dy \\
 &= \int_0^1 \int_0^{1-y} 4xy dx dy \\
 &= 4 \int_0^1 y \int_0^{1-y} x dx dy \\
 &= \frac{4}{2} \int_0^1 y (1-y)^2 dy \\
 &= \frac{4}{2} \cdot B(2,3) \\
 &= \frac{1}{2} \cdot \frac{\Gamma(2)\Gamma(3)}{\Gamma(5)} = \frac{2}{2 \cdot 24} = \frac{1}{24} \quad \square.
 \end{aligned}$$

[Q.11]

[Q.11(a)] The marginal PDF of  $x_1$  is  $(x_1, x_2)$  is

$$f_{x_1}(x_1) = \frac{1}{(2\pi)^{3/2}} \left[ e^{-\frac{1}{2}x_1^2} \right]_{-\infty}^{+\infty}$$

$$\begin{aligned}
 f_{x_1, x_2}(x_1, x_2) &= \frac{1}{(2\pi)^{3/2}} \left[ e^{-\frac{1}{2}(x_1^2 + x_2^2)} \int_{-\infty}^{+\infty} e^{-\frac{1}{2}x_3^2} dx_3 + \right. \\
 &\quad \left. x_1 x_2 e^{-\frac{1}{2}(x_1^2 + x_2^2)} \int_{-\infty}^{+\infty} x_3 e^{-\frac{1}{2}x_3^2} dx_3 \right]
 \end{aligned}$$

$$= \frac{1}{(2\pi)} e^{-\frac{1}{2}(x_1^2 + x_2^2)} \quad (x_1, x_2) \in \mathbb{R}^2,$$

The marginal PDF of  $x_1$  is

$$f_{x_1}(x_1) = \frac{1}{(2\pi)^{1/2}} e^{-\frac{x_1^2}{2}} \quad x_1 \in \mathbb{R}$$

The marginal PDF of  $x_2$  is

$$f_{x_2}(x_2) = \frac{1}{(2\pi)^{1/2}} e^{-\frac{x_2^2}{2}} \quad \text{if } x_2 \in \mathbb{R}$$

The marginal PDF of  $x_3$  is

$$f_{x_3}(x_3) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x_3^2}{2}} \quad \text{if } x_3 \in \mathbb{R}.$$

Hence for any  $(x_1, x_2, x_3) \neq (0, 0, 0)$ ,

$$f_{x_1, x_2, x_3}(x_1, x_2, x_3) \neq f_{x_1}(x_1) f_{x_2}(x_2) f_{x_3}(x_3).$$

$\Rightarrow x_1, x_2 \& x_3$  are not independent.  $\square$

**[Q11(b)]** As  $f_{x_1, x_2}(x_1, x_2) = f_{x_1}(x_1) f_{x_2}(x_2) \quad \forall (x_1, x_2) \in \mathbb{R}^2$ ,

$x_1 \& x_2$  are independent.

Similarly  $x_1 \& x_3$  are independent.

Similarly  $x_2 \& x_3$  are independent.  $\square$

$$\boxed{\text{[Q12]}} \quad \text{Corr}\left(\frac{x}{3} + \frac{2y}{3}, \frac{2x}{3} + \frac{y}{3}\right) = \frac{\text{Cov}\left(\frac{x}{3} + \frac{2y}{3}, \frac{2x}{3} + \frac{y}{3}\right)}{\sqrt{\text{Var}\left(\frac{x}{3} + \frac{2y}{3}\right)} \sqrt{\text{Var}\left(\frac{2x}{3} + \frac{y}{3}\right)}}$$

$$\text{Now } \text{Cov}\left(\frac{x}{3} + \frac{2y}{3}, \frac{2x}{3} + \frac{y}{3}\right) = \frac{2}{9} \text{Cov}(x, x) + \frac{1}{3} \text{Cov}(x, y) + \frac{1}{9} \text{Cov}(y, y) \\ + \frac{2}{9} \text{Var}(y) \quad \#$$

$$\text{Var}(x) = 2, \quad \text{Var}(y) = 2$$

$$\text{Corr}(x, y) = \frac{1}{3} \Rightarrow \text{Cov}(x, y) = \frac{1}{3} \times 2 = \frac{2}{3}.$$

$$\text{From } \textcircled{4}, \text{ Cov}\left(\frac{x}{3} + \frac{2y}{3}, \frac{2x}{3} + \frac{y}{3}\right)$$

$$= \frac{4}{9} + \frac{2}{27} + \frac{8}{27} + \frac{1}{9}$$

$$= \frac{34}{27}.$$

$$\text{Var}\left(\frac{x}{3} + \frac{2y}{3}\right) = \frac{1}{9} \text{Var}(x) + \frac{4}{9} \text{Cov}(x, y) + \frac{4}{9} \text{Var}(y)$$

$$= \frac{2}{9} + \frac{18}{27} + \frac{8}{9}$$

$$= \frac{38}{27}.$$

$$\text{Var}\left(\frac{2x}{3} + \frac{y}{3}\right) = \frac{4}{9} \text{Var}(x) + \frac{4}{9} \text{Cov}(x, y) + \frac{1}{9} \text{Var}(y)$$

$$= \frac{8}{9} + \frac{18}{27} + \frac{2}{9}$$

$$= \frac{38}{27}$$

Required correlation is  $\textcircled{4} \cdot \frac{17}{19}$ .

□

Q13 The JPDF of  $(x, y)$  is

$$f_{x,y}(x, y) = \begin{cases} 2 & \text{if } 0 < x < y < 1 \\ 0 & \text{O.W.} \end{cases}$$

$$\text{Cov}(x, y) = E(xy) - E(x)E(y)$$

$$E(xy) = 2 \int_0^1 \int_0^y xy dx dy = 2 \int_0^1 y^3 dy = \frac{1}{4}$$

$$E(x) = 2 \int_0^1 \int_0^y x dx dy = \frac{1}{3}, \quad E(y) = 2 \int_0^1 \int_y^1 y dx dy = \frac{2}{3}$$

$$\text{Cov}(x, y) = \frac{1}{36}$$

□.