Vectors and Matrices: A Gentle Introduction with Python

Introduction

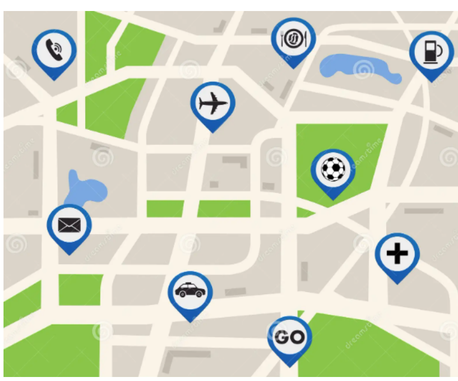
Welcome to the world of Vectors and Matrices! This book is designed to gently introduce you to these powerful mathematical concepts and how they're used in Python. You might be surprised to know that you've already encountered them in your daily life.

Location, QR Codes, and Images – Vectors and Matrices in the Real World

Imagine sharing your location with a friend. You likely use Google Maps or a similar app. When you share, you're essentially sending your GPS coordinates. These coordinates look something like this: 13.03° N, 77.56° E.

What is this? It's a set of two numbers that represent your position on the Earth's surface. These two numbers are your latitude and longitude. If we ignore the ”N” and ”E” and just focus on the numerical values, we have $(13.03, 77.56)$. This is a *point* defined by two values. It can be considered as a location data.

This point describes our location north of the Equator and east of the Prime Meridian. This simple example shows how two numbers can represent a location on a two-dimensional surface.



*GPS Location Marker on a Map

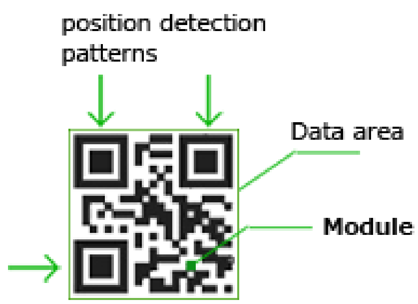
Now think about student data stored in a spreadsheet. Each row might represent a student, and each column a piece of information about them, like their class and percentage. For example:

Student	Class	Percentage
John	10	85
Jane	11	92

Each row of this data effectively represents a student using a series of values. These values define a data of that student.

Consider QR codes, which are everywhere today, linking us to websites or enabling payments. If we zoom in, we see that a QR code is made of small squares, each of which is either black or white.

Let's say we have a small QR code made up of 30 rows and 30 columns. This gives us 900 individual squares. Each square can be either black (represented by 0) or white (represented by 1). This QR code essentially stores information in a grid.



*Close-up of a QR Code

Finally, think about digital images. You've surely seen a lot of them. Images are composed of *pixels*, which are the smallest units of an image. Each pixel has a color value. So, an image can be represented as a grid of pixels, with each pixel having a specific color.

These examples demonstrate that data in the real world can be organized in different ways. Sometimes it is enough to have only one list of values to represent something. Sometimes you need data in the form of grids. These "lists" and "grids" are the basis of *vectors* and *matrices*, which we'll explore further in this book.

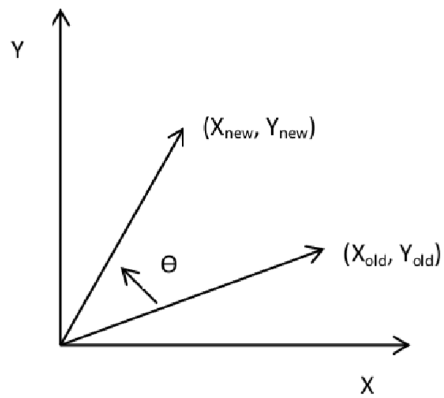
1 Vectors – One-Dimensional Data

A **vector** is essentially a list of numbers. Think of it as a way to represent a point in space. Let's start with a two-dimensional space, like a piece of graph paper.

Imagine a coordinate system with an X-axis and a Y-axis. The point where they meet is called the *origin*, represented as $(0, 0)$. Any point on this paper can be described by its X and Y coordinates.

For example, the point $(3, 4)$ means that you move 3 units along the X-axis and 4 units along the Y-axis from the origin.

Now, connect the origin $(0, 0)$ to the point $(3, 4)$ with a straight line. This line, with a direction from the origin to the point, is a **vector**.



*A Vector on a 2D Coordinate System

Mathematically, we represent a vector using its coordinates, often written in a column format:

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

This vector represents a direction and a distance from the origin to the point $(3, 4)$.

The point $(0, 0)$ itself can also be represented as a vector called the **zero vector**. A zero vector has no length and no specific direction.

Vector Operations

So, what can you *do* with vectors? You can perform arithmetic operations on them, similar to how you work with regular numbers.

1.1 Scalar Multiplication

Scalar multiplication means multiplying a vector by a single number (a scalar). This changes the length of the vector, scaling it up or down.

For instance, if we have a vector $\mathbf{v} = (2, 3)$ and multiply it by the scalar 2, we get a new vector:

$$2 \cdot \mathbf{v} = (2 \cdot 2, 2 \cdot 3) = (4, 6)$$

This new vector $(4, 6)$ is twice as long as the original vector $(2, 3)$ and points in the same direction. The point simply went further away from the origin.

Multiplying by a scalar less than 1 makes the vector shorter. Multiplying by a negative scalar reverses the direction of the vector.

1.2 Vector Addition

Vector addition means adding two vectors together. This combines their directions and distances.

For instance, if we have vectors $\mathbf{v} = (2, 3)$ and $\mathbf{w} = (6, 4)$, adding them together is done by adding the corresponding elements:

$$\mathbf{v} + \mathbf{w} = (2 + 6, 3 + 4) = (8, 7)$$

The resulting vector $(8, 7)$ represents a new point that you would reach by following the direction and distance of both original vectors in succession.

Geometrically, you can visualize this by placing the tail of vector $\mathbf{w}$ at the head of vector $\mathbf{v}$. The resulting vector $\mathbf{v} + \mathbf{w}$ is the vector from the origin to the head of $\mathbf{w}$.

2 Matrices – Two-Dimensional Data

A **matrix** is a grid of numbers arranged in rows and columns. Think of it as a table or a spreadsheet.

For example, a matrix $\mathbf{M}$ could look like this:

$$\mathbf{M} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

This is a 2×2 matrix because it has 2 rows and 2 columns. The numbers inside the matrix are called its *elements*.

Transforming Vectors with Matrices

One of the most powerful things you can do with matrices is to *transform* vectors. This means applying a matrix to a vector to change its direction or length.

To multiply a matrix by a vector, the number of columns in the matrix must equal the number of rows in the vector.

For example, let's multiply the matrix $\mathbf{M}$ from above by the vector $\mathbf{v} = (2, 3)$:

$$\mathbf{M} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad \mathbf{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

The result $\mathbf{r}$ is a new vector and can be found as follows:

$$\mathbf{r} = \mathbf{M} \cdot \mathbf{v} = \begin{bmatrix} 1 \cdot 2 + 2 \cdot 3 \\ 3 \cdot 2 + 4 \cdot 3 \end{bmatrix} = \begin{bmatrix} 8 \\ 18 \end{bmatrix}$$

The resulting vector is $(8, 18)$. The original vector $(2, 3)$ has been transformed into a new vector $(8, 18)$ by applying the matrix $\mathbf{M}$.

Multiplying with a matrix is also called **linear transformation**. The matrix $\mathbf{M}$ is known as **transformation matrix**.

3 Linear Transformations

Different matrices cause different transformations. Applying the *identity matrix*, for example, will result in the same vector.

Identity matrix:

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

In general, in the transformation matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

- The value of a controls scaling along the X-axis.
- The value of d controls scaling along the Y-axis.
- The value of b controls rotation with respect to the Y-axis.
- The value of c controls rotation with respect to the X-axis.

Different values for these parameters will result in different transformations of the original vector.

Why This Matters?

Vectors and matrices are fundamental building blocks in many fields, including:

- **Computer graphics**: Representing images, animations, and 3D models.
- **Machine learning**: Storing data, performing calculations, and building models.
- **Physics**: Modeling motion, forces, and other physical phenomena.