Vectors and Matrices

Q1. Understanding Vector Representation

A vector in two-dimensional space is represented as $v = (x, y)$. Which of the following statements is correct?

- **A)** The vector represents a point in space but does not have a direction.
- **B)** The vector is simply a number and cannot be visualized geometrically.
- **C)** The vector represents both a direction and a magnitude from the origin.
- **D)** The vector's length is always equal to the sum of its coordinates.

Approach:

1. **Define a Vector**: A vector represents **both magnitude and direction**, not just a point.
2. **Understand Magnitude Calculation**: The length of a vector is given by $\sqrt{x^2 + y^2}$, not simply $x + y$.
3. **Check Geometry**: Vectors have a geometric representation and direction, unlike scalars.

Q2. Scalar Multiplication Effect

If a vector $v = (4, -3)$ is multiplied by a scalar $k = -2$, what will be the new vector?

- **A)** $(2, -1.5)$
- **B)** $(-8, 6)$
- **C)** $(8, -6)$
- **D)** $(-2, 1.5)$

Approach:

1. **Recall Scalar Multiplication Rule**: Each component of the vector gets multiplied by the scalar.
2. **Perform Multiplication**: Multiply both x and y by -2.
3. **Check the Signs**: Ensure correct signs for both components.

Q3. Vector Addition

If two vectors are given as $v = (3, 5)$ and $w = (-2, 4)$, what is the result of $v + w$?

- **A)** $(5, 9)$
- **B)** $(1, 9)$
- **C)** $(-1, -1)$
- **D)** $(6, 1)$

Approach:

1. **Use the Component-wise Rule**: Add corresponding components of both vectors.
2. **Compute Individually**: $v_1 + w_1, v_2 + w_2$.
3. **Compare Options**: Check if the sum matches any answer choice.

Q4. Matrix-Vector Multiplication

Given the matrix:

$$M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

and vector:

$$v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

What is the resulting transformed vector $M \cdot v$?

- **A)** $(5, 11)$
- **B)** $(8, 18)$
- **C)** $(7, 10)$
- **D)** $(6, 15)$

Approach:

1. **Apply Matrix-Vector Multiplication**: Multiply each row of M by v using the **dot product**.

2. **Compute Each Component**: Use $(a_1, a_2) \cdot (x, y) = a_1x + a_2y$.
3. **Verify Computation**: Compare results with answer choices.

Q5. Properties of the Identity Matrix

Which of the following matrices will not change a vector when multiplied with it?

- **A)**

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- **B)**

$$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

- **C)**

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- **D)**

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

Approach:

1. **Recall Identity Matrix Property**: The **identity matrix** does not alter a vector ($I \cdot v = v$).
2. **Compare with Other Matrices**: Any matrix with non-1 diagonal elements modifies the vector.
3. **Check Options**: Identify the matrix that satisfies the identity property.

Q6. Linear Transformation Effect

A transformation matrix:

$$T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

is applied to a vector. What does this transformation do to the vector?

- **A)** Reflects it across the X-axis
- **B)** Reflects it across the Y-axis
- **C)** Rotates it 90° counterclockwise
- **D)** Rotates it 180°

Approach:

1. **Analyze the Effect**: Transformation matrices perform specific operations (rotation, reflection).
2. **Apply Transformation to a Sample Vector**:
   - Try $(1, 0)$ and $(0, 1)$ to see what happens.
3. **Recognize Rotation Pattern**:
   - A 90° counterclockwise rotation swaps and negates components ($(x, y) \to (-y, x)$).

Probability in AI

Q1. Sample Space for Rolling a Six-Sided Die

What is the sample space when rolling a six-sided die?

- **A)** {1, 2, 3, 4, 5, 6, 7}
- **B)** {1, 2, 3, 4, 5, 6}
- **C)** {1, 2, 3, 4, 5}
- **D)** {2, 4, 6}

Approach:

1. **Define Sample Space**: The sample space contains all possible outcomes.
2. **Identify Valid Options**: A fair six-sided die has **six** possible outcomes.
3. **Check Incorrect Answers**:
   - Any option missing numbers or adding extra values is incorrect.

Q2. Cromwell’s Rule

According to Cromwell’s Rule, which of the following probabilities is acceptable?

- **A)** 0%
- **B)** 100%
- **C)** 99.95%
- **D)** Both A and B

Approach:

1. **Understand Cromwell’s Rule**: Avoid assigning absolute 0% or 100% probabilities unless **logically certain**.
2. **Interpret Acceptable Probabilities**: Probabilities should generally remain **between** 0% and 100%.

Q3. Independent Events

Which of the following is an example of independent events?

- **A)** Drawing two cards from a deck without replacement
- **B)** Flipping a coin twice
- **C)** Drawing two cards from a deck with replacement
- **D)** Both B and C

Approach:

1. **Define Independent Events**: The outcome of one event **must not** affect the other.
2. **Examine Each Scenario**:
    - **Without replacement** → Dependent.
    - **With replacement** → Independent.
    - **Coin flips** → Always independent.

Q4. Probability of Drawing Two Red Cards (Without Replacement)

What is the probability of drawing two red cards in succession from a standard deck of 52 cards without replacement?

- **A)** 25/102
- **B)** 1/4
- **C)** 1/2
- **D)** 25/51

Approach:

1. **Total Red Cards in a Deck**: A standard deck has **26 red cards** (hearts and diamonds).
2. **Probability of First Red Card**: Since there are 26 red cards in a deck of 52, the probability is **26/52** (which simplifies to **1/2**).
3. **Probability of Second Red Card**: Since one red card has already been drawn, only **25 red cards remain out of 51**.
4. **Multiply Probabilities**: Since these events are **dependent**, compute the combined probability:

$$P(\text{red first}) \times P(\text{red second} \mid \text{red first})$$

5. **Compare with Answer Choices**: The computed probability should match one of the provided options.

Q5. Rule for Probability of Either Event A or Event B Occurring

Which rule is used to calculate the probability of either event A or event B occurring when the events may overlap?

- **A)** Multiplication Rule
- **B)** Addition Rule
- **C)** Subtraction Rule
- **D)** Division Rule

Approach:

1. **Recognize the Scenario**: The question asks about calculating the probability of **either event A or event B happening**.

2. **Recall Addition Rule**: The probability of **A or B** happening is:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

3. **Check for Overlap**: If events **can overlap**, we **subtract** the probability of both happening to avoid double-counting.

Q6. Sample Space in Probability Modeling for a Self-Driving Car

In probability modeling for a self-driving car, what best represents the sample space?

- **A)** All possible driving scenarios, including road conditions and obstacles.
- **B)** The car's speed and direction.
- **C)** The number of passengers in the car.
- **D)** The date and time of driving.

Approach:

1. **Define Sample Space**: In probability theory, the sample space consists of **all possible outcomes** of an experiment.
2. **Consider a Self-Driving Car**:
   - What are the **possible random events** it must handle?
   - Speed, passengers, or time might affect decisions but **do not fully describe** all possible driving conditions.
3. **Choose the Most Comprehensive Option**:
   - **Road conditions, traffic, and obstacles** define the uncertainty and decision-making scenarios for the car.

Q7. Probability of Getting Heads Twice in a Row

What is the probability of flipping a coin twice and getting heads both times?

- **A)** 1/4
- **B)** 1/2
- **C)** 1/8
- **D)** 1/16

Approach:

1. **Identify Probability per Flip**: A fair coin has a **1/2 probability** of landing on heads.
2. **Independent Events**: The result of one flip **does not affect** the next.
3. **Multiply Probabilities**: Since each flip is independent, use:

$$P(HH) = P(H) \times P(H)$$

4. **Compare Answer Choices**: Ensure the computed probability matches one of the given options.

Q8. Applications of Probability in AI

Which of the following is an application of probability in AI?

- **A)** Bayesian Networks
- **B)** Markov Chains
- **C)** Monte Carlo Simulations
- **D)** All of the above

Approach:

1. **Recall AI Techniques Using Probability**: Many AI models rely on probability for **uncertainty modeling, predictions, and decision-making**.
2. **Break Down Each Option**:
   - **Bayesian Networks** → Used for probabilistic reasoning (e.g., medical diagnosis).
   - **Markov Chains** → Model sequences where future states depend only on the current state (e.g., weather prediction, language modeling).
   - **Monte Carlo Simulations** → Used in reinforcement learning and risk assessment.
3. **Consider If More Than One Applies**: Since **all three** use probability, the correct answer should reflect that.

Q9. Purpose of Character Frequency Analysis in Text Files

What is the primary purpose of using character frequency analysis in text files?

- **A)** To predict word sequences
- **B)** To design efficient encoding schemes

- **C)** To translate languages
- **D)** To simulate complex systems

Approach:

1. **Understand Character Frequency Analysis**: This technique counts how often each **character** appears in a text.
2. **Compare with Common Applications**:
    - **Efficient Encoding**: Used in **data compression** (e.g., **Huffman coding**) to assign shorter codes to frequently used characters.
    - **Language Modeling**: While frequency analysis **can help** in language models, it is **not primarily used** for predicting sequences.