Pre-Session Notes

Topic: Regularization – The Secret to Taming Overfitting

1. Introduction to Regularization

What is Overfitting?

Overfitting occurs when a model performs well on training data but poorly on unseen test data. This happens because the model learns the noise in the training data instead of the general patterns.

Regularization – The Cure:

Regularization is a technique used to reduce overfitting by discouraging complex models. It does this by penalizing large coefficients in the model's loss function.

Key Idea:

Regularization adds a penalty term to the original cost function (like Mean Squared Error) to restrict the magnitude of model parameters (weights).

New Cost = Original Loss + Penalty (Regularization term)

2. Lasso (L1 Regularization)

Definition:

Lasso (Least Absolute Shrinkage and Selection Operator) adds a penalty equal to the absolute value of the coefficients (L1 norm).

Cost Function:

 $J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|J(\theta)=MSE+\lambda\sum_{j=1}^{n}|\theta_{j}|$

- Encourages sparsity: Some coefficients become exactly zero.
- Useful for feature selection.
- Tends to create simpler models by eliminating unimportant features.

Example Use Case:

Useful when you believe many features are irrelevant and should be ignored.

2 3. Ridge (L2 Regularization)

Definition:

Ridge Regression adds a penalty equal to the square of the magnitude of the coefficients (L2 norm).

Cost Function:

J(θ)=MSE+λ∑j=1nθj2J(\theta) = \text{MSE} + \lambda \sum_{j=1}^{n} \theta_j^2
Key Characteristics:

- Shrinks coefficients toward zero, but never exactly zero.
- Keeps all features, but reduces their impact.
- Stabilizes the model by avoiding large weights.

Example Use Case:

Effective when you suspect all features are useful but want to prevent overfitting.

4. Polynomial Regression and Regularization

Why it Matters:

Polynomial regression increases model complexity by adding higher-degree terms (e.g., x2x^2, x3x^3). While it may improve training accuracy, it is highly prone to overfitting.

Solution:

Apply L1 or L2 regularization to polynomial regression to:

- Control coefficient growth of high-degree terms.
- Keep the model generalizable to test data.

Example:

A degree-10 polynomial may fit the training set perfectly but can oscillate wildly on test data. Regularization helps tame this.

5. Choosing the Right Regularization Type

Criteria	Lasso (L1)	Ridge (L2)
Feature selection	✓ Good (sets some weights to 0)	X Keeps all features
Works well when	Many features are irrelevant	All features are relevant
Solution	Sparse	Smooth
Optimization	Less stable due to absolute value	Stable (differentiable)

Elastic Net:

A combination of L1 and L2:

 $J(\theta)=MSE+\lambda1\sum|\theta j|+\lambda2\sum\theta j2J(\theta)= \text{1 in $J(\theta)=MSE} + \lambda2\sum\theta j2J(\theta)= \text{1 in $J(\theta)=MSE} + \lambda2E_\theta + \lambda2E_\theta$

Use when you want both sparsity and regularization stability.

P Summary

- Regularization is essential to prevent overfitting.
- L1 (Lasso) leads to simpler models by feature elimination.
- L2 (Ridge) retains all features but keeps them in check.
- Regularization is especially important when using complex models, like polynomial regression.
- Choice depends on your dataset characteristics and goals.