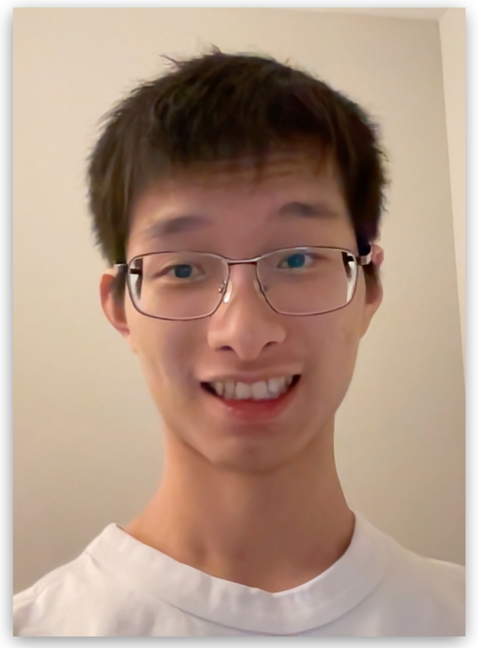
Poseidon: Automatic Profile-Guided Optimizations of Floating-Point Programs in Compilers



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LLVM Developers' Meeting 2024, October 24, 2024

Numerical Analysis Primer

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- Precision changes are not always the answer.
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  - **FP32 Reassociated:**
    $(1.0 \times 10^8 + 1.0) - 1.0 \times 10^8 = (1.0 \times 10^8 - 1.0 \times 10^8) + 1.0 = 1.0$

Existing Approaches

- Precision Tuning
  - `F32` $\rightarrow$ `BF16, F16, F64, F128, …`
- Floating-Point Expression Rewrites
  - $\sqrt{x+1} - \sqrt{x} \rightarrow \dfrac{1}{\sqrt{x+1} + \sqrt{x}}$
- Math Library Functions
  - $\sin(x) \approx x - \dfrac{x^3}{3!} + \dfrac{x^5}{5!}$

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All require human intervention to determine if they are valid, let alone performant!

Can we automate existing FP optimization techniques in compilers?

How much performance are we sacrificing due to *poor* choices of FP precision and expressions?

Poseidon

- An end-to-end system which automatically transforms floating-point expressions in programs to maximize program accuracy subject to a computation cost budget
- Implemented as a PGO-like two-phase compilation within LLVM
  - First Compilation: **Instrumentation Pass**
  - Second Compilation
    - **FP Subgraph & Expression Transformations**
    - **Dynamic Programming Solver**

Floats…

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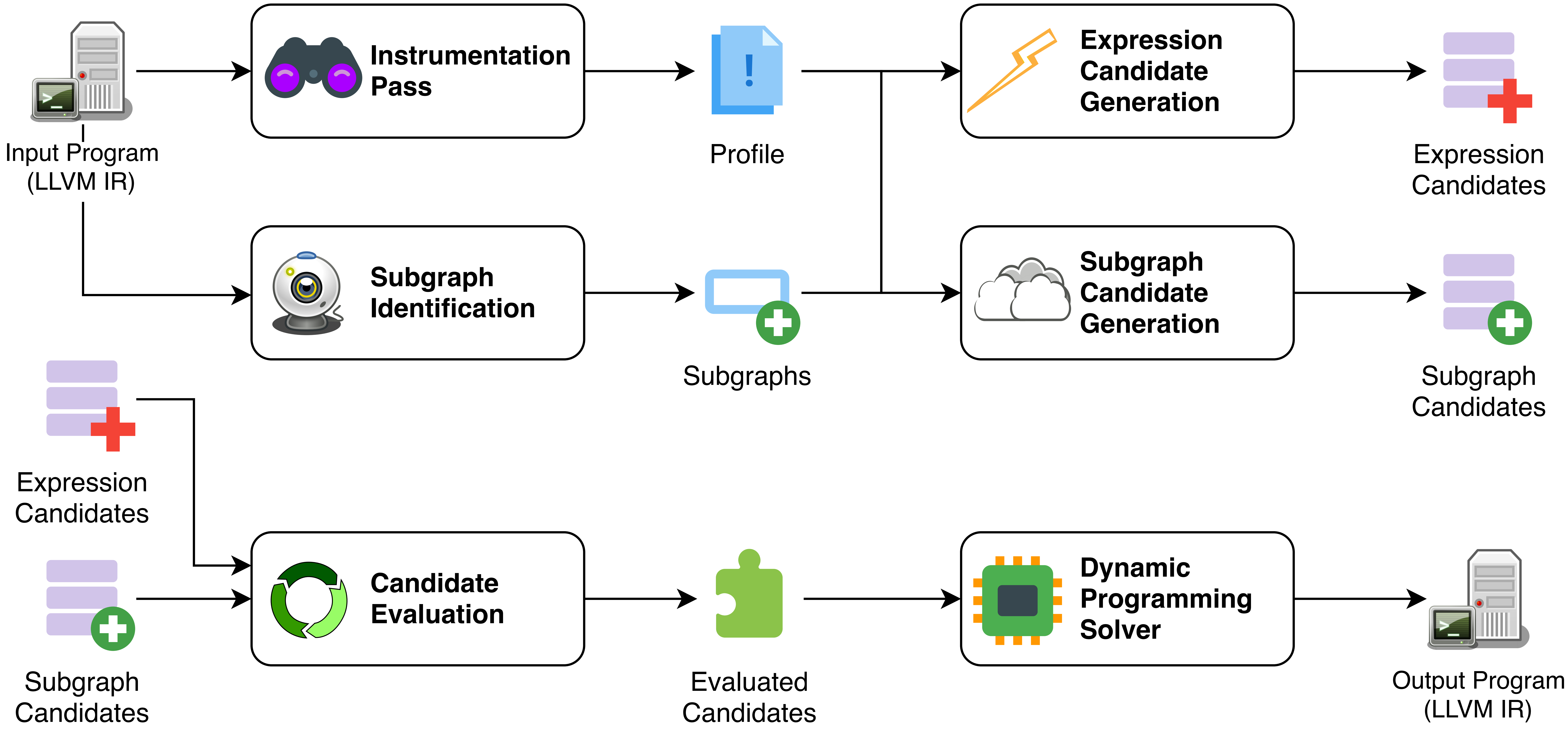
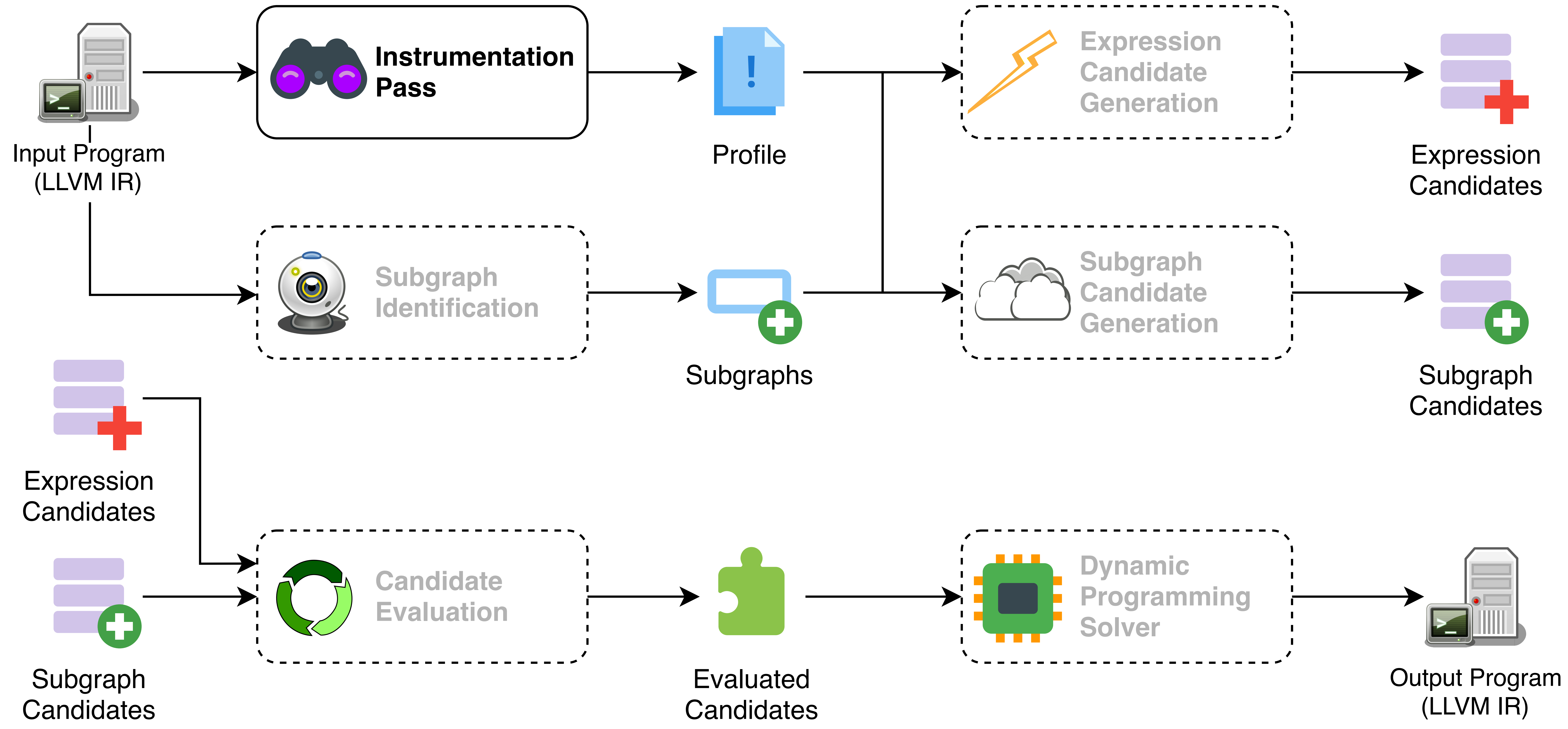
- Implemented on top of **Enzyme**, a High-Performance Automatic Differentiator of LLVM and MLIR

Floats…

Poseidon Overview



Phase 1: Instrumentation



Phase 1: Instrumentation

- Augment the input program
    - Logger function calls embedded in Enzyme-synthesized gradient
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```
double example(…) {
  double tmp;
  if (a < b)
    tmp = sin(a + c * 3.0);
  else
    tmp = cos(b + c * 4.0);
  return tmp;
}
```

Original computation

```
define double @diffe_example(…) {
  …
  %tmp = call @llvm.sin.f64(…)
  call void @enzymeLogValue(double %tmp, …)
  %grad = call @llvm.cos.f64(…)
  call void @enzymeLogGrad(double %grad, …)
  …
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Enzyme-synthesized gradient

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Enzyme-synthesized gradient   Logger function calls

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```
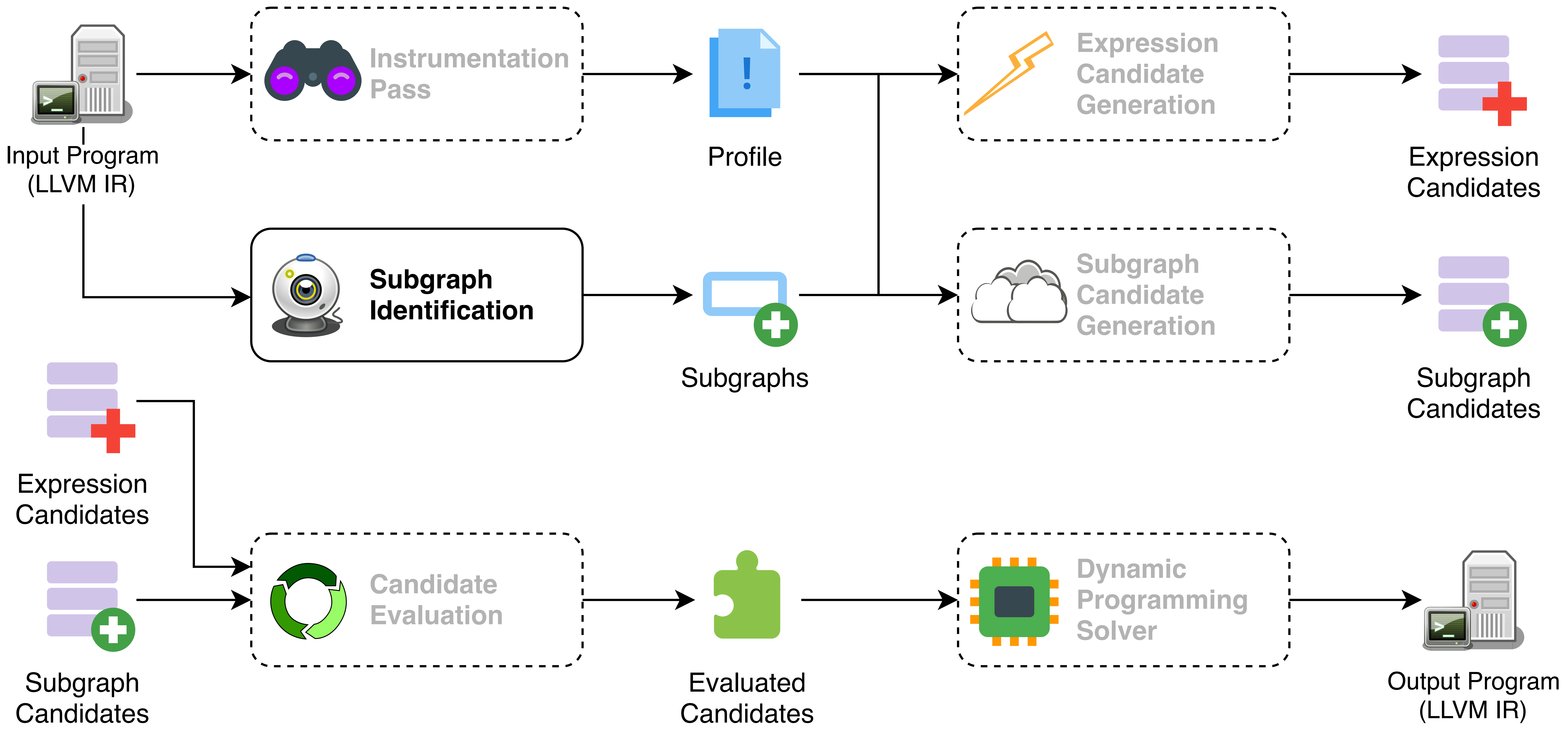

Original computation

```
Value:example:1:2
    Min = -0.999…
    Max = 0.999…
    Executions = 50053
    Geometric Average = 6.017…e-01
// Other Values…
Grad:example:1:2
    Geometric Average = 9.999…e-01
// Other Gradients…
```

Phase 2A: Subgraph Identification



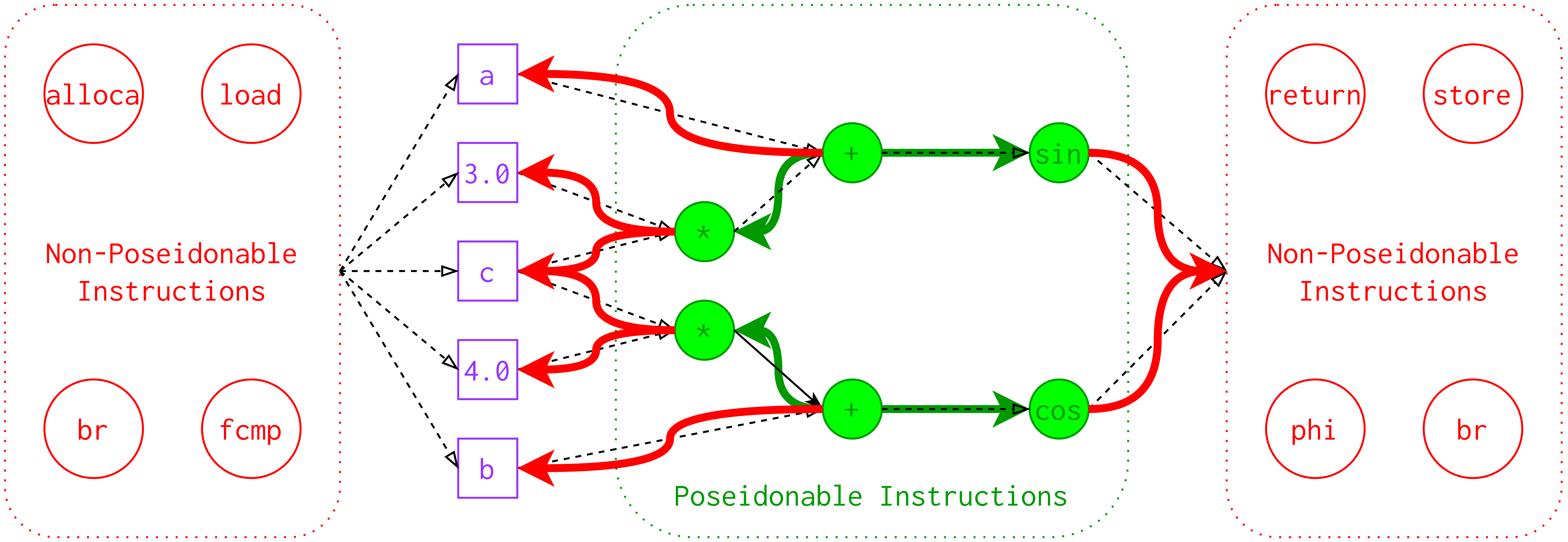
Phase 2A: Subgraph Identification

- Optimizable `llvm::Value` → *Poseidon-able* Values
- Poseidonable Values (FP Instructions)
  - `+, -, *, /, sin, cos, tan, exp, log, sqrt, cbrt, pow, fma, hypot, expm1, log1p, …`
- Non-Poseidonable Values
  - `load, store, br, ret, if*, ==*, <*, >*, <=*, >=*, !=*, and*, or*, not*, …`

* Opcode planned but not yet analyzable in Poseidon.

Phase 2A: Subgraph Identification

- Floodfill Algorithm → **DAGs of Poseidonable Instructions**


alloca
load
Non-Poseidonable Instructions
br
fcmp
a
3.0
c
4.0
b
+
sin
*
*
+
cos
Poseidonable Instructions
return
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phi
br

Phase 2A: Subgraph Identification

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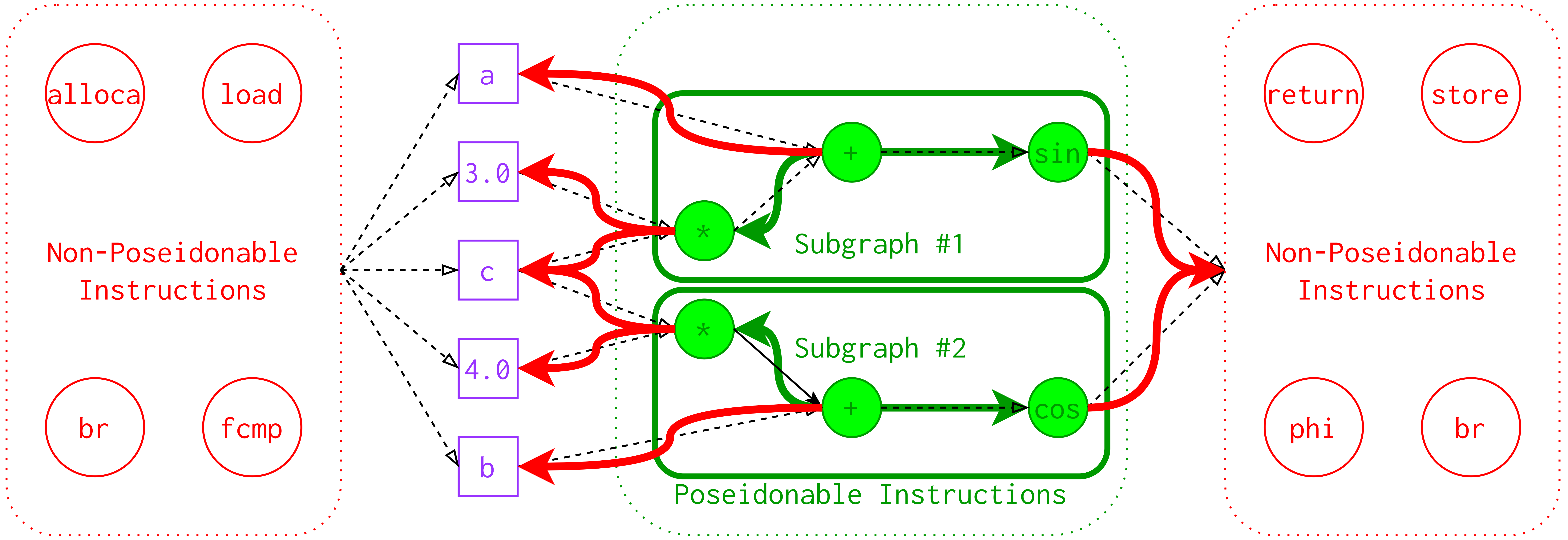
alloca
load
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Subgraph #1
*
Subgraph #2
+
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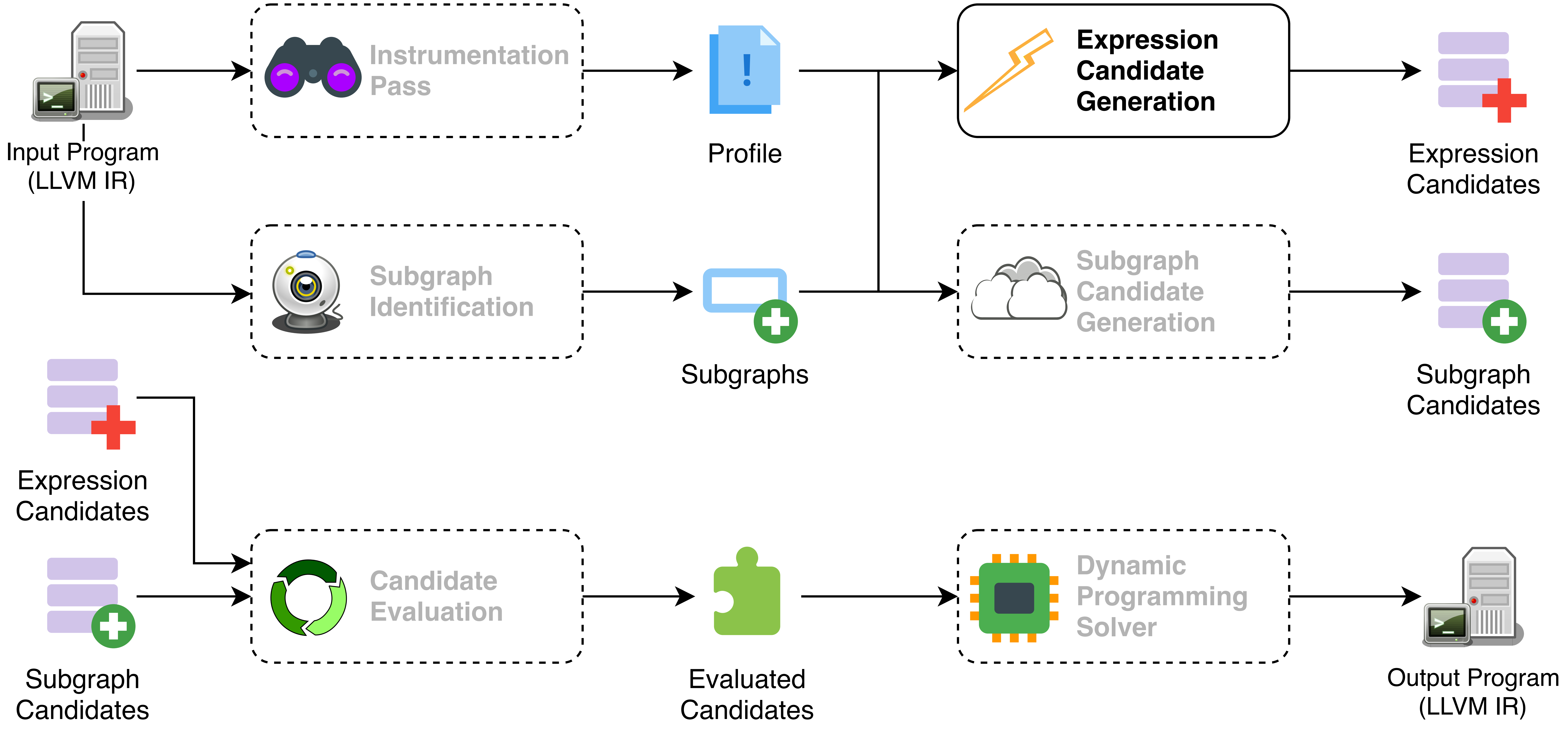
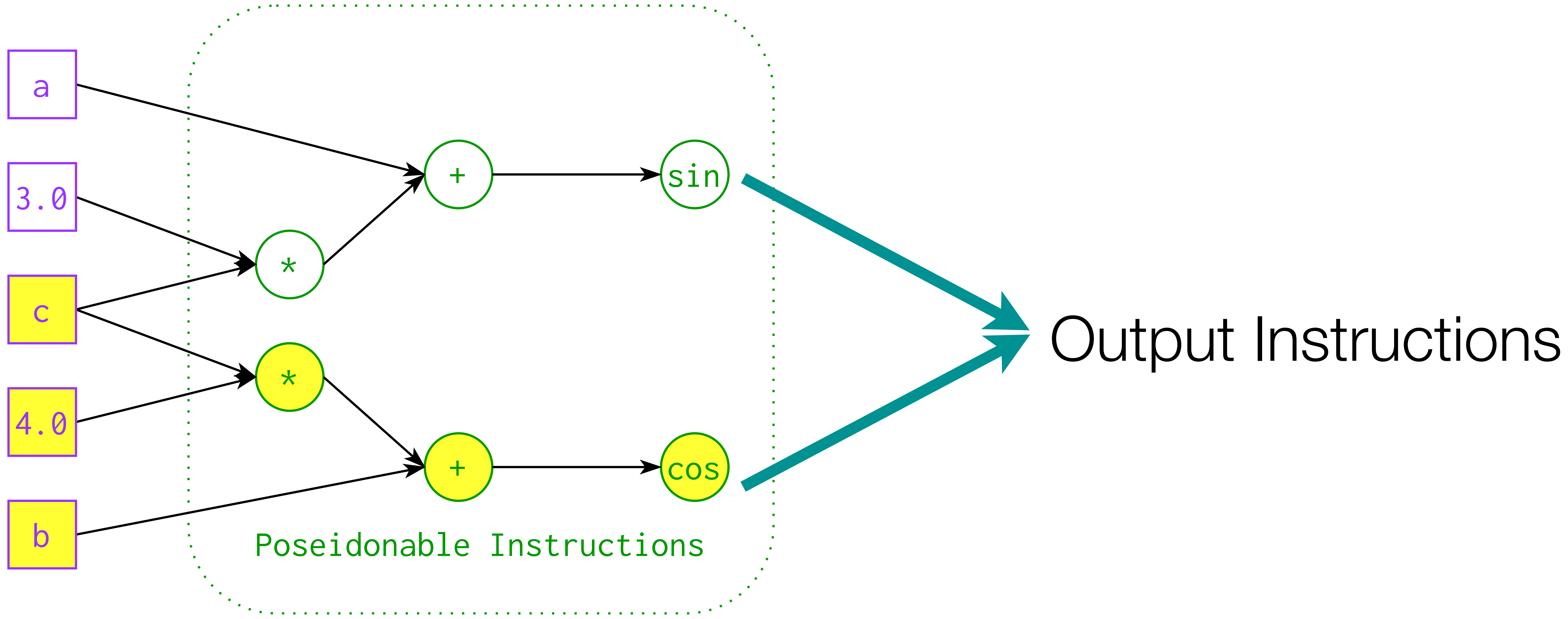
Poseidonable Instructions
return
store
Non-Poseidonable Instructions
phi
br

Phase 2B: Expression Candidate Generation



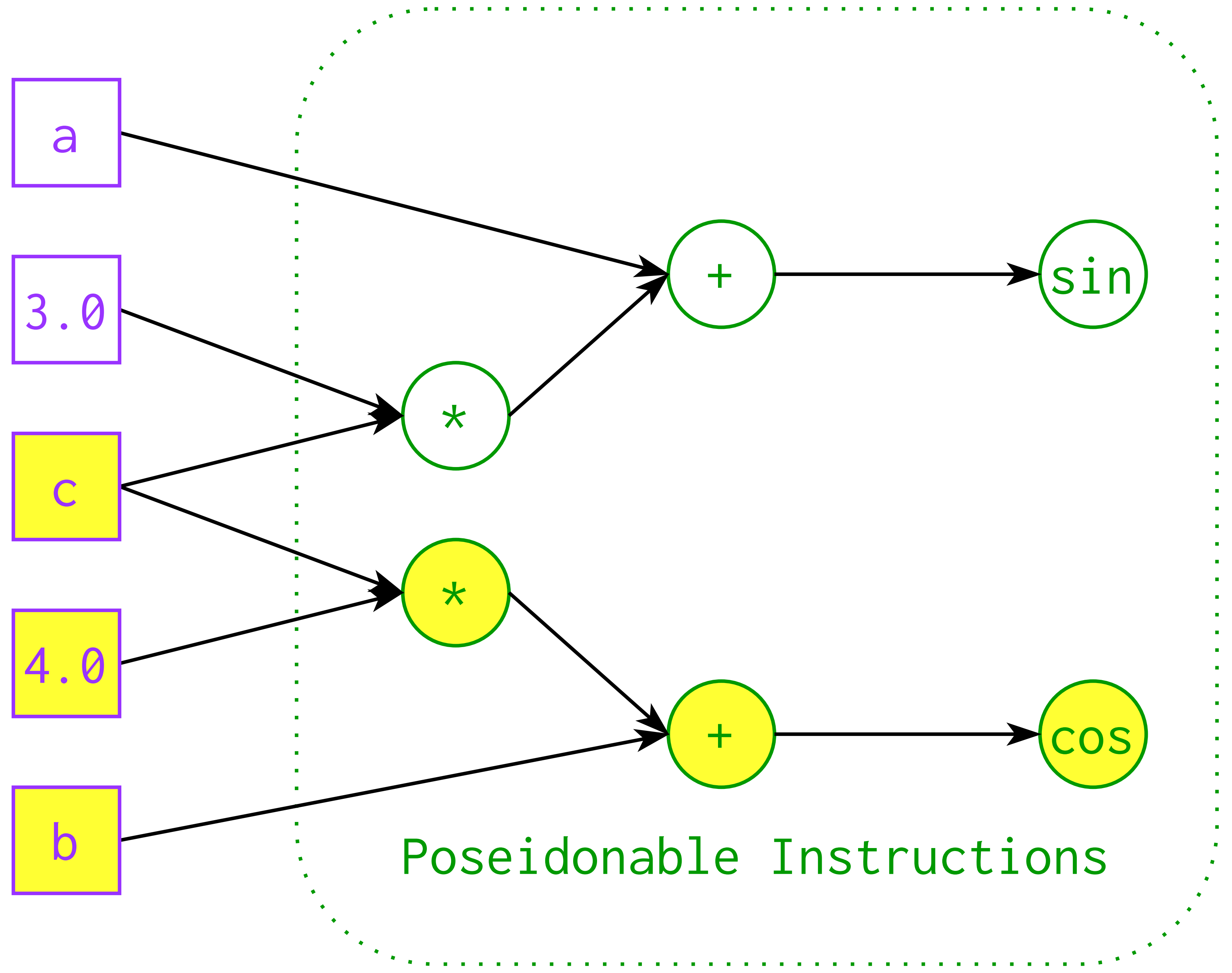
Phase 2B: Expression Candidate Generation

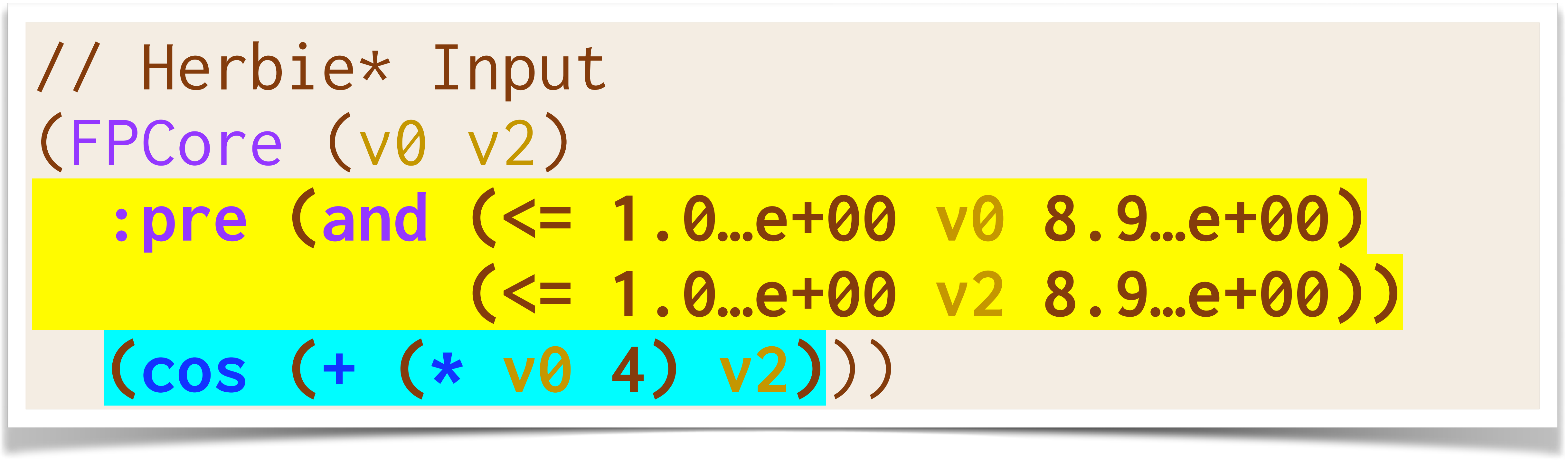
- *Output instructions* propagate values to non-poseidonable values



Phase 2B: Expression Candidate Generation

- *Output instructions* propagate values to non-poseidonable values
- Construct 𝔽 expressions from *output instructions* and use existing tools to produce expression candidates





```
// Herbie* Input
(FPCore (v0 v2)
  :pre (and (<= 1.0…e+00 v0 8.9…e+00)
            (<= 1.0…e+00 v2 8.9…e+00))
  (cos (+ (* v0 4) v2)))
```

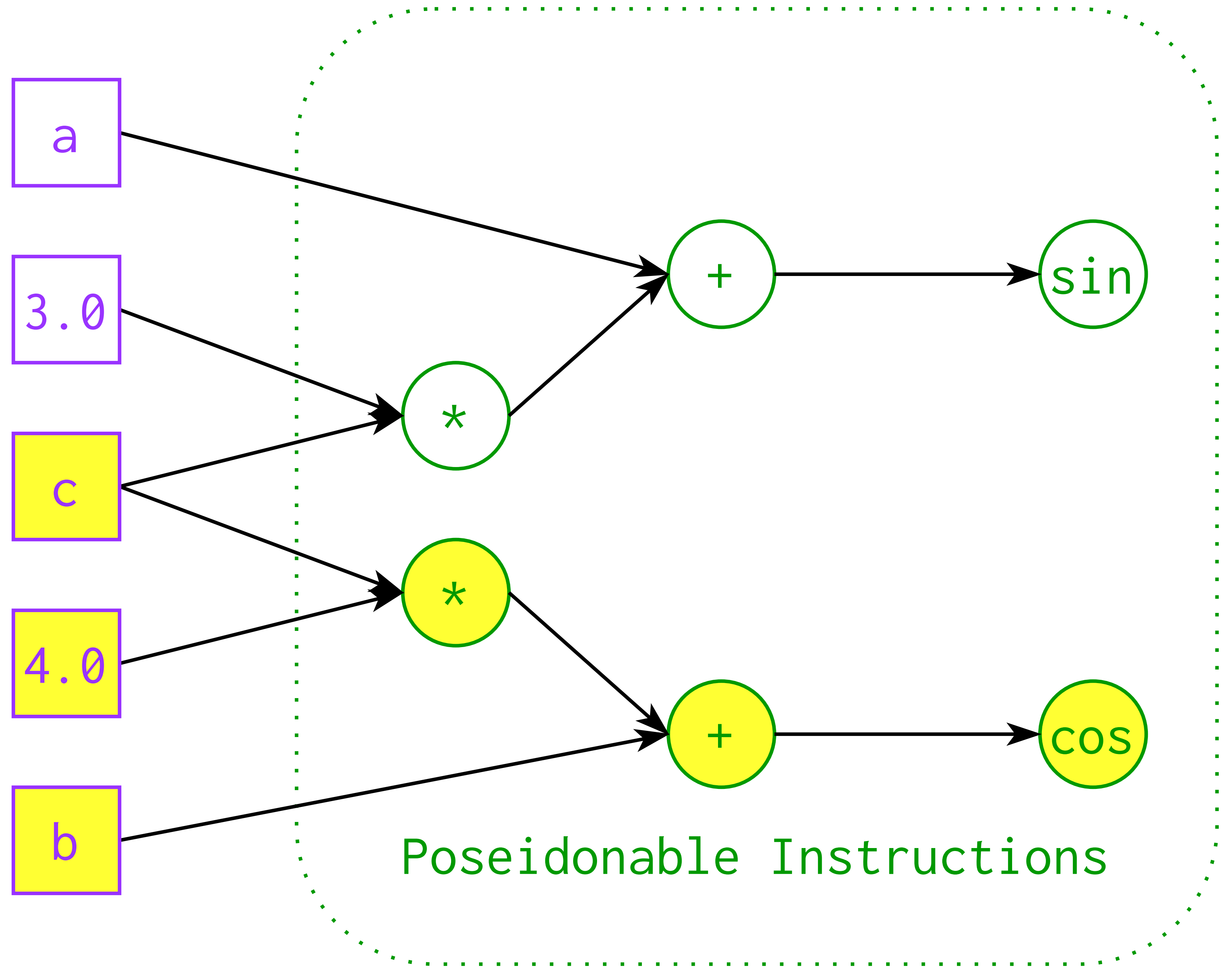
Bounds from *Phase 1: Instrumentation.*

Constructed expression

[*] Panchekha et al. '15

Phase 2B: Expression Candidate Generation

- *Output instructions* propagate values to non-poseidonable values
- Construct $\mathbb{F}$ expressions from *output instructions* and use existing tools to produce expression candidates



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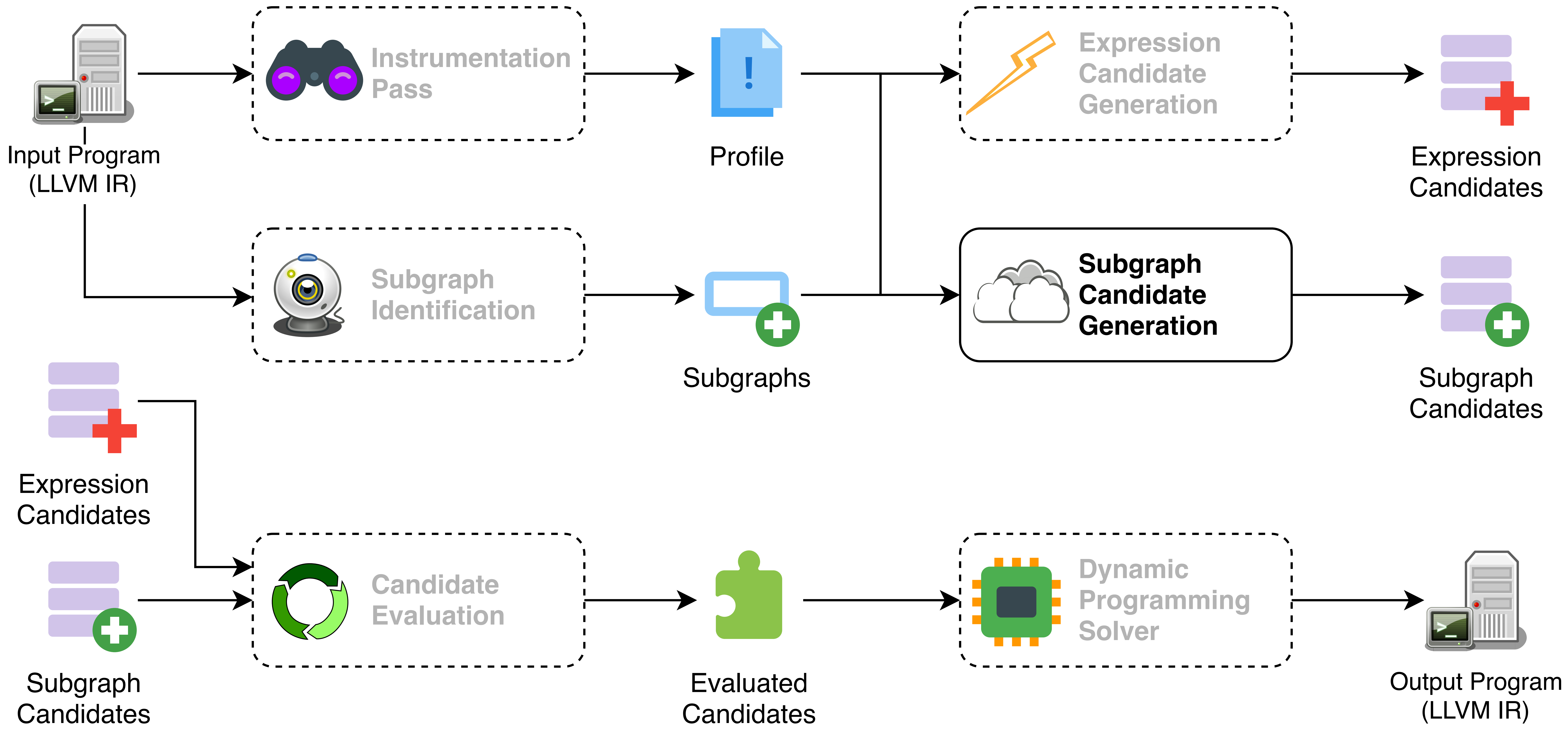
  (cos (+ (* v0 4) v2)))
```

```
// Expression Candidate #1
(- (* (cos (* v0 4)) (cos v2))
  (* (sin (* v0 4)) (sin v2)))
// Expression Candidate #2
(fma (cos (* v0 4)) (cos v2) (*
  (sin (* v0 4)) (sin (neg v2))))
```

Phase 2C: Subgraph Candidate Generation



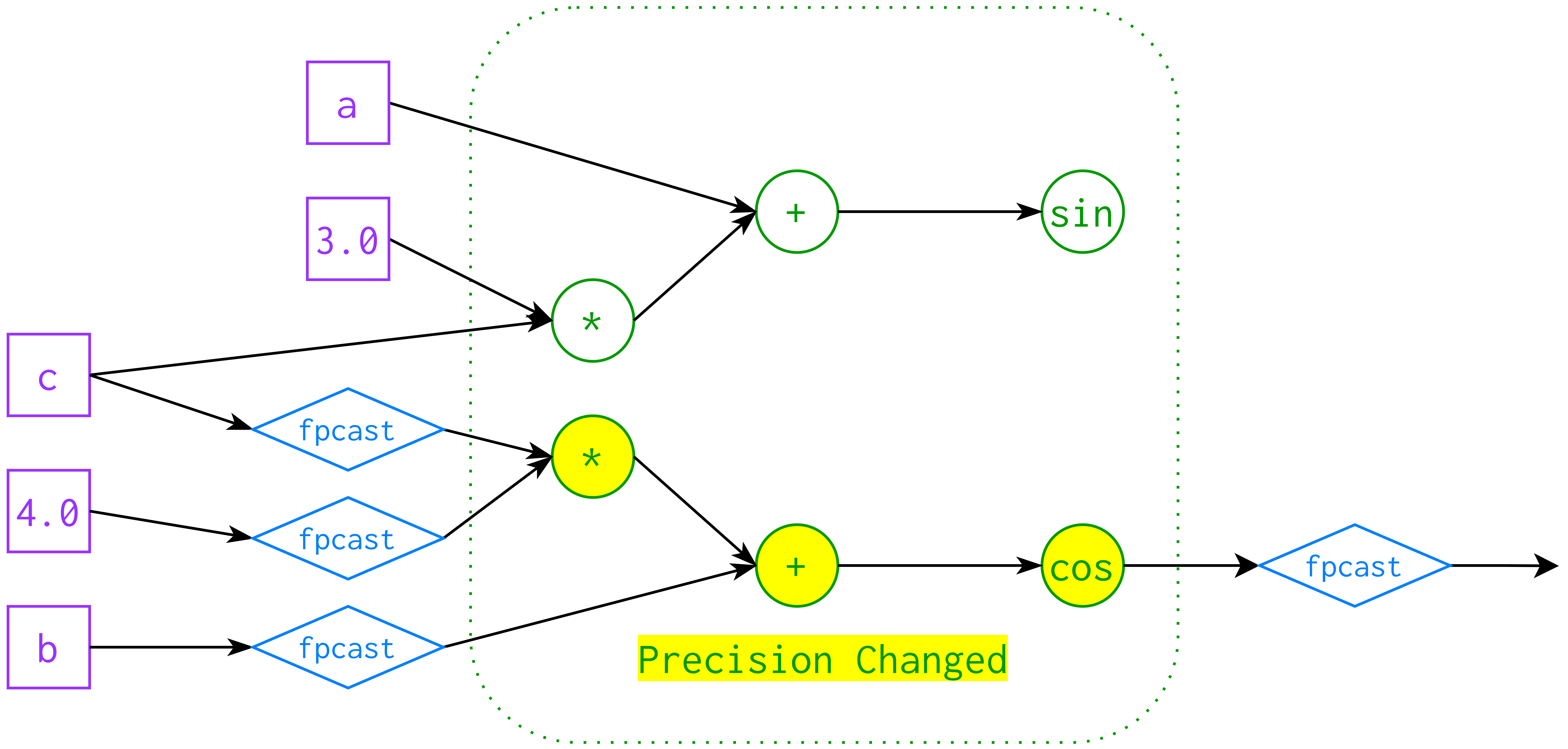
Phase 2C: Subgraph Candidate Generation

- **Sensitivity*** (How much changing an intermediate instruction impacts the final result)
    - Computed as $|x \cdot f'(x)|$
    - Values and gradients come from *Phase 1: Instrumentation*
- Gradient-Guided Precision Tuning

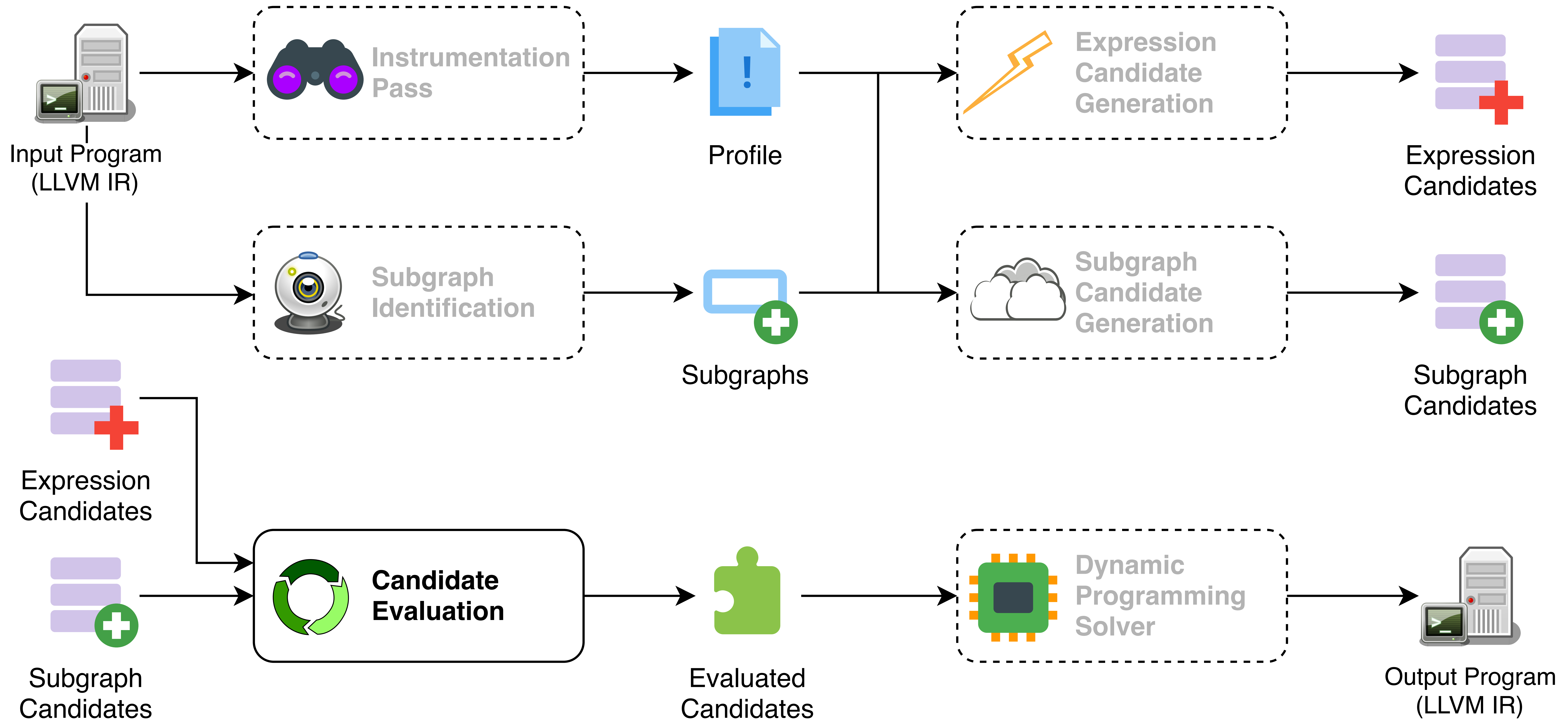
* From ADAPT (Menon et al. ’18).

Phase 2C: Subgraph Candidate Generation

- Gradient-Guided Precision Tuning



Phase 2C: Candidate Evaluation



Phase 2D: Candidate Evaluation

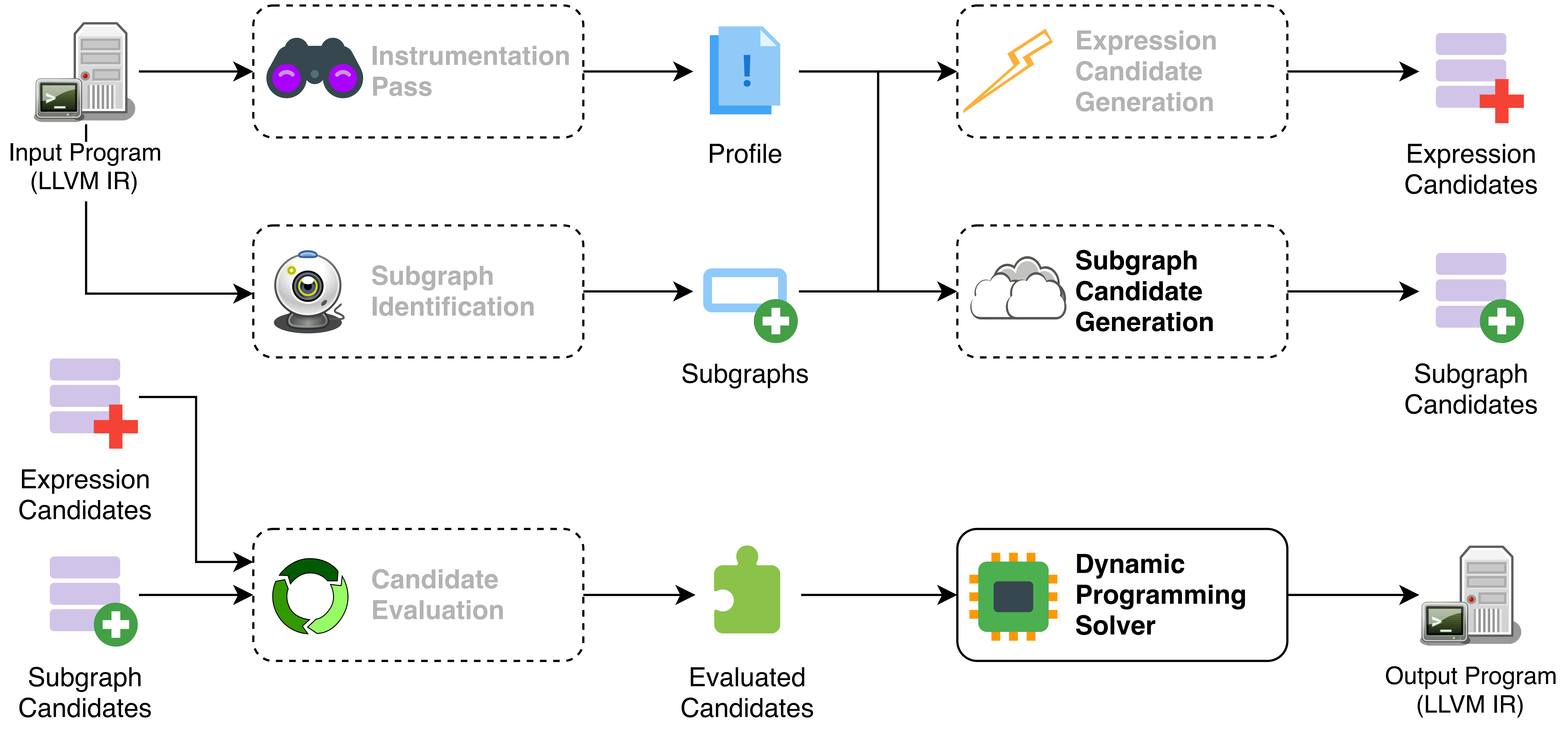
- How *local* transformations change *global* cost and accuracy?
- Cost Model
  - `llvm::TargetTransformInfo::getInstructionCost`
  - Custom Cost Model (Microbenchmarking LLVM Instructions)
  - $$\texttt{Cost} = \sum_i \texttt{ExecutionCount}(i) \times \texttt{InstructionCost}(i)$$

Phase 2D: Candidate Evaluation

- How *local* transformations change *global* cost and accuracy?
- Accuracy Model
  - Built-In MPFR Evaluator: **Ground Truth*** and **Emulated Results** of *Output Instructions*
  - $$\texttt{Global Error} = \sum_{o} \left| \texttt{Gradient}(o) \times (\texttt{GroundTruth}(o) - \texttt{EmulatedResult}(o)) \right|$$

* Similar to techniques in Herbie (Panchekha et al. '15).

Phase 3: Dynamic Programming Solver

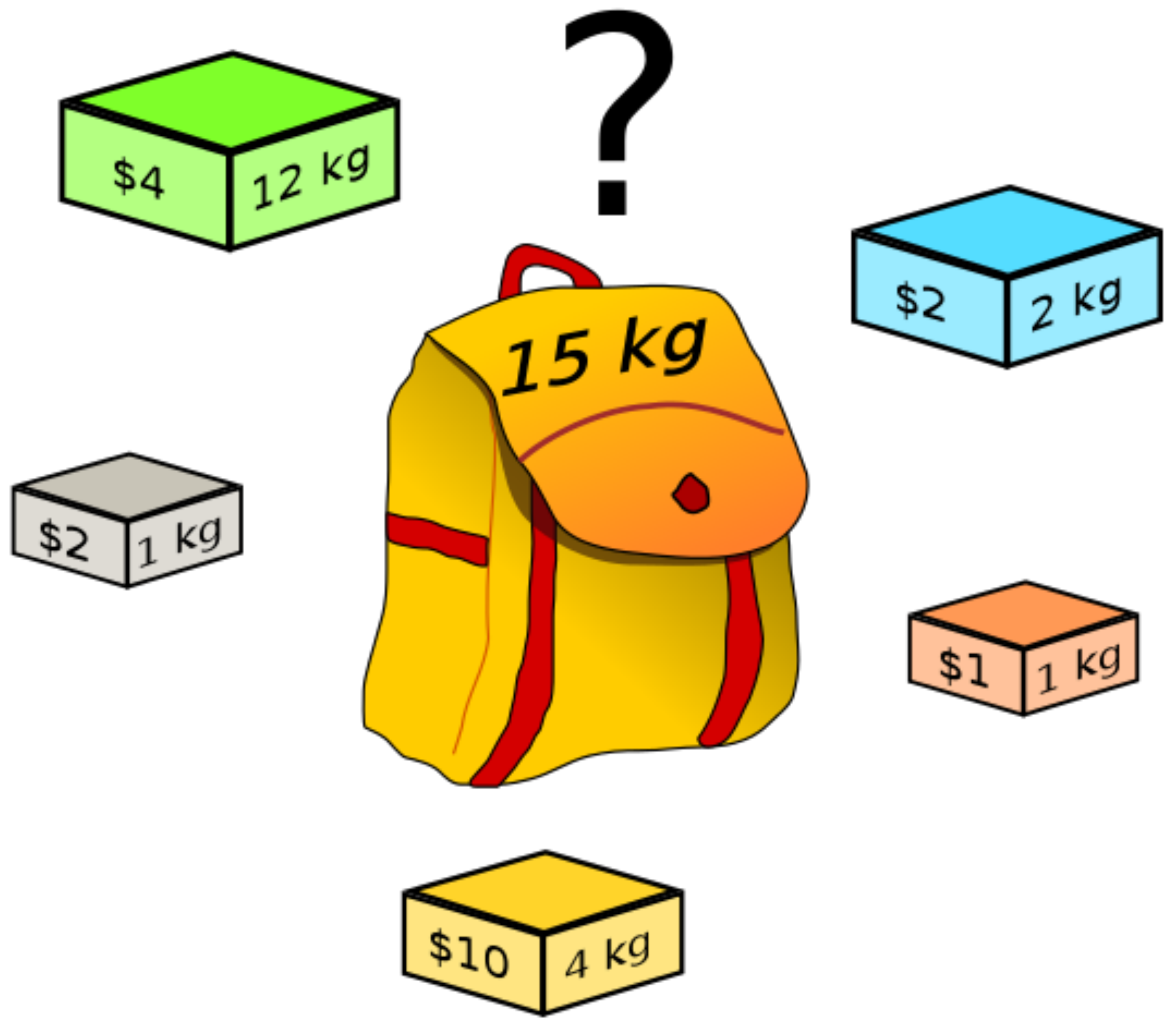


Phase 3: Dynamic Programming Solver

- Poseidon's Problem
  - Known: Computation Cost **Budget**, Expression Candidates (**Cost** & **Global Error**), Subgraph Candidates (**Cost** & **Global Error**).
  - Objective: $\text{argmin}_{\phi} \sum_{\phi} \texttt{GlobalError}(\phi)$
  - Constraints:
    - $\sum_{\phi} \texttt{Cost}(\phi) \leq \mathtt{\textbf{ComputationCostBudget}}$
    - Select only one candidate per $\mathbb{F}$ expression/subgraph

Phase 3: Dynamic Programming Solver

- 0/1 Knapsack Problem
  - Known: Knapsack **Capacity**, Items (**Weight** & **Profit**).
  - Objective: $\max_{\phi} \sum_{\phi} \texttt{Profit}(\phi)$
  - Constraint: $\sum_{\phi} \texttt{Weight}(\phi) \leq \texttt{Capacity}$
  - Dynamic Programming solver in pseudo-polynomial time



Phase 3: Dynamic Programming Solver

- Poseidon's Problem (Rounding all costs to nearest integers)
  - Known: Computation Cost **Budget**, Expression Candidates (**Cost** & **Global Error**), Subgraph Candidates (**Cost*** & **Global Error***).
  - Objective: $\operatorname{argmin}_{\phi} \sum_{\phi} \texttt{GlobalError}(\phi)$
  - Constraints:
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    - Select only one candidate per $\mathbb{F}$ expression/subgraph

* Adjusted in the solving process.

Evaluation: FPBench Microbenchmarks

- Small FP functions (exported to C): 46 immediately optimizable functions
- Permitting 0.0001% Relative Error → **3.8%** Average Runtime Improvement
- Permitting 0.01% Relative Error → **5.3%** Average Runtime Improvement
- Permitting 10% Relative Error → **9.4%** Average Runtime Improvement
- Up to **5** bits of accuracy improvements (**0.23** bit on average)

Evaluation: FPBench Microbenchmarks

```
double ex10(double x1, double
x2, double x3, double x4,
double x5, double x6) {
  return (((((((x1 * x4) *
(((((-x1 + x2) + x3) - x4) +
x5) + x6)) + ((x2 * x5) *
(((((x1 - x2) + x3) + x4) - x5)
+ x6))) + ((x3 * x6) * (((((x1
+ x2) - x3) + x4) + x5) - x6)))
- ((x2 * x3) * x4)) - ((x1 *
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- ((x4 * x5) * x6);
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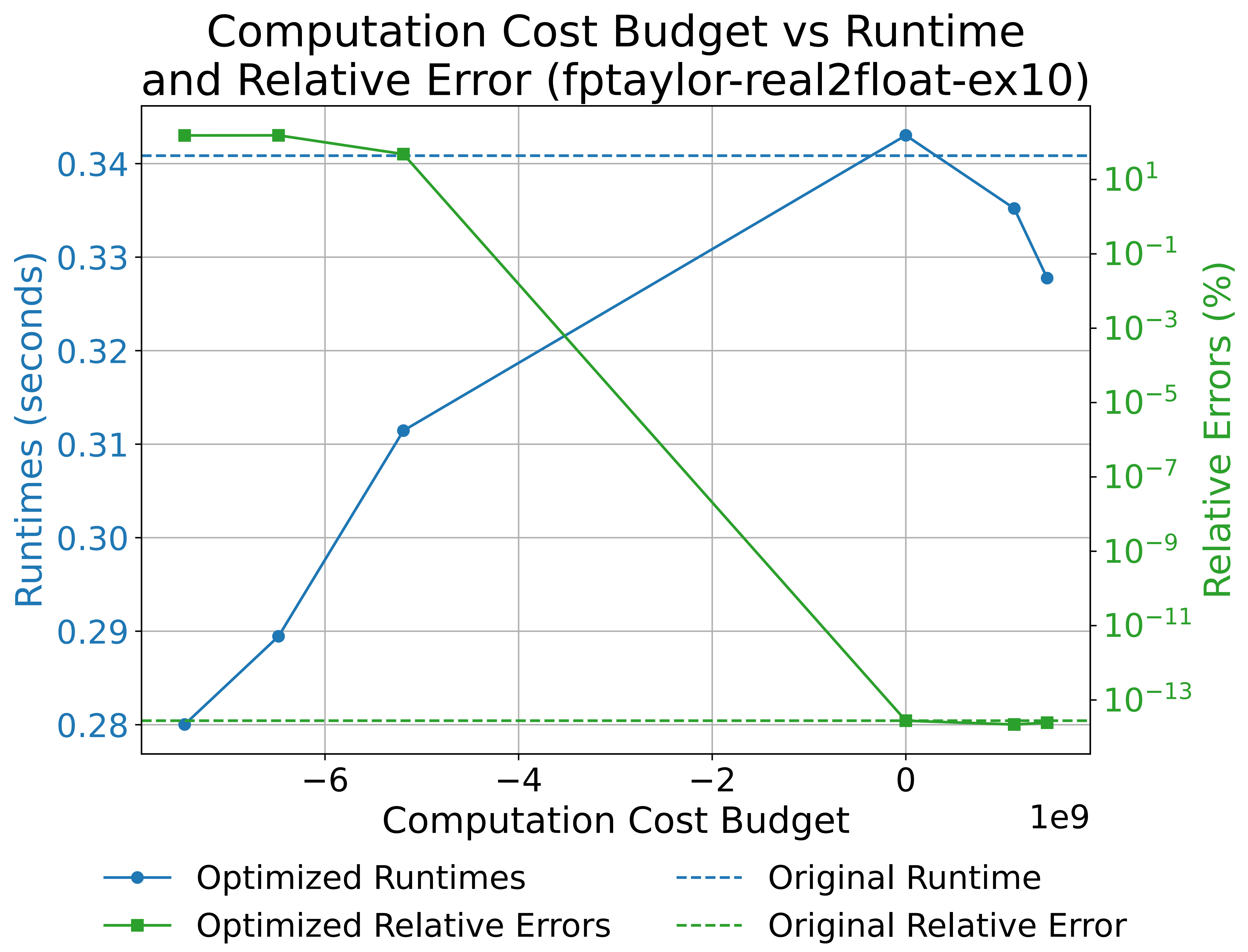
```



Lower is better

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```
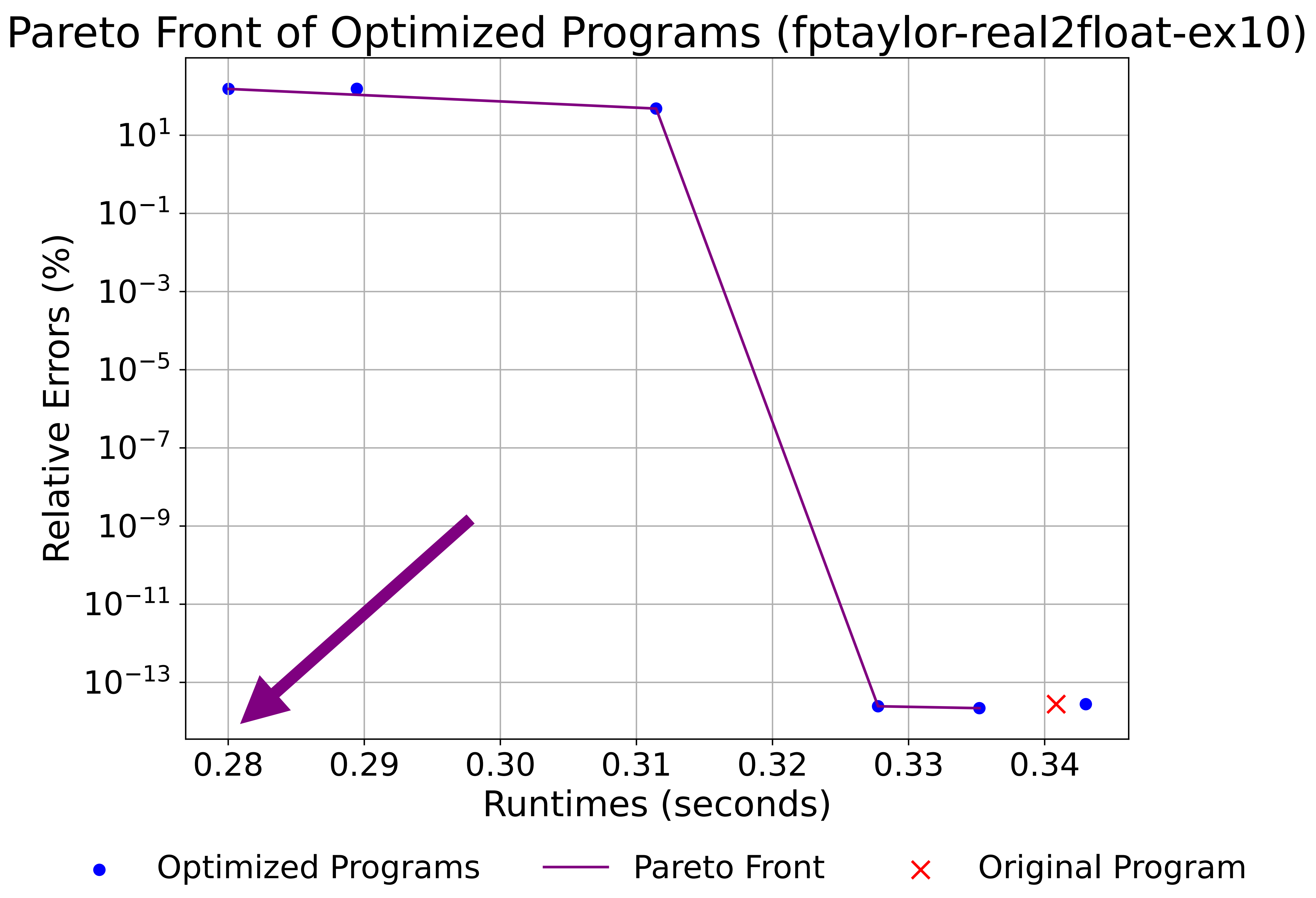



Lower-left is better

Poseidon (Open Source on GitHub: **EnzymeAD/Enzyme**)

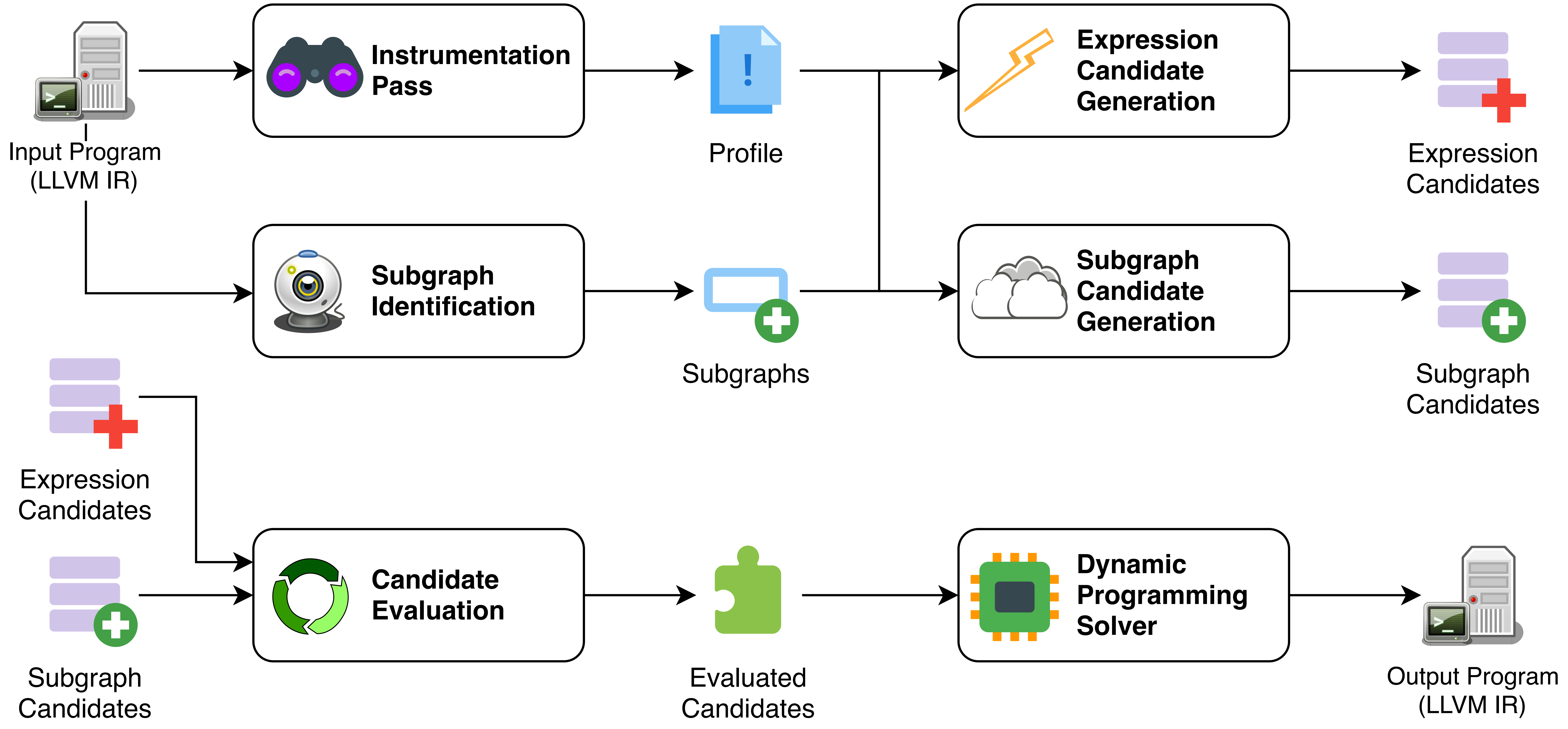
- An end-to-end system that automatically performs profile-guided two-phase optimizations of floating-point programs in compilers

- **Instrumentation Pass**

- **Expression/Subgraph Transformation Candidate Generation & Evaluation**

- **Dynamic Programming Solver**

- Permitting 0.01% Relative Error → 5.3% Average Runtime Improvement


Input Program (LLVM IR)
Instrumentation Pass
Profile
Expression Candidate Generation
Expression Candidates
Subgraph Identification
Subgraphs
Subgraph Candidate Generation
Subgraph Candidates
Expression Candidates
Subgraph Candidates
Candidate Evaluation
Evaluated Candidates
Dynamic Programming Solver
Output Program (LLVM IR)