Coded phase-shift 3D scanner

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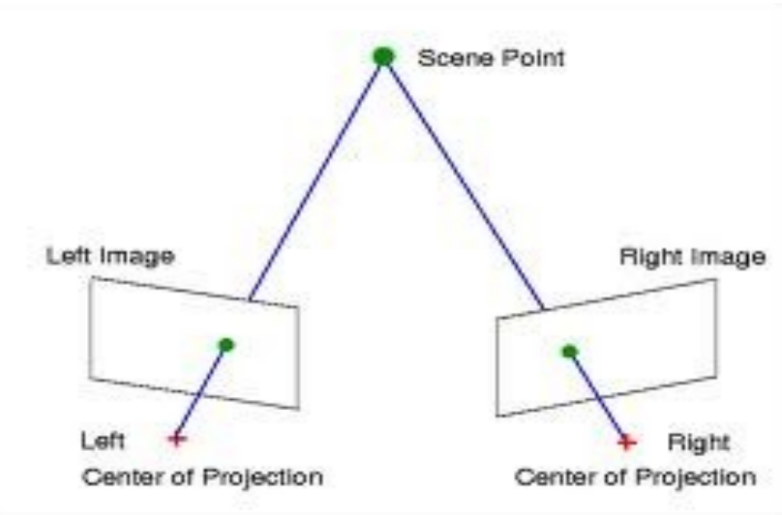
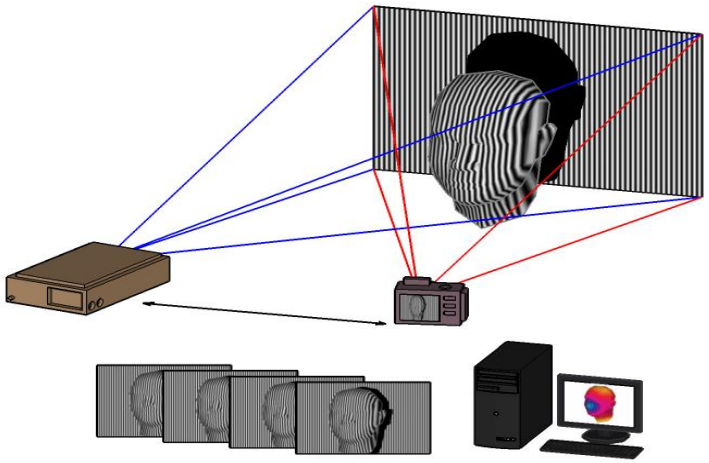
Working principle of the developed system

1. To perform triangulation *correspondence* between two views is required.
2. Phase-shift approach codifies the projector pixels by assigning a value of phase at every pixel unique only within a single period.
3. Binary patterns assist by assigning a unique period number to each period of the projected sinusoidal fringes,making every point *unique*.
4. Stereo-correspondence is estimated by recovering the phase at each point captured by camera.
5. Geometric calibration of camera and projector further allows for performing triangulation and translation of computed 3D coordinates into physical units.



(a) Triangulation



(b) Stereo-correspondence

Architecture of developed system

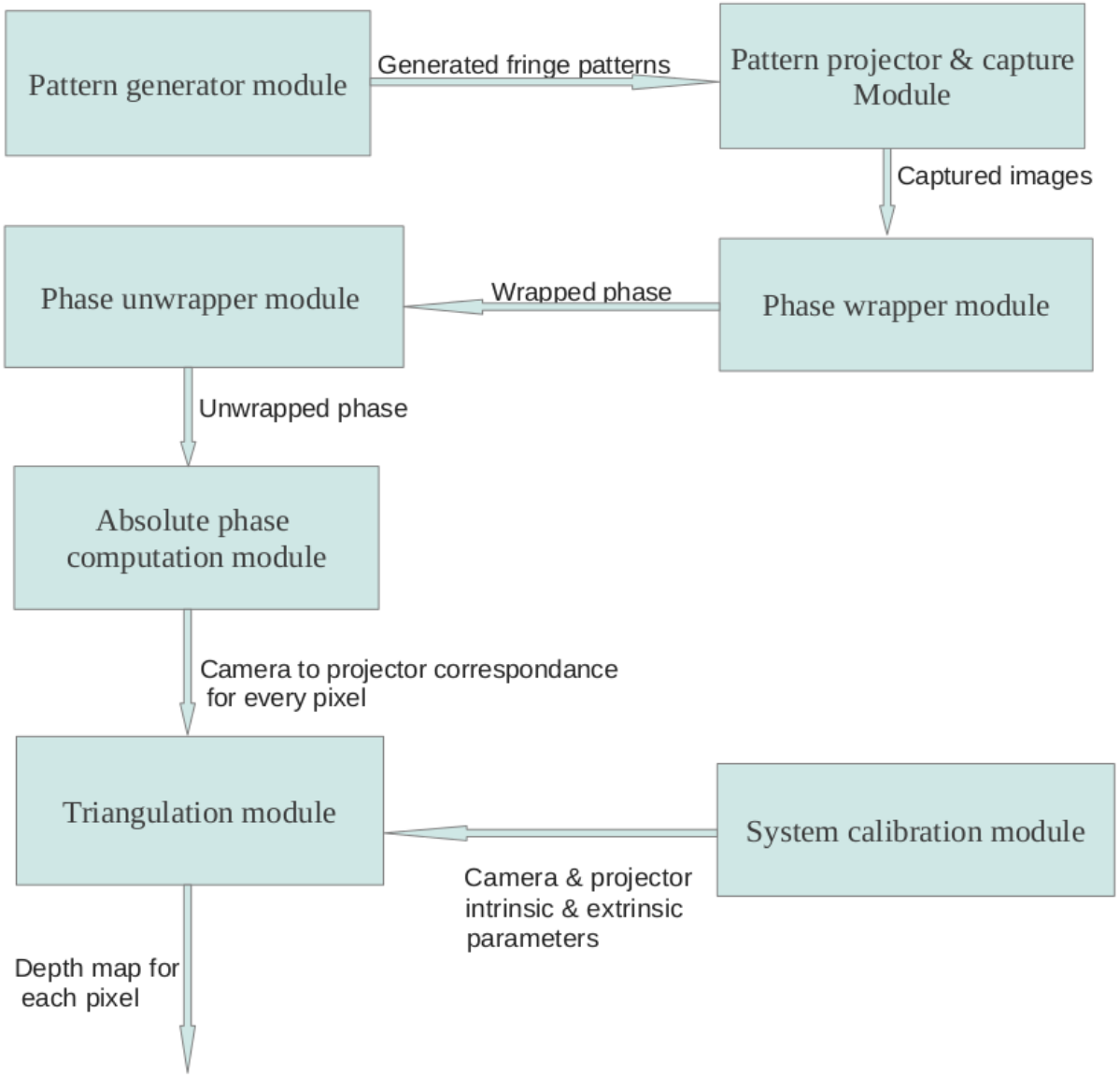


Figure: System architecture

System setup

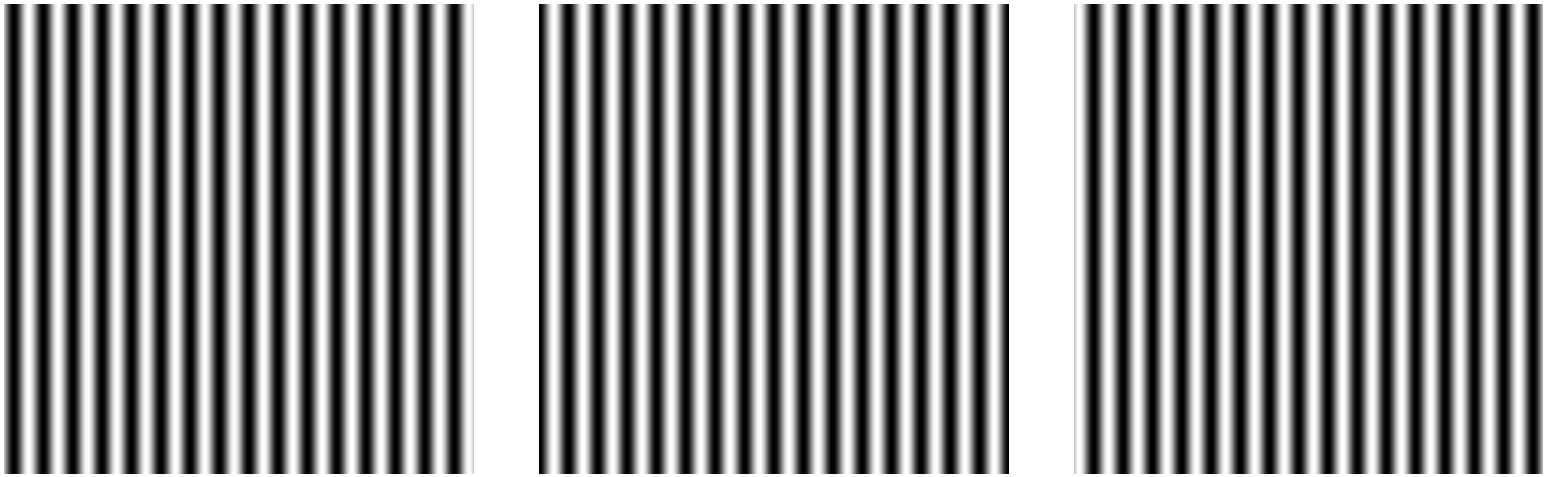
1. Sharp PG-F200X projector at 1024X768 resolution
2. Logitech Quickcam Sphere AF webcam at 1600X1200 resolution.
3. Developed system has been used and tested at Core i3-550 processor with 4GB RAM.

Pattern generator module

Vertical phase shifted patterns are defined by:

$$\begin{aligned} P_1^v &= A_v + B_v * cos(\theta_v - \alpha) \\ P_2^v &= A_v + B_v * cos(\theta_v) \\ P_3^v &= A_v + B_v * cos(\theta_v + \alpha) \end{aligned} \tag{1}$$

where $\theta_v = 2\pi \frac{x}{fringe\ \ width}$



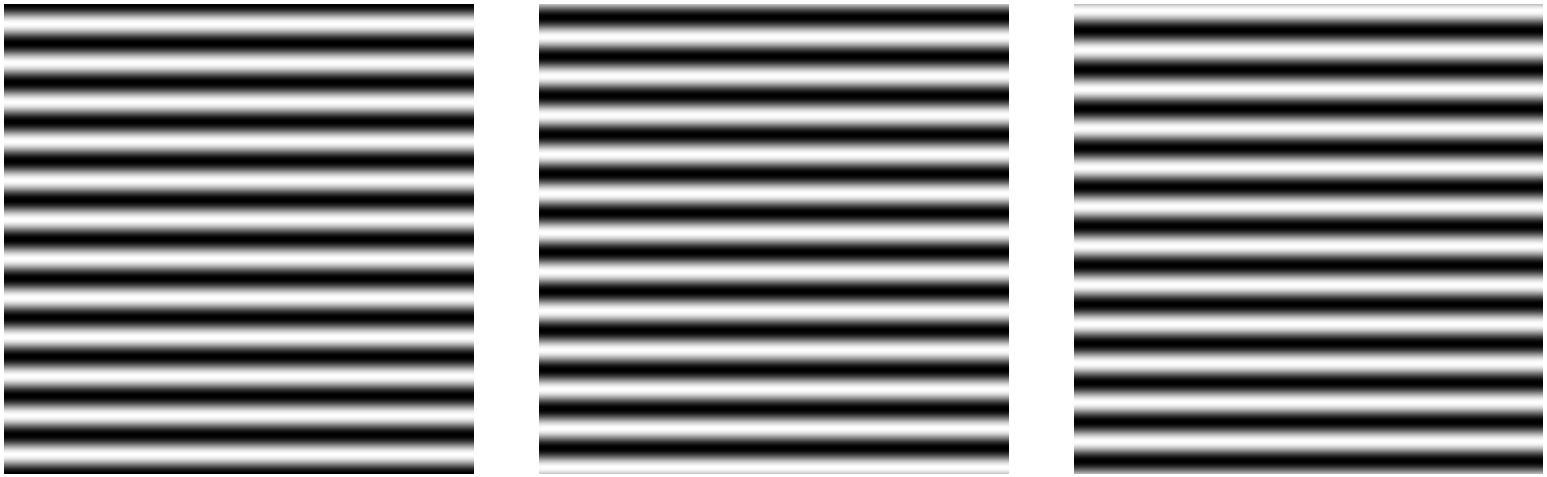
(a) (b) (c)

Figure: Vertical phase-shifted patterns

Horizontal phase-shifted patterns are defined by:

$$
\begin{aligned}
P_1^h &= A_h + B_h * cos(\theta_h - \alpha) \\
P_2^h &= A_h + B_h * cos(\theta_h) \\
P_3^h &= A_h + B_h * cos(\theta_h + \alpha)
\end{aligned}
\tag{2}
$$

where $\theta_h = 2\pi \frac{y}{fringe\ \ width}$



(a) (b) (c)

Figure: Horizontal phase-shifted patterns

Vertical binary coded patterns are defined by:

$$N_v^{codes} = \frac{W_{projector}}{w_{fringe}} \tag{3}$$

$$N_v^{patterns} = \log_2(N_v^{codes}) \tag{4}$$

$$Intensity_{(i,j)} = \lfloor i/(w_{fringe} * 2^{pattern\ \ number}) \rfloor \tag{5}$$

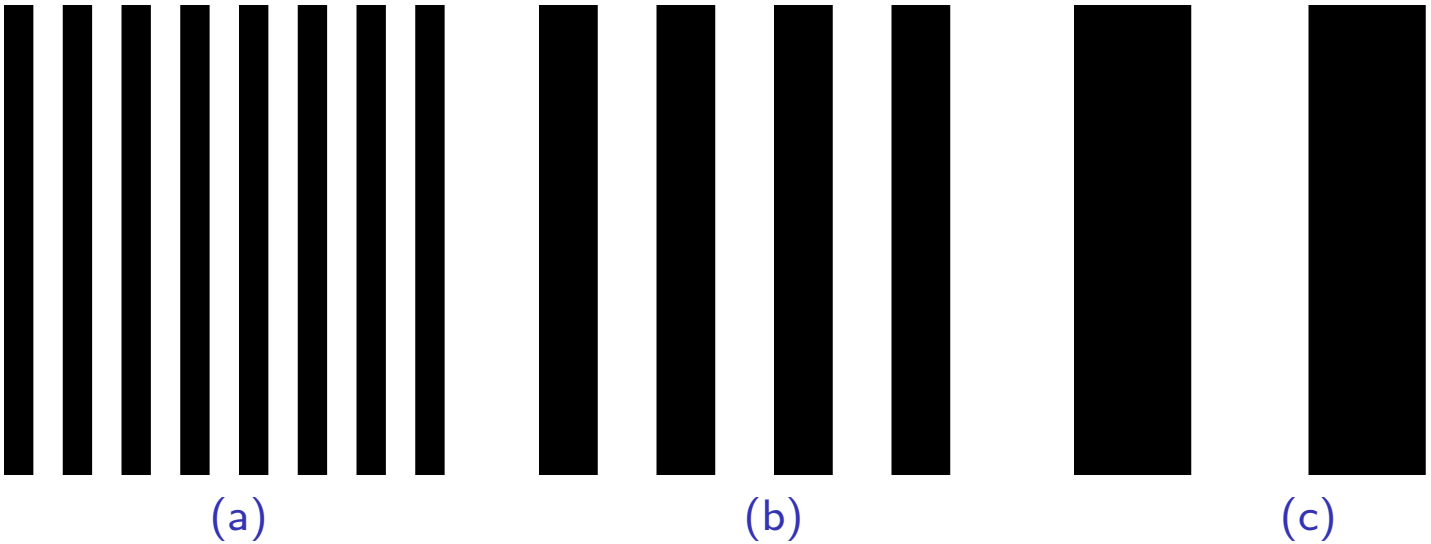


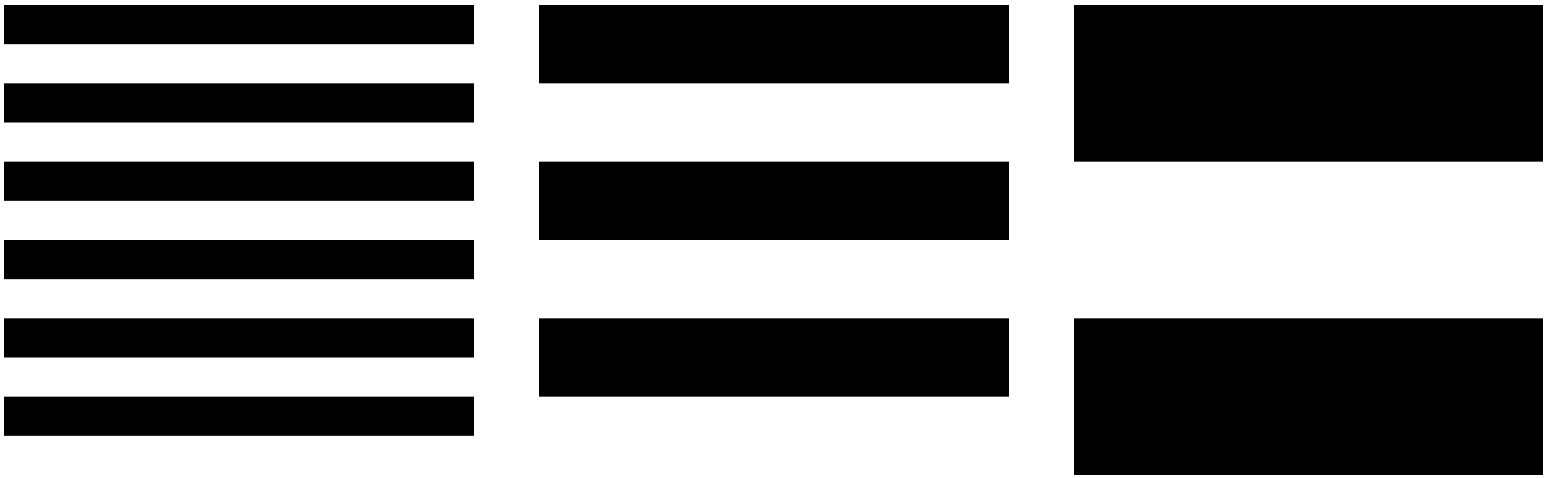
Figure: Vertical phase shifted patterns

Horizontal binary coded patterns are defined by:

$$N_h^{codes} = \frac{H_{projector}}{w_{fringe}} \tag{6}$$

$$N_h^{patterns} = \log_2(N_h^{codes}) \tag{7}$$

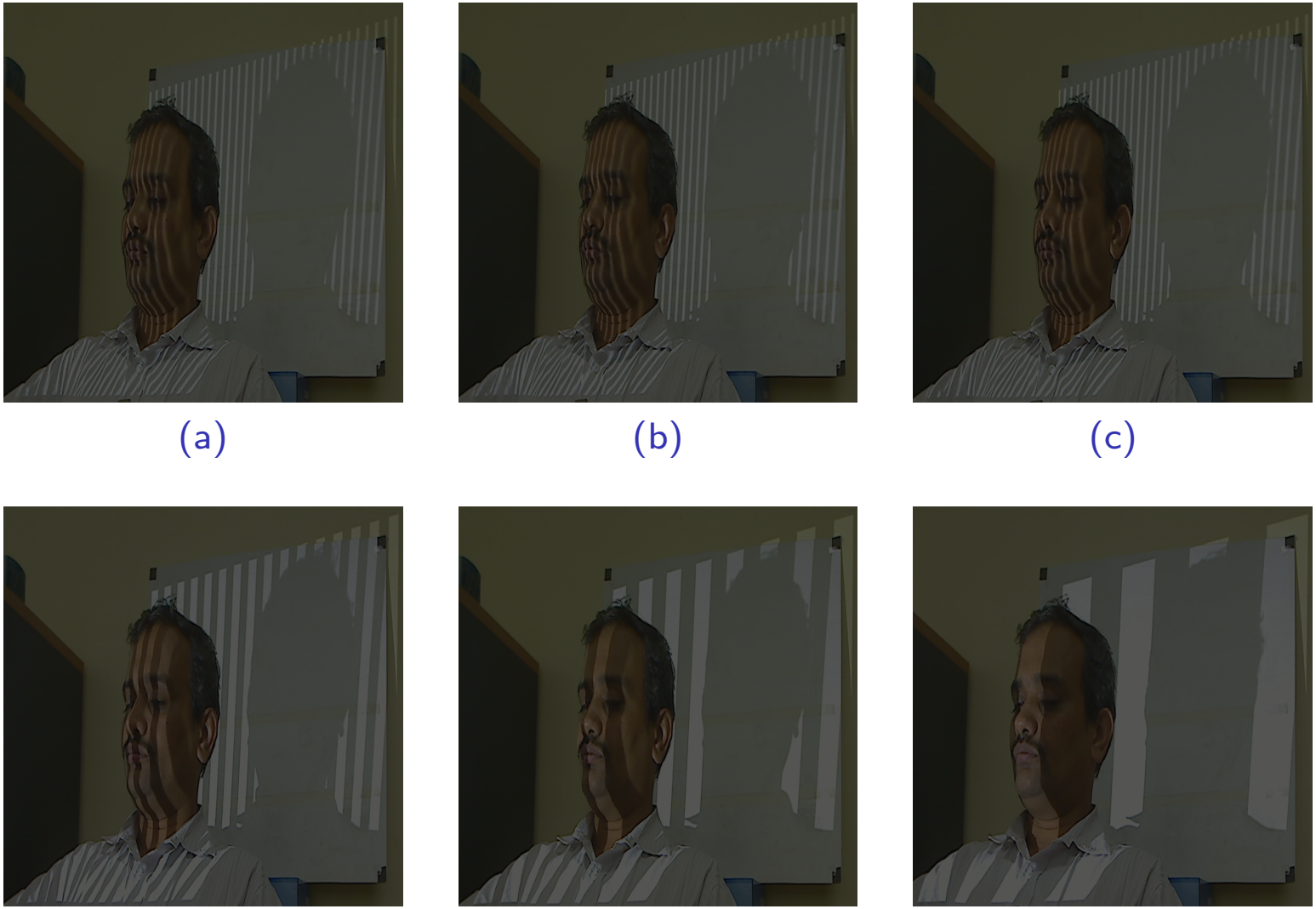
$$Intensity_{(i,j)} = \lfloor j/(w_{fringe} * 2^{pattern\ \ number}) \rfloor \tag{8}$$



(a) (b) (c)

Figure: Horizontal binary coded patterns

Pattern projection and capture module



(a) (b) (c)

(d) (e) (f)

Figure: Captured vertical phase-shifted and binary coded patterns

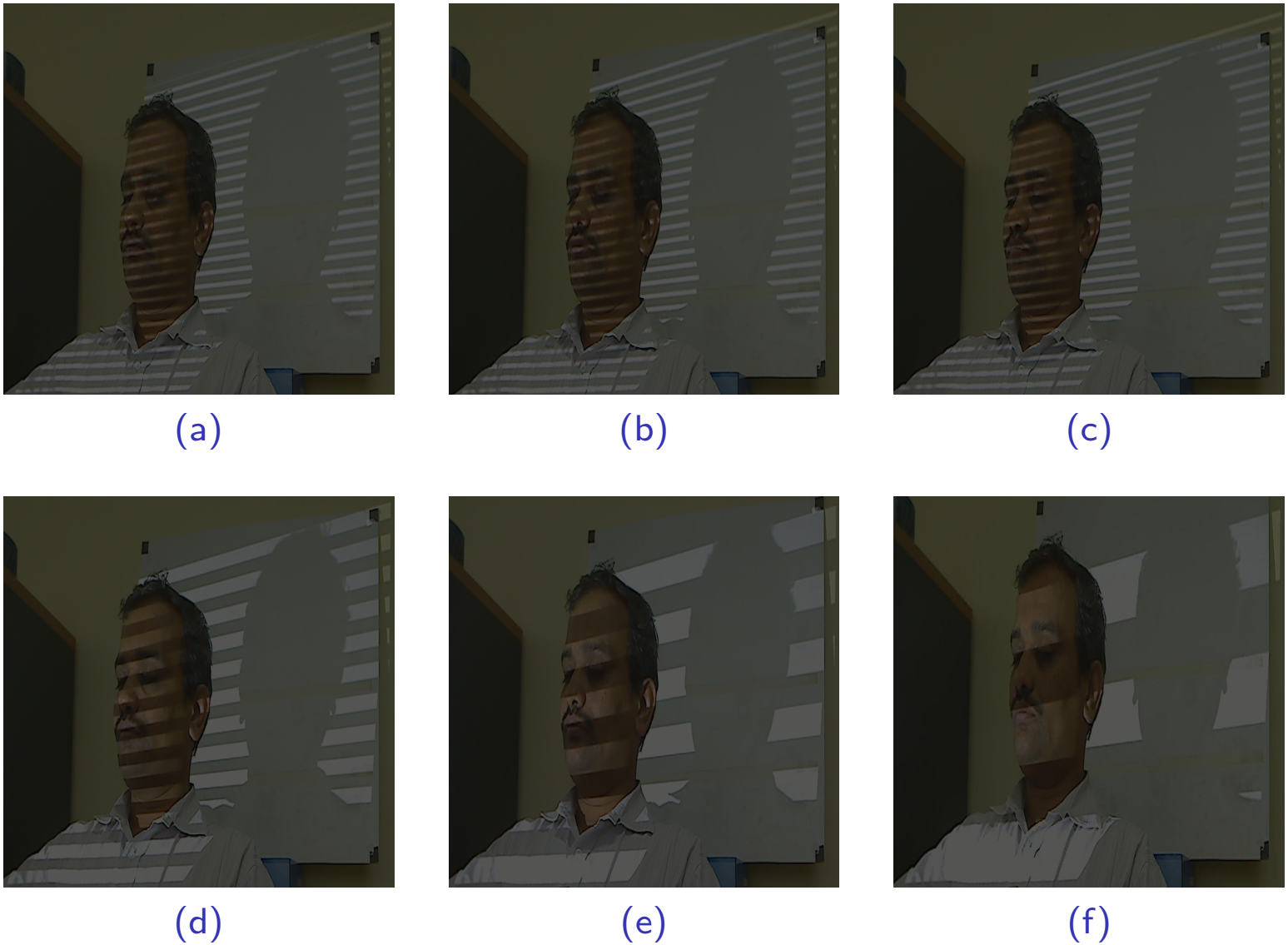


Figure: Captured horizontal phase-shifted and binary coded patterns

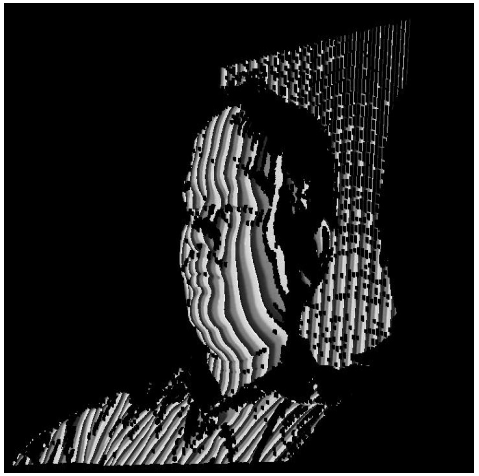
Phase wrapping module

Assumed illumination model for 3 phase shifted pattern approach:

$$\begin{aligned} I_1^{v/h} &= I_{dc}^{v/h} + I_{mod}^{v/h} * cos(\theta_{v/h} - \alpha) \\ I_2^{v/h} &= I_{dc}^{v/h} + I_{mod}^{v/h} * cos(\theta_{v/h}) \\ I_3^{v/h} &= I_{dc}^{v/h} + I_{mod}^{v/h} * cos(\theta_{v/h} + \alpha) \end{aligned} \tag{9}$$

Hence,

$$\theta_v = tan^{-1}\left[\frac{\sqrt[2]{3}(I_1^v - I_3^v)}{2I_2^v - I_1^v - I_3^v}\right], -\pi \leq \theta_v \leq \pi, \quad \theta_h = tan^{-1}\left[\frac{\sqrt[2]{3}(I_1^h - I_3^h)}{2I_2^h - I_1^h - I_3^h}\right], -\pi \leq \theta_h \leq \pi \tag{10}$$



(a) Vertical wrapped phase

(b) Horizontal wrapped phase

Figure: Computed wrapped phase

Phase unwrapping module

Unwrapped phase (ψ_v, ψ_h) maps (θ_v, θ_h) to its correct 2π multiple:

$$\begin{aligned} \psi_v &= \theta_v + 2\pi * C_v \\ \psi_h &= \theta_h + 2\pi * C_h \end{aligned} \tag{11}$$

where,
$C_v(x, y)$: Decoded vertical binary code(or *vertical period number*) at any pixel (x,y).
$C_h(x, y)$: Decoded horizontal binary code(or *horizontal period number*) at any pixel (x,y).

(a) Vertical unwrapped phase

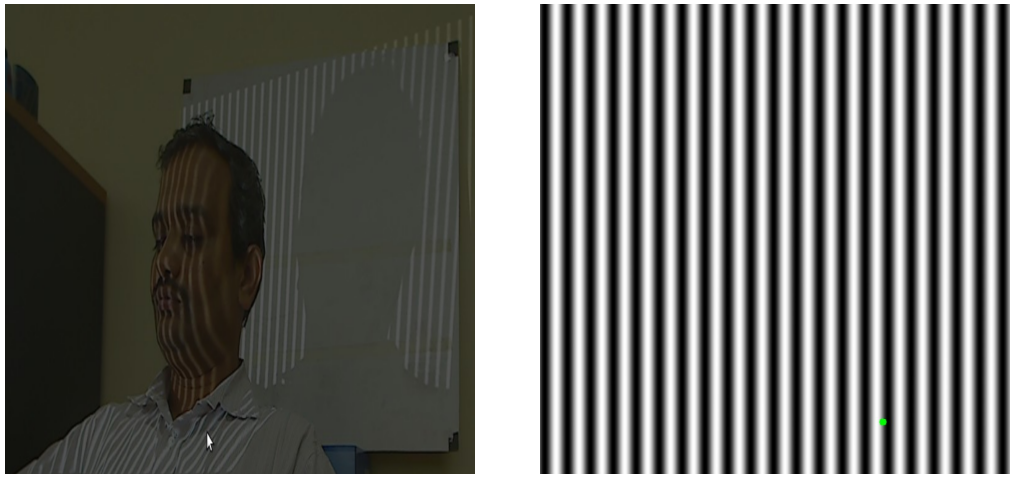
(b) Horizontal unwrapped phase

Figure: Computed unwrapped phase

Absolute phase computation module

Computing projector coordinates (X_p, Y_p) corresponding to a camera coordinates (X_c, Y_c)

$$X_p = \lfloor w_{fringe} * (\frac{\psi_v}{2\pi}) \rfloor, \quad Y_p = \lfloor w_{fringe} * (\frac{\psi_h}{2\pi}) \rfloor \tag{12}$$



(a) Camera image (b) Projector image

Figure: Stereo correspondence between camera and projector

System calibration module

Intrinsic parameters

f_x: focal length of lens expressed in number of pixels along X - axis
f_y: focal length of lens expressed in number of pixels along Y - axis
(c_x, c_y): Pixel coordinates of principal point
(k_1, k_2, k_3): Radial distortion coefficients
(p_1, p_2): Tangential distortion coefficients

Extrinsic parameters

(r_x, r_y, r_z): Rotation vector between camera(or projector) & world coordinate system
(t_x, t_y, t_z): Translation vector between camera(or projector) & world coordinate system

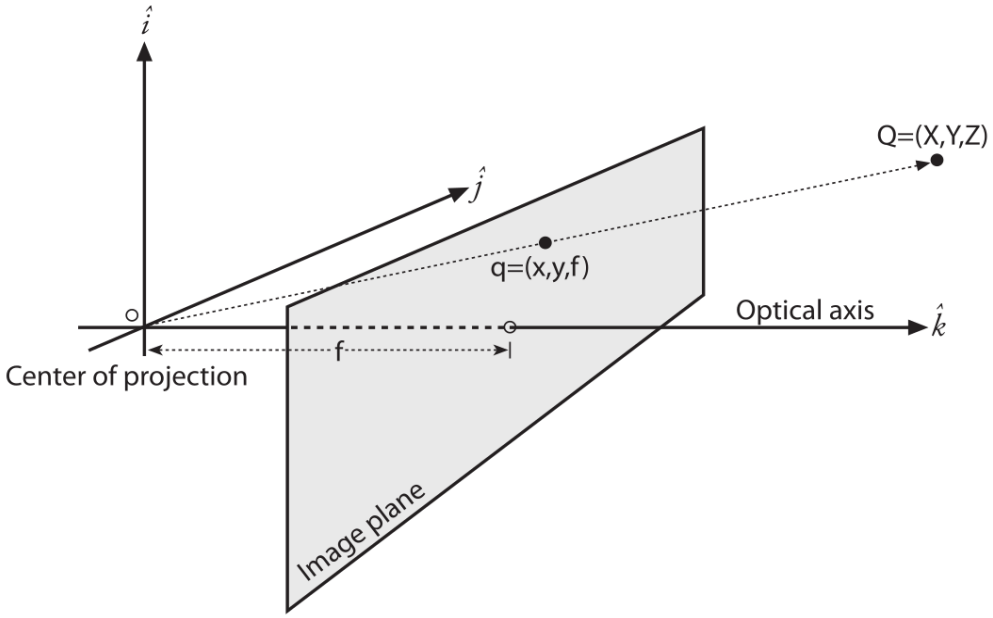


Figure: Pin-hole camera model

Mathematically,camera(or projector) model:

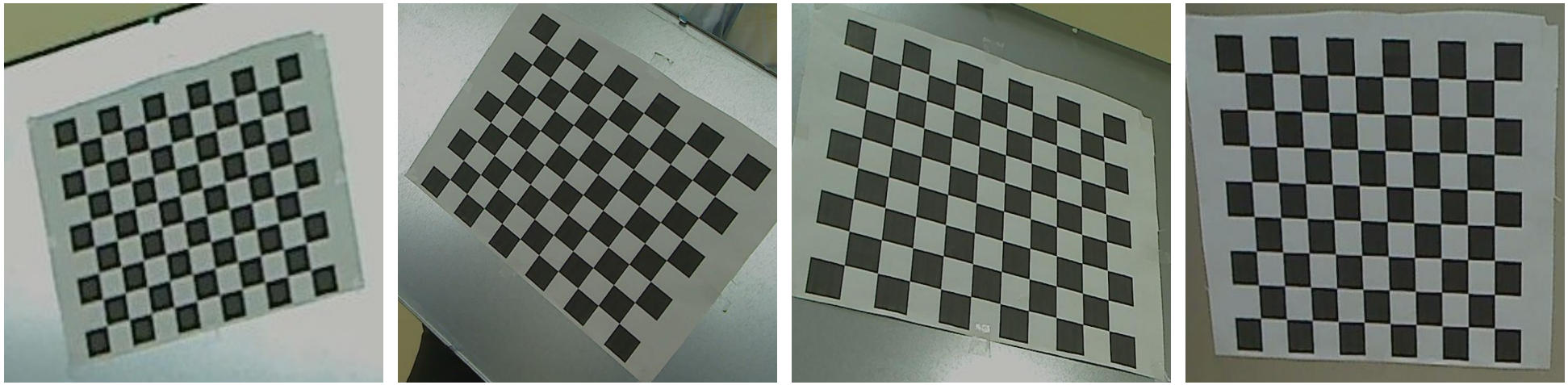
$$\begin{bmatrix} U_e^i \\ V_e^i \end{bmatrix} = A_c[R|T] \begin{bmatrix} X_w^i \\ Y_w^i \\ Z_w^i \end{bmatrix}$$
$$\begin{bmatrix} X_e^i \\ Y_e^i \end{bmatrix} = f(U_e^i, V_e^i) \tag{13}$$

OpenCV camera calibration algorithm is used which minimizes:

$$\varepsilon = \sum_{i=1}^{N} \left[\sqrt[2]{(observed_x - projected_x)^2 + (observed_y - projected_y)^2} \right] \tag{14}$$

Extrinsic parameters that minimize reprojection error are considered as the *optimal* pose(Rotation & translation) parameters.

Camera calibration



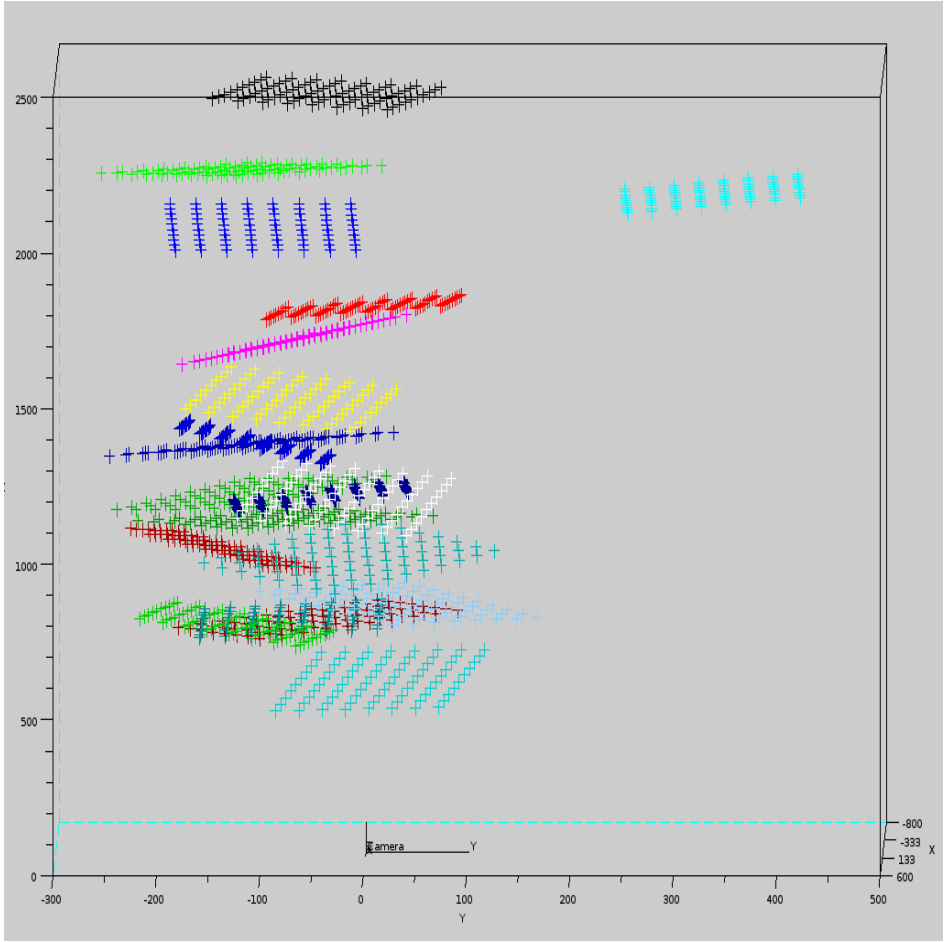
(a) (b) (c) (d)

Figure: Some views used for camera calibration

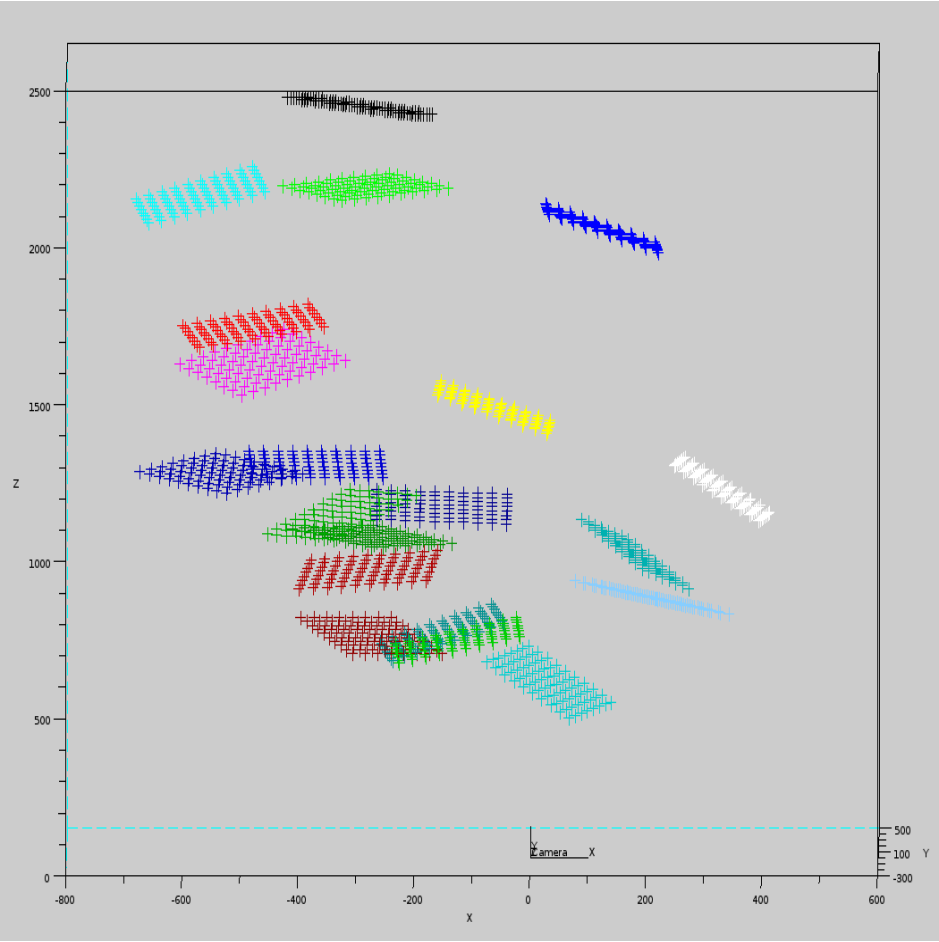
Parameter	Estimated value
f_x	1362.152
f_y	1372.189
c_x	803.884
c_y	590.066
k_1	0.073
k_2	-0.143
Reprojection error	0.195

Table: Estimated Camera intrinsic model parameters

SciLab script was developed to visualize calibration results for camera and projector calibration.



(a) View along X-axis

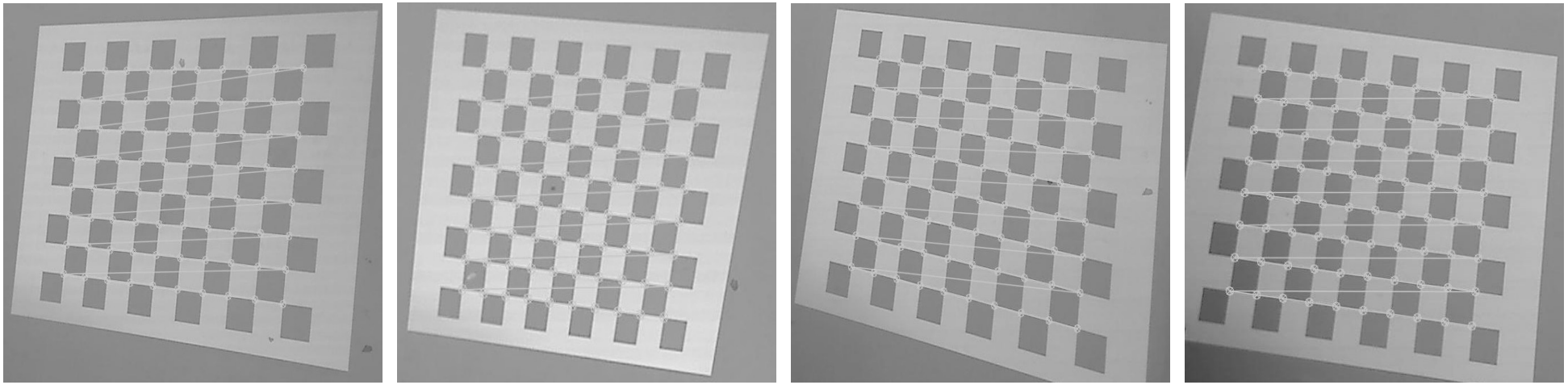


(b) View along Y-axis

Figure: Visualization of camera calibration results

Projector calibration

1. Projector assumed as *inverse camera*.
2. Camera used as a feedback device which will provide the 3D coordinates for the 2D features projected by the projector.

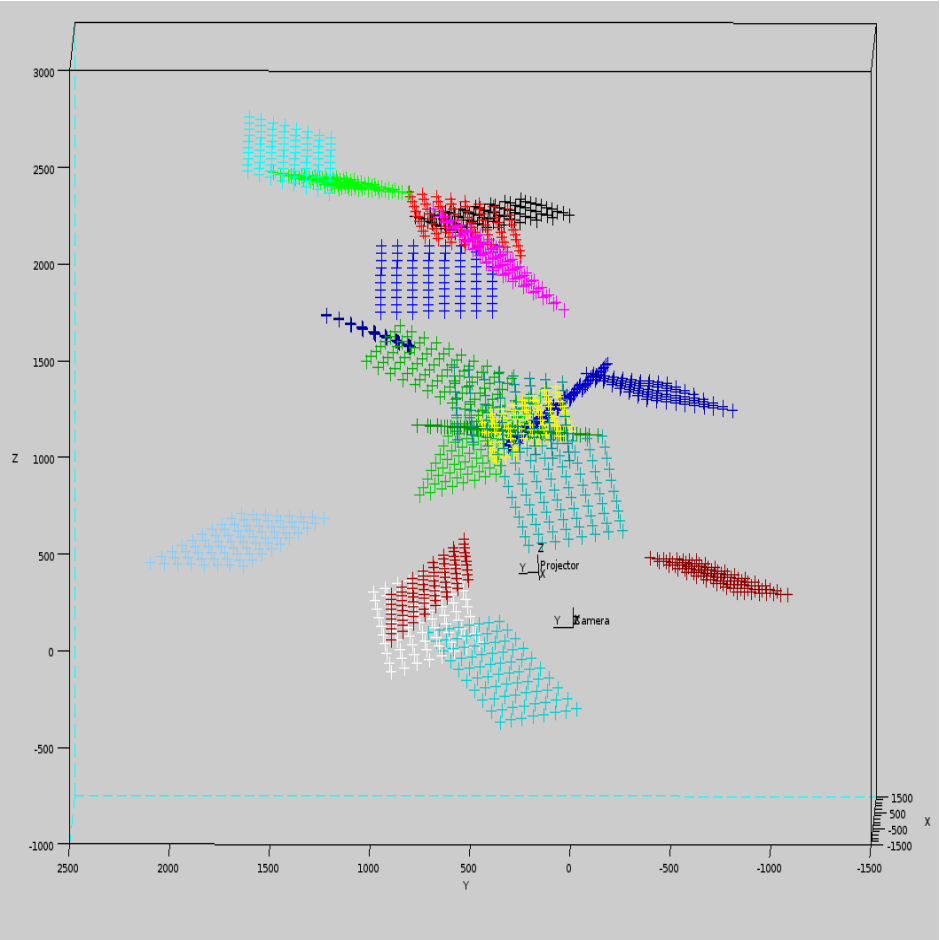


(a) (b) (c) (d)

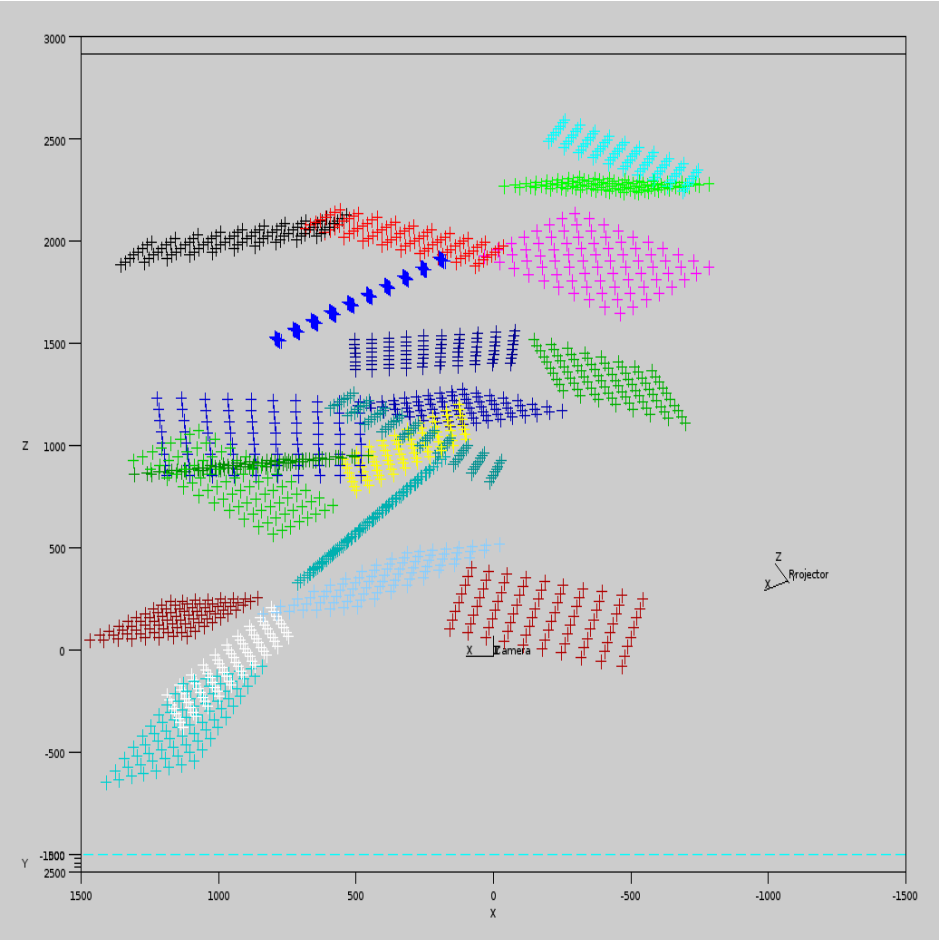
Figure: Some view used for projector calibration

Parameter	Estimated value
f_x	2261.710
f_y	2262.799
c_x	522.666
c_y	713.840

Table: Estimated Projector intrinsic model parameters



(a) View along X-axis



(b) View along Y-axis

Figure: Visualization of projector calibration results

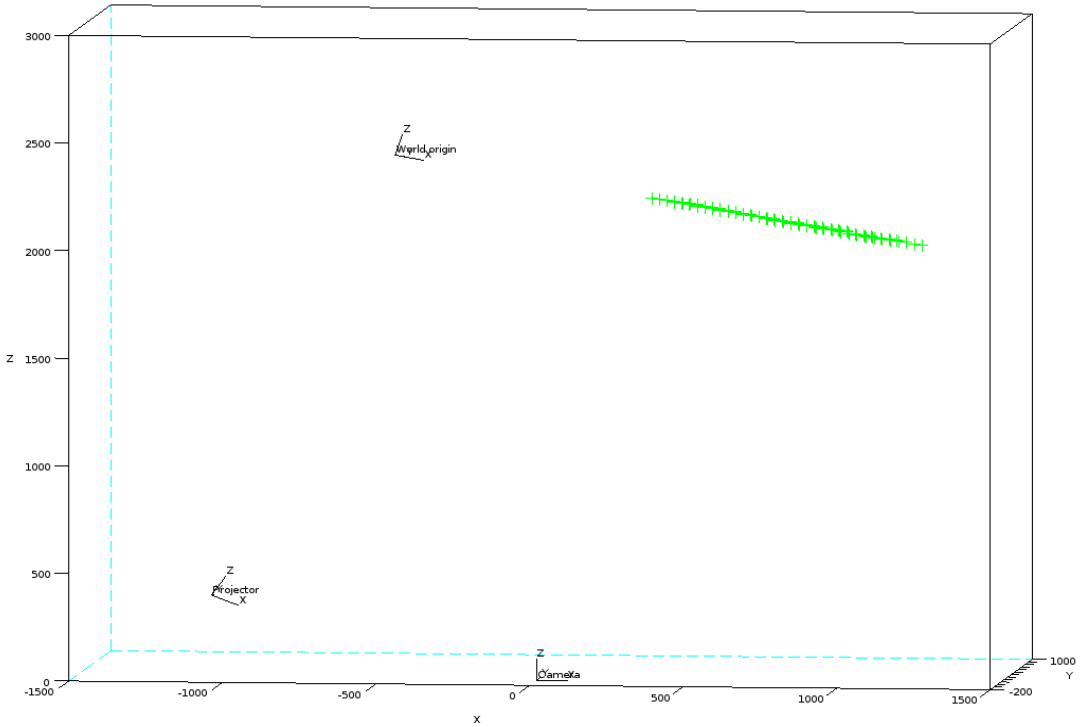
Projector-camera extrinsic calibration

Estimating rotation and translation vectors for transforming a point in projector coordinate system to camera coordinate system.

$$\begin{bmatrix} P_c \\ P_p \end{bmatrix} = \begin{bmatrix} R_{wc} * P_w \\ R_{wp} * P_w \end{bmatrix} + \begin{bmatrix} T_{wc} \\ T_{wp} \end{bmatrix} \tag{15}$$

hence,

$$P_c = \underbrace{(R_c * R_p^{-1})}_{R_{pc}} * P_p + \underbrace{(T_c - R_c * R_p^{-1} * P_p)}_{T_{pc}}$$



Triangulation module

To assign 3D coordinates to a point viewed by both camera and projector,
For camera,

$$\begin{bmatrix} w_c * u_i^c \\ w_c * v_i^c \\ w_c \end{bmatrix} = \begin{bmatrix} a_{1,1}^c & a_{1,2}^c & a_{1,3}^c & a_{1,4}^c \\ a_{2,1}^c & a_{2,2}^c & a_{2,3}^c & a_{2,4}^c \\ a_{3,1}^c & a_{3,2}^c & a_{3,3}^c & a_{3,4}^c \end{bmatrix} \begin{bmatrix} x_i^w \\ y_i^w \\ z_i^w \\ 1 \end{bmatrix} \tag{16}$$

Similarly,for projector,

$$\begin{bmatrix} w_c * u_i^p \\ w_c * v_i^p \\ w_p \end{bmatrix} = \begin{bmatrix} a_{1,1}^p & a_{1,2}^p & a_{1,3}^p & a_{1,4}^p \\ a_{2,1}^p & a_{2,2}^p & a_{2,3}^p & a_{2,4}^p \\ a_{3,1}^p & a_{3,2}^p & a_{3,3}^p & a_{3,4}^p \end{bmatrix} \begin{bmatrix} x_i^w \\ y_i^w \\ z_i^w \\ 1 \end{bmatrix} \tag{17}$$

In matrix form it can be written as,

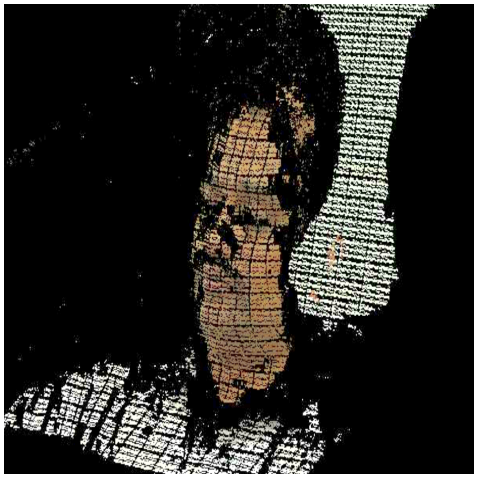
$$PV = F \tag{18}$$

where,

$$\text{P}=\begin{bmatrix} (a_{1,1}^c - u_i^c a_{3,1}^c) & (a_{1,2}^c - u_i^c a_{3,2}^c) & (a_{1,3}^c - u_i^c a_{3,3}^c) \\ (a_{2,1}^c - v_i^c a_{3,1}^c) & (a_{2,2}^c - v_i^c a_{3,2}^c) & (a_{2,3}^c - v_i^c a_{3,3}^c) \\ (a_{1,1}^p - u_i^p a_{3,1}^p) & (a_{1,2}^p - u_i^p a_{3,2}^p) & (a_{1,3}^p - u_i^p a_{3,3}^p) \\ (a_{2,1}^p - v_i^p a_{3,1}^p) & (a_{2,2}^p - v_i^p a_{3,2}^p) & (a_{2,3}^p - v_i^p a_{3,3}^p) \end{bmatrix}, V = \begin{bmatrix} x_i^w \\ y_i^w \\ z_i^w \end{bmatrix}, F = \begin{bmatrix} a_{3,4}^c u_i^c - a_{1,4}^c \\ a_{3,4}^c v_i^c - a_{2,4}^c \\ a_{3,4}^p u_i^p - a_{1,4}^p \\ a_{3,4}^p v_i^p - a_{2,4}^p \end{bmatrix}$$

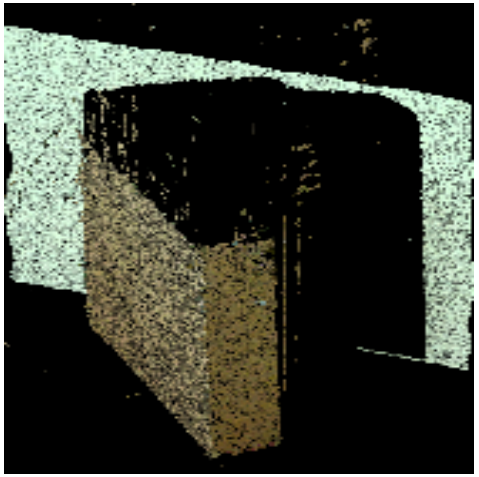
Results of applying equation $PV = F$,

(c) 2D face image



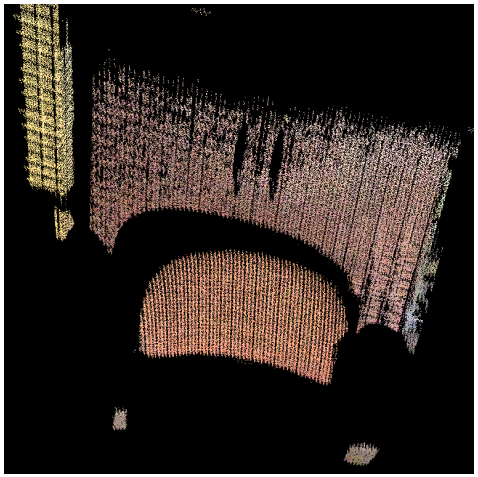
(d) 3D reconstruction of face

(e) 2D image of box in front of wall



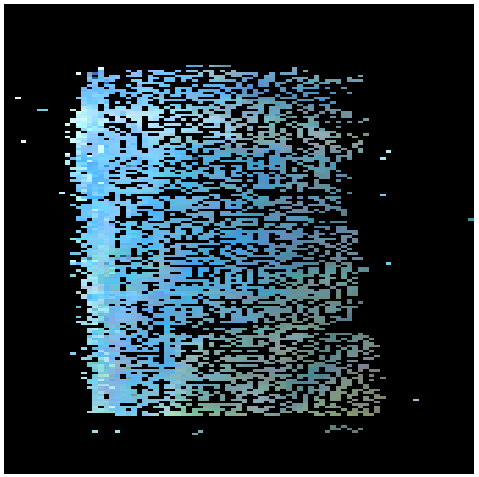
(f) 3D reconstruction of box in front of wall

(g) 2D image of a chair with background



(h) 3D reconstruction of chair with background

(i) A cup



(j) 3D reconstruction of cup

Figure: Some 3D reconstruction results