CS6360: ATML

Proof of Non-Triviality of ρ

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The (Very Simplified) Proof Statement

Theorem

If there exists group representation ρ, encoder h and decoder d that minimize the combined prediction and reconstruction losses, then ρ is non-trivial

The Losses

Definition (N-Step Prediction Loss)

$\mathcal{L}^N_{\text{pred}}(\rho, h) = \sum_{t=2}^{N+1} ||h(o_t) - (\prod_{i=1}^{t-1} \rho(g_i))h(o_0)||_2^2$

Definition (N-Step Reconstruction Loss)

$\mathcal{L}^N_{\text{rec}}(\rho, h, d) = \sum_{t=t}^{N+1} ||o_t - d((\prod_{i=1}^{t-1} \rho(g_i))h(o_1))||_2^2$

Definition (Combined Loss)

$\mathcal{L}^N_{\text{pred}} + \gamma \mathcal{L}_{\text{rec}}$ where $\gamma > 0$

The World-Statespace Assumption

There's one minor assumption, namely that the world state manifold W is diffeomorphic (via some function $m : W \to W*$) to a finite-dimensional real vector space $W*$, and that the group G has a group representation $\rho* : G \to GL(W*)$.

The Lie-Algebra - Lie-Group Correspondence

The Lie algebra of a lie group is its tangent space at the identity. One neat fact about this correspondence is that if we have a representation of the lie algebra in some vector space, then, through matrix exponentiation, we can arrive at the representation of the lie group on that same vector space. Thus, we can learn the representation of the group ρ by proxy, instead learning the representation for the algebra ϕ.
The intuition behind this is that the representation of the algebra is a vector space, while that of the group can only be guaranteed to be a manifold. Learning a vector space through backpropagation is, the authors assert, easier than learning a manifold.

Proof Outline

- If (ρ, h) minimizes only the prediction loss and h is allowed to be non-injective, then ρ can be trivial.
- If (ρ, h, d) minimizes the $0-$step reconstruction loss, then h *must* be injective.
- If (ρ, h, d) minimizes the combined loss, then the $0-$step reconstruction loss must be zero.

Proof

Proof