# HOMEWORK 12

1. **Given a graph G and two vertex sets A and B, let E(A,B) denote the set of edges with one endpoint in A and one endpoint in B. The Max Equal Cut problem is defined as follows: Given an undirected graph G(V, E), where V has an even number of vertices, find an equal partition of V into two sets A and B, maximizing the size of E(A,B). Provide a factor 2-approximation algorithm for solving the Max Equal Cut problem and prove the approximation ratio for the algorithm. (15 points)**
   **Hint: Iteratively build A and B. At each step consider a pair of vertices, and put one in each of A and B to ensure they are equal. Decide which of the two vertices goes to which side to greedily maximize E(A, B) in that step.**
   **Ans:**
   Here, we start by dividing the vertices of the graph into two sets A and B. Now, consider a pair of unassigned vertices. Assign one pair to set A and other pair to set B to ensure that they are equal sized. This should maximize the number of edges between A and B. Check which pair does that. Repeat this till all vertices are assigned.

   If Ea is the number of edges in our algorithm and Eo is number of edges in the optimal solution, we are choosing a pair of vertices and assigning one to set A and the other to set B to maximize E(A,B). This means that for each pair of vertices, we are adding at least one edge to Ea. Hence, when the algorithm terminates, Ea is at least half the size of the optimal solution. Hence Ea >= (Eo)/2 or Eo = 2Ea.
   Hence, the approximation ratio for our algorithm is 2.

2. **650 students in the "Analysis of Algorithms" class in 2024 Spring take the exams onsite. The university provided 7 classrooms for exam use, each classroom i can contain Ci (capacity) students. The safety level of a classroom is defined as αi(Ci − Si), where αi is the known parameter for classroom i, and Si is the actual number of students placed to take the exams in the classroom. We want to maximize the total safety level of all the classrooms. Besides, to guarantee good spacing, the number of students in a classroom should not exceed half of the capacity of each classroom.**
   **Express the problem as a integer linear programming problem to obtain the number of students to be placed in each room. You DO NOT need to numerically solve the program. (15 points)**
   **a) Define your variables.**
   **Ans:**
   Here, the variables are Si (actual number of students placed to take the exams in the classroom) ranging from 1 to 650.

   **b) Write down the objective function.**
   **Ans:**
   Maximize $\Sigma^{7}_{i=1}$ αi(Ci − Si).

**c) Write the constraints.**

**Ans:**

1. Total number of students should be greater than or equal to 0:

$$Si >= 0$$

2. Total number of students placed in all classrooms cannot exceed the total number of students:

$$\Sigma^{7}_{i=1} Si = 650$$

3. Total number of students should not exceed half of the capacity of each classroom:

$$Si <= Ci/2$$

**3. Write down the problem of finding a Min-s-t-Cut of a directed network with source s and sink t as problem as an Integer Linear Program. (20 pts)**

**a) Define your variables.**

**Ans:**

Here, the variables are c(i), where c(i) is the flow over the edge i for all edges in the graph flowing from s or into t.

**b) Write down the objective function.**

**Ans:**

Minimize $\Sigma_i$ c(i).

**c) Write the constraints.**

**Ans:**

1. Total flow should be conserved:

$$(\Sigma_i \ c(i) \text{ from S}) – (\Sigma_i \ c(i) \text{ into T}) = 0$$

2. Lower bound:

   If Li is the lower bound, then c(i) >= Li for all edges i.