Pre-report for ELE456:
"Resolving photon number states in a superconducting circuit"

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This is the pre-report for the course ELE456 about the project on understanding the paper[1].

I. UNDERSTANDING THE PAPER

A. The system of the experiment

The most important result in this paper is the evidence of discreteness of the electromagnetic field energy (photon) on a system of coupled superconducting qubit and wave-guide cavity. The Hamiltonian in the paper is:

$$H_0 = \hbar\omega_r(a^\dagger a + 1/2) + \hbar\omega_a\sigma_z/2 + \hbar\chi(a^\dagger a + 1/2)\sigma_z \tag{1}$$

Which is actually the diagonalized form of Jaynes-Cummings (JC) model, with $2\chi = 2g_0^2/\Delta$:

$$H_{\mathrm{JC}} = \hbar\omega_r(a^\dagger a + 1/2) + \hbar\omega_a\sigma_z/2 + \hbar g_0(a^\dagger\sigma^- + a\sigma^+) \tag{2}$$

in the large detuning limit ($\Delta = |\omega_r - \omega_a| \gg g_0$), with transformation matrix:

$$U = \exp\left[\frac{g_0}{\Delta}(a\sigma^+ - a^\dagger\sigma^-)\right] \tag{3}$$

(Eq. (12) in [2], $H_0 = UH_{\mathrm{JC}}U^\dagger$).

The Hamiltonian in Eq. (1) commutes with "photon number" operator $a^\dagger a$ and σ_z [1]. This means that energy eigenstates are also eigenstates of "photon number" and spin state. For given "photon number" $a^\dagger a\,|n\rangle = n\,|n\rangle$, the Hamiltonian is reduced to $H = \hbar[\omega_a + (2n + 1)\chi]\sigma_z/2$. This can can be viewed as a photon-number dependent frequency shift in the qubit's transition frequency: $\Delta\omega = (2n+1)\chi$. Thus, photon number discreteness can be shown from the discreteness of frequency shift.

[1] Strictly speaking they are not actually photon number and Pauli matrix for the spin, but $Ua^\dagger aU^\dagger$ and $U\sigma_zU^\dagger$ in JC model, though the difference is small in the large detuning limit.

B. Pumping and readout

A more precise description of the system should also include the decay terms (described by the coupling of the qubit and the cavity mode with reservoir modes), which can be calculated from master equations. To make up for the dissipation, a coherent external pumping field is required, described by:

$$H_{\rm rf} = \hbar\varepsilon_{\rm rf}(a^{\dagger}{\rm e}^{-{\rm i}\omega_{\rm rf}t} + a{\rm e}^{{\rm i}\omega_{\rm rf}t}) \tag{4}$$

This is part of the bare Hamiltonian (should be added to Eq. (2)). In the basis of Eq. (1) this becomes $UH_{\rm rf}U^{\dagger}$, and is actually coupled to both the "cavity" and the "qubit". However, the frequency $\omega_{\rm rf}$ is only near resonance with ω_r, making it effectively coupled to the cavity only.

To measure the spectrum of the system, a sweeping signal (at the frequency ω_s) is required. This measuring signal behaves just like Eq. (4) except for the signal consists of pulses ($\varepsilon_s(t)$):

$$H_s = \hbar\varepsilon_s(t)(a^{\dagger}{\rm e}^{-{\rm i}\omega_s t} + a{\rm e}^{{\rm i}\omega_s t}) \tag{5}$$

And the sweeping frequency ω_s range covers the effective "qubit" frequencies we are interested in $\omega_a + (2n+1)\chi$. In this way, the spectrum of the system can be readout by observing the reduction of the transmitted sweeping signal due to the coupling of the signal and the system (Details are shown in Section VI in [2]).

II. KEY RESULTS TO REPRODUCE

We plan to reproduce the numerical result in Fig. 3 (red lines), and try to see if we can reproduce the difference shown in Fig. 4 for thermal and coherent distributions.

III. METHODS FROM THE CLASS

The method we shall use for the project includes:

1. The Jaynes-Cummings model to describe the system

2. Quantized electromagnetic field to describe the cavity mode

3. Semi-classical coupling of electromagnetic field pumping signal and sweeping signal) and the system

4. Master equation to describe the dissipation and the linewidth

IV. ADDITIONAL CONCEPTS

To fully understand the experiment system, it is necessary to lean about the Josephson junction and superconducting qubit.

$$H = 4E_C(n - n_g)^2 - E_J \cos\phi; \qquad \text{with}[n, \phi] = \mathrm{i} \tag{6}$$

The difference of this superconducting qubit Hamiltonian and two-level system is discussed in the comments of Fig. 2 in [1].

To understand the transmitted amplitude in the measurement, we probably need to explore the couple-mode theory.

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[1] DI Schuster, AA Houck, JA Schreier, A Wallraff, JM Gambetta, A Blais, L Frunzio, J Majer, B Johnson, MH Devoret, et al. Resolving photon number states in a superconducting circuit. *Nature*, 445(7127):515–518, 2007.

[2] Alexandre Blais, Ren-Shou Huang, Andreas Wallraff, Steven M Girvin, and R Jun Schoelkopf. Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation. *Physical Review A*, 69(6):062320, 2004.