Resolving photon number states in a superconducting circuit

Ming Lyu, Elena de la Hoz Lopez-Collado

Final projects for ELE456 at Princeton

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Outline

- Resolve photon number states in a circuit QED
- System: superconducting qubit + microwave transmission line
- Strong dispersive regime
- Spectroscopic measurements:
  Qubit's spectral lines different for each photon number state

The system: circuit QED + cavity QED

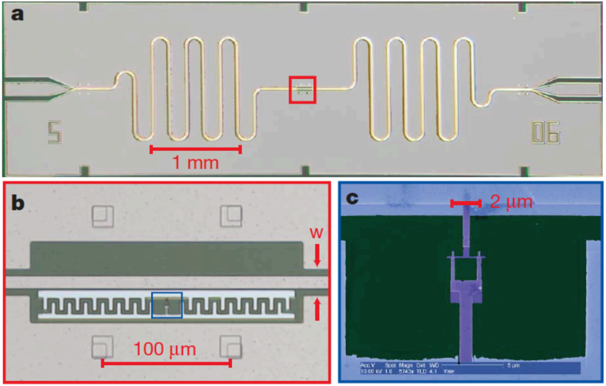


Image from Schuster, D. I., et al. "Resolving photon number states in a superconducting circuit." Nature 445.7127 (2007): 515-518.[3]

The system: simplified

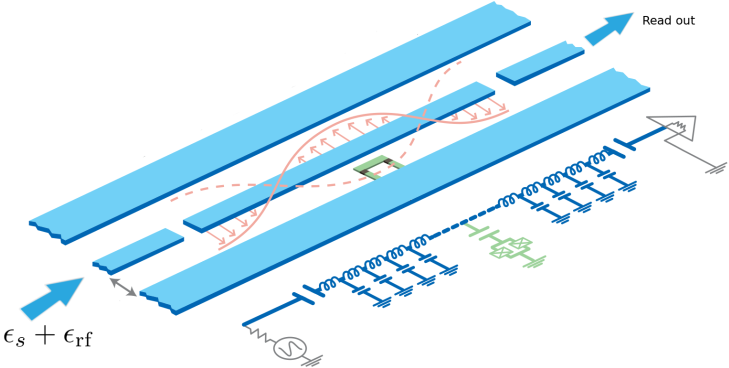


Image from Blais, Alexandre, et al. "Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation." Physical Review A 69.6 (2004): 062320.[1]

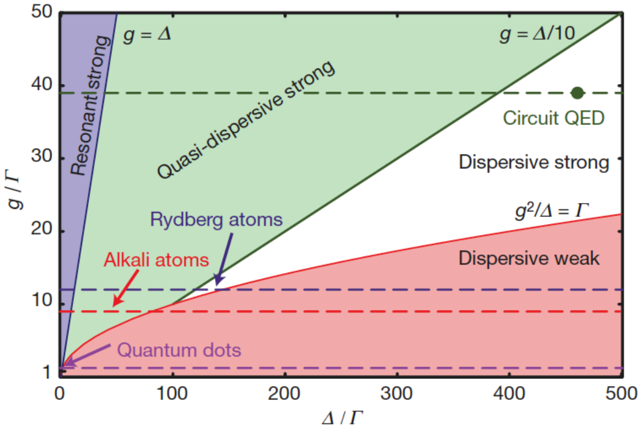
Cavity QED: the Hamiltonian

Hamiltonian

$$H = \omega_r \left(a^\dagger a + \frac{1}{2}\right) + \omega_a \frac{\sigma^z}{2} + g\left(a^\dagger \sigma^- + a\sigma^+\right)$$

- ω_r: cavity resonance frequency
- ω_a: qubit transition frequency
- g: strength qubit-photon coupling
- $\Delta = \omega_r - \omega_a$: detuning between qubit and cavity

Strong Dispersive Regime



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Strong dispersive Regime: Diagonalization

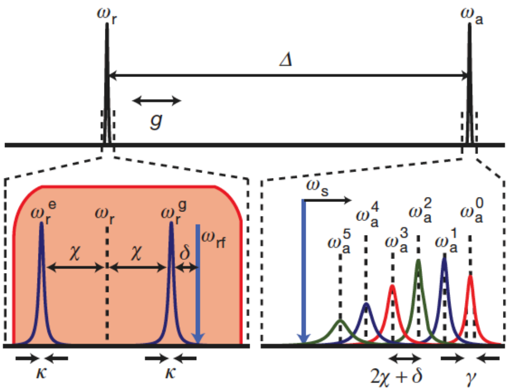
- Transformation:

$$U = \exp\left(\frac{g}{\Delta}\left(a\sigma^{+} - a^{\dagger}\sigma^{-}\right)\right)$$

- Hamiltonian to first order in $\dfrac{g}{\Delta}$ (dispersive regime):

$$\begin{aligned} H_0 &= U\,H\,U^{\dagger} \\ &\simeq \omega_r\left(a^{\dagger}a + \frac{1}{2}\right) + \omega_a\frac{\sigma^z}{2} + \chi\left(a^{\dagger}a + \frac{1}{2}\right)\frac{\sigma^z}{2} \end{aligned}$$

where $\chi = g/\Delta^2$



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Driving terms

- To conduct a measurement we first drive the cavity:

$$H_{\mathsf{rf}} = \epsilon_{\mathsf{rf}} \left(a^\dagger \mathrm{e}^{-\mathrm{i}\omega_{\mathsf{rf}} t} + a \mathrm{e}^{\mathrm{i}\omega_{\mathsf{rf}} t} \right)$$

  with ω_{rf} near ω_r

- The frequency shift of the qubit measured with a sweeping signal

$$H_s = \epsilon_s \left(a^\dagger \mathrm{e}^{-\mathrm{i}\omega_s t} + a \mathrm{e}^{\mathrm{i}\omega_s t} \right)$$

  with ω_s near ω_a

- Note that relative strength of ϵ_s is not mentioned. We treat it as a perturbation.

Rotating frame and Rotating wave approximation

- Applying the transformation

$$U = \exp\left[\frac{g}{\Delta}\left(a\sigma^{+} - a^{\dagger}\sigma^{-}\right)\right]$$

- And moving to the rotating frame:

$$U_I = \exp\left[\mathrm{i}t\left(\omega_{\mathsf{rf}}a^{\dagger}a + \omega_s\sigma^{z}/2\right)\right]$$

Under rotating frame, H_{rf} and H_s are (with RWA):

$$H_{\mathsf{rf}} = \epsilon_{\mathsf{rf}}\left(a^{\dagger} + a\right)$$
$$H_s = \left(\frac{g}{\Delta}\right)\epsilon_s\left(\sigma^{+} + \sigma^{-}\right)$$

Final Hamiltonian and collapse operators

- Full Hamiltonian:

$$
\begin{aligned}
H = & \omega_r \left(a^\dagger a + \frac{1}{2} \right) + \omega_a \frac{\sigma^z}{2} + \chi \left(a^\dagger a + \frac{1}{2} \right) \frac{\sigma^z}{2} \\
& - \left(\omega_{\text{rf}} a^\dagger a + \omega_s \frac{\sigma^z}{2} \right) + \epsilon_{\text{rf}} \left(a^\dagger + a \right) + \epsilon_s \frac{g}{\Delta} \left(\sigma^+ + \sigma^- \right)
\end{aligned}
$$

- Collapse operator:
  - Collapse operators cavity: $\sqrt{\kappa\left(1+n_{\text{th}}\right)}a$, $\sqrt{\kappa n_{\text{th}}}a^\dagger$
  - Collapse operator qubit: $\sqrt{\gamma}\sigma^-$
  - Dephasing: $\sqrt{\gamma_\phi}\sigma^z$

Measurement

- In the experiment, the transmitted amplitude at frequency ω_{rf} is the main observable under steady state.

Steady state

$$\dot{\rho}_s = 0 = -\mathrm{i}[H, \rho_s] + \sum_n \left(2C_n \rho_s C_n^\dagger - \{\rho_s, C_n^\dagger C_n\}\right)$$

- What they really measure is the expectation of the electrical field $E \propto \langle a + a^\dagger \rangle$ [2] on a given frequency

$$E \propto \langle a + a^\dagger \rangle = \mathrm{Tr}[\rho_s (a + a^\dagger)]$$

Property of the cavity: Analytical

- Without the qubit, the cavity state is equivalently a damped harmonic oscillator with driving

$$H = \delta a^\dagger a + \epsilon(a + a^\dagger)$$

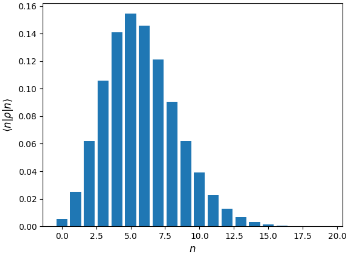
Collapse operators: $\sqrt{\kappa(n_{\text{th}} + 1)}a$ and $\sqrt{\kappa n_{\text{th}}}a^\dagger$

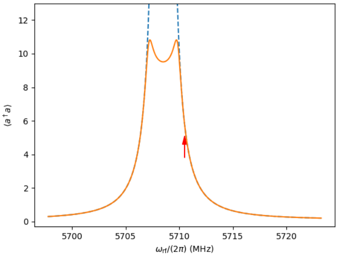
- When it's off resonant, its steady state is not but approximately a coherent state
- Analytically the photon number expectation value is

$$\bar{n} = \frac{\epsilon^2}{\delta^2 + \kappa^2/4} + n_{\text{th}}$$

Property of the cavity: Numerical

- Numerically, a truncate on Fock space is needed
- To check the validity of the truncate, we plot the photon distribution and frequency response of the cavity.



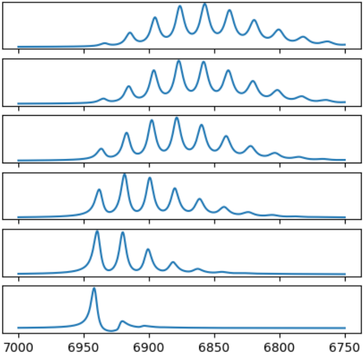


Direct spectroscopic observation of quantized cavity photon number

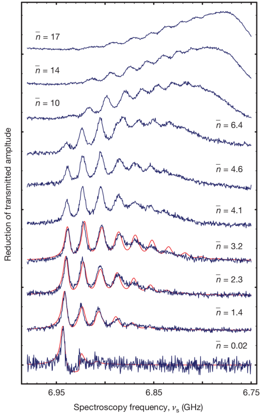
For a fixed driving ϵ_{rf}, plot the reduction $V_0 - \langle a^\dagger + a \rangle_{ss}$ v.s. ω_s.

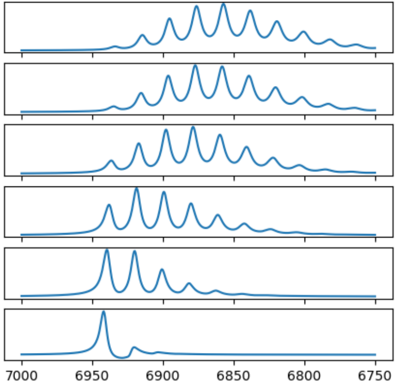
ϵ_{rf} is labeled by $\bar{n}$ with relationship:

$$\bar{n} = n_{\text{th}} + \frac{\epsilon_{\text{rf}}^2}{\delta^2 + \kappa^2/4}$$



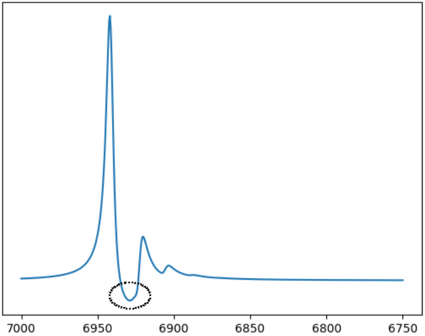
Direct spectroscopic observation of quantized cavity photon number: compare





- Fits well with small $\bar{n}$, but other noise becomes significant for larger $\bar{n}$

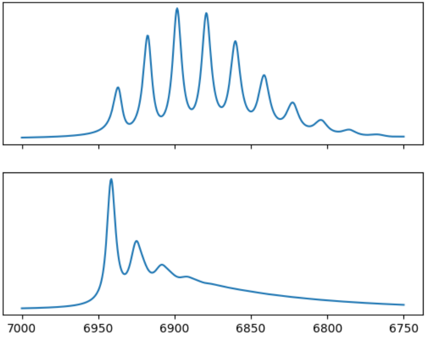
Strengthen?



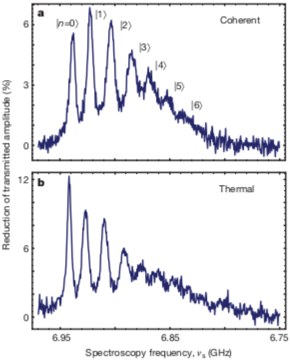
For small signal, there's a range where the transmitted amplitude is increased. We'll explain it later.

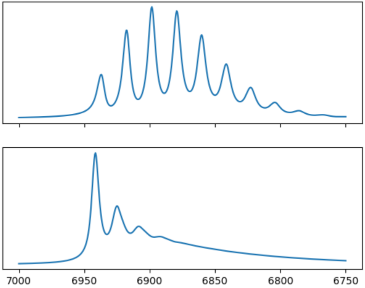
Thermal Drive

- Thermal Drive is equivalent to setting n_{th} in collapse operator to the driving average, with small ϵ_{rf} to show the phase lock-in at the given frequency.



Thermal Drive: compare





- Note that there's no thermal drive theory fitting. Our results tracks fewer peaks, but this depends on how they do the measurement, which is not mentioned in the paper.

Discussion: The picture of what happens

- The peaks shows discreteness in the photon state in the cavity.
- Exciting the qubit making the cavity off-resonance, which results in the reduction?

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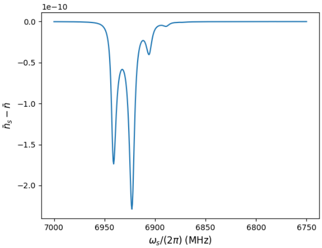
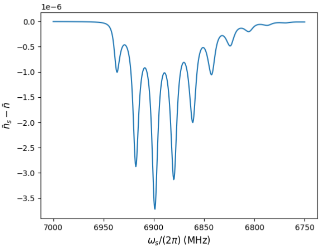
Discussion: The picture of what happens

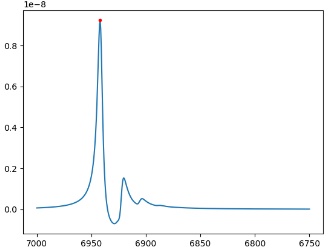
- The peaks shows discreteness in the photon state in the cavity.
- Exciting the qubit making the cavity off-resonance, which results in the reduction? **NOT TRUE**
- Expected photon number increases at the peaks!

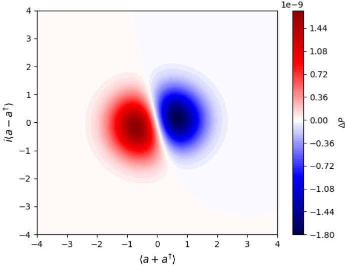




What happens

- Excitation of the qubit is not the dominant effect, but the polarization of the qubit, which twists the cavity photon state.
- This can be shown from the difference of the Wigner function (quasiprobability distribution on phase diagram) with/without the signal field.





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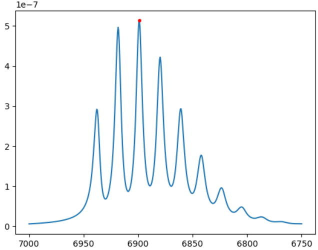

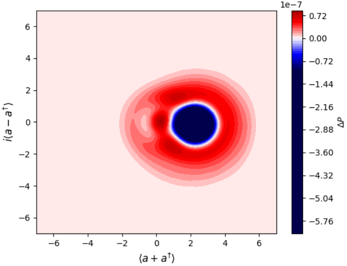

Conclusion

- Existence of photons in the cavity shifts the qubit frequency, which can be read out by applying the sweeping signal to see the qubit spectrum
- The way the qubit state affects the cavity state is trick: more like polarization of qubit affect the wave function

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- The way the qubit state affects the cavity state is trick: more like polarization of qubit affect the wave function

- "Approximately" the peak hight can be interpreted as the photon number distribution: "Resolving" photon number
- Potential application of quantum nondemolition measurement (QND)

Reference

- Alexandre Blais, Ren-Shou Huang, Andreas Wallraff, Steven M Girvin, and R Jun Schoelkopf.

  Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation.

  Physical Review A, 69(6):062320, 2004.

- David Isaac Schuster.

  Circuit quantum electrodynamics.

  Yale University, 2007.

- DI Schuster, AA Houck, JA Schreier, A Wallraff, JM Gambetta, A Blais, L Frunzio, J Majer, B Johnson, MH Devoret, et al.

  Resolving photon number states in a superconducting circuit.

  Nature, 445(7127):515–518, 2007.

The End...

Thank you for listening!

Q & A

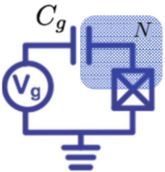
Josephson junction and superconducting circuit

Nature 445, 515-518

Ming, Elena

- The Hamiltonian

$$H = E_c(N - N_g)^2 - E_J \cos\delta$$

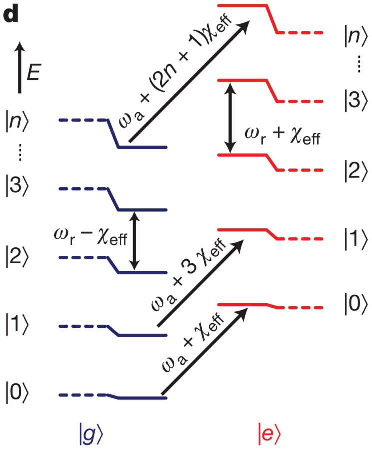


- Commutation relationship: $[\delta, N] = \mathrm{i}$, this means $\mathrm{e}^{\pm \mathrm{i}\delta} |n\rangle = |n \pm 1\rangle$
  - Approximately two-level system: $0 \leq N_g \leq 1$, $N = 0, 1$:

$$H = -E_c(1 - 2N_g)\sigma^z - \frac{1}{2} E_J \sigma^x$$

  - With coupling, $N_g \longrightarrow N_g + CV_0(a + a^\dagger)/2e$
  - Choose eigen basis at degeneracy point ($N_g = 1/2$), we can have JC model up to some constants.

Energy levels



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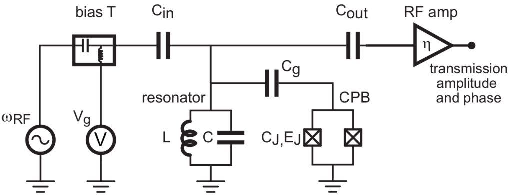
Measurement

In the experiment, the transmitted amplitude at frequency ω_{rf} is the main observable. The exact way to measure can be found in Schuster's thesis [2]:



- What we really measure is the expectation of the voltage, or electrical field $E \propto \langle a + a^\dagger \rangle$

Wigner function (Wigner quasiprobability distribution)

- Wigner function is an analogue of classical probability distribution on phase space

Definition: Wigner function

$$
\begin{aligned}
P(x,p) &\equiv \frac{1}{(2\pi\hbar)^n}\int \mathrm{d}^n y\, \psi(x-y/2)\psi^*(x+y/2)\mathrm{e}^{\mathrm{i}p\cdot y/\hbar} \\
&= \frac{1}{(2\pi\hbar)^n}\int \mathrm{d}^n y\, \langle x-y/2|\rho|x+y/2\rangle\, \mathrm{e}^{\mathrm{i}p\cdot y/\hbar}
\end{aligned}
$$

- Marginals:

$$
\int \mathrm{d}^n p\, P(x,p) = \langle x|\rho|x\rangle
$$

$$
\int \mathrm{d}^n x\, P(x,p) = \langle p|\rho|p\rangle
$$

Wigner function: properties

- Inner product → overlap:

$$|\langle\psi|\varphi\rangle|^2 = 2\pi\hbar \int \mathrm{d}^n x \, \mathrm{d}^n p \, P_\psi(x,p) P_\varphi(x,p)$$

- Operator Wigner transformation and expectation values:

$$g(x,p) \equiv \int \mathrm{d}^n y \, \langle x - y/2 | G | x + y/2 \rangle \, \mathrm{e}^{\mathrm{i}p\cdot y/\hbar}$$

$$\mathrm{Tr}[\rho G] = \int \mathrm{d}^n x \, \mathrm{d}^n p \, P(x,p) g(x,p)$$

- Cauchy inequality for pure state

$$-\frac{2}{h} \leq P(x,p) \leq \frac{2}{h}$$