Resolving photon number states in a superconducting circuit

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Final projects for ELE456 at Princeton

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Outline

- System sensitive to number of photons
- System: superconducting qubit + microwave transmission line
- Strong dispersive regime
- Spectroscopic measurements: Qubit's spectral lines different for each photon number state

Cavity QED

- Cavity QED (cQED) → interaction electromagnetic field modes with atoms (or qubits)

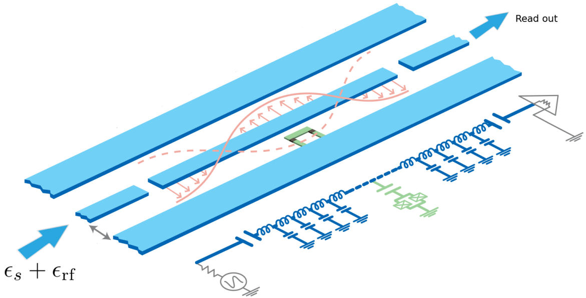


Image from Blais, Alexandre, et al. "Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation." Physical Review A 69.6 (2004): 062320.[1]

Cavity QED: the Hamiltonian

Hamiltonian

$$H = \omega_r \left(a^\dagger a + \frac{1}{2} \right) + \omega_q \frac{\sigma^z}{2} + g \left(a^\dagger \sigma^- + a \sigma^+ \right)$$

- ω_r: cavity resonance frequency
- ω_q: qubit transition frequency
- g: strength qubit-photon coupling

Strong Dispersive Regime

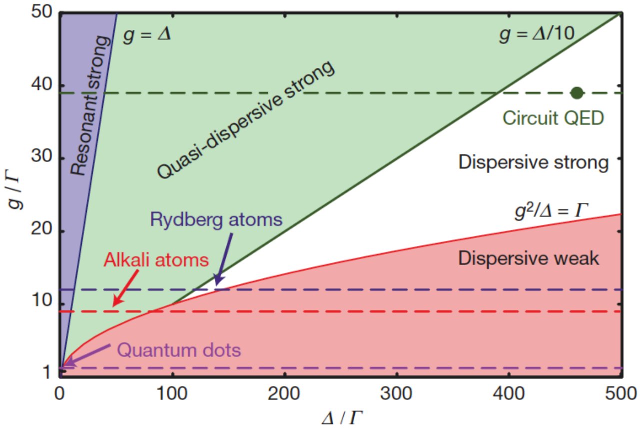


Image from Schuster, D. I., et al. "Resolving photon number states in a superconducting circuit." Nature 445.7127 (2007): 515-518.[2]

Strong dispersive Regime: Diagonalization

- Transformation:

$$U = \exp\left(\frac{g}{\Delta}\left(a\sigma^{+} - a^{\dagger}\sigma^{-}\right)\right)$$

- Hamiltonian to first order in $\dfrac{g}{\Delta}$:

$$\begin{aligned} H_0 &= U\,H\,U^{\dagger} \\ &\simeq \omega_r\left(a^{\dagger}a + \frac{1}{2}\right) + \omega_q\frac{\sigma^z}{2} + \chi\left(a^{\dagger}a + \frac{1}{2}\right)\frac{\sigma^z}{2} \end{aligned}$$

where $\chi = g/\Delta^2$

Driving terms

- To conduct a measurement we first drive the cavity:

$$H_{\mathrm{rf}} = \epsilon_{\mathrm{rf}} \left(a^\dagger e^{-i\omega_{\mathrm{rf}} t} + a e^{i\omega_{\mathrm{rf}} t} \right)$$

  with ω_{rf} near ω_r

- The frequency shift of the qubit measured with a sweeping signal

$$H_s = \epsilon_s \left(a^\dagger e^{-i\omega_s t} + a e^{i\omega_s t} \right)$$

  with ω_s near ω_q

- Note that relative strength of ϵ_s is not mentioned. We treat it as a perturbation.

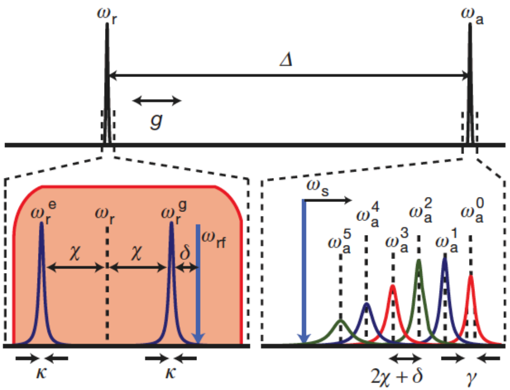


Image from Schuster, D. I., et al. "Resolving photon number states in a superconducting circuit." Nature 445.7127 (2007): 515-518.[2]

Rotating frame and Rotating wave approximation

- Applying the transformation

$$U = \exp\left[\frac{g}{\Delta}\left(a\sigma^{+} - a^{\dagger}\sigma^{-}\right)\right]$$

- And moving to the rotating frame:

$$U_I = \exp\left[i\left(\omega_{\mathsf{rf}} a^{\dagger}a + \omega_s \sigma^z/2\right)t\right]$$

H_{rf} and H_s are:

$$H_{\mathsf{rf}} = \epsilon_{\mathsf{rf}}\left(a^{\dagger} + a\right)$$

$$H_s = \left(\frac{g}{\Delta}\right)\epsilon_s\left(\sigma^{+} + \sigma^{-}\right)$$

Final Hamiltonian and collapse operators

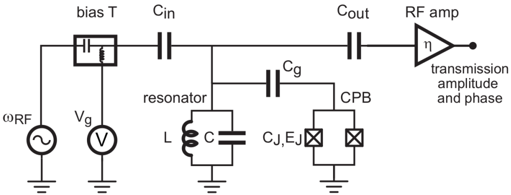
- Full Hamiltonian:

$$H = \omega_r \left(a^\dagger a + \frac{1}{2}\right) + \omega_q \frac{\sigma^z}{2} + \chi \left(a^\dagger a + \frac{1}{2}\right) \frac{\sigma^z}{2} - \left(\omega_{\mathrm{rf}} a^\dagger a + \omega_s \frac{\sigma^z}{2}\right) + \epsilon_{\mathrm{rf}} \left(a^\dagger + a\right) + \epsilon_s \frac{g}{\Delta} \left(\sigma^+ + \sigma^-\right)$$

- Collapse operator:
  - Collapse operators cavity: $\sqrt{\kappa\,(1 + n_{\mathrm{th}})}a$, $\sqrt{\kappa\,(n_{\mathrm{th}})}a^\dagger$
  - Collapse operator qubit: $\sqrt{\gamma}\sigma^-$
  - Dephasing: $\sqrt{\gamma_\phi}\sigma^z$

Measurement

In the experiment, the transmitted amplitude at frequency $\omega_{\rm rf}$ is the main observable. The exact way to measure can be found in Schuster's thesis [3]:



- What we really measure is the expectation of the voltage, or electrical field $E \propto \langle a + a^\dagger \rangle$

Property of the cavity: Analytical

- Without the qubit, the cavity state is equivalently a damped harmonic oscillator with driving

$$H = \delta a^\dagger a + \epsilon(a + a^\dagger)$$

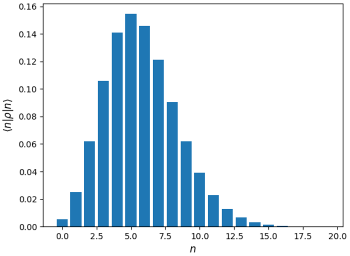
Collapse operators: $\sqrt{\kappa(n_{\text{th}} + 1)}a$ and $\sqrt{\kappa n_{\text{th}}}a^\dagger$

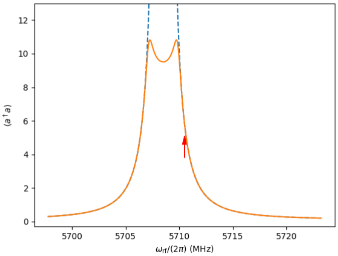
- When it's off resonant, its steady state is not but approximately a coherent state
- Analytically the photon number expectation value is

$$\bar{n} = \frac{\epsilon^2}{\delta^2 + \kappa^2/4} + n_{\text{th}}$$

Property of the cavity: Numerical

- Numerically, a truncate on Fock space is needed
- To check the validity of the truncate, we plot the photon distribution and frequency response of the cavity.



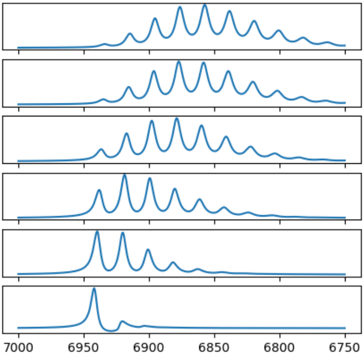


Direct spectroscopic observation of quantized cavity photon number

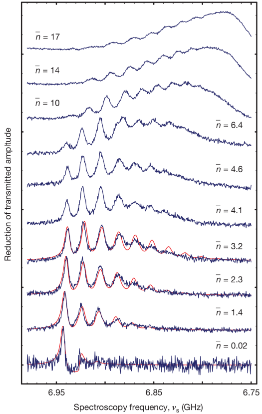
For a fixed driving ϵ_{rf}, plot the reduction $V_0 - \langle a^\dagger + a \rangle_{ss}$ v.s. ω_s.

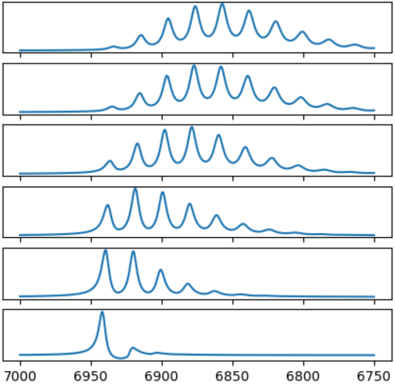
ϵ_{rf} is labeled by $\bar{n}$ with relationship:

$$\bar{n} = n_{\text{th}} + \frac{\epsilon_{\text{rf}}^2}{\delta^2 + \kappa^2/4}$$



Direct spectroscopic observation of quantized cavity photon number: compare



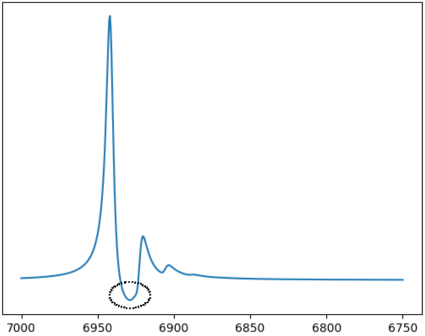


- Fits well with small $\bar{n}$, but other noise becomes significant for larger $\bar{n}$

Strengthen?

Nature 445, 515-518

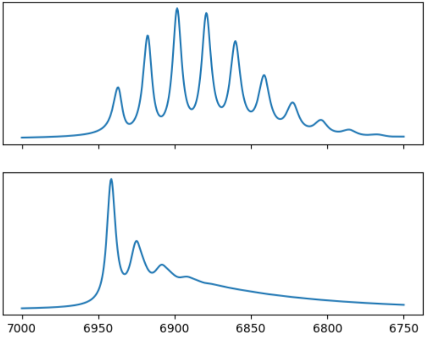
Ming, Elena



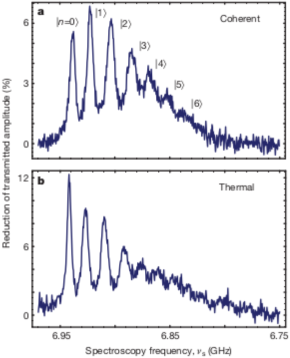
For small signal, there's a range where the transmitted amplitude is increased. We'll explain it later.

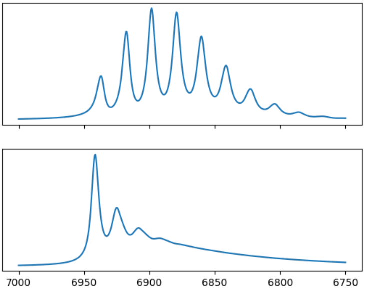
Thermal Drive

- Thermal Drive is equivalent to setting n_{th} in collapse operator to the driving average, with small ϵ_{rf} to show the phase lock-in at the given frequency.



Thermal Drive: compare





- Note that there's no thermal drive theory fitting. Our results tracks fewer peaks, but this depends on how they do the measurement, which is not mentioned in the paper.

Discussion: The picture of what happens

- The peaks shows discreteness in the photon state in the cavity.
- Exciting the qubit making the cavity off-resonance, which results in the reduction?

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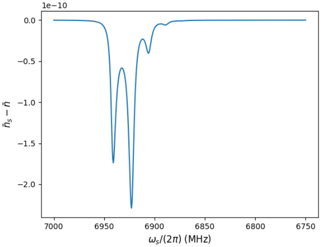
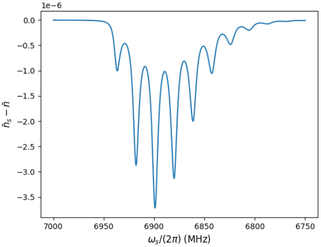
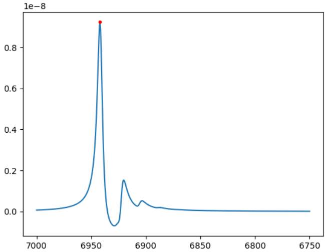
- The peaks shows discreteness in the photon state in the cavity.
- Exciting the qubit making the cavity off-resonance, which results in the reduction? **NOT TRUE**
- Expected photon number increases at the peaks!

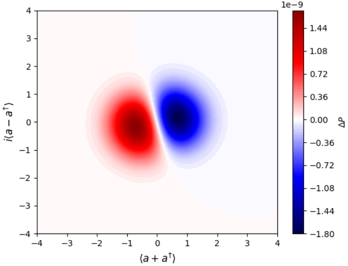




What happens

- Excitation of the qubit is not the dominant effect, but the polarization of the qubit, which twists the cavity photon state.
- This can be shown from the difference of the Wigner function (quasiprobability distribution on phase diagram) with/without the signal field.





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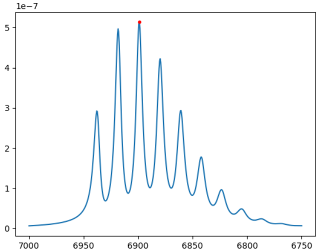

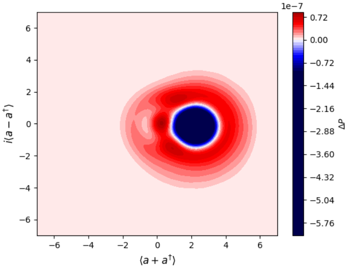

Reference I

Alexandre Blais, Ren-Shou Huang, Andreas Wallraff, Steven M Girvin, and R Jun Schoelkopf.
Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation.
Physical Review A, 69(6):062320, 2004.

DI Schuster, AA Houck, JA Schreier, A Wallraff, JM Gambetta, A Blais, L Frunzio, J Majer, B Johnson, MH Devoret, et al.
Resolving photon number states in a superconducting circuit.
Nature, 445(7127):515–518, 2007.

Reference II

David Isaac Schuster.
Circuit quantum electrodynamics.
Yale University, 2007.

Circuit Cavity QED

Cavity

- 1D transmission line resonator
- Full-wave section of superconducting coplanar waveguides

Qubit

- Cooper pair box
- Superconducting mesoscopic island connected via a Josephson Junction to a reservoir

Circuit Cavity QED

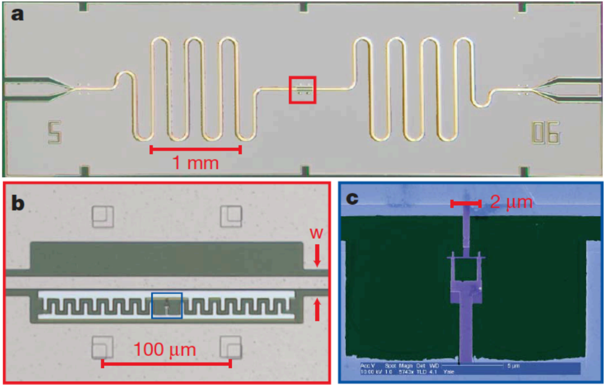

Figure: Cooper pair box inside a cavity, and spectral features of the circuit QED system.

Image from Schuster, D. I., et al. "Resolving photon number states in a superconducting circuit." Nature 445.7127 (2007): 515-518.