



Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

Resolving photon number states in a superconducting circuit

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Final projects for ELE456 at Princeton

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Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- 1 Introduction
- 2 Experiment implementation
- 3 The model
 - Cavity QED
 - Driving terms
 - Measurement
- 4 Numerical simulation
 - Property of the cavity
 - Reproduce results
- 5 Discussion
- 6 Circuit cQED



Outline

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- System sensitive to number of photons
- System: superconducting qubit + microwave transmission line
- Strong dispersive regime
- Spectroscopic measurements: Qubit's spectral lines different for each photon number state



Cavity QED

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED

Driving terms
Measurement

Numerical
simulation

Property of the
cavity

Reproduce results

Discussion

Circuit cQED

- Cavity QED (cQED) → interaction electromagnetic field modes with atoms (or qubits)

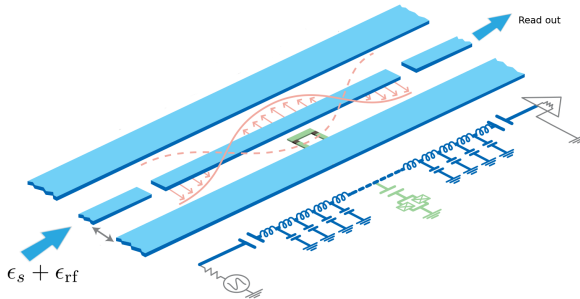


Image from Blais, Alexandre, et al. "Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation." Physical Review A 69.6 (2004): 062320.[1]



Cavity QED: the Hamiltonian

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

Hamiltonian

$$H = \omega_r \left(a^\dagger a + \frac{1}{2} \right) + \omega_q \frac{\sigma^z}{2} + g \left(a^\dagger \sigma^- + a \sigma^+ \right)$$

- ω_r : cavity resonance frequency
- ω_q : qubit transition frequency
- g : strength qubit-photon coupling



Strong Dispersive Regime

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

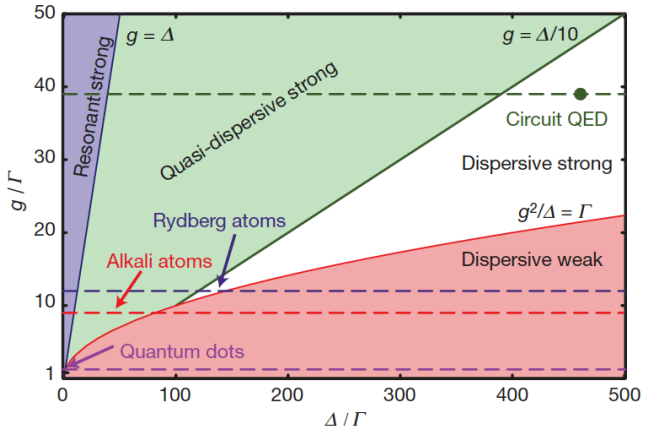


Image from Schuster, D. I., et al. "Resolving photon number states in a superconducting circuit." Nature 445.7127 (2007): 515-518.[2]



Strong dispersive Regime: Diagonalization

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- Transformation:

$$U = \exp \left(\frac{g}{\Delta} \left(a \sigma^+ - a^\dagger \sigma^- \right) \right)$$

- Hamiltonian to first order in $\frac{g}{\Delta}$:

$$\begin{aligned} H_0 &= U H U^\dagger \\ &\simeq \omega_r \left(a^\dagger a + \frac{1}{2} \right) + \omega_q \frac{\sigma^z}{2} + \chi \left(a^\dagger a + \frac{1}{2} \right) \frac{\sigma^z}{2} \end{aligned}$$

where $\chi = \frac{g}{\Delta^2}$



Driving terms

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED

Driving terms

Measurement

Numerical
simulation

Property of the
cavity

Reproduce results

Discussion

Circuit cQED

- To conduct a measurement we first drive the cavity:

$$H_{\text{rf}} = \epsilon_{\text{rf}} \left(a^\dagger e^{-i\omega_{\text{rf}} t} + a e^{i\omega_{\text{rf}} t} \right)$$

with ω_{rf} near ω_r

- The frequency shift of the qubit measured with a sweeping signal

$$H_s = \epsilon_s \left(a^\dagger e^{-i\omega_s t} + a e^{i\omega_s t} \right)$$

with ω_s near ω_q



Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

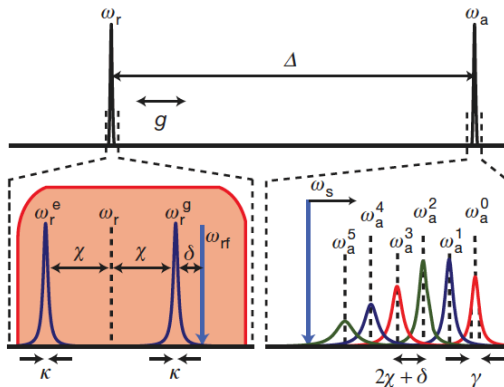


Image from Schuster, D. I., et al. "Resolving photon number states in a superconducting circuit." Nature 445.7127 (2007): 515-518.[2]



Rotating frame and Rotating wave approximation

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED

Driving terms

Measurement

Numerical
simulation

Property of the
cavity

Reproduce results

Discussion

Circuit cQED

- Applying the transformation

$$U = \exp \left[\frac{g}{\Delta} (a\sigma^+ - a^\dagger\sigma^-) \right]$$

- And moving to the rotating frame:

$$U_I = \exp \left[i \left(\omega_{\text{rf}} a^\dagger a + \omega_s \sigma^z / 2 \right) t \right]$$

H_{rf} and H_s are:

$$H_{\text{rf}} = \epsilon_{\text{rf}} (a^\dagger + a)$$

$$H_s = \left(\frac{g}{\Delta} \right) \epsilon_s (\sigma^+ + \sigma^-)$$



Final Hamiltonian and collapse operators

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED

Driving terms

Measurement

Numerical
simulation

Property of the
cavity

Reproduce results

Discussion

Circuit cQED

- Full Hamiltonian:

$$H = \omega_r \left(a^\dagger a + \frac{1}{2} \right) + \omega_q \frac{\sigma^z}{2} + \chi \left(a^\dagger a + \frac{1}{2} \right) \frac{\sigma^z}{2} \\ - \left(\omega_{rf} a^\dagger a + \omega_s \frac{\sigma^z}{2} \right) + \epsilon_{rf} (a^\dagger + a) + \epsilon_s \frac{g}{\Delta} (\sigma^+ + \sigma^-)$$

- Collapse operator:

- Collapse operators cavity: $\sqrt{\kappa(1+n_{th})}a$, $\sqrt{\kappa(n_{th})}a^\dagger$
- Collapse operator qubit: $\sqrt{\gamma}\sigma^-$
- Dephasing: $\sqrt{\gamma_\phi}\sigma^z$



Measurement

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

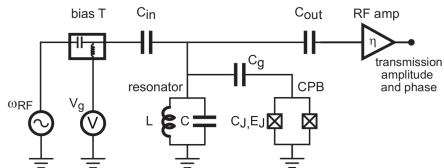
Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

In the experiment, the transmitted amplitude at frequency ω_{rf} is the main observable. The exact way to measure it is not mentioned in this paper, but can be check in Schuster's thesis [3]:



- What we really measure is the expectation of the voltage, or electrical field $E \propto \langle a + a^\dagger \rangle$



Property of the cavity: Analytical

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- Without the qubit, the cavity state is equivalently a damped harmonic oscillator with driving

$$H = \delta a^\dagger a + \epsilon(a + a^\dagger)$$

Collapse operators: $\sqrt{\kappa(n_{\text{th}} + 1)}a$ and $\sqrt{\kappa n_{\text{th}}}a^\dagger$

- When it's off resonant, its steady state is not but approximately a coherent state
- Analytically the photon number expectation value is

$$\bar{n} = \frac{\epsilon^2}{\delta^2 + \kappa^2/4} + n_{\text{th}}$$



Property of the cavity: Numerical

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

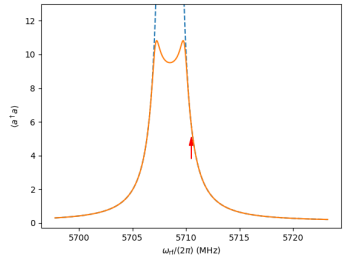
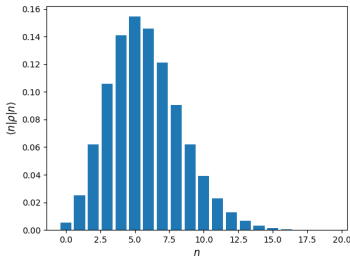
Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- Numerically, a truncate on Fock space is needed
- To check the validity of the truncate, we plot the photon distribution and frequency response of the cavity.





Direct spectroscopic observation of quantized cavity photon number

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

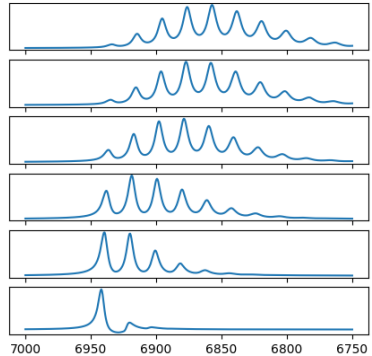
Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- For a fixed driving ϵ_{rf} , plot the reduction $V_0 - \langle a^\dagger + a \rangle_{ss}$ v.s. ω_s .

$$\bar{n} = n_{\text{th}} + \frac{\epsilon_{\text{rf}}^2}{\delta^2 + \kappa^2/4}$$





Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementation

The model

Cavity QED
Driving terms
Measurement

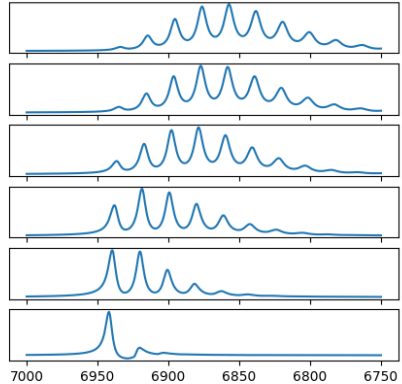
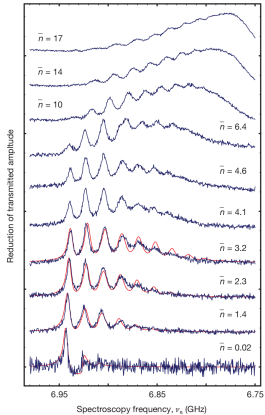
Numerical
simulation

Property of the
cavity

Reproduce results

Discussion

Circuit cQED



- The way \bar{n} is defined is larger than ours.



Thermal Drive

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

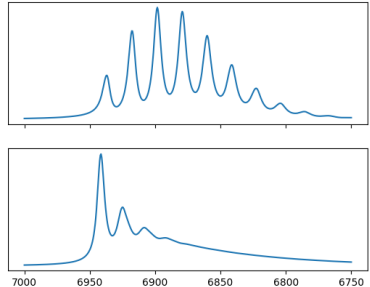
Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- Thermal Drive is equivalent to setting n_{th} in collapse operator to the driving average, with small ϵ_{rf} to show the phase lock-in at the given frequency.





Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

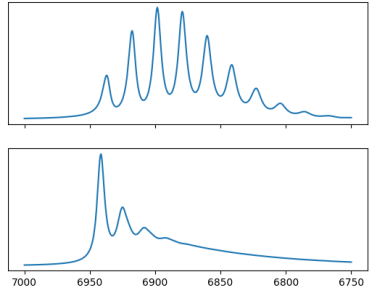
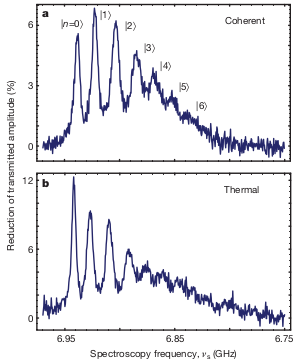
Numerical
simulation

Property of the
cavity

Reproduce results

Discussion

Circuit cQED



- Note that there's no thermal drive theory fitting. Our results tracks fewer peaks, but this depends on how they do the measurement, which is not mentioned in the paper.



Discussion: The picture of what happens

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- The peaks shows discreteness in the photon state in the cavity.
- Exciting the qubit making the cavity off-resonance, which results in the reduction?



Discussion: The picture of what happens

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- The peaks shows discreteness in the photon state in the cavity.
- Exciting the qubit making the cavity off-resonance, which results in the reduction? **NOT TRUE**



Discussion: The picture of what happens

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

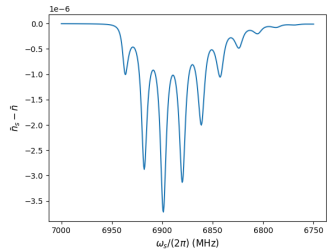
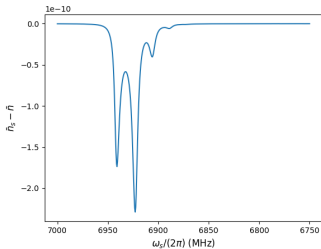
Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- The peaks shows discreteness in the photon state in the cavity.
- Exciting the qubit making the cavity off-resonance, which results in the reduction? **NOT TRUE**
- Expected photon number increases at the peaks!





What happens

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

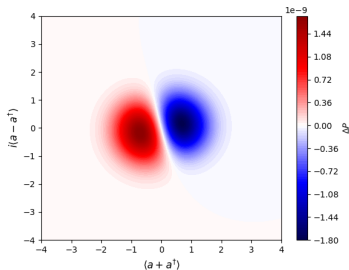
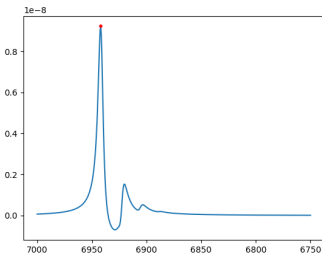
Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

- Excitation of the qubit is not the dominant effect, but the polarization of the qubit, which twists the cavity photon state.
- This can be shown from the difference of the Wigner function (quasiprobability distribution on phase diagram) with/without the signal field.





What happens

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

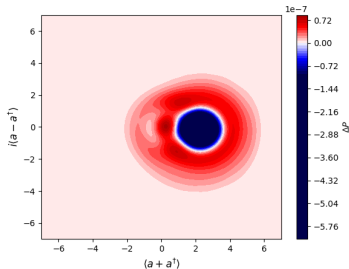
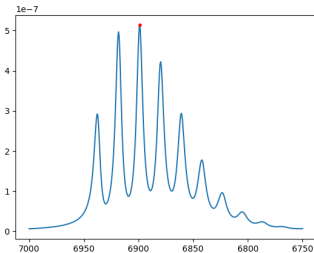
Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

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Reference I

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED



Alexandre Blais, Ren-Shou Huang, Andreas Wallraff, Steven M Girvin, and R Jun Schoelkopf.

Cavity quantum electrodynamics for superconducting electrical circuits: An architecture for quantum computation.

Physical Review A, 69(6):062320, 2004.



DI Schuster, AA Houck, JA Schreier, A Wallraff, JM Gambetta, A Blais, L Frunzio, J Majer, B Johnson, MH Devoret, et al.

Resolving photon number states in a superconducting circuit.

Nature, 445(7127):515–518, 2007.



Reference II

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED



David Isaac Schuster.

Circuit quantum electrodynamics.

Yale University, 2007.



Circuit Cavity QED

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

Cavity

- 1D transmission line resonator
- Full-wave section of superconducting coplanar waveguides

Qubit

- Cooper pair box
- Superconducting mesoscopic island connected via a Josephson Junction to a reservoir



Circuit Cavity QED

Nature 445,
515-518

Ming, Elena

Introduction

Experiment
implementa-
tion

The model

Cavity QED
Driving terms
Measurement

Numerical
simulation

Property of the
cavity
Reproduce results

Discussion

Circuit cQED

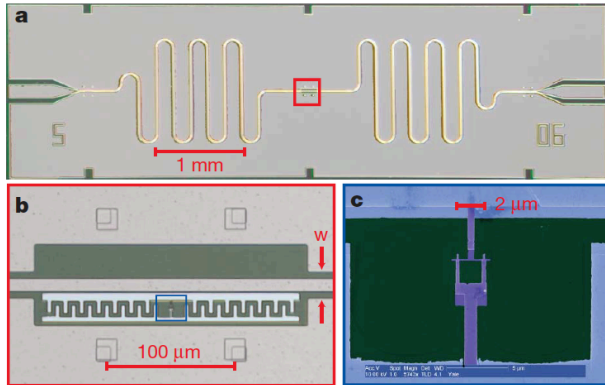


Figure: Cooper pair box inside a cavity, and spectral features of the circuit QED system.

Image from Schuster, D. I., et al. "Resolving photon number states in a superconducting circuit." Nature 445.7127 (2007): 515-518.