Unit 4. Random Variable & Probability distribution

Random Variable: Real value of a random experim is called random variable.

ex. Toss of 2 coins simultaneously.

$S = \{HH, TH, HT, TT\}$

0, 1 & 2 are real values of random experim

Discrete Random Variable: Real value in random variable is finite and ~~om~~ countable.

Continuous Random Variable: Real values in random variable are infinite.

$$a \leq x \leq b$$

Probability mass function: If x is a discrete random variable with distinct values $X = (x_1, x_2 \cdots x_n)$ With respectively probabilities

$P(x = x_1) = p_1$

$P(x = x_2) = p_2$

$\vdots$

$P(x = x_n) = p_n$

then

$p_i = p(x_i)$ is known as probability mass function if it satisfies following properties.

① $p(x_i) \geq 0$

② $\sum_{i=1}^{n} (Px_i) = 1$.

The set $(x_i, p(x_i))$

X			
P(X)			

is called probability distribution of random variable discrete r.v → dmf (Imp)

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Probability density function: (Pdf)

Let X(P) a continuous random variable. The probability density function of x is denoted by $f(x)$ or $f_x(x)$ satisfies the following properties.

① $f(x) \geq 0 \quad \forall\, x.$

② $\int_{-\infty}^{\infty} f(x)\,dx = 1$

③ $P(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx$

Imp ⟹ Pdf → continuous Random Variables

Binomial Distribution:

If an experiment results only in two ways, success (p) and failure (q) and the experiment is repeated n times, then the probability of r successes is given by.

$$\boxed{P(X = r) = {}^{n}C_{r}\ p^{r}\ q^{n-r}}$$

Remark: Binomial probability distribution is discrete probability distribution. Since X takes only integral values.

In binomial distribution, the number of trials n are finite. Only two outcomes either success (p) and failure (q).

$$\boxed{p + q = 1}$$

(3) All the trials are independent.

x = binomial variable
p = success
q = failure
n = no. of trials
r = required probrability

Mean of Binomial distribution = np

Variance of Binomial distribution = npq

S.D. of Binomial distribution = $\sqrt{npq}$

In binomial distribution, p & q always remain constant.

Q1) If 10% of bolts are produced by a machine are defective. Determine the probability that out of 10 bolts chosen at random.

i) two will be defective
ii) At most two will be defective.

Soln → let x be the defective boalts.

At most $\Rightarrow \leq$

At least $\Rightarrow \geq$

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$p = \frac{10}{100} = 0.1$

$p + q = 1$

$q = 1 - 0.1$

$\boxed{q = 0.9}$

$\boxed{n = 10}$

$P(X = r) = {}^{10}C_r \, p^r \, q^{10-r}$

i] $P(X = 2) = {}^{10}C_2 \, (0.1)^2 \, (0.9)^8$
$= 0.193$

ii] $P(X \leq 2) = P(0) + P(1) + P(2)$
$= {}^{10}C_0 \, (0.1)^0 \, (0.9)^{10} + {}^{10}C_1 \, (0.1)^1 (0.9)^9$
${}^{10}C_2 \, (0.1)^2 \, (0.9)^8$
$= 0.9298$

Q: A die is thrown 8 times. It is required to find the probability that 3 will show

i] exactly two times.
ii] At most seven times.
iii) At least once.

A discrete random variable x . the poisson

$$P(x=r) = \frac{e^{-z} z^{r}}{r!}$$

x is mean of poissons distribution

① mean = $z = np = \frac{\Sigma xf}{\Sigma f}$

② Variance = $z = np$ ③ $S.D = \sqrt{np}$

Q1) It is known that in a certain plant there are on an average 4 individual accondents per month. Find the probability than in the given month there will be less than 4 accidents.

⇒ Let x denote the no. of accidents

$z = 4$

For poisons dist.

$$P(x=r) = \frac{e^{-z} z^{r}}{r!} = \frac{e^{-4} 4^{r}}{r!}$$

Probability that there will be less than 4

$$P(x<4) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$$

$$= \frac{e^{-4}4^{0}}{0!} + \frac{e^{-4}4^{1}}{1!} + \frac{e^{-4}4^{2}}{2!} + \frac{e^{-4}4^{3}}{3!}$$

$=$

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Q1) A firm produces articals 0.1 % of which are defective. It packs them in cases containing 500 articals. If a wholesar purchases 100 such cases how many cases can be expected ① to be free from defective. ② to have one defective.

⇒ Let x = the defective article.

$n = 500$.

$N = 100$.

Probability of defective articles $= 0.1\% = p$

$= 0.001$. $\frac{0.1}{100}$

Mean $= np = 500 \times 0.001$

$\boxed{z = 0.5}$

$$P(X=r) = \frac{e^{-z} z^{r}}{r!} = \frac{e^{-0.5}(0.5)^{r}}{r!}$$

1] Probability of no defective article

$$P(X=0) = \frac{e^{-0.5}(0.5)^{0}}{0!} =$$

2] The no. of cases free from defects

$= N \cdot P(X=0) = 100 \times 0.6065$

$= 60.65$

$\simeq 61$

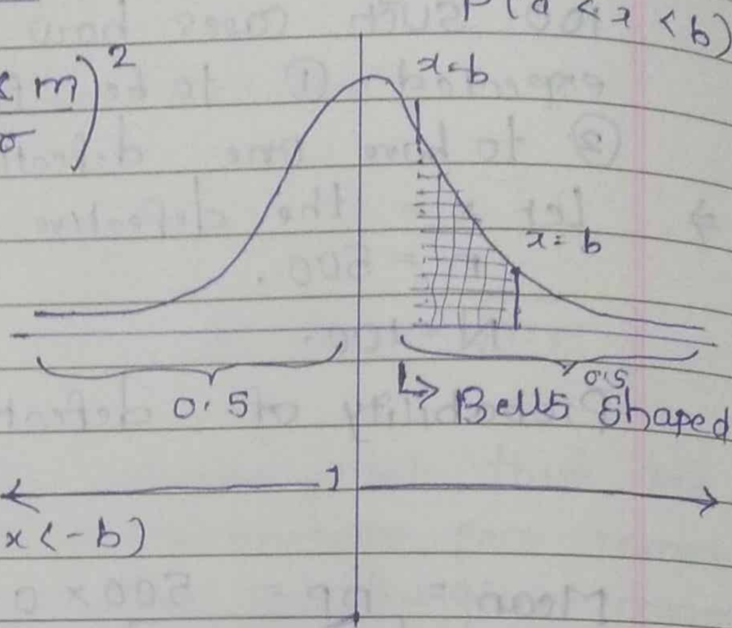
$$P(X=1) = \frac{e^{-0.5}(0.5)^{1}}{1!} = 0.3032$$

No. of cases $100 \times 0.3032 = \simeq 30$.

Normal distribution:

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x\ m}{\sigma}\right)^2}$$

m = mean
σ = S.D.



* $p(a < x < b) = p(-a < x < -b)$

$\Rightarrow$ $z = \frac{x-m}{\sigma}$; $x = 0$

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-z^2/2}$$ $\longrightarrow$ Standard normal distribution

$\rightarrow x = m \Rightarrow z = 0$

Q: Daily income of workers follows normal distribution with mean 1000 Rs. & S.D 100 Rs. Find probability of income less than 1100 Rs.

Soln $\Rightarrow$

$\mu/m = 1000$

$\sigma = 100$

$x = 1100$

we calculate z for given x

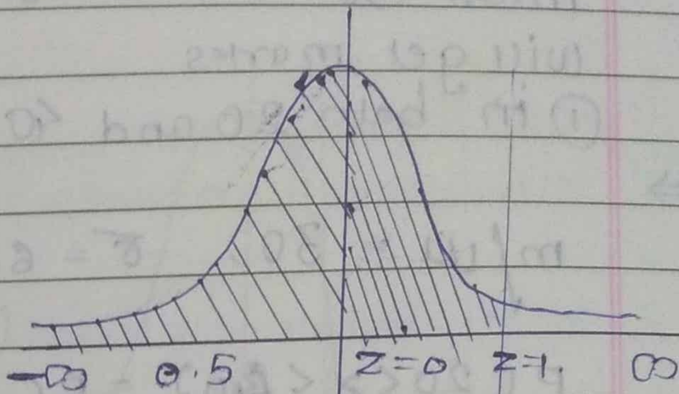
$$z = \frac{1100 - 1000}{100} = 1$$

$P(x < 1100) = P(z < 1)$

$= P(z = 1) + 0.5$

$= 0.3413 + 0.5$

$= 0.8413$



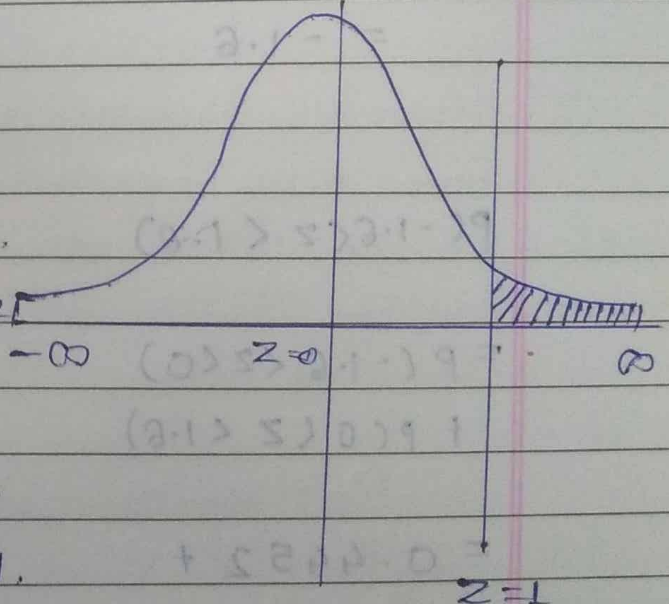
2] more than 1100

$x = 1100; \quad z = 1$

$P(x > 1100) = P(z > 1)$

$P(z > 1) = 0.5 - P(z = 1)$

$= 0.5 - 0.3413$



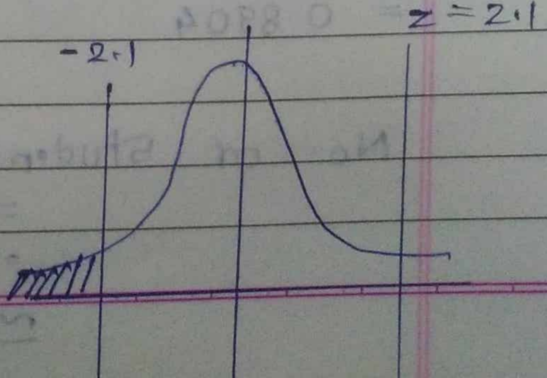
(3) Prob x less than 790.

given that $(z = 2.1 = 0.4821$

$x = 790 \ ; \quad z = ?$

$$z = \frac{790 - 1000}{100} = -2.1.$$

$P(x < 790) = P(z < -2.1)$



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$$P(z < -2.1) = 0.5 - P(z = -2.1)$$
$$= 0.5 - 0.4821$$

Q: 2) 2000 Students appeared for exam. Distribution of marks is assumed to be normal with mean 30 · S.D is 6.25. How many students will get marks
① in betn 20 and 40. $P(z = 1.6) = 0.4$.

Ans ⇒

$m/\mu = 30$, $\sigma = 6.25$

$$P(20 < x < 40) = P(\quad < z < \quad)$$

$x_1 = 20$ $\qquad$ $x_2 = 40$

$z_1 = \dfrac{20-30}{6.25}$ $\qquad$ $z_2 = \dfrac{40-30}{6.25}$

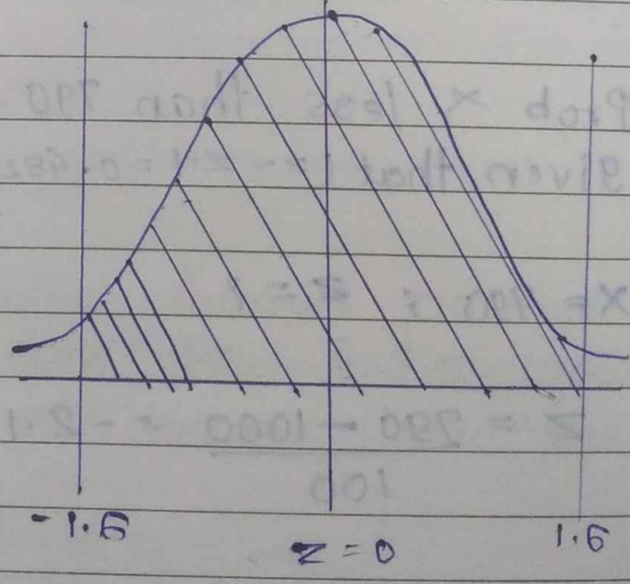
$= -1.6$ $\qquad$ $= 1.6$

$P(-1.6 < z < 1.6)$

$= P(-1.6 < z < 0)$
$+ P(0 < z < 1.6)$

$= 0.4452 +$
0.4452

$= 0.8904$



No. of students betn 20 to 40.
$= 2000 \times 0.8904$
$= 1708.8$
$\simeq 1781$

(4.5, 4.6, 4.7) Sum solve.

ii) 35 to 40

$x_1 = 35$

$z_1 = \frac{35-30}{6.25}$

$= 0.8$

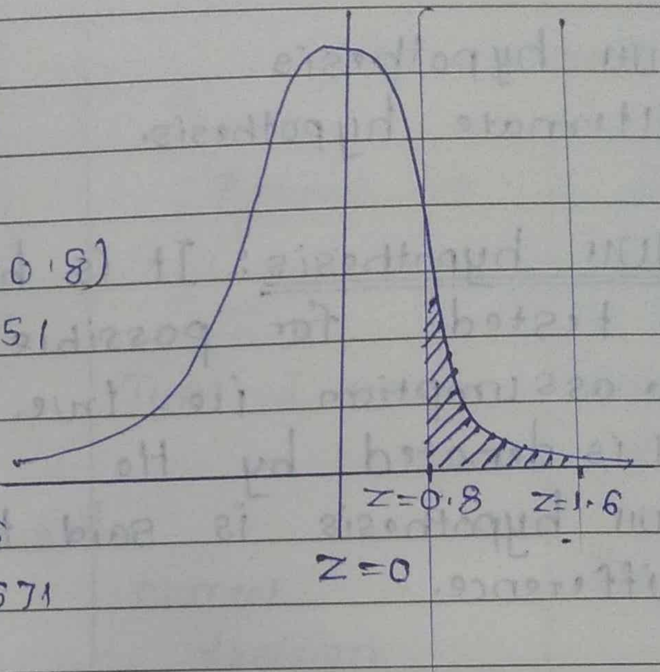
$x_2 = 40$

$z_2 = \frac{40-30}{6.25}$

$= 1.6$

$P(35 < x < 40)$

$= P(0.8 < z < 1.6)$



$= P(z=1.6) - P(z=0.8)$

$= 0.4452 - 0.2551$

$= 0.1571.$

$\therefore$ No. of students

$= 2000 \times 0.1571$

$\simeq$