Synchronization

Ragini V. Meshram
Department of Electrical Engineering
Veermata Jijabai Technological Institute

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Various Control Techniques for Three Phase Drives: Field Oriented Control of Induction Motor

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Ragini V. Meshram
(Roll No. 132071020)

Under the guidance of

Dr. Sushama R. Wagh Dr Navdeep Singh

Department of Electrical Engineering
Veermata Jijabai Technological Institute, Mumbai - 400 019
(Autonomous Institute Affiliated to University of Mumbai)

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Chapter 1

Introduction

Variable speed drive systems are essential in many industrial applications. In the past, DC motors were used extensively in areas where variable speed operation was required, since their flux and torque could be controlled easily by the field and armature current. DC motors have certain disadvantages, which are due to the existence of the commutator and the brushes. That is, they require periodic maintenance; they cannot be used in explosive or corrosive environments and they have limited commutator capability under high speed, high voltage operational conditions. These problems can be overcome by the application of alternating current motors, which can have simple and rugged structure, high maintainability and economy; they are also robust and immune to heavy overloading. These advantages have recently made induction machines widely used in industrial applications. However, the speed or torque control of induction motors is more difficult than DC motors due to their nonlinear and complex structure. The torque of the DC motors can be controlled by two independent orthogonal variables, stator current and rotor flux, where such a decoupling does not exist in induction motors.

Recent years have seen the evolution of a new control strategy for AC motors called Field Oriented Control (Vector Control), which has made a fundamental change in this picture of AC motor drives in regard to dynamic performance. Vector control makes it possible to control an AC motor in a manner similar to the control of a separately excited DC motor, and achieve the same quality of dynamic performance. As for DC machines, torque control in AC machines is achieved by controlling the motor currents. However, in contrast to a DC machine, in AC machine, both the phase angle and the modulus of the current has to be controlled, or in other words, the current vector has to be controlled. This is the reason for the terminology vector control. The high quality of the dynamic performance of the separately excited DC motor is a consequence of the fact that its armature circuit and the field circuit are magnetically decoupled. In a DC motor, the mmf produced by the field current and the mmf produced by the armature current are spatially in quadrature. Therefore there is no magnetic coupling between the field circuit and the armature circuit. Because of the repetitive switching action of the commutator on the rotor coils as the rotor rotates, this decoupling continues to exist irrespective of the angular position and speed of the rotor. This makes it possible to effect fast current changes in the armature circuit, without being hampered in this by the large inductance of the field circuit. Since the armature current can change rapidly, the machine can develop torque and accelerate or decelerate very quickly when speed changes are called for, attain the demanded speed in the fastest manner possible. As in the DC motors, in AC motors also, the torque production is the result of the interaction of a current and a flux. But in the AC induction motor, in which the power is fed on the stator side only, the current responsible for the torque and the current responsible for producing flux are not easily separable. The underlying principle of vector control is to separate out the component of the motor current responsible for producing the torque and the component responsible for producing the flux in such a way that they are magnetically decoupled, and then control each independently, in the same way as is done in a separately excited DC motor.

In the case with an AC machine in which, torque and flux control is exercised using standard two axis machine concepts. For decoupling the torque and flux controlling components of current, motor parameters should be known. All inductance parameters are known to be nonlinear functions of the motor flux level and hence may change either intentionally. Electrical parameters of the machine should be well known for field orientation. Parameters are also required for the design of the flux, speed and current controllers. Generally, the settling response of the torque and field controllers is dependent on the stator and rotor time constants, respectively (for the DC machine these are dependent on armature and field time constant which are quite easy to measure). The settling response of the speed controller is, of course, load dependent and is thus normally catered for by variable PI controllers.

1.1 Overview of the chapters

This thesis is organised as foliows:

The literature review is given in Chapter 2. The review mainly focused on field oriented control, sensorless control and state observers such as EKF, UKF and MRAS. The previous works are discussed briefly and compared with each other.

Chapter 3 presents principle of vector orientation and its orientation condition for induction motor also explain direct and indirect rotor flux orientation.

Chapter 4 presents a generalized dynamic mathematical model of the induction motor which can be used to construct equivalent circuit models in stationary reference frame.

Chapter 5 presents PI controller design for flux, speed and current controller, also given Internal Model Control technique to find gains of controller.

Chapter 6 is devoted to sensorless control technique. First the Luenberger observer technique is given in detail with the linear state space model of induction motor. Second is a interconnected observer technique where nonlinear state space model is consider with controller is explain in detail, also includes stator resistance calculation for low speed operation.

Chapter 7 presents implementation of Indirect vector control and Luenberger observer. The performance of each technique is confirmed by simulation results.

Chapter 8 summarizes the thesis and concludes with the contributions associated with the observation techniques employed in FOC.

Chapter 2

Literature Review

In induction machine, a power converter and a controller are the three major components of an induction motor drive system. Some of the disciplines related to these components are electric machine design, electric machine modeling, sensing and measurement techniques, signal processing, power electronic design and electric machine control. It is beyond the scope of this research to address all of these areas: it will primarily focus on the issue related to the induction machine control. A conventional low cost volts per hertz or a high performance field oriented controller can be used to control the machine. This chapter reviews the principles of the field orientation control of the induction machines and outline major problems in its design and implementation.

2.1 Induction machine control

The controllers required for induction motor drives can be divided into two major types: a conventional low cost volts per hertz v/f controller and torque controller [**?**]-[**?**]. In v/f control, the magnitudes of the voltage and frequency are kept in proportion. The performance of the v/f control is not satisfactory, because the rate of change of voltage and frequency has to be low. A sudden acceleration or deceleration of the voltage and frequency can cause a transient change in the current, which can result in drastic problems. Some efforts were made to improve v/f control performance, but none of these improvements could yield a v/f torque controlled drive systems and this made DC motors a prominent choice for variable speed applications. This began to change when the theory of field orientation was introduced by Hasse and Blaschke. Field orientation control is considerably more complicated than DC motor control. The most popular class of the successful controllers uses the vector control technique because it controls both the amplitude and phase of AC excitation. This technique results in an orthogonal spatial orientation of the electromagnetic field and torque, commonly known as Field Oriented Control (FOC).

2.2 Field orientation control (FOC) of induction machine

The concept of field orientation control is used to accomplish a decoupled control of flux and torque. This concept is identical to a dc machine direct torque control that has following requirements [**?**]:

- An independent control of armature current to overcome the effects of armature winding resistance, leakage inductance and induced voltage
- An independent control of constant value of flux

If all of these requirements are met at every instant of time, the torque will follow the current, allowing an immediate torque control and decoupled flux and torque regulation.

Next, a two phase d-q model of an induction machine rotating at the synchronous speed is introduced which will help to carry out this decoupled control concept to the induction machine. This model can be summarized by the following equations (see chapter 4 for detail):

$$V_{ds}^{\omega} = R_s i_{ds}^{\omega} + \frac{d}{dt}\psi_{ds}^{\omega} - \omega_e \psi_{qs}^{\omega} \tag{2.1}$$

$$V_{qs}^{\omega} = R_s i_{qs}^{\omega} + \frac{d}{dt}\psi_{qs}^{\omega} + \omega_e \psi_{qs}^{\omega} \tag{2.2}$$

$$0 = R_r i_{dr}^{\omega} + \frac{d}{dt}\psi dr^{\omega} - (\omega_e - \omega_r)\psi_{qr}^{\omega} \tag{2.3}$$

$$0 = R_r i_{qr}^{\omega} + \frac{d}{dt}\psi qr^{\omega} + (\omega_e - \omega_r)\psi_{dr}^{\omega} \tag{2.4}$$

$$T_e = 3p\frac{L_m}{L_r}(\psi_{dr}^{\omega} i_{qs}^{\omega} - \psi_{qr}^{\omega} i_{ds}^{\omega}) \tag{2.5}$$

and it is quite significant to synthesize the concept of field-oriented control. In this model it can be seen from the torque expression (2.5) that, if the flux along the q-axis can be made zero then all the flux is aligned along the d-axis and, therefore, the torque can be instantaneously controlled by controlling the current along q-axis.

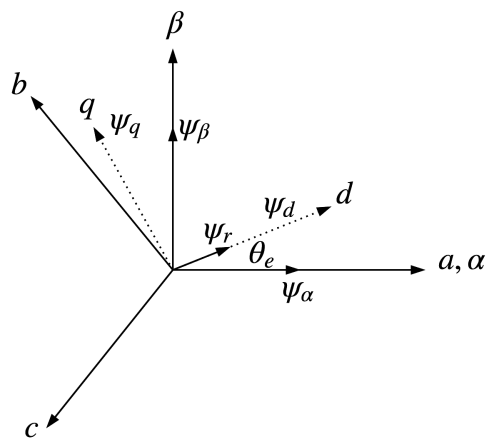

Figure 2.1: Phasor diagram of the field oriented drive system

Then the question will be how it can be guaranteed that all the flux is aligned along the d-axis of the machine. When three-phase voltages are applied to the machine, they produce three-phase fluxes both in the stator and the rotor. The three phase fluxes can be represented in a two phase stationary (α-β) frame. If these two phase fluxes along (α-β) axes are represented by a single-vector then all the machine flux will be aligned along that vector. This vector is commonly specified as d-axis which makes an angle θ_e with the stationary frame α-axis, as shown in **??**. The q-axis is set perpendicular to the d-axis. The flux along the q-axis in this case will be obviously zero. The phasor diagram **??** presents these axes. When the machine input currents change sinusoidally in time, the angle θ_e keeps changing. Thus the problem is to know the angle θ_e accurately, so that the d-axis of the d-q frame is locked with the flux vector.

The control inputs can be specified in two phase synchronously rotating d-q frame as i_{ds}^{ω} and i_{qs}^{ω} such that i_{ds}^{ω} being aligned with the d-axis or the flux vector. These two phase synchronous control inputs are converted into two phase stationary quantities and then to three phase stationary control inputs. To accomplish this the flux angle θ_e must be known precisely. The angle θ_e can be found either by Direct Field Orientation control (DFO) or by Indirect Field Orientation control (IFO). The controller implemented in this fashion that can achieve a decoupled control of the flux and the torque is known as field oriented controller. The block diagram is shown in the **??** In the field-oriented controller the flux can be regulated in the stator, air-gap or rotor flux orientation [**?**]-[**?**].

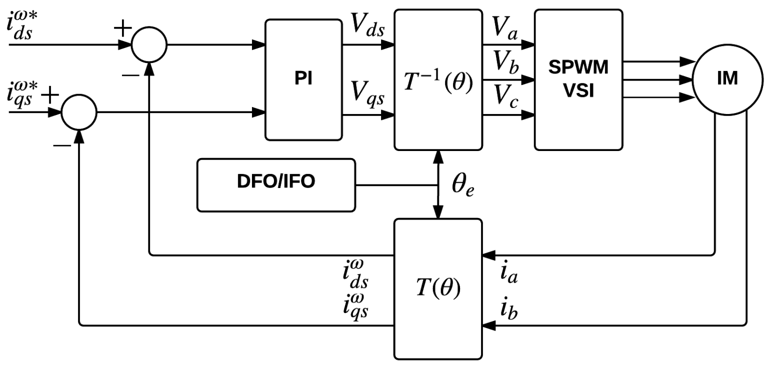

Figure 2.2: Field oriented induction motor drive system

The control algorithm for calculation of the rotor flux angle θ_e using IFO control is shown in the **??**. This algorithm is based on the assumption that, the flux along the q-axis is zero, which forces the command slip velocity to be $\omega_{sl} = i_{qs}^{\omega}/(\tau_r i_{ds}^{\omega})$ as a necessary and sufficient condition to guarantee that all the flux is aligned with d-axis and the flux along q-axis is zero. The angle θ_e can then be determined as the sum of the slip and the rotor angles after integrating the respective velocities. This slip angle includes the necessary and sufficient condition for decoupled control of flux and torque. The rotor speed can be measured directly by using an encoder or can be estimated. In case the rotor speed is estimated, the control technique is known as sensorless control. This concept will be studied in detail in the following chapters. **??** shows the control algorithm block diagram for DFO control. In this technique the flux angle θ_e is classically calculated by sensing the air-gap flux through the use of flux sensing coils, or can be calculated by estimating the flux along the d-q axes using the voltage and current signals.

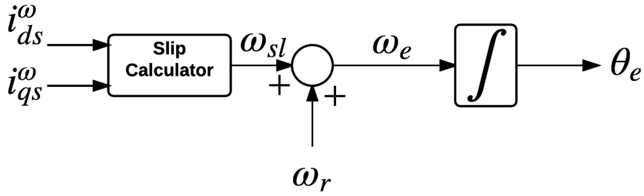

Figure 2.3: Indirect field oriented drive system

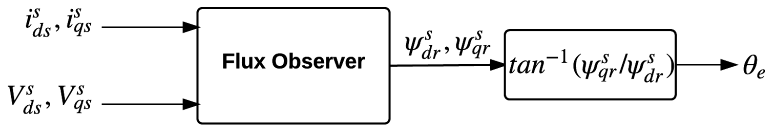

Figure 2.4: Direct field oriented drive system

2.2.1 Direct field orientation (DFO)

The DFO control and sensorless control rely heavily on accurate flux estimation. DFOC is most often used for sensorless control, because the flux observer used to estimate the synchronous speed or angle can also be used to estimate the machine speed. Investigation of ways to estimate the flux and speed of the induction machine has also been extensively studied in the past two decades. Classically, the rotor flux was measured by using a special sensing element, such as Hall effect sensors placed in the air-gap. An advantage of this method is that additional required parameters, L_{lr}, L_m, and L_r are not significantly affected by changes in temperature and flux level. However, the disadvantage of this method is that a flux sensor is expensive and needs special installation and maintenance. Another flux and speed estimation technique is saliency based with fundamental or high frequency signal injection. One advantage of saliency technique is that the saliency is not sensitive to actual motor parameters, but this method fails at low and zero speed level. When applied with high frequency signal injection [**?**], the method may cause torque ripples, and mechanical problems.

Gabriel [**?**] avoided the special flux sensors and coils by estimating the rotor flux from the terminal quantities (stator voltages and currents). This technique requires the knowledge of the stator resistance along with the stator, rotor leakage inductances and magnetizing inductance. This method is commonly known as the Voltage Model Flux Observer (VMFO). The stator flux in the stationary frame estimated by the equations:

$$\psi_{ds}^s = V_{ds}^s - R_s i_{ds}^s \tag{2.6}$$
$$\psi_{qs}^s = V_{qs}^s - R_s i_{qs}^s \tag{2.7}$$

Then the rotor flux can be expressed as:

$$\psi_{dr}^s = \frac{L_r}{L_m}(\psi_{ds}^s - \sigma i_{ds}^s) \tag{2.8}$$

$$\psi_{qr}^s = \frac{L_r}{L_m}(\psi_{qs}^s - \sigma i_{qs}^s) \tag{2.9}$$

In this model, integration of the low frequency signals, dominance of stator resistance voltage drop at low speed and leakage inductance variation result in a less precise flux estimation. Integration at low frequency is studied by [?] and three different alternatives are given. Estimation of rotor flux from the terminal quantities depends on parameters such as stator resistance and leakage inductance. The study of parameter sensitivity shows that the leakage inductance can significantly affect the system performance such as stability, dynamic response, and utilizations of the machine and the inverter.

The Current Model Flux Observer (CMFO) is an alternative approach to overcome the problems of leakage inductance and stator resistance at low speed. In this model flux can be estimated as:

$$\psi_{dr}^s = -\frac{1}{\tau_r}\psi_{dr}^s - \omega_r\psi_{qr}^s + \frac{L_m}{\tau_r}i_{ds}^s \tag{2.10}$$

$$\psi_{qr}^s = -\frac{1}{\tau_r}\psi_{qr}^s - \omega_r\psi_{dr}^s + \frac{L_m}{\tau_r}i_{qs}^s \tag{2.11}$$

However, it does not work well at high speed due to its sensitivity to the rotor resistance. Jansen [?] did an extensive study on VMFO and CMFO based direct field orientation control, discussed the design and accuracy assessment of various flux observers, compared them, and analyzed the alternative flux observers. To further improve the observer performance, closed-loop rotor flux observers are proposed which use the estimated stator current error [? ?] or the estimated stator voltage error [?] to estimate the rotor flux. Furthermore, Lennart [?] proposed reduced order observers for this task.

2.2.2 Indirect field orientation control (IFOC)

In indirect field orientation, the synchronous speed ω_e is the same as the instantaneous speed of the rotor flux vector ψ_{dr}^{ω} and the d-axis of the d-q coordinate system is exactly locked on the rotor flux vector (rotor flux vector orientation). This facilities the flux control through the magnetizing current i_{ds}^{ω} by aligning all the flux with the d-axis while aligning the torque producing component of the current with the q-axis. After decoupling the rotor flux and torque producing component of the current components, the torque can be instantaneously controlled by controlling the current i_{qs}^{ω}. The requirement to align the rotor flux with the d-axis of the d-q coordinate system means that the flux along the q-axis must be zero. This means that the current through the q-axis of the mutual inductance is zero.

Based on this restriction ω_{sl} is :

$$\omega_{sl} = \frac{i_{qs}^{\omega}}{(\tau_r i_{ds}^{\omega})} \tag{2.12}$$

These relations suggest that flux and torque can be controlled independently by specifying d-q axis currents provided the slip frequency is satisfied (2.12) at all instants.

The concept of indirect field oriented control developed in the past has been widely studied by researchers during the last two decades. The rotor flux orientation is both the original and usual choice for the indirect orientation control. Also the IFO control can be implemented in the stator and air-gap flux orientation as well. De Doncker [**?**] introduced this concept in his universal field oriented controller. In the air-gap flux the slip and flux relations are coupled equations and the d-axis current does not independently control the flux as it does in the rotor flux orientation. For the constant air-gap flux orientation, the maximum of the produced torque is %20 less than that of the other two methods [**?**]. In the stator flux orientation, the transient reactance is a coupling factor and it varies with the operating conditions of the machine. In addition, Mircea [**?**] shows that among these methods, rotor flux oriented control has linear torque curve. Therefore, the most commonly used choice for IFO is the rotor flux orientation.

The IFOC is an open loop, feed forward control in which the slip frequency is fed forward guaranteeing the field orientation. This feed forward control is very sensitive to the rotor open circuit time constant τ_r. Therefore, τ_r must be known in order to achieve a decoupled control of torque and flux components by controlling i_{ds}^{ω} and i_{qs}^{ω} , respectively. When τ_r is not set correctly, the machine is said to be detuned and the performance will become sluggish due to loss of decoupled control of torque and flux. The measurement of the rotor time constant, its effects on the system performance and its adaptive tuning to the variations resulting during the operation of the machine have been studied extensively in the literature [?]-[?]. Lorenz, Krishnan and Novotny [?]-[?] studied the effect of temperature and saturation level on the rotor time constant and concluded that it can reduce the torque capability of the machine and torque/amps of the machine. The detuning effect becomes more severe in the field-weakening region. Also, it results in a steady-state error and, transient oscillations in the rotor flux and torque. Some of the advanced control techniques such as estimation theory tools and adaptive control tools are also studied to estimate rotor time constant and other motor parameters [?]-[?].

2.3 Variable speed control using advance control algorithms

There are two issues in motion control using field oriented controlled (FOC) induction machine drives. One is to make the resulting drive system and the controller robust against parameter deviations and disturbances. The other is to make the system intelligent to adjust the control system itself to environment changes and task requirements. If the speed regulation loop fails to produce the command current correctly, than the desired torque response will not be produced by the induction machine. In addition, such a failure may cause the degradation of slip command. As a result, a satisfactory speed regulation is extremely important not only to produce desired torque performance from the induction machine but also to guarantee the decoupling between control of torque and flux.

Conventionally, a PI controller has been used for the speed regulation to generate a command current for last two decades, and accepted by industry because of its simplicity. Even though, a well tuned PI controller performs satisfactorily for a field oriented induction machine during steady state. The speed response of the machine at transient, especially for the variable speed tracking, may sometimes be problematic. In last two decades, alternative control algorithms for the speed regulation were investigated. Among these, fuzzy logic, sliding mode, and adaptive nonlinear control algorithms gained much attention.

A traditional rotor flux oriented induction machine drive offers a better control performance but it often requires additional sensors on the machine. This adds to the cost and complexity of the drive system. To avoid these sensors on the machine, many different algorithms are proposed for the last three decades to estimate the rotor flux vector and rotor shaft speed. The recent trend in field oriented control is to use such algorithms based on the terminal quantities of the machine for the estimation of the fluxes and speed. They can easily be applied to any induction machine. Therefore, our focus in this study is also on these algorithms.

Before looking into individual approaches, the common problems of the speed and flux estimation are discussed briefly for general field orientation and state estimation algorithms.

- **Parameter sensitivity:** One of the important problems of the sensorless control algorithms for the field oriented induction machine drives is the insufficient information about the machine parameters which yield the estimation of some machine parameters along with the sensorless structure. Among these parameters stator resistance, rotor resistance and rotor time constant play more important role than the other parameters since these values are more sensitive to temperature changes. The knowledge of the correct stator resistance R_s is important to widen the operation region toward the lower speed range. Since at low speeds the induced voltage is low and stator resistance voltage drop becomes dominant, a mismatching stator resistance induces instability in the system. On the other hand, errors made in determining the actual value of the rotor resistance R_r, may cause both instability of the system and speed estimation error proportional to R_r [**?**]. Also, correct τ_r value is vital decoupling factor in IFOC

- **Pure Integration:** The other important issue regarding many of the topologies is the integration process inherited from the induction machine dynamics where an integration process is needed to calculate the state variables of the system. However, it is difficult both to decide on the initial value, and prevent the drift of the output of a pure integrator. Usually, to overcome this problem a low-pass filter replaces the integrator.

- **Overlapping loop Problems:** In a sensorless control system, the control loop and the speed estimation loop may overlap and these loops influence each other. As a result, outputs of both of these loops may not be designed independently, in some bad cases this dependency may influence the stability or performance of the overall system.

The algorithms, where terminal quantities of the machine are used to estimate the fluxes and speed of the machine are categorized in two basic groups. First one is "the open loop observers," in a sense that the on-line model of the machine does not use the feedback correction. Second one is "the closed loop observers" where the feedback correction is used along with the machine model itself to improve the estimation accuracy. These two basic groups can also be divided further into subgroups based on the control method used. These can be summarized as:

Open loop observers based on:
- Current model
- Voltage model
- Full-order observer

Closed loop observers based on:
- Model Reference Adaptive Systems (MRAS)
- Kalman filter techniques
- Adaptive observers based on both voltage and current model
- Neural network flux and speed estimators
- Sliding mode flux and speed estimators

Current model based open loop observers use the measured stator currents and rotor velocity. The velocity dependency of the current model is very important since this means that although using the estimated flux eliminates the flux sensor, the position sensor is still required. On the other hand, voltage model based open loop observers [**?**]-[**?**] use the measured stator voltage and current as inputs. A full-order open loop observer can be formed using only the measured stator voltage and rotor velocity as inputs where the stator current appears as an estimated quantity. Because of its dependency on the stator current estimation, the full order observer will not exhibit better performance than the current model. Furthermore, parameter sensitivity and observer gain are the problems to be tuned in a full order observer design [**?**]. These open-loop observer structures are all based on the induction machine model, and they do not employ any feedback. Therefore, they are quite sensitive to parameter variations, which yield the estimation of some machine parameters along with the sensorless structure.

To produce more robust structures to parameter variations some kind of feedback may be helpful. For this purpose many closed loop topologies are proposed using different induction machine models and control methods. Among these MRAS attracts attention and several different algorithms are produced. In MRAS a comparison is made between the outputs of two estimators. The estimator which does not contain the quantity to be estimated can be considered as a reference model of the induction machine. The error between these two estimators is used as an input to an adaptation mechanism. The estimated rotor speed in the adjustable model is changed in such a way that the difference between two estimators converges to zero asymptotically, and the estimated rotor speed will be equal to actual rotor speed. The basics of the analysis and design of MRAS are discussed in [**?** **?**]. In [**?** **?**]. In [**?**] similar speed estimators are proposed based on the MRAS, and a secondary variable is introduced as the reference quantity by letting the rotor flux through a first-order delay instead of a pure integration to nullify the offset. However, their algorithms produce inaccurate estimated speed if the excitation frequency goes below certain level. In addition these algorithms suffer from the machine parameter uncertainties since the parameter variation in the reference model cannot be corrected. Zhen [**?**] proposed an interesting MRAS structure that is built with two mutual MRAS schemes. In this structure, the reference model and the adjustable models are interchangeable. For rotor speed estimation, one model is used as reference model and other model is used as adjustable model. The pure integration is removed from reference model. [**?**] supported the MRAS scheme with ANN using its training and modeling of non-linear systems. MRAS scheme is also used for the on-line adaptation of the motor parameters in field oriented control techniques [**?** **?**].

Kalman filter (KF) is another method employed to identify the speed and rotor flux of an induction machine based on the measured quantities such as stator current and voltage [**?**]. Kalman filter approach is based on the system model and a mathematical model describing the induction motor dynamics for the use of Kalman filter application. Parameter deviations and measurement disturbance are taken into consideration in KF covariance matrices of the KF must be properly initialized. KF works for linear systems and for non linear induction motor model extended Kalman filter (EKF) is used. However, KF approach is computationally intensive and depends on the accuracy of the model of the motor. In the EKF model proposed by one can estimate rotor fluxes and rotor speed which makes the field orientation. EKF is also used for online parameter estimation of induction motor [**?** **?**]. Reduced order models are also proposed to shorten and speed up the complex EKF algorithm [?]. A new KF technique for non-linear systems, Unscented Kalman Filter (UKF), is applied to induction machine state estimatio, which is a derivative free KF technique which avoids costly calculation of Jacobian matrix, linearization of the estimates [?].

Another method used for the sensorless control of induction motor is the neural network technique, which is based on a learning process. It has the advantage of tolerating machine parameter uncertainties. For speed estimation, a two-layered neural network, based on back propagation technique, is used and the neural network outputs are compared with the actual measurement values and error then back- propagated to adjust the weights such that the estimated speed converges to actual one. The neural network based sensorless control algorithms have the advantages of fault-tolerant characteristics. However, because of the neural network learning process these algorithms may suffer from the computational intensity.

Another approach is sliding mode control for FOC of induction machine. In the sliding mode technique, the control action is very strong and being switched into either "on" or "off" at high frequency. The command signals control directly the power devices. This type of control is also favorable because "on-off" is the only admissible mode of operation for the power converters. Therefore, it seems more natural to employ the algorithm towards discontinuous control.

In addition to the algorithms mentioned above, some of the proposed work is hard to classify because of their combined structure. In [?], a nonlinear high- gain observer structure is proposed, and it is claimed that with the exact knowledge of stator resistance, flux and speed estimation convergence is guaranteed.

2.4 Conclusion

The literature review of DFOC, IFOC, flux, position and speed estimation and speed control can be summarized as:

- The DFOC and IFOC are the methods for instantaneous torque and speed control of an induction motor drive system. These methods can be implemented with or without a speed sensor. An IFOC is synthesized by properly controlled slip frequency which is necessary for the field-orientation

- The main problem of an IFO drive system is the rotor time constant deviation. The drive system torque control performance decreases if the rotor time constant is not set precisely. Therefore, on-line estimation is necessary and is one of the main challenges for better performance of an IFOC. Most of the techniques proposed so far either need some special hardware or are very complex with respect to the software and require intensive calculations which put extra burden on the processor.

- Voltage model and current model flux observers are the two most common ways to estimate the flux using the terminal quantities. The voltage model flux observer is dominated by stator IR drop at low speed, whereas the current model flux observer has problems of rotor time constant variations. Also the current model flux observer requires the rotor speed. Therefore, if the flux observer is being used for the sensorless control, an error in the estimated speed will be fed back in to the system. Thus will affect the observer accuracy.

- The proposed open-loop observers can be simple in the structure but they are susceptible to variety of errors that become specially detrimental at low stator frequencies, including measurement, noise digital approximation errors, parameter detuning and DC offset in measurements, which ultimately may drive the observer instability.

- For the time varying system model problems, closed-loop observers are proposed here feedback correction is used along with the machine model itself to improve the estimation accuracy. The algorithmic complexity and calculation intensity looks higher when compared with former solutions but the recent processors are fast enough to solve these algorithms in real-time applications. They also require a strong mathematical background to deal with.

Chapter 3

Field Oriented Control of Induction Motor

3.1 Introduction

The dynamics of a power system is extremely complex which involve interactions between its three components: generation, transmission and distribution. The interactions between the mechanical power input and the electrical power demand determine the dynamics of the generator which is governed by the swing equation. Solutions of this equation are analyzed to check for any violations in the rotor angles of the generators. The equal area criterion is a tool used for single machine infinite bus structure whereas assessment of the stability of **multi-machine power systems (MMPS)** involves two approaches: the time domain solutions and the Lyapunov based direct methods. The former approach fails to provide sound stability margins and tools for suitable sensitivity analysis [**?**]. Though this drawback is overcome by the direct methods, constructing good Lyapunov functions(LF) for MMPS has been challenging. Often, one has to use experience or physical insights (e.g., the energy function (EF) for electrical and mechanical systems) to search for an appropriate LF [**?**]. There have been several papers which have used the concept of EF to assess rotor angle stability of MMPS. Though the EF has served as the LF for two machines, extension to N-machine case is elusive owing to the presence of transfer conductances (TC).

For MMPS, formation of an EF is possible only when TC are negligible. Presence of TC makes the integral of the potential energy term path dependent whereas, formation of EF will be possible only if this integral is path independent. EF formulations are possible only when the dynamical equations governing them are integrable and presence of TC makes the EF non-integrable. The notion of integrability implies that the governing dynamical equations need to be solved in order to achieve global solutions and the most natural procedure to come up with solutions is to find the constants of motion (CoM). A N-dimensional system needs (N-1) CoM for integrability. As it's not feasible to find the required number of CoM for $N > 2$, such systems are non-integrable. A motivating parallel can be drawn from the N-body problem of celestial mechanics. As quoted in [**?**], solutions to the N-body problem exist. However, analytically solving the dynamical equations is difficult unless integrability helps in.

Apart from assessing stability of system's equilibrium point, the purpose of forming LF is to also find suitable controllers to attain stability, thereby aiding in the enhancement of the domain of attraction. Many control approaches have asserted that the integrability condition needs to be satisfied so as to derive globally convergent stabilizing controllers for MMPS [**?**]. However, satisfying the integrability condition will remain a holy grail problem for lossy MMPS when the tool to evaluate the stability of its equilibrium points is the EF. Though researchers have attempted to extend the EF formulations for two machines and beyond, the issue of TC hampers generalization to N-machines [**?**]. Despite the prolonged efforts made by authors to address non-integrability, there has been no analytical proof to show that N-machine systems are non-integrable. This brings in the first contribution of the paper, wherein, using the Watanabe-Strotgatz (WS) theory, non-integrability of N-machine system is proved by considering the entire gamut of dynamics of N-machines.

The inability to account for TC is due to the limitations in modeling of loads which simplifies the formation of the LF. In network reduced models (NRM), the loads are treated as constant impedances and are merged into a network where only generator internal nodes are retained. The load impedances appear as TC in the reduced network which hamper the evaluation of EF beyond two machines. Though the structure preserving models (SPM) include detailed load and generator dynamics, this approach also uses certain assumptions in evaluating the EF [**?**]. Hence, forming EF for lossy MMPS will remain pervasive. Irrespective of whether EF are integrable or non-integrable, MMPS continue to exist in synchronism. A supporting analogy can be drawn from the N-body problem of celestial mechanics, wherein the celestial bodies continue to exist in synchrony. The second contribution of this work is to reiterate that even though the EF for lossy MMPS is not integrable, N-machines can be synchronized. As proposed in [**?**], there is no relation between line losses and system instability. **Integrability is not mandatory for synchronization**. Global solutions to such systems are infeasible, however, numerical procedures to assess stability domains exist.

The existence of synchrony of N-machines despite the presence of TC urges to explore a different approach in understanding the underlying phenomenon. In literature, this has been envisaged by modeling of N-machines as an ensemble of Kuramoto oscillators [**?**]. The Kuramoto model symbolizes the transient stability problem as a synchronization problem by modeling the machines as phase oscillators. The region of interest for rotor angle stability is the cohesive region, wherein, the oscillators would be phase locked with a region of arc on the unit circle (which corresponds to relative rotor angles $< 180^0$). The model assesses the stability of N-machine with TC and explains that there exists a critical value K_{cr} of the coupling co-efficient K below which the system remains in asynchrony and as $K \to 1$, oscillators are phase locked. The value of K_{cr} is proportional to the spread of the phase angles of the oscillators and is dependent on the value of TC. Their relationship is explored to show the effect of using a conservative approach in forming a LF and its impact on the domain of attraction which brings in the third contribution of the paper.

The paper is organized as follows: Section II reviews the modeling issues in assessment of transient stability of MMPS using EF as a tool. A brief note on integrability is presented. Section III revisits the synchronization approach of the Kuramoto model and analyzes the effect of TC on K_{cr}. Section IV describes the behavioural properties of Kuramoto model and describes the general procedure for forming CoM for N oscillators with the help of WS theory, thereby, illustrating the regions of integrability. Section V summarizes the discussion and proposes the conclusive remarks and future scope.

3.2 Transient stability assessment: Modeling issues

The assessment of rotor angle stability is extremely crucial to measure the effect of large disturbances on the system. In a power network comprising of n generator nodes, and m load nodes, for transient stability assessment, rotor dynamics of generators need to be monitored, which can be represented by swing equations of generator i as:

$$M_i\ddot{\delta}_i = P_{mi} - E_i^2 G_{ii} - D_i\dot{\delta}_i - P_{ei}, \quad for\ i \in \{1, \cdots, n\} \tag{3.1}$$

where, internal voltage of generator $E_i > 0$, mechanical power input $P_{mi} > 0$, inertia $M_i > 0$, damping constant $D_i > 0$. The electrical power output P_{ei} is given as:

$$\begin{aligned} P_{ei} &= \sum_{j=1}^{n} |E_i||E_j|[Re(Y_{ij})cos(\delta_i - \delta_j) + Im(Y_{ij})[sin(\delta_i - \delta_j)] \\ &= \sum_{j=1}^{n} |E_i||E_j||Y_{ij}|sin(\delta_i - \delta_j) + \psi_{ij}] \end{aligned} \tag{3.2}$$

$$where\ |Y_{ij}| = \sqrt{G_{ij}^2 + B_{ij}^2},\ \ \psi_{ij} = arctan(G_{ij}/B_{ij}) \tag{3.3}$$

For SPM, P_{ei} would represent the power flow in the transmission lines having admittance of $G_{ij} + jB_{ij}$. For NPM, Kron reduction is employed wherein only generator nodes are retained and all the load nodes are merged in the reduced admittance matrix, $Y_{red}(orY_{ij})$. G_{ij} represents the TC between generator i and generator j. If $G_{ij} = 0, (\psi_{ij} = 0)$, then the line is treated as lossless, else it is a lossy line. P_{ei} would then represent the power flow exchanged between the machines which is termed as generator flows (GF). Owing to the different modeling techniques of the loads, evaluation of P_{ei} serves as a hindrance in formation of EF.

In NRM, the load impedances appear as ψ_{ij} which account for the TC between the generators. The major difficulty in the analysis of systems with TC is that a closed form expression for the total system energy cannot be obtained. The SPM incorporates detailed dynamic models of loads, yet path dependent integrals are neglected in evaluation of the EF. Though, EF is a powerful tool in analyzing stability, the inherent modeling of power system in evaluating the first swing poses a problem in their evaluation as lossy systems are not integrable. Formulating an EF or Hamiltonian with TC for a N-machine is an un-tractable problem similar to the N-body problem owing to its non-integrability. The concept of integrability and its relation to synchronization is explained in the next section.

3.2.1 Integrability and Synchronization

A separate section on integrability is implored to address the obsession of control community to the idea of existence of integrability for formation of suitable control law in order to achieve stability. Given a dynamical system governed by

$$\dot{X} = f(X) \tag{3.4}$$

with an initial condition $X(0)$, integraility implies solving these equations to obtain the state of the system at time t, i.e. $X(t)$ given $X(0)$. This indicates that integrability involves finding global solutions. According to Poincare, integrating a differential equation is finding a finite expression for the general solution, possibly multivalued, in a finite number of functions [**?**]. The word finite indicates that integrability is related to a global rather than local knowledge of the solution. Integrability is the property of equations for which all local or global solutions can be obtained either explicitly from the solutions or implicitly from the CoM [**?**]. Since an EF approach is generally followed to assess stability, a N-body system needs $(N-1)$ CoM for integrability. With dissipative nature of systems, solving dynamical equations analytically is a challenge beyond the two body problem as its difficult to find the required number of CoM for $N > 2$. Hence, its said that such systems are non-integrable. The authors propose to reiterate that solutions exist for three machines and beyond but they cannot be found by using EF as a tool.

Integrability is a rare phenomenon, a typical dynamical system is non-integrable [**?**]. In the context of power systems, integrability is associated with the EF used to assess stability of MMPS. The effect of the presence of TC is to act as a perturbation to the otherwise, conserved system. Analytically procedures of solving this integral for lossy MMPS fail as the integral becomes path dependent and its value would change with change in the fault location. Hence, energy integrals fail to provide global solutions and systems with TC are non-integrable. Nevertheless, in a realistic scenario, TC exist in the system and yet, a N-machine system remains synchronized. This raises questions on the relation between synchronization and integrability. If a system is not integrable, does it imply that it is unstable? Probably not, as supported by the discussion above. Maybe, a different modeling approach is needed to understand the underlying phenomena. This is achieved by modeling N-machines as oscillators, an approach, which has widely been used and is briefed in the next section.

3.3 Machines as Kuramoto oscillators

The dynamics of the rotor of the generator described by the swing equation, gives a balance between the mechanical output of the rotor and the coupling of this output to the other nodes of the grid. The rotational properties of generators help in visualizing them as oscillators whose dynamics are governed by the Kuramoto model. The analogy between the swing equation and the oscillator dynamical equation is derived from the fact that both are a consequence of Newton's second law and are related to the active power flow. Solutions to Kuramoto model results in estimating synchronization of oscillators whereas solutions to swing equations involve determination of rotor angle stability. The motivation to symbolize machines as oscillators is to view phase locking of oscillators as synchronization of the rotor angles. The equivalence between the two dynamical equations has been explored by [**?**].

Each oscillator oscillates independently at its natural frequency while the coupling tends to synchronize it to all others. When the coupling is weak, oscillators run incoherently whereas for coupling strength beyond a certain threshold, oscillators are synchronized with each other. The governing equations of the coupled Kuramoto oscillator as shown in Fig.**??** are:

$$\dot{\theta}_i = \omega_i - \sum_{j=1}^{N} K_{ij} sin(\theta_i - \theta_j), for \ i \in \{1, \cdots, N\} \tag{3.5}$$

model in power system parlance, the relationship between the power network model and
a first-order model of coupled oscillators is exploited. The singular perturbation analysis
K_{ij} is a matrix comprising of the coupling weights. In order to apply the Kuramoto
is applied to show the congruity between (**??**) and (**??**), assuming that the generators are overdamped possibly due to local excitation controllers. Using the relation (**??**) and (**??**) to represent the effective power input to generator i and the coupling weights representative of power transferred between generator i and j respectively, (**??**) would get modified to (**??**)

$$w_i \equiv (P_{mi} - E_i^2(G_{ii})) \tag{3.6}$$

$$P_{ij} = |E_i||E_j||Y_{ij}| \quad with \quad P_{ii} = 0 \tag{3.7}$$

$$M_i\ddot{\delta}_i = -D_i\dot{\delta}_i + \omega_i - \sum_{j=1}^{n} P_{ij} sin(\delta_i - \delta_j + \psi_{ij}) \tag{3.8}$$

For small inertia over damping ratio ($\frac{M_i}{Di}$) of generators, singular perturbation can be applied to seperate slow and fast dynamics of the system, then (**??**) reduces to,

$$D_i\dot{\delta}_i = \omega_i - \sum_{j=1}^{n} P_{ij} sin(\delta_i - \delta_j + \psi_{ij}) \tag{3.9}$$

which captures the power system dynamics sufficiently well during first swing. A corelation between (**??**) and (**??**) reveals that with $\psi_{ij} = 0$, they both are alike, where P_{ij} plays the same role as that of the coupling function K_{ij}. The relationship between ψ_{ij} and K_{ij} is described in this section whereas the role of ψ_{ij} in integrability will be discussed in Section IV.

An intuitive representation of synchronization is brought about by the Kuramoto mean field model (KMFM), by taking $K_{ij} = K/N > 0$, resulting in:

$$\dot{\theta}_i = \omega_i - \frac{K}{N}\sum_{j=1}^{N} sin(\theta_i - \theta_j), for\ i \in \{1, \cdots, N\} \tag{3.10}$$

(??) can be written in a more convenient form using the order parameter (OP) r, the centroid of the oscillators, which is a natural measure of synchronization and defined as:

$$r(e^{j\phi t}) = \frac{1}{N}\sum_{k=1}^{N} e^{j\theta_k(t)} \tag{3.11}$$

where r measures phase coherence of the oscillators and ϕ measures the average phase. Kuramoto model (??) represented in terms of OP as:

$$\dot{\theta}_i = \omega_i - Krsin(\phi - \theta_i), for\ i \in \{1, \cdots, N\} \tag{3.12}$$

The equivalent of TC in the KMFM is brought about by adding a term β in the coupling function of (??). It denotes the averaged value of the TC for oscillators.

$$\dot{\theta}_i = \omega_i - Krsin(\phi - \theta_i - \beta), for\ i \in \{1, \cdots, N\} \tag{3.13}$$

In literature, (??) is also known as the non-uniform Kuramoto model. The effect of change in K on r is depicted in Fig.?? which shows that as β increases, more coupling strength is needed to synchronize oscillators, resulting in an increase in K_{cr} which corresponds in a way, to the stability margin/ the region of attraction of the LF. The effect of increasing β on K_{cr} is shown in Fig.??. K_{cr1} is the critical coupling strength at $\beta_1 = 0$ and K_{cr2} is the critical coupling strength at β_2 and so on. With an increase in β, K_{cr} also increases. The inferences drawn from Fig.?? are:

leftmirgin=* Beyond 2 machine, forming a LF with TC is infeasible, i.e., LF cannot be formed at β_2, β_3....

leftmiirgiin=* Hence, from a practical point of view, LF is always formed at $\beta_1 = 0$.

leftmiiirgiiin=* For e.g., if the system is operating at β_2 and its stability is analyzed at $\beta_1 = 0$, then this would result in an underestimate of the K_{cr}, i.e. the system sees K_{cr1} and not K_{cr2}, resulting in a shrinking in the etimate of domain of attraction.

leftmivrgivn=* The larger the β, the more would be the error in the estimate of domain of attraction.

This discussion emphasizes that LF can be formed by neglecting the TC but this assumption would be at the cost of sacrificing the estimates of region of attraction.

3.4 Behaviour and Integrability of Kuramoto oscillators

The periodic solutions of (**??**) fall into three categories: in-phase, splay and incoherent. Fig.**??** depicts these states followed by their definitions:

leftmirgin=* In-phase state/ synchronous state: A state in which the interaction between the oscillators is attractive and tends to lock them in-phase. If all the oscillators are in the generating mode, then they are said to be phase cohesive.

leftmiirgiin=* Incoherent/ Asynchronous state: A state in which the interaction between the oscillators is repulsive and tends to lock them in anti-phase

leftmiiirgiiin=* Splay state: A state which is the most stable of the asynchronous state.

Fig.**??**a shows N oscillators which are synchronized and are in the synchronous region. Fig.**??**b depicts the splay (outspread) states which are the most stable of the incoherent states, Fig.**??**c. The significance of these states is highlighted as:

[leftmargin=*]The whole in-phase state is not conducive to synchronization. The oscillators would exist in the generating mode only above K_{cr} which is the phase cohesive region. As TC increases, the oscillators start falling apart and spread over the unit circle, well within the phase cohesive region. If TC increases beyond K_{cr}, then the oscillators start spreading out and move towards the incoherent state. As the application is to assess rotor angle stability, the dynamics of the oscillators when they are in the synchronous state have only been examined. Though splay states have been discussed in literature, as they are not relevant from machine stability point of view, they have not been explored. However, the dynamics at the splay states give lot of insights into the integrability of N-machine systems as proved by the WS theory.

Exploring the whole spectrum of the solutions of an ensemble of N oscillators gives rise to a varied dynamics which help in exposing the domains of integrability. The WS theory aids in providing a full dynamical description of the Kuramoto model of identical oscillators by reducing the dynamics to that of three macroscopic constants along with CoM. As the notion of integrability involves finding CoM, the WS theory finds (N-3) CoM for an N-dimensional system, thereby reducing the dynamical equations to three dimension. This would aid in evaluating the system analytically. Forming a LF, L, using these reduced co-ordinates aids in deciding the stability of the coherent and the incoherent regions. It can be observed that (**??**) can be written in the generalized form [**?**]:

$$\dot{\theta}_i = g(t)cos\theta_i + h(t)sin\theta_i \tag{3.14}$$

With respect to (**??**), the functions $g(t)$ and $h(t)$ in (**??**) are $g(t) = Krsin(\phi + \beta)$ and $h(t) = -Krcos(\phi + \beta)$. The ensemble of (**??**) can be reduced to three time dependent variables γ, Φ and Ψ and $(N - 3)$ constants:

$$\begin{aligned} \dot{\gamma} &= -(1-\gamma^2)(g(t)sin\Phi - h(t)cos\Phi) \\ \gamma\dot{\Psi} &= -\sqrt{(}(1-\gamma^2)))(g(t)cos\Phi + h(t)sin\Phi) \\ \gamma\dot{\Phi} &= (g(t)cos\Phi - h(t)sin\Phi) \end{aligned} \tag{3.15}$$

where Ψ and Φ are global variables, and $0 \leq \gamma \leq 1$ is the amplitude of the harmonic force. The reduction of N-dimensional system to three dimensional is achieved using the transformation:

$$\begin{aligned} \theta_i = \Phi(t) + 2arctan\Big[\frac{\sqrt{(}1+\gamma(t))}{\sqrt{(}1-\gamma(t))}tan[\frac{\varphi_k - \Psi(t)}{2}]\Big] \\ i \in \{1, \cdots, N\} \end{aligned} \tag{3.16}$$

where φ_k are constants. WS have demonstrated that the set of constants φ_k together with the solutions of (**??**) yields a solution of (**??**) via transformation (**??**). Obtaining the initial values of γ, Ψ and Φ and the constants φ_k from the initial conditions θ_i are explained in [**?**]. Two additional constraints in the incoherent state are imposed on the constants φ_k :

$$\sum_{k=1}^{N} cos\varphi_k = \sum_{k=1}^{N} sin\varphi_k = 0 \tag{3.17}$$

These two constants (**??**) along with the (N-3) CoM imposed in the incoherent manifold make the splay state integrable. The graphical representation of (**??**) is shown in Fig.**??** [**?**]. The reduced co-ordinate variables γ and Φ are analogous to the amplitude of the mean field r and its average phase ϕ [**?**]. It can be seen from Fig.**??** that $\gamma = 0$ corresponds to the incoherent manifold where the phase of the oscillator is equal to the mean field. Note that as γ increases from 0 to 1, the phases of the oscillators move from asynchronous to synchronous regime. A similar phenomena is also observed in the cascade failures in power grid. The line flows under a healthy grid condition follow a Gaussian distribution (GD) which is an assumption. Owing to distrurbances, as lines get overloaded and tripped, the cumulative distribution of line flows collapses from Gaussian to non-Gaussian (NG) distribution which indicates decrease in coherence or movement from the synchronous to the asynchronous mode. Such phenomenon has been discussed in [?] wherein, the GD of line flows is based on the assumptions that the line flows follow the central limit theorem. The analysis of the reduced coordinates of the original system using the WS theory rather, justifies this assumption. For the analysis of (??) it is convenient to introduce quantities S and T

$$S(\gamma, \Phi) = rsin(\phi - \Phi),\ T(\gamma, \Phi) = -rcos(\phi - \Phi) \tag{3.18}$$

which gives $r^2 = S^2 + T^2$. They can also be represented via derivatives of a LF $L(\gamma, \Phi)$ in polar coordinates (γ and Ψ):

$$\begin{aligned} L(\gamma, \Phi) &= \frac{1}{N}\sum_{k=1}^{N} log\Big(\frac{1 - \gamma cos(\varphi - \Psi)}{\sqrt{(1-\gamma^2)}}\Big), \\ as\ T &= (1-\gamma^2)\frac{\partial L}{\partial \gamma};\ S = -\frac{\sqrt{(1-\gamma^2)}}{\gamma}\frac{\partial L}{\partial \Psi} \end{aligned} \tag{3.19}$$

The time derivative of L is given by:

$$\dot{L} = RKr^2 cos\beta \tag{3.20}$$

Depending on the sign of $\dot{L}$, the stability of the reduced system can be analyzed. For the case of linear coupling, R, K and β are constant, then (??) implies that if:

leftmirgin=* • $cos\beta < 0 \Rightarrow \beta > \pi/2$, L decreases, $0 < \gamma << 1$ and an incoherent state with zero mean field $r = 0$ sets in.

leftmiirgiin=* $cos\beta > 0 \Rightarrow \beta < \pi/2$, L grows and a fully synchronous state with $\gamma \rightarrow 1$ establishes.

leftmiiirgiiin=* $cos\beta = 0$, $\beta = \pi/2$, splay state sets in which is the most stable of the incoherent states. This corresponds to a full lossy case where the system is integrable.

leftmivrgivn=* L has a minimum $L = 0$ at the origin $\gamma = 0$, and tends to infinity on the unit circle $\gamma = 1$.

Fig.**??** shows the regions at which synchronous, asynchronous and integrable behaviour can be obtained. It illustrates the transition from asynchrony to synchrony by virtue of variation in γ. The repulsive region comprises of two modes: the chaotic incoherent state and the stable splay state. As $\gamma \approx 1$ and the coupling increases beyond K_{cr}, frequency synchronization and phase co-hesiveness of the oscillators is achieved. At the border between attraction and repulsion $cos(\beta) = 0$ (assuming $K \neq 0$) the system becomes integrable.

3.5 Conclusions and future scope

The focus of this work is to elucidate the limitations in the assessment of rotor angle stability of lossy MMPS with EF formulations as a tool. Global solutions to such systems are infeasible, however, numerical procedures to assess stability domains exist. The discussions help in putting to rest the notion of finding integrable energy functions for lossy MMPS. The formation of LF and thereby the control action by neglecting the TC would result in a shrinking in the region of attraction. The whole endeavour is to draw parallel between the stability of the N-body problem and the lossy MMPS, which reveals that lossy MMPS continue to remain in equilibrium irrespective of the presence of TC. The WS theory provides a concrete proof that lossy MMPS are non-integrable and efforts to find energy function for such systems would be unproductive.

kura.png

Figure 3.1: Kuramoto model of N coupled oscillators

0.2 fig_rK.png

Figure 3.2: r vs K

0.35 fig_BK.png

Figure 3.3: Effect of β on K_{cr}

Figure 3.4: Relation of r and β with K

fig11.png

Figure 3.5: Periodic solution of Generalized Kuramoto oscillators

fig2.png

Figure 3.6: Variation in coherence with change in γ and Φ

figK1.png

Figure 3.7: Stability of the incoherent manifold

Chapter 4

Application of Adaptive Network Approach for Power Grid Operations: Synchronization, Clustering And Self-organized Criticality

4.1 Introduction

The study of complex systems involves interactions between its sub-systems resulting in the emergence of collective behaviour, attempts of perceiving which have been the driving force for developments in science and engineering. Examples of complex systems in real-life are the world wide web, social networks, biological protein networks, transportation networks, power grid etc. Numerous ways of modeling these systems help in analyzing their structure and behaviour. They exhibit unique spatio-temporal characteristics, which have rendered them as highly attractive targets of exploration. However, the research carried out to comprehend the structure and the dynamics of complex systems have been carried out in isolation. Combining these two under one modeling paradigm brings in a new persepctive of understanding several existing phenomenon. To this end, modeling of complex systems as an "adaptive network"will prove to be very insightful.

A network is treated as a structure comprising of nodes and links, which are interacting based on certain rules. If the network remains static while the states of the dynamical system are evolving, then such networks are known as dynamics on networks ($DoNN$) as shown in Fig **??**. An important application that is studied within this framework include synchronization of the individual dynamical systems [**?**]. The other line of

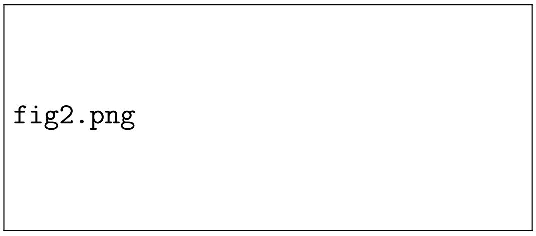

Figure 4.1: Dynamics on Network (DoNN)

research is concerned with the dynamics of networks ($DoFN$) as shown in Fig **??**. Here, the topology of the network itself is treated as a dynamical system, which keeps evolving with time according to certain rules or laws. Such an evolution would lead to peculiar network topologies with special properties. Notable examples include the formation of small world and scale-free networks.

For the purpose of analysis, the ($DoFN$) and ($DoNN$) are treated exclusive of each

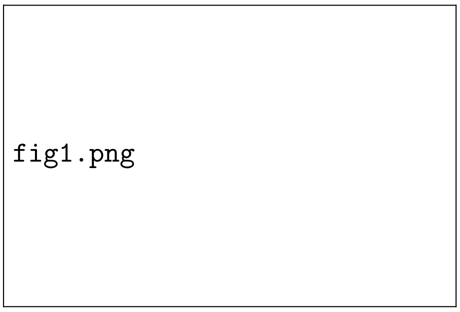

Figure 4.2: Dynamics of Network (DoFN)

other in literature, as they evolve at different time scales. However, considering both the dynamics together can drive the system to a steady state that differs from the one obtained when the two processes are considered separately [**?**]. Combining the two results in equilibria which are an outcome of the dynamical interplay between the state and the topology of the network. The main motive of this work is to view certain mechanisms in the power grid as a consequence of interactions between the dynamics of the network and the topology. Certain traits like self-organized criticality, synchronization

and clustering can be explained better by considering the coevolving dynamics between the topology and the behaviour of the system. An adaptive network modeling forms a strong link in undertsanding these mechanisms.
The paper is organized as follows: Section II discusses the general framework of an adaptive network and the dynamical equations governing them. Section III discusses the classical swing equation and the flux-decay model in the context of an adaptive synchronization using the Kuramoto model. The self-organized criticality (SOC) and application of adaptive network theory to it is discussed in Section IV. Section V depicts how adaptive coupling introduced by variation in coupling strength will result occurence of clustesr in a power network. Section VI summarizes the discussion and proposes the conclusive remarks and future scope.

4.2 Adaptive network theory

This section focuses on how researchers have modeled power grid as DoNN and DoFN. Many systems in nature can be described by models of complex networks, which are structures consisting of nodes and edges. Initially, a complex network was modelled as regular shape such as an Euclidian lattice. In the 1950s, Erdos and Renyi (ER) described a complex network as a random graph, though intuition tells us that many real life complex networks are neither completely random nor completely regular. Later on, with increase in computational facilities and the size of the networks, two discoveries to model complex networks evolved: the small world model and the scale free model. In order to describe the transition from a regular lattice to a random graph, Watts and Strogatz (WS) introduced the concept of small world networks. A common feature of the ER and the WS models is that the connectivity distribution of a network peaks at an average value and decays exponentially: hence theyre called exponential or homogeneous networks, because each node has about the same number of edge connections. Another model has evolved from the observation that many real large scale complex networks are scale free i.e. their connectivity distributions are in a power law form i.e. most nodes have very few link connections and yet a few nodes have many connections: inhomogeneous in nature.

In the past few years, a number of large blackouts have led to an increasing interest in the study of bulk power grid. The topology of a power grid is critical in assessing its vulnerability to cascading failures. As a type of a complex network, many power grids show the characteristic of a scale free network. In a power grid, there are always a few central buses with high economic or geographic significance; in the planning, these buses are supposed to hold more links than the other buses. The characteristics of evolution of power grid contributes to few nodes with high degree. This scale free characteristic of a power grid makes it robust to random attack but fragile to intentional attack. The methods of modeling discussed so far fall under the category of DoFN. Topological metrics such as average path length, clustering co-efficient, degree distribution etc are crucial in deciding the emergence of a blackout.

The power grid is modeled as DoNN when trying to address issues of transient stability assessment. The dynamics of the rotor of the generator described by the swing equation, gives a balance between the mechanical output of the rotor and the coupling of this output to the other nodes of the grid. The rotational properties of generators help in visualizing them as oscillators whose dynamics are governed by the Kuramoto model. The analogy between the swing equation and the oscillator dynamical equation is derived from the fact that both are a consequence of Newton's second law and are related to the active power flow. Solutions to Kuramoto model results in estimating synchronization of oscillators whereas solutions to swing equations involve determination of rotor angle stability. The motivation to symbolize machines as oscillators is to view phase locking of oscillators as synchronization of the rotor angles. The equivalence between the two dynamical equations has been explored by [**?**].

This paper wishes to propose that both the modeling approaches: of an evolving network and the transient stability assessment fall under the much broader framework of an adaptative network. An adaptive network is treated as a dynamical system with three functions f, h and g, depicting the dynamics of the node, the relation showing the interaction between nodes and the adaptation rule respectively [**?**].

$$\dot{x} = f(x, t, u, A) \tag{4.1}$$

$$\dot{a} = g(t, a, v, x) \tag{4.2}$$

where A is the time varying adajecency matrix describing the network topology and $u \epsilon \Re^m$ is the input on the network nodes, $v \epsilon \Re^l$ is the input on interconnetion matrix described by the vector a of nonzero elements of A (*i.e.* the set of active edges of the network). According to this framework, denoting σ_{ij} as the adaptive gain associated with non-zero elements of A, the adaptive synchronization problem can be written as:

$$\begin{aligned}\dot{x} &= f(x_i, t) + \sum_{j \epsilon \varepsilon_i} \sigma_{ij}(t)[h(x_j) - h(x_i)] \\ \dot{\sigma}_{ij} &= g(t, \sigma_{ij}, x_i, x_j)\end{aligned} \tag{4.3}$$

where ε_i is the set of links beginning with i. The choice of the functions f, g and h would depend on the application of interest.
This work proposes to view power systems as adaptive networks, from three different perspectives:

1. Synchronization
2. Self-Organized Criticality
3. Clustering or island formation

4.3 Adaptive Approach to Synchronization

The phenomenon of synchronization is ubiquitous in nature and a very widely explored area from the point of view of applications in sciences and engineering. As stated in [**?**], synchronization is the alignment of rythms of oscillating objects due to their weak interactions. There are two notions in perceiving the mechanism of synchronization: the first in which the dynamics of the system evolve to a particular solution due to their interactions with either the internal parameters or with the environment. And the second in which a desired trajectory has to be arrived upon, for which certain states of the system have to be controlled. The former is where the systems aligns in an organized way and in the process could attain a critical transition point. The latter approach has been widely applicable to tracking problems wherein different control strategies are proposed to synchronize the system. Therefore, phase transition and an organized behaviour are indigenous to synchronization.
The modeling of a physical system into an oscillator involves certain assumptions, of which, the notion that they are identical is practically unrealistic. A group of such non-identical oscillators will experience interacting forces which are weak. [**?**] exemplify that the key to synchronization are the weak interactions between the non- identical oscillating objects. The work of Winfree and Kuramoto have used models with weak interaction between oscillators to achieve phase synchronization.
To achieve this, the underlying dynamical system has to be converted to a phase equa-

tion model which depicts the dynamics between the inherent oscillations of each oscillator and the interactions from the other oscillators. The Kuramoto model of synchronization is a very successful model to understand the synchronization of an ensemble of oscillators, wherein the coupling coefficient is assumed to be constant. However, variation in the coupling strength leads to an adaptive approach to synchronization as proposed in [**?**]. The Kuramoto model has been applied to address the transient stability problem of power systems, which is by large treated as a subclass of the synchronization problem.
The assessment of rotor angle stability is extremely crucial to measure the effect of large disturbances on the system. There have been two schema of bringing in synchronization of rotor angles, one is the series control (using FACTS devices like UPFC, TCSC) and the other is using excitation control at the generator nodes. The earlier method achieves global synchronization, whereas the latter ensures only enhancement in synchronization. Both the schemes are discussed in the following sections.

4.3.1 The Classical Model : Kuramoto oscillators

In a power network comprising of n generator nodes, and m load nodes, for transient stability assessment, rotor dynamics of generators need to be monitored, which can be represented by swing equations of generator i as:

$$M_i\ddot{\delta}_i = P_{mi} - E_i^2 G_{ii} - D_i\dot{\delta}_i - P_{ei}, \quad for\ i \in \{1, \cdots, n\} \tag{4.4}$$

where, internal voltage of generator $E_i > 0$, mechanical power input $P_{mi} > 0$, inertia $M_i > 0$, damping constant $D_i > 0$. The electrical power output P_{ei} is given as:

$$\begin{aligned} P_{ei} = \sum_{j=1}^{n} &|E_i||E_j|[Re(Y_{ij})cos(\delta_i - \delta_j) + \\ &Im(Y_{ij})[sin(\delta_i - \delta_j)] \\ = \sum_{j=1}^{n} &|E_i||E_j||Y_{ij}|sin(\delta_i - \delta_j) + \psi_{ij}] \end{aligned} \tag{4.5}$$

$$where\ |Y_{ij}| = \sqrt{G_{ij}^2 + B_{ij}^2}, \ \ \psi_{ij} = arctan(G_{ij}/B_{ij}) \tag{4.6}$$

Each oscillator oscillates independently at its natural frequency while the coupling tends to synchronize it to all others. When the coupling is weak, oscillators run incoherently whereas for coupling strength beyond a certain threshold, oscillators are synchronized with each other. The governing equations of the coupled Kuramoto oscillator as shown in Fig **??** are:

kura.png

Figure 4.3: Coupled Oscillators

$$\dot{\theta}_i = \omega_i - \sum_{j=1}^{N} K_{ij} sin(\theta_i - \theta_j), for \; i \in \{1, \cdots, N\} \tag{4.7}$$

K_{ij} is a matrix comprising of the coupling weights. In order to apply the Kuramoto model in power system parlance, the relationship between the power network model and a first-order model of coupled oscillators is exploited. The singular perturbation analysis is applied to show the congruity between (**??**) and (**??**), assuming that the generators are overdamped possibly due to local excitation controllers. Using the relation (**??**) and (**??**) to represent the effective power input to generator i and the coupling weights representative of power transferred between generator i and j respectively, (**??**) would get modified to (**??**)

$$w_i \equiv (P_{mi} - E_i^2(G_{ii})) \tag{4.8}$$

$$P_{ij} = |E_i||E_j||Y_{ij}| \quad with \quad P_{ii} = 0 \tag{4.9}$$

$$M_i\ddot{\delta}_i = -D_i\dot{\delta}_i + \omega_i - \sum_{j=1}^{n} P_{ij} sin(\delta_i - \delta_j + \psi_{ij}) \tag{4.10}$$

For small inertia over damping ratio ($\frac{M_i}{Di}$) of generators, singular perturbation can be applied to seperate slow and fast dynamics of the system, then (**??**) reduces to,

$$D_i\dot{\delta}_i = \omega_i - \sum_{j=1}^{n} P_{ij} sin(\delta_i - \delta_j + \psi_{ij}) \tag{4.11}$$

which captures the power system dynamics sufficiently well during first swing. A corelation between (??) and (??) reveals that with $\psi_{ij} = 0$, they both are alike, where P_{ij} plays the same role as that of the coupling function K_{ij}. An intuitive representation of synchronization is brought about by the Kuramoto mean field model (KMFM), by taking $K_{ij} = K/N > 0$, resulting in:

$$\dot{\theta}_i = \omega_i - \frac{K}{N}\sum_{j=1}^{N} sin(\theta_i - \theta_j), for\ i \in \{1, \cdots, N\} \tag{4.12}$$

Equation (??) can be written in a more convenient form using the order parameter (OP) r, the centroid of the oscillators, which is a natural measure of synchronization and defined as:

$$r(e^{j\phi t}) = \frac{1}{N}\sum_{k=1}^{N} e^{j\theta_k(t)} \tag{4.13}$$

where r measures phase coherence of the oscillators and ϕ measures the average phase. Kuramoto model (??) represented in terms of OP as:

$$\dot{\theta}_i = \omega_i - Krsin(\phi - \theta_i), for\ i \in \{1, \cdots, N\} \tag{4.14}$$

As $K \to 0$, oscillators oscillate with their own natural frequencies, resulting in OP $r \to 0$ as $t \to \inf$, implying incoherent oscillators. When $K \to 1$, the oscillators are synchronized to their mean phase and $r \to 1$. For OP between $0 < r < 1$, a few of the oscillators are phase locked and remaining are spread around the circle.
When a fault occurs in the system, (??) is analyzed to assess the violation of rotor angles and calculate suitable control. This is achieved by inserting FACTS devices, which leads to modifying the Y_{ij} in (??). Eventually, this would result in change in line flows, thereby, ensuring synchronization of rotor angles. This is an adaptive approach to synchronization, where the change in topology changes the dynamics of the states.

4.3.2 One axis Generator model with exciters

This model includes one circuit for the field winding of the rotor, i.e., this model considers the effects of field flux decay [?]. As a result, the voltage behind the direct transient reactance is no longer a constant. The adaptive network paradigm can also be extended to the flux decay model as in [?]. The third equation governing the dynamics of the exciter can be considered, in addition to (**??**):

$$\dot{E}_i = -a_i E_i + b_i \sum_{j=1, j\neq i} E_j cos(\delta_i - \delta_j) + E_{fij} + u_i \tag{4.15}$$

where

$$b_i = \frac{1}{T_{di}}(x_{di} - x'_{di})Y_{ij}; a_i = \frac{1}{T_{di}}[1 - B_{ii}(x_{di} - x'_{di})]$$

The reader is suggested to refer [**?**] for notations and symbols used in (**??**).
It can be observed that as E_i changes, eventually, with respect to **??**, P_{ij} can be varied. Therefore, a feedback mechanism from E_i is changing the coupling weights of the network, thereby altering the dynamics of the oscillator, aiding in the enhancement of synchronization. Hence, the process of achieving rotor angle stability is inherently an adaptive phenomenon, irrespective of the control method used.

4.4 Self-Organized Criticality

The (SOC) is a consequence of the adaptive nature of a complex network, wherein, it sets in a state of equilibrium which is marginally stable. As the system evolves from some intial condition, its interactions with the environment or its internal parameters results in a feedback mechanism driving the system to its critical value. Such a state is indicative of appearance of phase transitions, which could lead to the emergence of fractals. At a phenomenological level, SOC aims at explaining the tendency of open dissipative system to rearrange themselves in such a way to develop long-range temporal and spatial correlations [**?**]. Even the slightest perturbation at the critical point would lead the system into an avalanche. Therefore, modeling of systems in the adaptive framework provide a better platform to perceive the spontaneous manifestation of the topological properties characterized by complex networks.
A power network is a continuously evolving network, the equilibrium of which keeps changing, based on the change in demand, generation or in the network topology. Either of these changes would eventually result in dynamics which affect both the connecting nodes as well as the dynamics of the network. As the load changes, the extra demand has to be met by the generator by changing the power angle at the machine. In this process, there could be a possibility that some of the lines may trip due to overload, thereby resulting in change in line flows, eventually leading to variation in the phase angles of the connecting buses. This mechanism of change in load is possible until the

steady state power limit is reached, beyond which the system cannot take any further load and it collapses.

It has to be noted that the evolution mechanism of a power grid occurs at different time scales. Depending on the control motive, either the power system engineer is interested in solving the faster evolving power flow equations or the slow evolving rotor angles based on the swing equations. However, it would be of interest to consider the interaction between the two dynamics, as shown in Fig **??**. The slow dynamics, which

power_evolution.png

Figure 4.4: Evolution of power system

are long-term, are characterized by the existence of multiple equilibrium points and the system transits from one operating point to the other. The system at each juncture, tries to maintain a balance between the external ebergy supplied, the output delivered and the system losses. Therefore, its always in a transient state. However, the fast dynamics are governed by the constraints which are intrinsic to the system and can lead to catastrophic behaviour owing to presence of uncertainty. Therefore, the slow and the fast dynamics cumulatively drive the system into a state of SOC.

The adaptive framework to model SOC in a power grid can be explained by the following Differential-Difference-Algebraic equations [**?**]:

$$\dot{x} = f(x, y, s, u, w) \tag{4.16a}$$

$$0 = g(x, y, s, u, w) \tag{4.16b}$$

$$s(t_{k+1}) = \phi(x, y, s_k, u, w) \tag{4.16c}$$

$$\tag{4.16d}$$

It is to be noted that 16a indicate the transient dynamics of generators and loads, 16b denote the fast dynamics which are a set of algebraic equations governed by the real and reactive power flow constraints; and the set of difference equations 16c describe slow dynamics including the load increase and power grid constructions. The state x comprises of the rotor angles and frequencies of the generators, the output y includes the bus voltages and the line flows through the transmission lines; the discrete parameters s are determined by the slowly changing dynamics of the system; the control u consists of the adjustments of generation outputs, relay actions, switches, and other preventive

and emergency control actions; the disturbances w include short-circuit faults, incorrect operation of dispatchers and misoperation of relays [**?**].
The combined set of (16) indicate how the power grid model evolves continuously, suggesting an adaptive mechanism involved in the emergence of an organized and a critical behaviour.

4.5 Cluster Formation

The third application of adaptive network theory is with respect to the role played by adaptive coupling in the formation of clusters in a network of coupled oscillators. As proved in [**?**], the introduction of adaptive coupling can induce the occurrence of multi-stable states (MSS) in a system of coupled oscillators. MSS implies: existence of two cluster state and a desynchronized or a splay state. It has also found that the weight of the coupling strength and the plasticity of the coupling play a crucial role in controlling the occurrence of MSS. From power system point of view, the existence of the two-cluster state is more relevant to corelate the formation of islands or clusters after a cascade failure happens. The analysis in this section helps in deducing that introducing adaptive coupling to the basic Kuramoto model results in a faster attainment of the two-cluster state.
The standard Kuramoto model of phase oscillators:

$$\dot{\theta}_i = \omega_i - \frac{K}{N}\sum_{j=1}^{N} f(\theta_i - \theta_j), for\ i \in \{1, \cdots, N\} \tag{4.17}$$

R_1 is the coherence parameter which determines the coupling strength of the whole system.

$$R_1 = \frac{1}{N}\sum_{k=1}^{N} e^{j\theta_k(t)} \tag{4.18}$$

In case of adaptive coupling, $K = \sigma K_i$, where σ is the strength of coupling and K_i are the coupling weights of oscillators. The coupling weights K_i are considered to be dynamic, governed by the following dynamical equation:

$$\dot{K}_i = \frac{\eta}{N}\sum_{j=1}^{N} g(\theta_i - \theta_j) \tag{4.19}$$

where g is a 2π periodic function called the plasticity function which determines how the coupling weigths depend on the relative timing of oscillators and η is called the plasticity parameter. The choice of the functions f and g are the simple sin and cos functions which modifies the dynamical equations to:

$$\dot{\theta}_i = \omega_i - \frac{K}{N}\sum_{j=1}^{N} sin(\theta_i - \theta_j), \tag{4.20}$$

$$\dot{K}_i = \frac{\eta}{N}\sum_{j=1}^{N} cos(\theta_i - \theta_j)$$

Before proceeding further, a few definitions are in order:

mss.png

Figure 4.5: Synchronization regimes with variation in order parameters R_1 and R_2

1. Synchronous state: A state in which the interaction between the oscillators is attractive and tends to lock them in-phase. Fig **??**a and Fig **??**b correspond to the phase synchronized and phase cohesive states. If all the oscillators are in the generating mode, then they are said to be phase cohesive.
2. Partial synchronous state: A few oscillators are locked together while a few have moved out of the coupling as in Fig **??**c.
3. Splay state: A state wherein all the oscillators are spread uniformly along the circle as in Fig **??**d.

Here, $K = \frac{1}{N}\sum_i K_i$ is defined as the average coupling strength. Fig **??** shows how the variation in order parameters induces different modes of synchronization. Fig **??**a and **??**d showcase extreme conditions of order parameter, R_1, indicating phase synchronization and a splay state respectively. As one moves from Fig **??**a to **??**d, the coupling strength k decreases. Fig **??**c depicts a condition of partial synchronization, or a critical state, depicting the phase transition stage. As $K < K_{cr}$, R_1 further decreases and would result in a splay state, wherein the oscillators are distributed all around the circle. However, the presence of plasticity parameter η helps in formation of a two cluster state which is governed by the order parameter R_2, given by:

$$R_2 = \frac{1}{N}\sum_{j=1}^{N} e^{2i\theta_j} \tag{4.21}$$

It has been proved in [**?**] that R_2 has to be equal to 1 so as to result in formation of the two cluster state. With $R_2 = 1$, if $R_1 = 0$, then each cluster has equal number of oscillators whereas if $R_1 \neq 0$, then each cluster has unequal number of oscillators. It can be clearly seen from the figures that when the system splits up into islands, the rotor angles of the machines are perturebed from Fig **??**c and the presence of R_2 would aid in forming a two cluster state, rather than a splay state. Therefore, if the original system has to be restored, increase in R_1 results in a faster movement from Fig **??**e to **??**c, thereby resulting in a rapid attainment of synchronization. In short, R_1 controls the synchronization of the system, whereas the value of R_2 regulates oscillators to be in a group of two. This discussion emphasizes the role of adaptive coupling in the attainment of synchronization.

4.6 Conclusions and future scope

The adaptive approach of modeling complex systems helps in explaining various phenomenonin a power network. The adaptation between the slow and fast dynamics of the power grid makes it a resilient network. However, increase in load or generator outages drives the system into a self-organized state, perturbations to which are followed by blackouts. The paper has brought in a different view point to the traditional problem of transient stability assessment, stating that the control efforts to stabilize the system after a fault, are in a way, a consequence of an adaptive phenomenon. The mathematical framework that is central to this concept is the standard Kuramoto model. Another interesting application is that the presence of adaptive coupling would aid in cluster formation, which could result in saving the system from a complete collapse. *The future scope of this work is two fold: first is to extend these concepts and interpret their relevance in the context of a microgird . And secondly, it is envisaged that the paradigm of adaptive coupling would serve as a modeling framework to comprehend

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