Sector VAD processing

1)Define the following cost function

$$L = \sum_n \left(\mathbf{u} \cdot \hat{\mathbf{r}}_n - u_{r_n} \right)^2$$

where

$\hat{r}_n = \sin(az)\cos(el)\hat{\mathbf{x}} + \cos(az)\cos(el)\hat{\mathbf{y}} + \sin(el)\hat{\mathbf{z}}$, Radial unit vector from the lidar to a point on the "range ring."

$u_{r_n} =$ Radial velocity measured by the lidar

$\mathbf{u} = u\hat{\mathbf{x}} + v\hat{\mathbf{y}}$, this is the vector we're solving for.

$\sum_n$ = summation over all the radial velocity measurements within the range ring

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2) Minimize L with respect to u and v. Compute

$$\frac{\partial L}{\partial u} = 2\sum_n \left(\mathbf{u}\cdot\hat{\mathbf{r}}_n - u_{r_n}\right)\sin(az_n)\cos(el_n)$$

$$\frac{\partial L}{\partial v} = 2\sum_n \left(\mathbf{u}\cdot\hat{\mathbf{r}}_n - u_{r_n}\right)\cos(az_n)\cos(el_n)$$

Setting $\frac{\partial L}{\partial u} = 0$ and $\frac{\partial L}{\partial v} = 0$ gives a 2x2 linear system. Solve it for u and v.

3) Build the vertical profile of u and v by repeating steps (1) and (2) for each range ring. The height of each range ring is z=rsin(el).