Analog Modulation and Demodulation

EE340: Prelab Reading Material for Experiment 2

AUTUMN 2022

Modulation is the process of manipulation of a carrier wave to add message signal to it. The carrier is generally a high frequency periodic signal, whereas the message signal is typically a lower frequency signal occupying a finite non-zero bandwidth. Generally, the message signals are referred to as base-band signals and are low pass in nature. On the other hand, the modulated signals are pass band in nature, which makes it easier to transmit them over communication channels, such as the wireless channel. The process of *demodulation* is the inverse of the modulation process, which basically involves recovery of the message signal from the received modulated carrier. At a very basic level, the analog modulation techniques can be classified as amplitude modulation (AM), phase modulation (PM) and frequency modulation (FM). In Experiment 2, we will study modulation and demodulation techniques for AM and FM signals, which are commonly used for transmitting analog signals.

1 Amplitude Modulation

In amplitude modulation (AM), the amplitude of the carrier wave, c(t) = cos(2πfct), is varied with the amplitude of the message signal x(t) to obtain the desired modulated signal s(t). In Experiment 2, we will study the following two AM schemes, which are commonly used for transmission of analog message signals:

- **DSB-FC (double side-band with full carrier):** In DSB-FC, the carrier is multiplied by the message signal to obtain the desired modulated signal, but after adding an offset to the signal before multiplication, so that the envelope of the signal never crosses zero. Therefore, we can write

$$\text{s(t)} = [1 + m.x(t)]cos(2\pi f_c t)$$

  where m is called the modulation index, which is chosen to ensure that $|m.x(t)| < 1$ always, so that the envelope never crosses zero (as shown in Fig.1 for the DSB-FC signal)

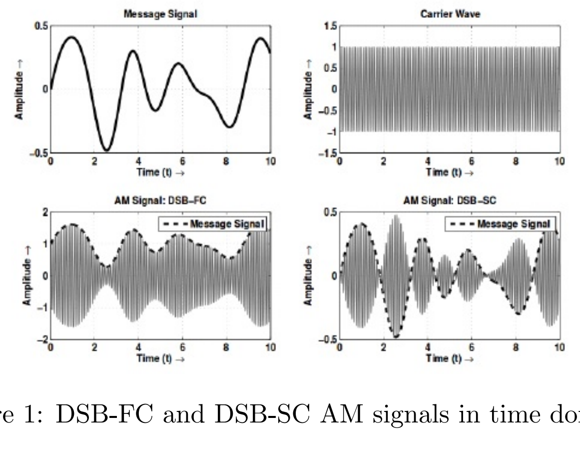

Figure 1: DSB-FC and DSB-SC AM signals in time domain

- **DSB-SC (double side-band with suppressed carrier):** In DSB-SC, the carrier is directly multiplied by the message signal to obtain the desired modulated signal. For DSB-SC, we can write the modulated signal as

$$\text{s(t)} = x(t)cos(2\pi fct)$$

  The time domain and frequency domain representations of the DSB-FC and DSB-SC AM signals are shown in Figures 1 and 2, respectively. Transmission of DSB-SC signals is more power efficient as no power is spent in transmission of the carrier. However, it is easier to demodulate a DSB-FC signal as it requires a simple envelope detector and does not require precise knowledge of the carrier frequency.

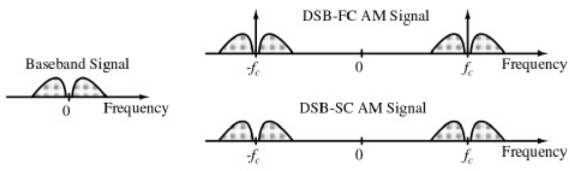

Figure 2: Baseband message signal and its corresponding DSB-FC and DSB-SC AM signals in frequency domain

1.1 Demodulation of AM signals

As can be observed from Fig. 1, the message signal can be demodulated from the DSB-FC signal by simply using an envelope detector, i.e. by passing the signal through a rectifier (or taking its absolute value), and then passing the resultant waveform through a low-pass filter. However, this approach cannot be used for demodulating the DSB-SC signals as the zero crossings in the message signal result in sign ambiguity because the envelope detector output is always positive. For demodulating a DSB-SC signal, the modulated signal is first multiplied by the carrier signal cos(2 πf_ct). The resultant waveform is passed through a low pass filter (LPF) to remove the $2f_c$ frequency component from it to obtain the message signal as the demodulated output:

$$x(t).cos(2\pi f_c t).cos(2\pi f_c t) = \frac{x(t)}{2} + \frac{1}{2}cos(4\pi f_c t)\underrightarrow{LPF}\frac{x(t)}{2}$$

For this operation to work successfully, the carrier frequency f_c has to be estimated precisely at the receiver, which is not very straight-forward.

2 SSB Modulation

The DSBSC modulated signal has two sidebands. Since, the two sidebands carry the same information, there is no need to transmit both sidebands. We can eliminate one sideband. The process of suppressing one of the sidebands along with the carrier and transmitting a single sideband is called as Single Sideband Suppressed Carrier system or simply SSBSC.
There are two approaches to eliminating one of the sidebands, one is the filter method and the other is the phasing method. The process of selective filtering of the upper or lower sideband is difficult due to the stringent filters required, especially if there's signal content close to DC. So we prefer to use the phasing method, which uses a Hilbert Transformer to implement SSB Modulation.

2.1 Hilbert Transform

Intuitively, the Hilbert transform of a signal applies a phase change of 90 degees. Formally the Hilbert transform $\hat{x}(t)$ of a signal x(t) is the output corresponding to the input x(t) of the LSI system defined by the following frequency response.

$$H(f) = \begin{cases} -j & f > 0 \\ 0 & f = 0 \\ j & f < 0 \end{cases}$$

Convince yourself that the Hilbert transform corresponding to cos(t) is sin(t).
Q: What is the Fourier transform of $x(t) + j\hat{x}(t)$?
Q: What is the Fourier transform of $x(t) - j\hat{x}(t)$?

2.2 Hilbert Transform in GNURadio

The Hilbert Transform block in GNURadio generates the complex output $x(t)+j\hat{x}(t)$ corresponding to the input x(t). In other words, for input real x(t), the imaginary part of the block output is the actual Hilbert transform of x(t).
Q: Is the Hilbert transform of a real signal guaranteed to be real?

2.3 SSB Modulation using Hilbert Transform

The SSB modulated signal (containing the USB part), s(t) can be written as

$$s(t) = \Re(x_c(t)e^{j2\pi f_c t})$$

where $x_c(t) = x(t) + j\hat{x}(t)$ is the complex signal for the message signal $x(t)$.
Expanding the equation and taking the real part we get

$$s(t) = x(t)\cos 2\pi f_c t - j\hat{x}(t)\sin 2\pi f_c t)$$

Similarly, the SSB modulated signal (containing the LSB part), s(t) can be written as

$$s(t) = \Re(x_c(t)e^{-j2\pi f_c t})$$

resulting in

$$s(t) = x(t)\cos 2\pi f_c t + j\hat{x}(t)\sin 2\pi f_c t)$$

3 IQ Modulator and demodulator

Any arbitrary passband signal $s_p(t)$ with center frequency f_c can be written as

$$s_p(t) = \Re(s(t)e^{j2\pi f_c t}) = s_I(t)\cos 2\pi f_c t - s_Q(t)\sin 2\pi f_c t$$

where $s_p(t) = s_I(t) + js_Q(t)$ is a complex baseband signal consisting of the two independent real baseband signals $s_I(t)$ and $s_Q(t)$. The signal $s(t)$ is also called the complex envelope of $s_p(t)$.Therefore, an IQ modulator is typically used for upconverting a complex baseband signal to a passband IF (intermediate frequency) or RF (radio frequency) signal, as shown in Fig. 3a. In a similar way, an IQ demodulator is used for downconverting as passband RF or IF signal to the complex baseband signal, as shown in Fig.3b .

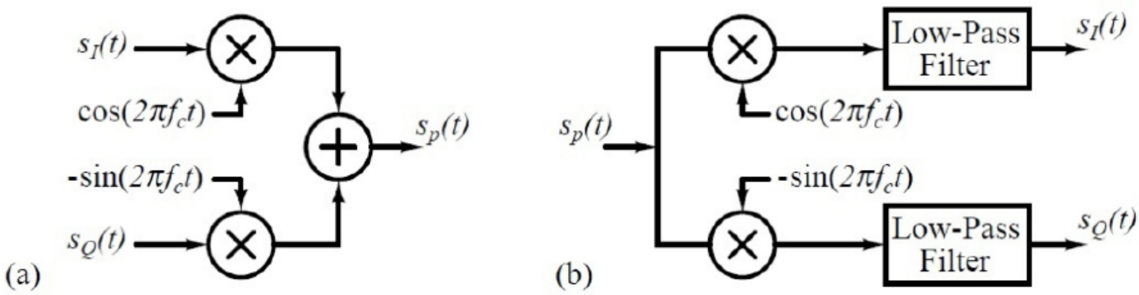

Figure 3: Flowgraph of (a) an IQ modulator, and (b) an IQ demodulator