VEHICLE DYNAMICS

# Torque Vectoring

**Report by :**

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## Introduction

Since the eighties, several advanced chassis control systems have been researched and developed, for instance active steering, direct yaw moment control (DYC) and active roll control. Active steering system provides additional steer angle to driver commands based on the difference between the expected and actual vehicle response to maintain the vehicle in its desired path. Vehicle lateral stability can be improved using direct yaw moment control by applying differential braking and traction. However, individual chassis control systems have certain limitations, for instance, in the near saturation region; active steering control cannot generate more tire lateral forces.

### Torque vectoring system:

By controlling the motors independently, we have direct control over the torque that can be applied at each wheel. By utilising the theory of Ackerman Steering we can calculate the distance travelled by each wheel for any turning circle of radius r. We will then use a state-based system to approximate these calculations in order to reduce computation time for the on-board processor. This state-based system will scale the output torque to each wheel depending on desired driver response.

The torque vectoring system is used to modify the lateral vehicle dynamics, improve the stability and maneuverability of the vehicle. These situations are usually not critical. On the other hand, the electronic stability system controls the vehicle stability in dangerous situations, where some severe damage might be caused.

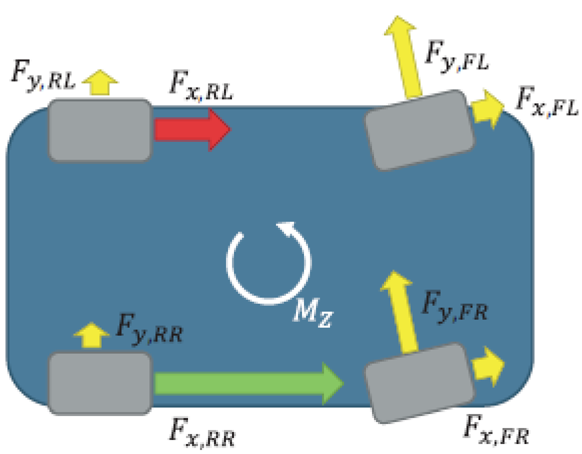
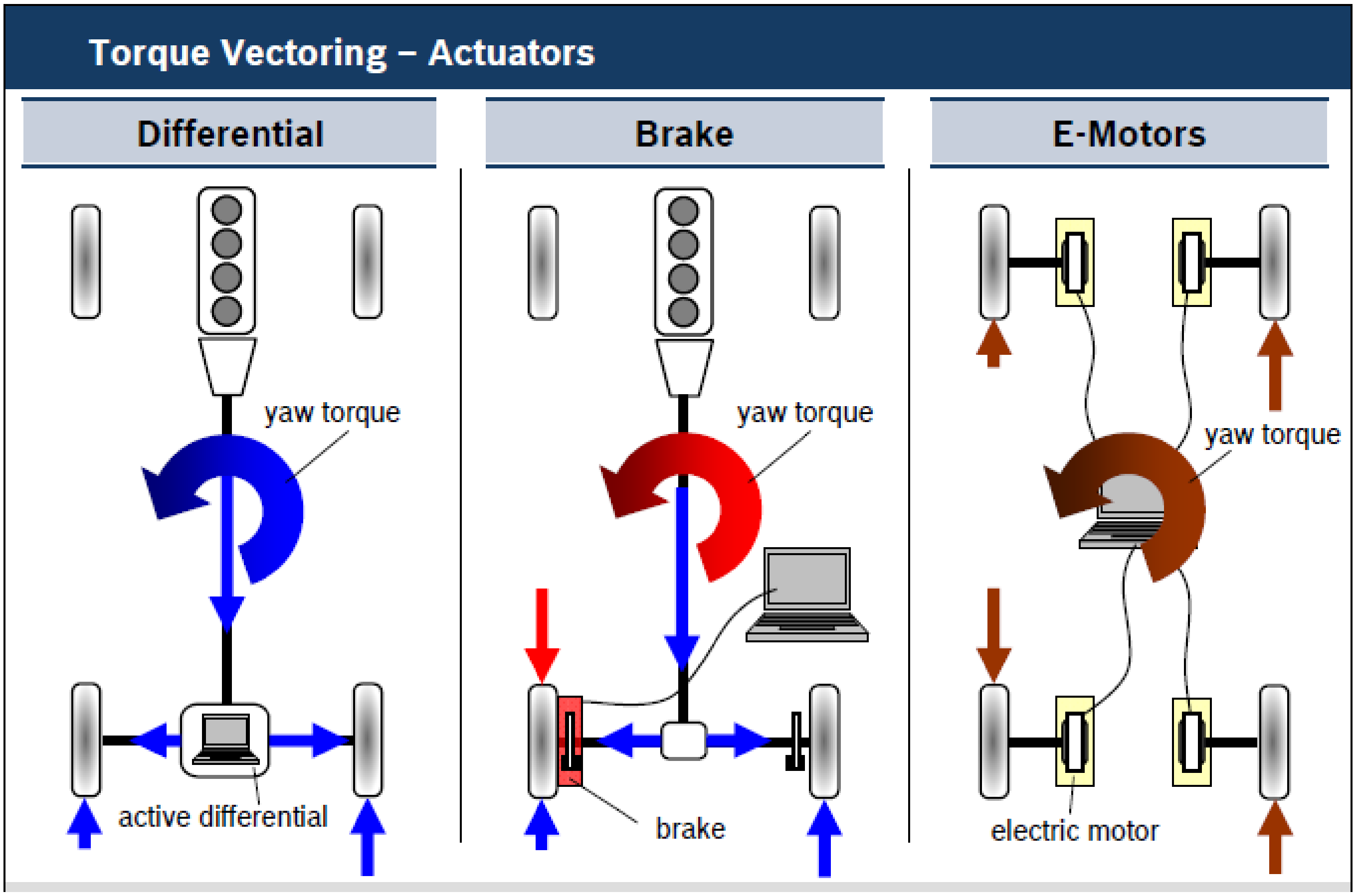



**Figure 1.4:** The difference in the rear wheel torque creates additional vehicle yaw moment, which can be used for the control of the vehicle stability.



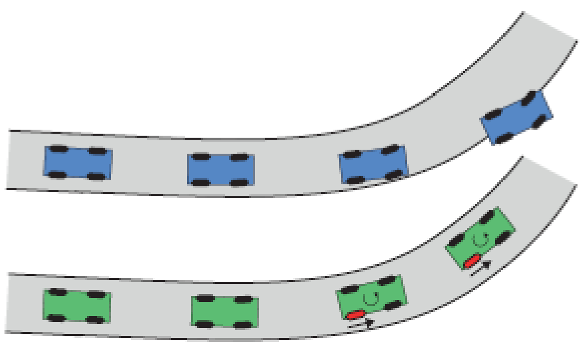



**Figure 1.5:** The comparison of the understeer vehicle maneuver without and with torque vectoring control system. The control system increases the torque on the wheel with the red color. This action creates additional vehicle yaw moment, which stabilizes the vehicle.

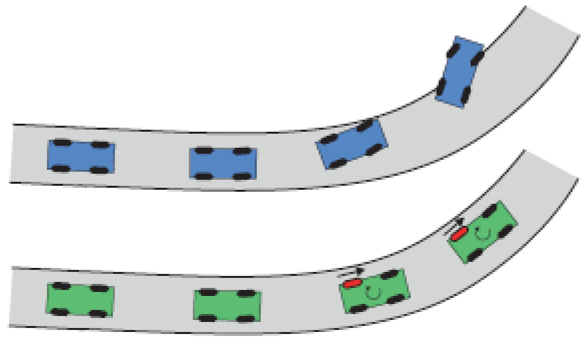


**Figure 1.6:** The comparison of the oversteer vehicle maneuver without and with torque vectoring control system. The control system increases the torque on the wheel with the red color. This action creates additional vehicle yaw moment, which stabilizes the vehicle.

## **Electronic stability program** (ESP)

It is a control system, which uses the vehicle braking system as the control actuator for vehicle stabilization. This system usually brakes one wheel to achieve higher or lower vehicle yaw rate depending on the vehicle understeer or oversteer behavior. This control action should provide the vehicle stability during all critical situations. An example of the ESP control of the understeer and oversteer vehicles is presented in figure 1.1 and 1.2.

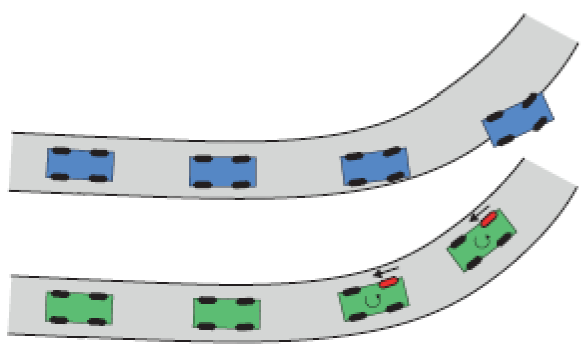


**Figure 1.1:** The comparison of the understeer vehicle maneuver without and with ESP control systems. The red wheel is slowed down by the control system and creates additional vehicle yaw moment, which stabilizes the vehicle. The term understeer means the tendency of the vehicle to steer less than the driver wants to.

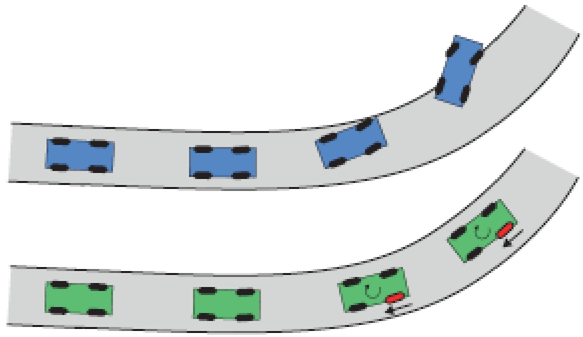


**Figure 1.2:** The comparison of the oversteer vehicle maneuver without and with ESP control systems. The red wheel is slowed down by the control system and creates additional vehicle yaw moment, which stabilizes the vehicle. The term oversteer means the tendency of the vehicle to steer more than the driver wants to.

## Torque vectoring system and Electronic stability program:

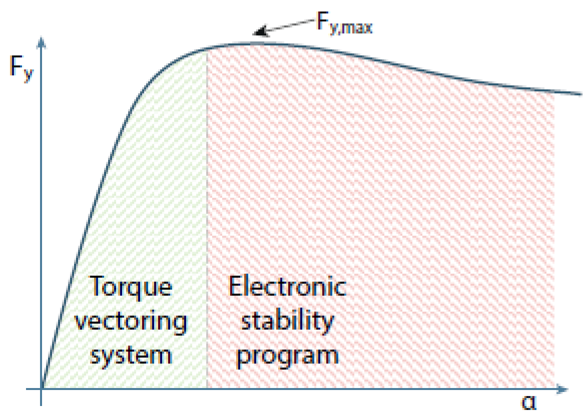



**Figure 1.3:** Area of lateral sideslip characteristics, where torque vectoring is used to control and modify the vehicle dynamics.

# The vehicle modeling

## Tire models:

Side-slip angle $\alpha$ and longitudinal slip $\lambda$ are defined as follows:

$$\lambda = \left|\frac{v_x - v_c}{v_x}\right|$$

$$\alpha = \arctan(\frac{v_y}{v_x})$$

where $vc$ is the circumferential velocity of the tire, $vx$ is longitudinal velocity of the tire center and $vy$ is lateral tire velocity of the tire center, both with respect to the ground.

Tire forces and slip ratio and wheel side slip angle relationship

$$F_x(\lambda) = \begin{cases} C_{\lambda 0}\lambda & \text{when} \quad C_{\lambda 0}\lambda \leq F_z\mu_{max} \\ F_z\mu_{max} & \text{when} \quad C_{\lambda 0}\lambda > F_z\mu_{max} \end{cases},$$

$$F_y(\alpha) = \begin{cases} C_{\alpha 0}\alpha & \text{when} \quad C_{\alpha 0}\alpha \leq F_z\mu_{max} \\ F_z\mu_{max} & \text{when} \quad C_{\alpha 0}\alpha > F_z\mu_{max} \end{cases},$$

## Pacejka's 'Magic' formula

$$F_y(\alpha) = D \cdot \sin\left(C \cdot \arctan\left(B\alpha - E(\,B\alpha - \arctan\left(B\alpha\right)\,)\right)\right),$$

## Kinematic vehicle model

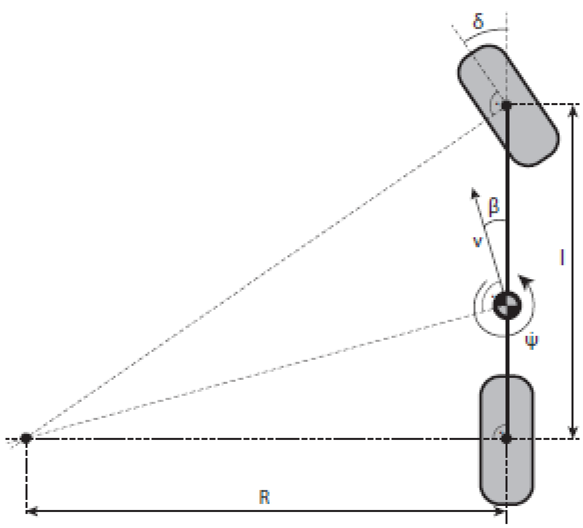



**Figure 2.6:** Mathematical vehicle model valid only for lower vehicle speeds.

$$\dot{\psi} = \frac{v}{R} = \frac{v}{l_r}\tan(\beta),$$

$$\beta = \tan^{-1}\left(\frac{l_r}{l_r + l_f}\tan(\delta_f)\right),$$

where $v$ is the vehicle speed, $R$ is the cornering radius, $\delta_f$ is the front tire steer angle, $l_f$ and $l_r$ are distances of the centre of gravity of the car to the front and rear tire respectively, $\dot{\psi}$ is the vehicle yaw rate and finally $\beta$ is the side-slip angle.

## Single track vehicle model

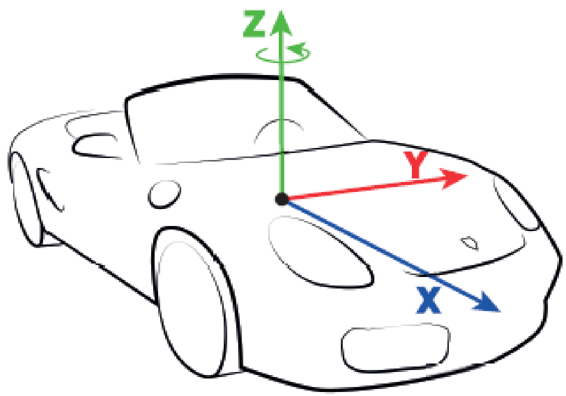



Figure 2.7: Vehicle coordinate system used within this thesis.

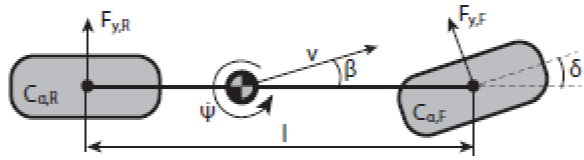



Figure 2.8: Single track model of the vehicle.

$$\beta = \arctan(\frac{v_y}{v_x})$$

Where $v_x$ and $v_y$ are the vehicle velocities in $x$ and $y$ direction of the vehicle coordinate frame respectively.

An angle between the longitudinal vehicle axis $x$ and fixed global axis $x_0$ is called vehicle yaw. Vehicle acceleration $\dot{v}$ has the same direction as the vehicle velocity $v$ if the vehicle motion is linear. If the vehicle is moving on curved track, we observe centripetal acceleration $a_c$ defined as

$$a_c = \frac{v^2}{R} = v(\dot{\beta} + \dot{\psi})$$

where $v$ is the instantaneous vehicle velocity, $R$ instantaneous radius of the curved track, $\dot{\beta}$ is angular rate of the vehicle side-slip angle and $\dot{\psi}$ is the vehicle yaw rate about the vehicle $z$ axis. However, the instantaneous radius of the motion is often unknown and changes frequently, thus only the second part of the equation with vehicle side-slip angle rate and yaw rate is normally used to compute the vehicle side acceleration.

$$-m\dot{v}\cos(\beta) + mv(\dot{\beta} + \dot{\psi})\sin(\beta) - F_{y,F}\sin(\delta) + F_{x,F}\cos(\delta) + F_{x,R} = 0$$

$$-m\dot{v}\sin(\beta) - mv(\dot{\beta} + \dot{\psi})\cos(\beta) + F_{y,F}\cos(\delta) + F_{x,F}\sin(\delta) + F_{y,R} = 0$$

$$-I_z\ddot{\psi} + F_{y,F}l_f\cos(\delta) - F_{y,R}l_r + F_{x,F}l_f\sin(\delta) = 0$$

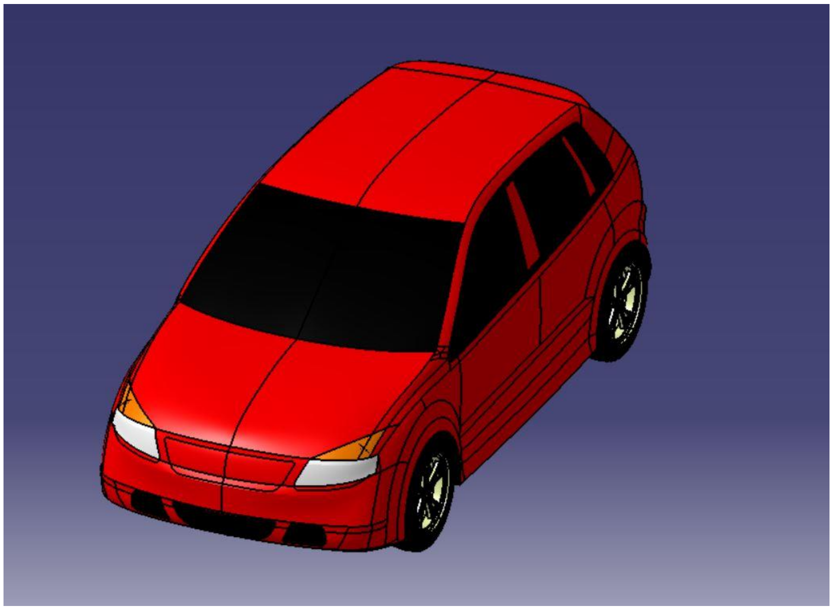
## Linear vehicle model with constant velocity

$$-m\dot{v} + F_{x,F} + F_{x,R} = 0,$$

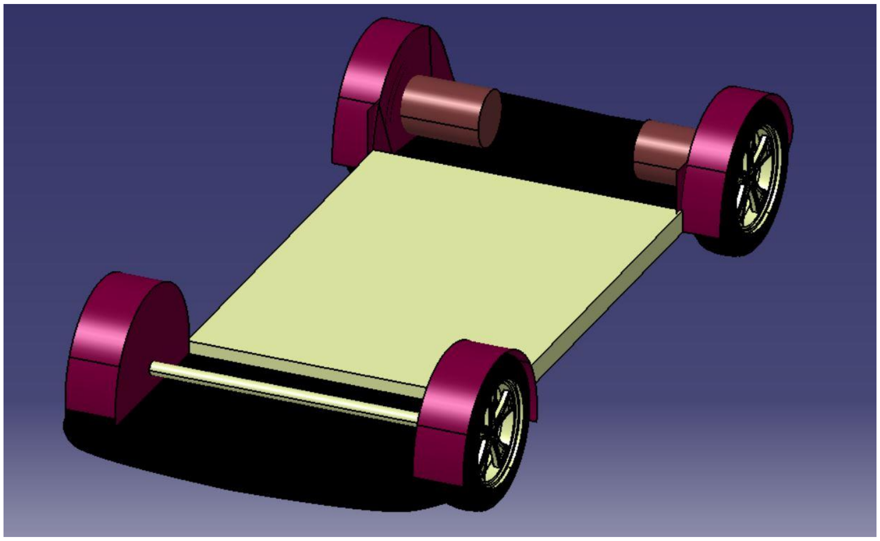
$$-mv(\dot{\beta} + \dot{\psi}) + F_{y,F} + F_{y,R} = 0 \quad \text{and}$$

$$-I_z\ddot{\psi} + F_{y,F}l_f - F_{y,R}l_r = 0.$$

## Car Model



## Rear Wheel Drive having individual motors



Governing Equations

- Considering only cornering forces at steady state conditions.
- $For\ small\ \beta\ and\ \delta_i,\ the\ terms\ containing\ the\ longitundinal\ forces\ of\ the\ tires\ can\ be\ neglected\ and\ the\ equilibrium\ equations\ reduce\ to$

$$\Sigma F_{y_i} = \frac{mV^2}{R}$$

$$\Sigma F_{y_i} x_i = 0$$

- Governing equation for handling for high speed cornering

$$mV(\dot{\beta} + r) + mV\beta = Y_\beta \beta + \dot{Y}_r r + Y_\delta + F_{ye}$$
$$J_z \dot{r} = N_\beta \beta + N_r r + N_\delta + M_{ze}$$

$m$ – *mass of the vehicle*
$V$ – *net speed of the vehicle*
$\beta$ – *side slip angle of the vehicle*
$r$ – *yaw rate or velocity*
$Y_\beta, Y_r, Y_\delta, N_\beta, N_r, N_\delta$ *are the stability derivatives*
$F_{ye}$ *and* $M_{ze}$ *are the external lateral force and external moment respectively*

- We obtain two first order differential equations which when in written in the state space form

$$\dot{z} = Az + B_c u_c + B_e u_e$$

where the state and input vectors $z,\ u_c$ *and* $u_e$ *are*

$$z = \begin{Bmatrix} \beta \\ r \end{Bmatrix},\ u_c = \delta \text{ and } u_e = \begin{Bmatrix} F_{ye} \\ M_{ze} \end{Bmatrix}$$

- $A = \begin{bmatrix} \frac{Y_\beta}{mV} - \frac{\dot{V}}{V} & \frac{Y_r}{mV} - 1 \\ \frac{N_\beta}{J_z} & \frac{N_r}{J_z} \end{bmatrix}$ *is the dynamic matrix*

- *And the input gain matrices are*

$$B_c = \begin{bmatrix} \dfrac{Y_\delta}{mV} \\ \dfrac{N_\delta}{J_z} \end{bmatrix} \text{ and } B_e = \begin{bmatrix} \dfrac{1}{mV} & 0 \\ 0 & \dfrac{1}{J_z} \end{bmatrix}$$

- *Stability is decided from the eigen values of the dynamic matrix* $A$.

$$|A - \lambda I| = 0$$

## Why can a monotrack model be used?

- $\alpha_i = \beta + \frac{x_i}{V} r - \delta_i$

- There is no y value dependence of the side slip angle of the wheels which implies that the side slips are the same for the two wheels of the same axle, neglecting the difference in the steering angles of the two wheels.
- Side slips are small, so constant cornering stiffness is assumed C0, linear region in the slip-mu curve.

## Toe in and load transfer

- Nor is allowance made for toe in and transversal load transfer. If the dependence of the cornering stiffness were linear with the load Fz, this would be correct since the increase of cornering stiffness of the more loaded wheel would exactly compensate for the decrease of the other wheel. As this is not exactly the case, the load transfer causes a decrease of the cornering stiffness of each axle, but this effect is usually considered negligible, at least for lateral accelerations lower than 0.5g.
- Toe in causes in increase of the cornering stiffness of the axle if it is positive, a decrease if it is negative.

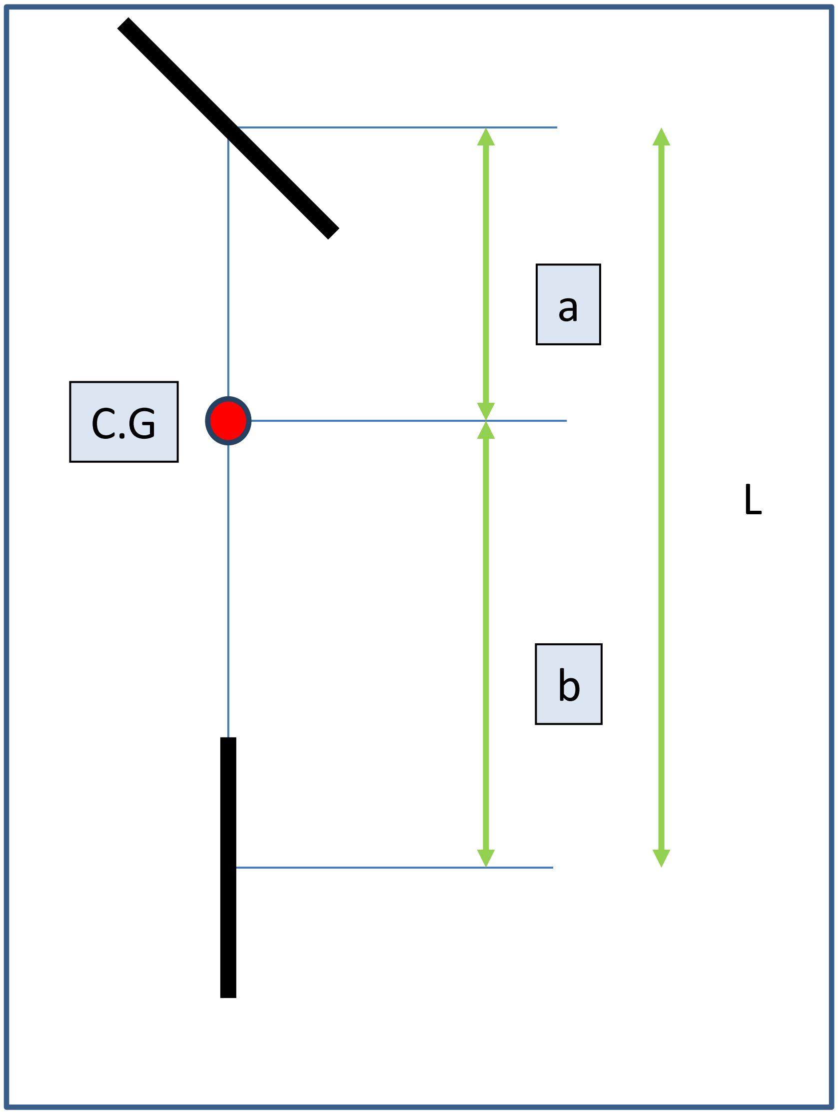
## Aerodynamic Yawing Moment

- The aerodynamic yaw moment produces a strong effect. If the derivative of Cz wrt beta is negative, the effect is increasing oversteer or decreasing understeer, at increasing speed. If a critical speed exists, such an aerodynamic effect lowers it and has an overall destabilising effect, increasing with the absolute value of Cmz, beta. Opposite occurs if Cmz, beta is positive.

## Feedforward Control System

- In this a control the output is known and assumed to be unchanging. A feedforward signal is input to the system and no output is fed back to any controller. Therefore it does not consider the response of the system which is a sort of a passive system. The error between the desired parameter and the response is not considered in such systems to modify the system response.

Single Track / Bicycle Model




**VEHICLE PARAMETERS USED:**

m = 1500; L = 2160 mm; a = 870 mm; b = 1290 mm
w = 1560 mm; Rw = 300 mm;
Cornering Stiffnesses :

$$C_f = C_1 = 67369\frac{N}{rad} C_r = C_2 = 63411\frac{N}{rad}$$

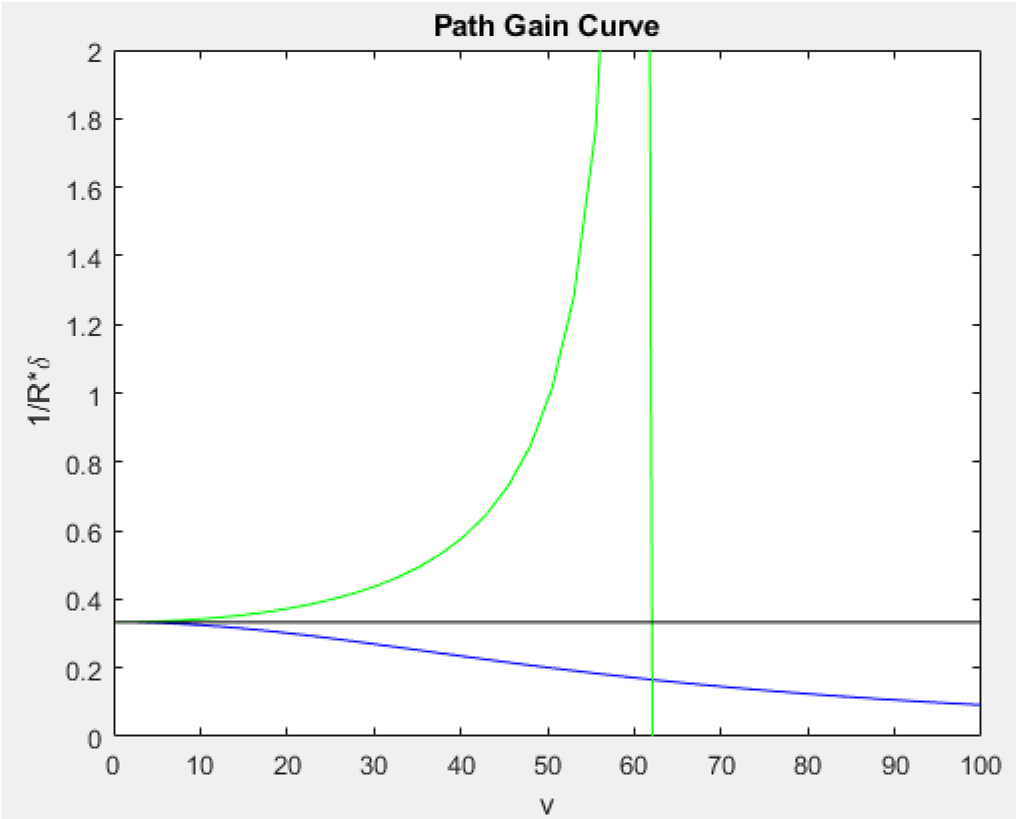
$$S = 1.7\ m^2; C_{y,\beta=-2.2}; C_{mz,\beta} = 0.6;$$

$$mz1_{,\alpha} = 2010; mz2_{,\alpha} = 1366;$$

$$\rho = 1.225\frac{kg}{m^3}; J_z = I = 2000\ kgm^2$$

(Obtained from *The Automotive Chassis Volume2 : System Design , Giancarlo Genta and Lorenzo Morello*

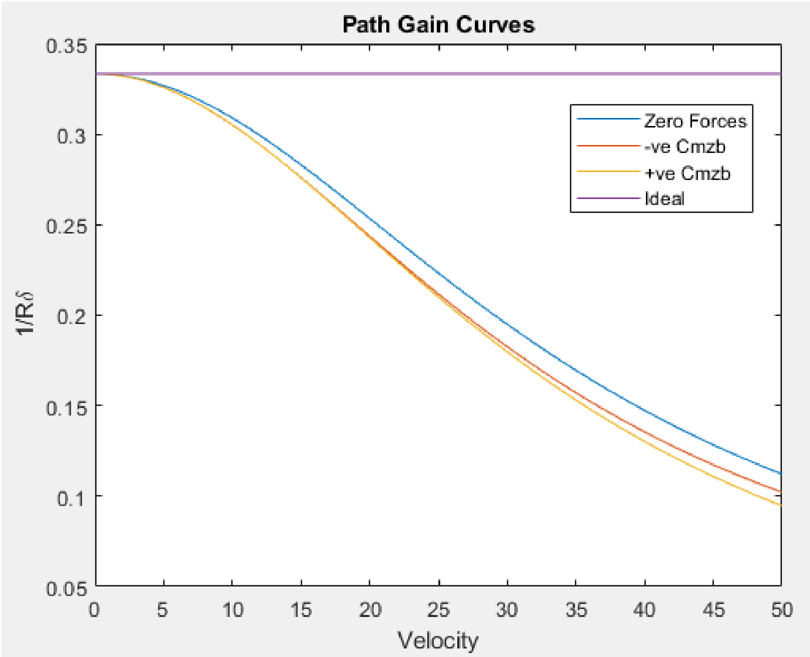
## Path Gain Curves




where

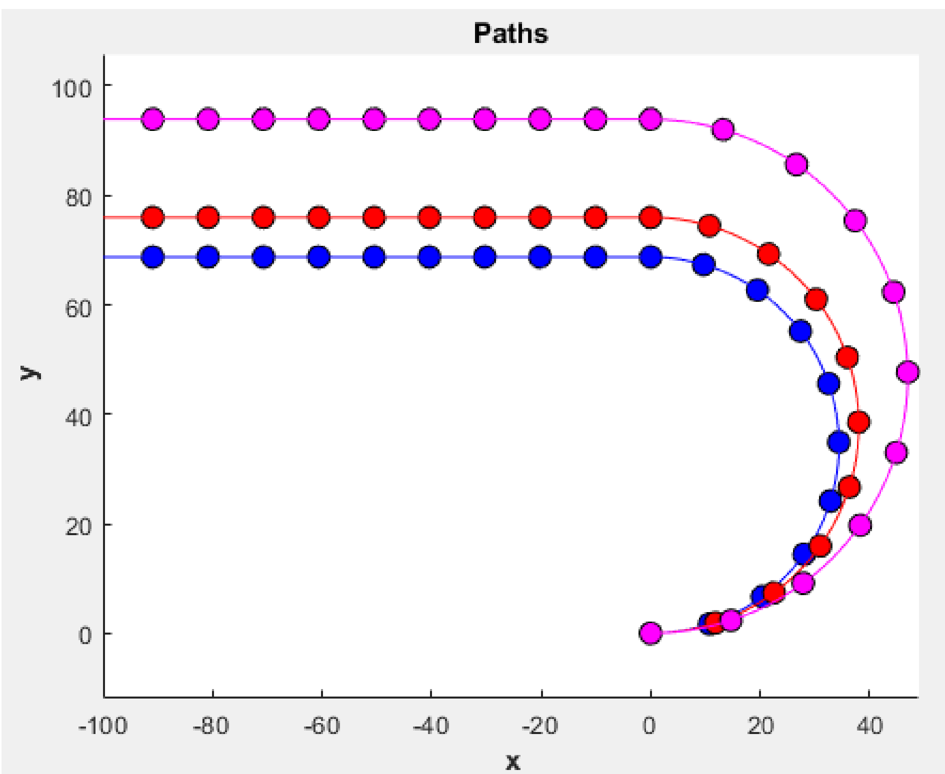
$$\frac{1}{R\delta} = \frac{1}{l} * \frac{1}{1 + \frac{\mathrm{K_{us}V^2}}{\mathrm{gl}}}$$

Where the above curves were obtained considering only the cornering forces and neglecting the aerodynamic forces and the aligning torques, which is the simplified approach. The below curve is obtained by the consideration of the same forces where all the derivatives associated with drag forces, side wind forces, aligning torques are all non-zero. In all the cases no consideration has been given to the load transfer in the lateral or longitudinal directions. The figure below also compares the understeer characteristic with varying parameters particularly the longitudinal drag coefficient derivative changing from positive to negative. The blue line represents the consideration of only the cornering forces which approximate the gains obtained by the previous consideration where the understeer gradient Kus was defined.




$$\frac{1}{R\delta} = \frac{y_\delta n_\beta - n_\delta y_\beta}{n_\beta(V^2 m - V y_r) + V n_r y_\beta}$$

## Paths Followed by Vehicles with understeers for different considerations



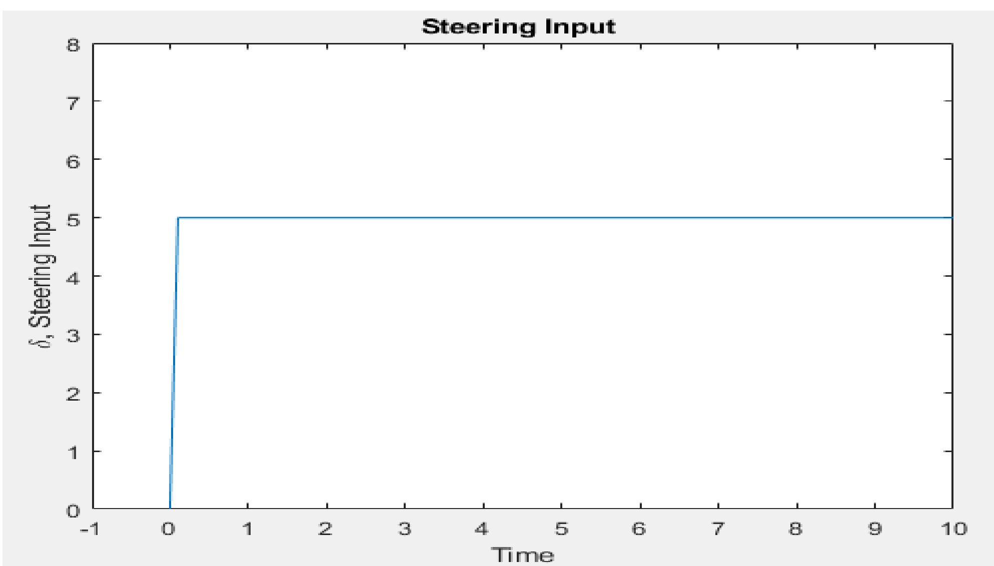
Under Kinematic Steering

Under Simplified Conditions With Only Cornering Forces

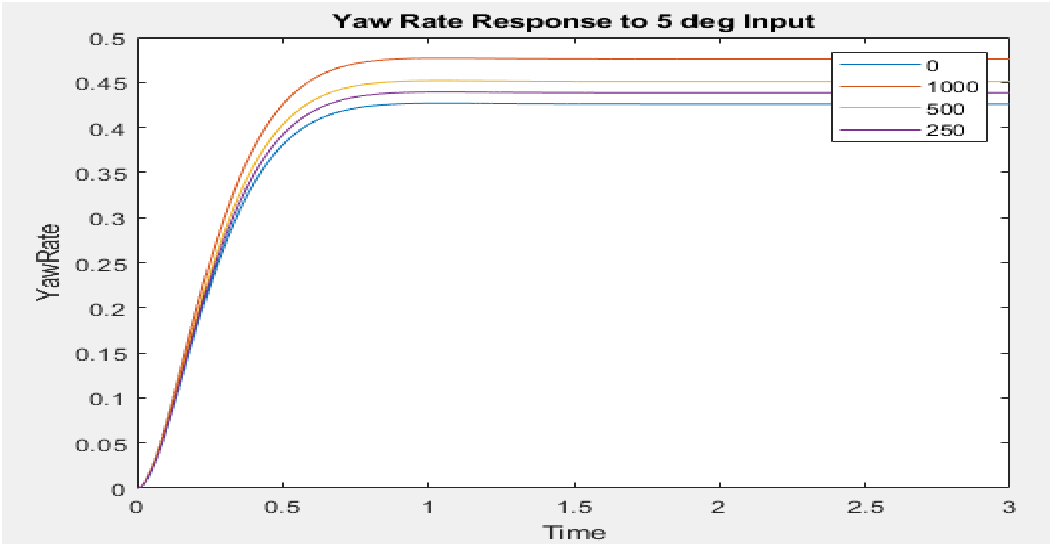
By considering cornering forces, aerodynamic forces, aligning torques.

The above figure represents the understeer characteristics of the chosen vehicle for various considerations as indicated in the legends. The curves form the locus of the locations of the vehicle with time. The kinematic steering is considered to be the ideal case of steering, the simplified approach is an understeer version where only the cornering forces are the acting forces considered and the radius of the turn is larger than the desired one. The third case depicts the consideration of all the forces like the aero forces and the aligning torques acting at the wheels, where the radius of the turn is further larger than the previous two cases and hence the understeer characteristic is larger.

### Yaw Rate Responses at Various Yaw Moments to 5-degree step steering Input



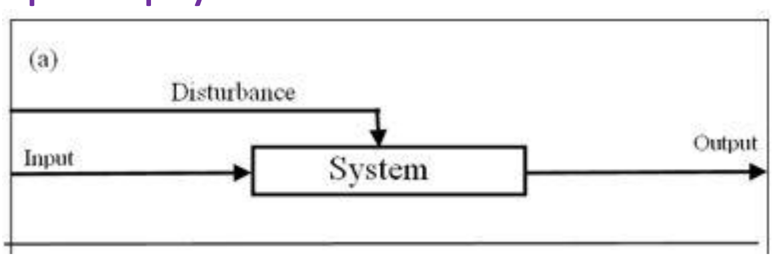

The study done in this term project is based on a vehicle initially taking a turn by a steering input of 5 degrees at the wheel (the main focus) and then the vehicle follows a straight path.



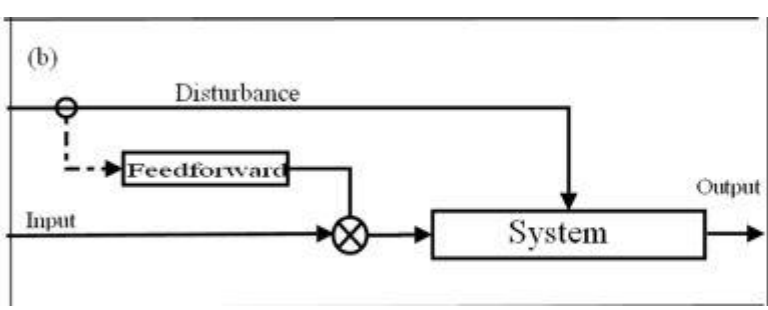

**The understeering of the vehicle ensues the lower yaw rate of the vehicle when compared the desired one. Therefore at any instant the understeer vehicle does not cover the desired yaw and follows a larger radius as the longitudinal velocity is kept constant in all the cases. The above plots were obtained as the responses to varying yaw moments which forms the major actuation in the torque vectoring system. As the yaw moment is increased it can be seen that the yaw rate increases and approaches the desired value (for a velocity of 20 m/s and steering input of 5 deg).**

## Types of Control Systems

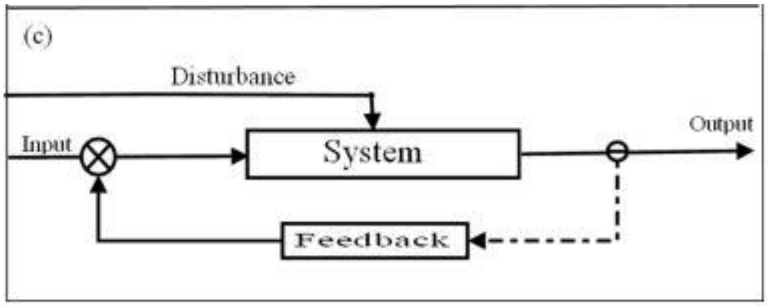
### Open loop System



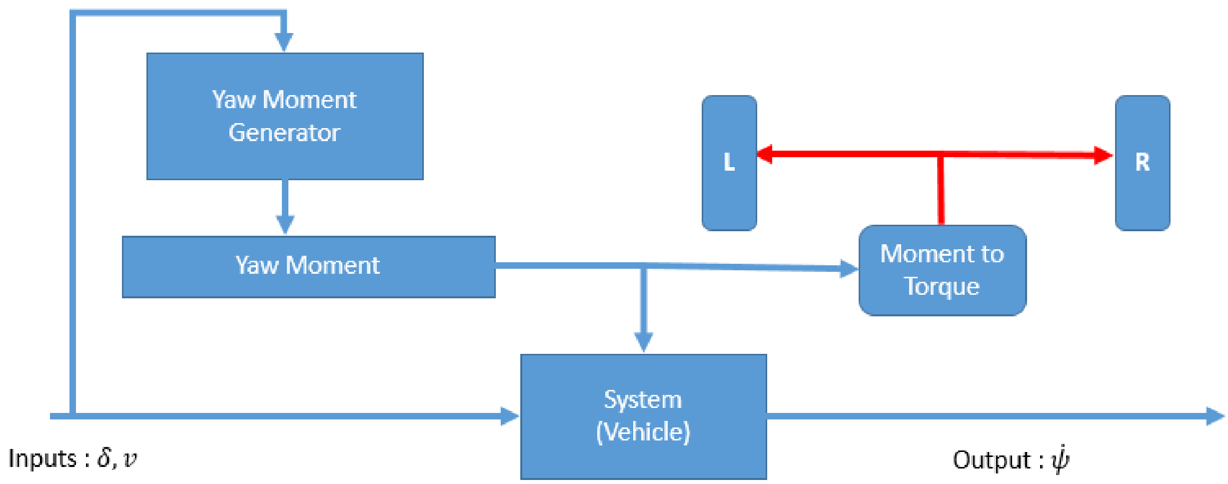

### Feedforward Control System



## Feedback Control System (Closed Loop)

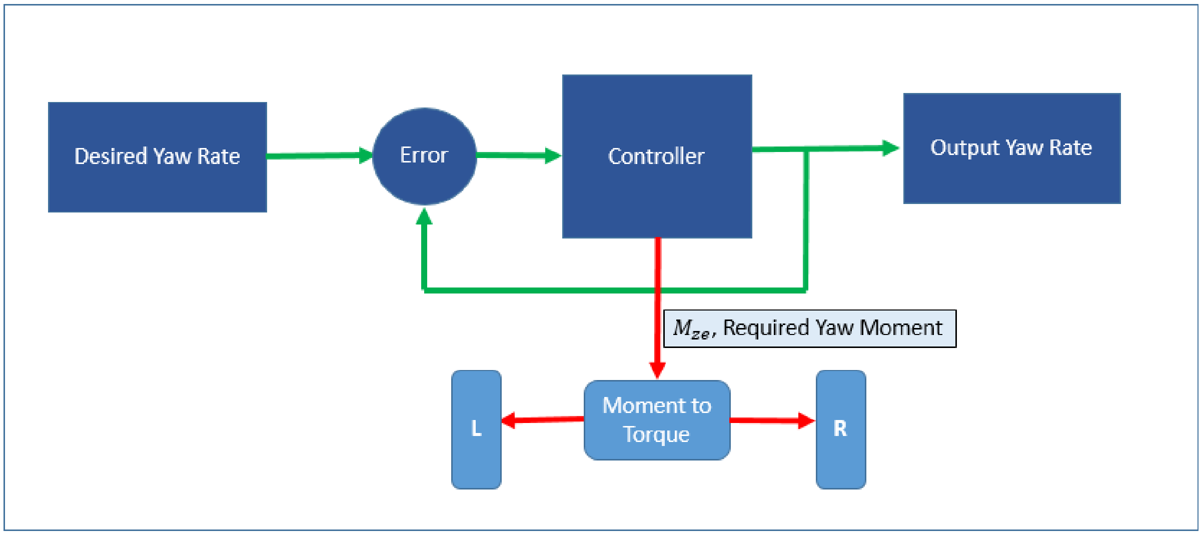



Simple Feedforward Control With Yaw Moment as Input



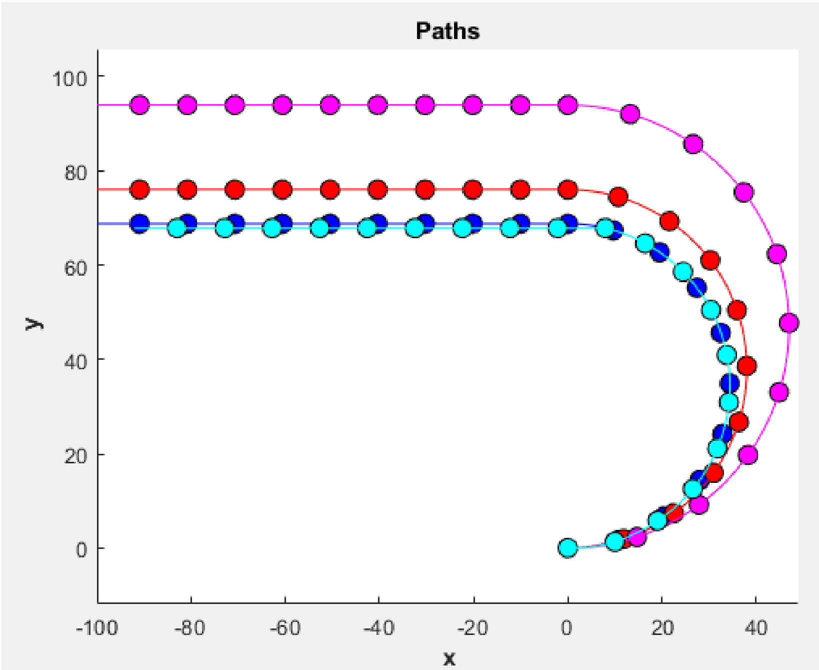

The above is a representation of the feedforward control of the system where there is no feedback taken from the output of the system. A mathematical model is stored in the memory of the control system and the driver inputs are fed into the system which determines the desired path and applies the required yaw moment based on the understeer/oversteer characteristic of the vehicle. The yaw moment is converted to torque and supplied to the wheels.

Feedback Control Model

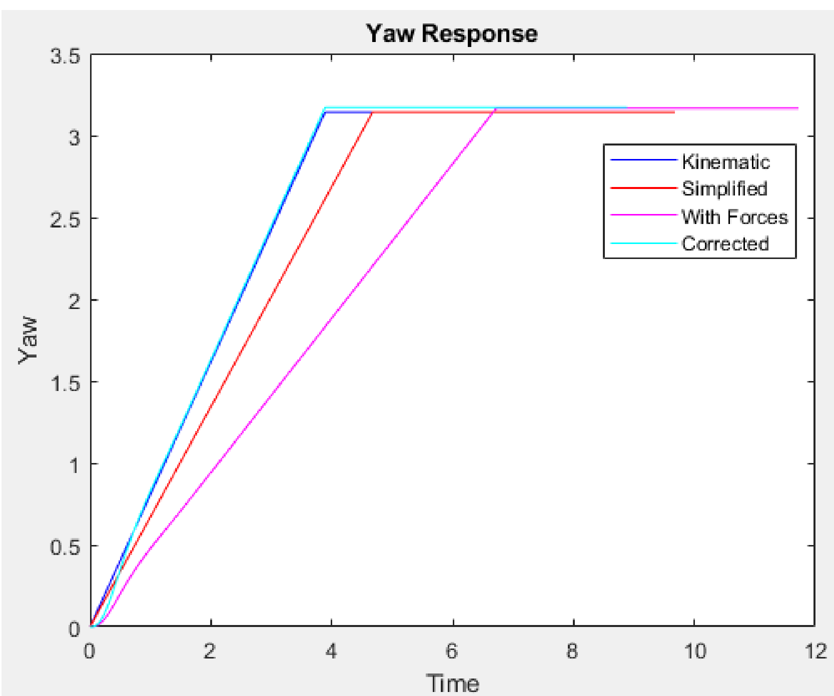


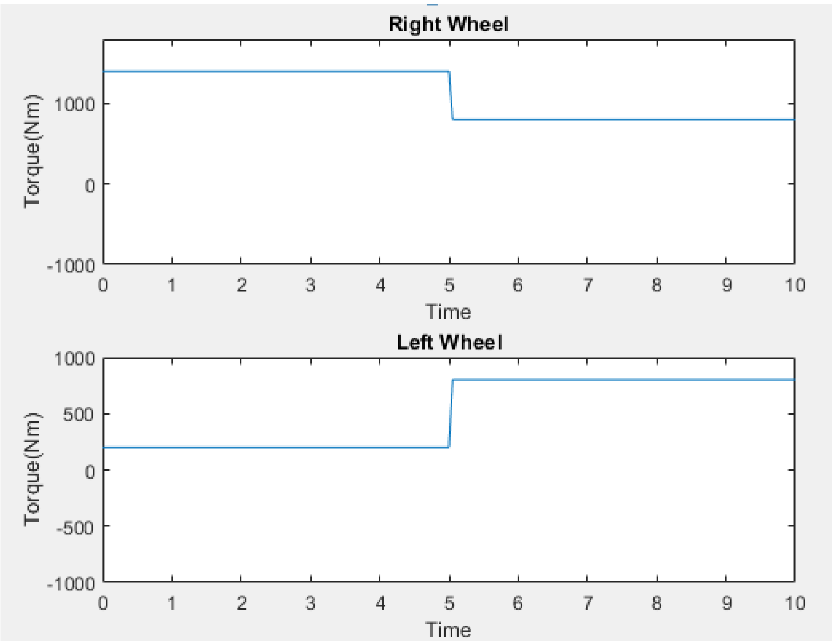

**The feedback control system is a closed loop control system which works distinctly different from an open loop or a feed forward control system in that it takes feedback from the output from the system and compares with a desired value and calculates the error. A PID or a PD or a PI controller then tries to minimise the error and varies the inputs accordingly (in this case the steering input can be kept constant and yaw moments can be generated and applied to the system) . The yaw moment which produces the least error between the desired and the actual values is given to the wheels in the form of torque. The feedback control can work based on the yaw rate feedback, the lateral acceleration feedback or the difference in the wheel slip angles feedback. The slip angle difference is zero for an understeer vehicle, front slip greater than the rear slip for understeer cases and rear slip greater than the front slip for oversteer cases.**

## Corrected Path of Vehicle at 72 kmph

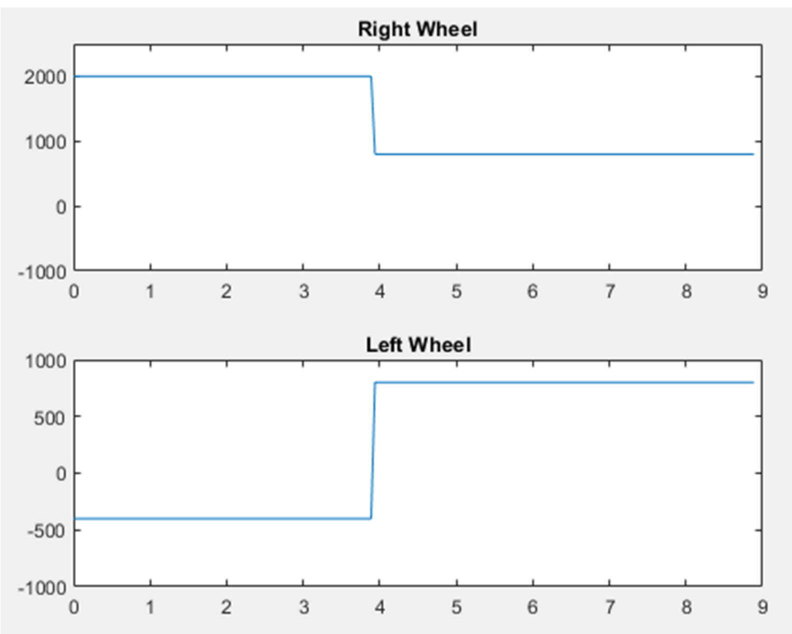



Path Followed by the Vehicle






## Distributed Torque to Left and Right Wheels at 100 kmph




**The above graphs show the corrected paths taken by the vehicle using the inputs from the feedforward control. The cyan path is the corrected path which closely approaches the ideal path in this case which is because the required yaw moment was generated from the governing equations which closely generates the dynamic response of the vehicle. The final figure shows the redistributed torque where the torque is taken from the left wheel and given to the right wheel (as in this case the turn is towards the**

**left). The changes in the torque apply to the desired torque which depends on the throttle input of the driver.**

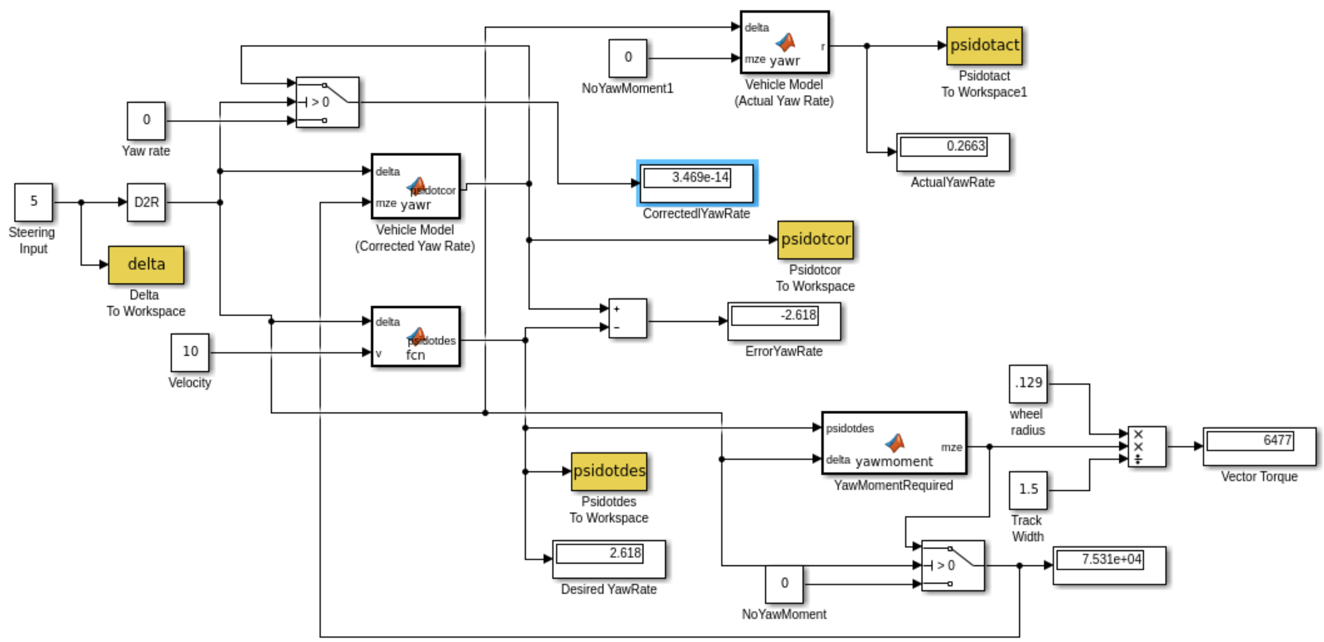
## Required Yaw moment converted to Torque:

$$\Delta T_{TV} = \frac{M_{ze}}{w} r_w$$

$$T_{RW} = T_{req} + \Delta T_{TV}$$

$$T_{LW} = T_{req} - \Delta T_{TV}$$

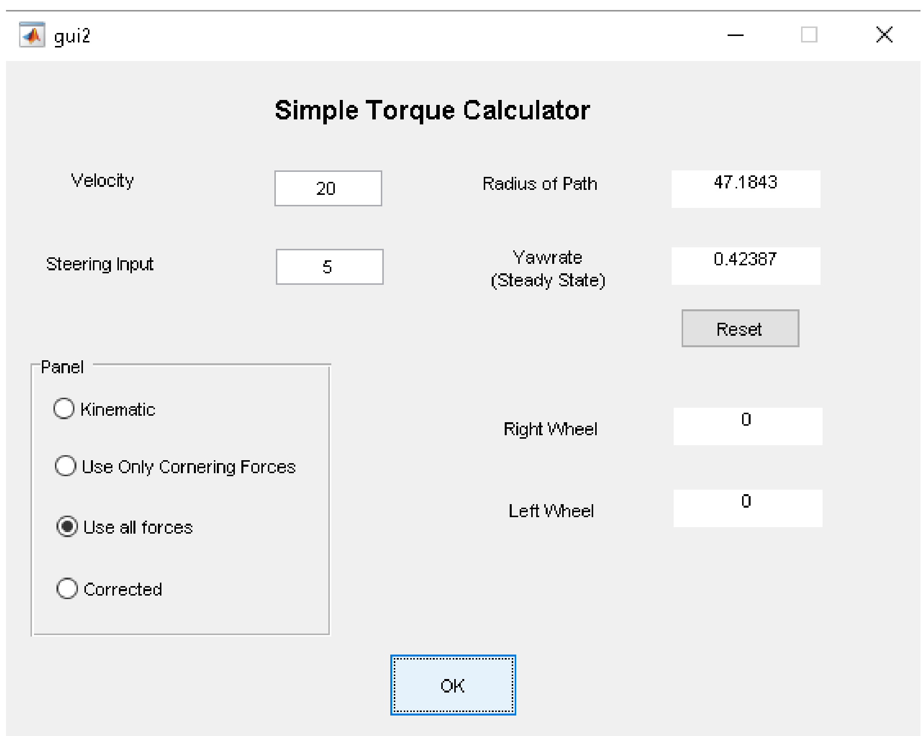
## A Simple Simulink Feedforward Control



## Special Implementations

- Active Yaw Control by Mitsubishi
- Torque Vectoring Differential by Lexus
- Dynamic Performance by BMW
- Quattro with Torque Vectoring by Audi
- Torque Vectoring by Mercedes Benz

## A Simple GUI for Torque Calculation




- Uses a set of the vehicle parameters in this study to obtain the yaw rate and the path radius.
- For a step input of steering which can be entered to the application along with the velocity of the vehicle which formed the inputs to the system.
- Scope for further additions like a variable steering input and an all-wheel torque distribution for stable vehicle dynamics, the purpose served by the torque vectoring system.

## Control Systems Integration

- The required torques are calculated from the inputs from the throttle.
- The completed control system software developed using the Matlab-Simulink is compiled into C code. The code also contains the instructions necessary to communicate the information with the module on the vehicle.

## References


1. Active Torque Vectoring Systems for Electric Drive Vehicles, Main Thesis, Martin Mondek, Czech Technical University in Prague.

2. A Torque Vectoring Strategy for Improving the Performance of a Rear Wheel Drive Electric Vehicle, Jyotishman Ghosh et al, Turin, Italy, 2015.

3. The Automotive Chassis Volume 2 : System Design, Giancarlo Genta and Lorenzo Morello.

4. Vehicle Stability by Dean Karnopp.

5. Video Ref : https://www.youtube.com/watch?v=AgqWcsivlAA by Protean Electric In_Wheel Drive Motors