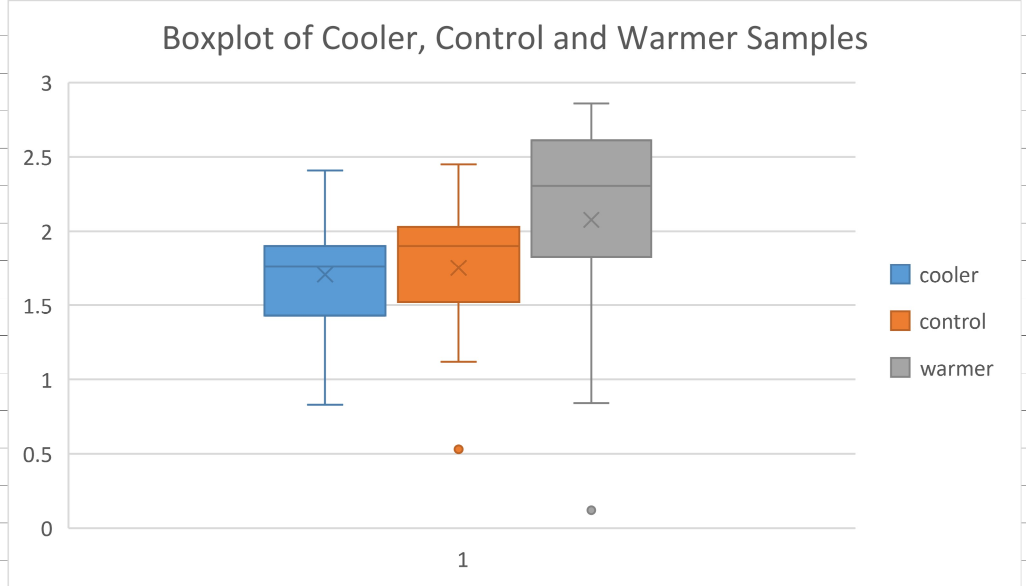
Pranav Pillai - SMLE HW 1

Q1	SMLE HW1		Pranav Pillai		
Chapter 1					d)
Question 46		cooler	control	warmer	
		1.59	1.92	2.57	
		1.43	2	2.6	
		1.88	2.19	1.93	
		1.26	1.12	1.58	
		1.91	1.78	2.3	
		1.86	1.84	0.84	
		1.9	2.45	2.65	
		1.57	2.03	0.12	
		1.79	1.52	2.74	
		1.72	0.53	2.53	
		2.41	1.9	2.13	
		2.34		2.86	
		0.83		2.31	
		1.34		1.91	
		1.76			
a)	mean	1.706	1.752727273	2.076429	
	median	1.76	1.9	2.305	
	mode	#N/A	#N/A	#N/A	
b)	sd	0.400835	0.531320824	0.777873	
c)	Q1	1.5	1.65	1.915	
	Q3	1.89	2.015	2.5925	
	Fourth Spread	0.39	0.365	0.6775	



In a) the mean and median is in the order of warmer> control> cooler .For b) the standard deviation is in the order for warmer>control>cooler. This means that the samples in warmer has a greater disparity from average compared to control and cooler so warmer has a greater measure of variability. For c) The fourth spread is not in agreement with the standard deviation as the fourth spread is in order of warmer>cooler> control which is different in comparison to standard deviation.

For d) it can be noticed that there are outliers for control and warmer data both at the lower end and the range is in the order of warmer > cooler > control. Cooler has no outliers. All three data are negatively skewed

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		2.41	1.9	2.13
		2.34		2.86
		0.83		2.31
		1.34		1.91
		1.76		
a)	mean	=AVERAGE(C4:C18)	=AVERAGE(D4:D14)	=AVERAGE(E4:E17)
	median	=MEDIAN(C4:C18)	=MEDIAN(D4:D14)	=MEDIAN(E4:E17)
	mode	=MODE.SNGL(C4:C18)	=MODE.SNGL(D4:D14)	=MODE.SNGL(E4:E17)
b)	sd	=STDEV.S(C4:C18)	=STDEV.S(D4:D14)	=STDEV.S(E4:E17)
c)	Q1	=QUARTILE.INC(C4:C18,1)	=QUARTILE.INC(D4:D14,1)	=QUARTILE.INC(E4:E17,1)
	Q3	=QUARTILE.INC(C4:C18,3)	=QUARTILE.INC(D4:D14,3)	=QUARTILE.INC(E4:E17,3)
	Fourth Spread	=C24-C23	=D24-D23	=E24-E23

Chapter 4 problem 4

Q2 a) To be a legitimate PDF → (1) $f(x) \geq 0$ for all x.

for $f(x;\theta) = \begin{cases} \frac{x}{\theta^2} \cdot e^{-x^2/(2\theta^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$ → $f(x) \geq 0$ as

the function concerned is always positive for $f(x) \geq 0$ when $x > 0$. This is because this is an exponential function for $x > 0$ leading to $f(x) \geq 0$.

(2) $\int_{-\infty}^{\infty} f(x)\,dx = 1 \rightarrow \int_0^{\infty} \frac{x}{\theta^2} \cdot e^{-x^2/2\theta^2}\,dx$.

Let $+x^2/2\theta^2 = u \rightarrow dx = \frac{2\theta^2}{+2x}\,du$

$\frac{+2x}{2\theta^2}\,dx = du$ $\qquad dx = \frac{+\theta^2}{x}\,du$

$$\int_0^{\infty} \frac{\cancel{x}}{\cancel{\theta^2}} \cdot e^{-u} \cdot \frac{+\cancel{\theta^2}}{\cancel{x}}\,du \rightarrow \int_0^{\infty} +e^{-u}\,du$$

$$\rightarrow \left[-e^{-u}\right]_0^{\infty} \rightarrow \left[\frac{-1}{e^u}\right]_0^{\infty} = \left[-\frac{1}{e^{\infty}} + \frac{1}{e^0}\right]$$

$= [0+1] = 1 \therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$ has been proved.

b) $\theta = 100$.

i) $P(X \leq 200) = P(X < 200)$

$$= \int_0^{200} \frac{x}{(100)^2} \cdot e^{-x^2/(2\times100^2)} = \int_0^{200} \frac{x}{10{,}000} \cdot e^{-x^2/20{,}000}.$$

Solving using wolfram Alpha solve, we get.

$P(X \leq 200)$ and $P(X < 200) = 0.86466.$

iii) $P(X \geq 200)$

$$= \int_{200}^{\infty} \frac{x}{(100)^2} \cdot e^{-x^2/(2\times 100^2)}$$

$$1 - P(X \leq 200) = P(X \geq 200)$$

$$1 - 0.86466 = 0.13534.$$

c) $P(100 \leq X \leq 200)$

$$\int_{100}^{200} \frac{x}{(100)^2} \cdot e^{-x^2/(2\times 100)^2}$$

Solving using wolfram alpha solver, we obtain

$= 0.47120$

d) $P(X \leq x)$

$$= \int_0^x \frac{x}{\theta^2} \cdot e^{-x^2/2\theta^2} dx = \left[-e^{-x^2/2\theta^2}\right]_0^x$$

$$= 1 - e^{-x^2/2\theta^2}$$

Chapter 8 Problem 25

Q3 $H_0 = \mu = 2$ sec $\quad H_a = \mu < 2$ sec.

As sample size $> 30 \rightarrow$ we use Central Limit theorem.

$n = 52$, $\sigma = 0.25$, $\mu = 1.95$ s.

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{1.95 - 2}{0.2/\sqrt{52}} = -1.803 \approx -1.8$$

Reading off the table for $z \rightarrow P(z = -1.8) = 0.0359$.

As we are testing against $\alpha = 0.01$ and $P(z) = 0.0359 > 0.01$

It appears that the average drive is less than 2 sec and hence. The null hypothesis should be fail to reject.

Chapter 8 Problem 40

Q4 $H_0 = \mu_0 = 48$ MPa, $H_0 = \mu_0 \equiv 48$ MPa. $\quad n = 10$

$H_a = \mu_0 > 48$ MPa. $\quad \sigma = 1.2$

as samples involved < 30, we use a t-test. $\bar{x} = 51.3$

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} = \frac{51.3 - 48}{1.2/\sqrt{10}} = 8.69626 \approx 8.69$$

$r = n - 1 = 10 - 1 = 9$ df

Referrering to the t-table we get $\rightarrow$

The P value for the one tailed test under the 9 df is to the right of 8.69 ~ 8.70. The area under the t curve to the right of 4.0 is 0.50 the area to the right of 8.69/8.70 is definitely 0. $P(t = 8.69) = 0$.

The P value obtained argues strongly for rejection of H_0 at any significance level and the difference between sample mean and expected value, when H_0 is true cannot be explained through chance variation. The true strength is something less than

48 MPa so the composition of fiber may affect the fiber strengths

Q5. Chapter 9 Problem 2

a) $\mu_1 = 64.9$ $n_1 = 866$ $SE_1 = 0.09$

$\mu_2 = 63.1$ $n_2 = 934$. $SE_2 = 0.11$

Confidence intervals for younger and older women at 95% →

$$64.9 - 63.1 \pm 1.96 \sqrt{\frac{(0.09)^2}{\cancel{866}} + \frac{(0.11)^2}{\cancel{934}}}$$

$$= 1.8 \pm 1.96 \left(\cancel{4.7232 \times 10^{-3}}\right)(0.14213)$$

$$= \left(\cancel{1.79074}, \cancel{1.80926}\right) \left(\cancel{-0.9857, 4.5857}\right)$$

or $1.8 \pm 0.2786 \rightarrow (1.5214, 2.079)$

At 95% CI → $\cancel{1.79074}$ 1.5214 $< \mu_1 - \mu_2 <$ $\cancel{1.80926}$ 2.079, one can be highly confident that any heights of young and/or old women exceeds by between 1.79074 and 1.80926 m. It does not include 0 so for chosen CI, 0 is not a plausible value for $\mu_1 - \mu_2$.

b), c) $H_0: \mu_1 - \mu_2 = 1$, $H_a: \mu_1 - \mu_2 > 1$

$$SE = \frac{\sigma}{\sqrt{n}}$$

In this formula SE is squared for it to be σ^2/n →

$$12 = \frac{\bar{x} - \bar{y} - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{m} + \frac{s_2^2}{n}}}$$

$\alpha = 0.001$

Q5 b),c) continued.

$$z = \frac{\bar{x} - \bar{y} - (\mu_1 - \mu_2)}{\sqrt{SE_1^2 + SE_2^2}}$$

$$= \frac{64.9 - 63.1 - 1}{\sqrt{0.09^2 + 0.11^2}} \approx 5.62878$$

$P(z = 5.62878)$, the $>$ sign in Ha suggests that a one-tailed test is appropriate.

The p value is $\rightarrow [1 - \phi(5.62878)] \simeq 0.00001$

As the p value $\simeq 0.00001 \leq 0.001 = a$, Ho is rejected at level 0.001 in favor of conclusion that $\mu_1 - \mu_2 > 1$. With a P-value so small the null hypothesis would be rejected at any significance level. The sample data strongly suggests that the true average height for ages 20-39 differs from that of ages 60 and older.

d) Ho: $\mu_2 - \mu_1 = 1$ inch, Ha: $\mu_2 - \mu_1 < 1$

Q6 Chapter 9, Problem 36 Ho: $\mu_D = 0$; Ha: $\mu_D > 0$.

Uses Paired t-test $n = 8 \rightarrow df = 8 - 1 = 7$

$D = U - A \rightarrow 7.9, 35, 5.5, 4.2, 6.7, -3.7, -0.9, 3.3$

$\mu_d = 7.25$. $\alpha = 0.01$.

$\sigma_d = 11.86279$.

$$t = \frac{\bar{d}}{S_D/\sqrt{n}} = \frac{7.25}{11.86279/\sqrt{8}} = 1.728612 \approx 1.7$$

Referring to the T table:

$p(t = 1.7) \rightarrow 0.065 = P$ value.

right of the t curve

at $1.7 = t$ and $v = 9$

as $0.065 > 0.01$, the null hypotheses can be failed to reject at significance level 0.01. It does seem apparent that the true average difference on the breaking load is something other than zero, the true average time for abraded and unabraded is much different.

Q2b) 1,2

WolframAlpha computational intelligence.

integral of x/(100^2)*e^((-x^2)/(2*100^2))) from 0 to 200

NATURAL LANGUAGE MATH INPUT EXTENDED KEYBOARD EXAMPLES UPLOAD RANDOM compute input

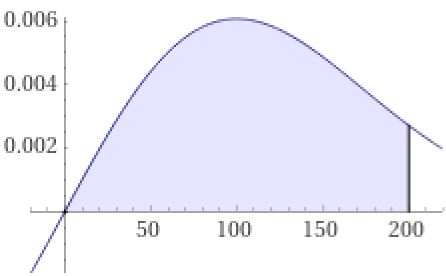
An attempt was made to fix mismatched parentheses, brackets, or braces.

Definite integral

More digits Step-by-step solution

$$\int_0^{200} \frac{x\, e^{-x^2/(2\times 100^2)}}{100^2}\, dx = 1 - \frac{1}{e^2} \approx 0.86466$$

Visual representation of the integral



Indefinite integral

Approximate form Step-by-step solution

$$\int \frac{x\, e^{-x^2/(2\times 100^2)}}{100^2}\, dx = -e^{-x^2/20000} + \text{constant}$$

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Q2c)

WolframAlpha computational intelligence.

integral of x/(100^2)*e^((-x^2)/(2*100^2))) from 100 to 200

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Definite integral

More digits | Step-by-step solution

$$\int_{100}^{200} \frac{x\, e^{-x^2/(2\times 100^2)}}{100^2}\, dx = \frac{e^{3/2}-1}{e^2} \approx 0.47120$$

Indefinite integral

Approximate form | Step-by-step solution

$$\int \frac{x\, e^{-x^2/(2\times 100^2)}}{100^2}\, dx = -e^{-x^2/20000} + \text{constant}$$

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