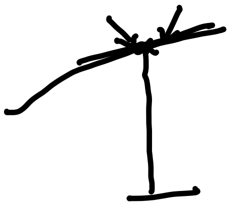
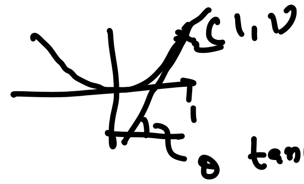




$\tan\theta = 2\times 1$
$= 2$ //

$y = x^2$, At any x RDL $\boxed{2x}$ //

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h}$$

$$\lim_{h\to 0} 2x = 2x \text{ //}$$

LDL $\lim_{h\to 0} \frac{(x-h)^2 - x^2}{-h}$

$$= \lim_{h\to 0} \frac{\cancel{x^2} - 2hx + h^2 - \cancel{x^2}}{-h}$$

$= 2x$ //

LDL = RDL $= \frac{dy}{dx} = 2x$

$\therefore \frac{d}{dx}(x^2) = 2x$ | First principle method
w.r.t x. | abinitio method //

$\frac{d}{dx} x^n = n x^{n-1}$.

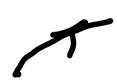
$$f'(x^+) = \lim_{h\to 0} \frac{(x+h)^n - x^n}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{x^n} + n x^{n-1} h + {}^nC_2 x^{n-2} h^2 + \dots + h^n - \cancel{x^n}}{h}$$

$\frac{d}{dx} x^n = \boxed{n x^{n-1}}$ ✓

$= n x^{n-1}$

LCDD



$$\frac{d}{dx} e^x = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h}$$

$$= \lim_{h\to 0} \frac{e^x (e^h - 1)}{h}$$

$$= e^x \lim_{h\to 0} \frac{e^h - 1}{h} \to 1 = e^x$$

$$\frac{d}{dx} \ln x = \lim_{h\to 0} \frac{\ln(x+h) - \ln x}{h} = \lim_{h\to 0} \frac{\ln\left(\frac{x+h}{x}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\ln\left(1+\frac{h}{x}\right)}{h} = \lim_{\substack{h\to 0 \\ \frac{h}{x}\to 0}} \frac{\ln\left(1+\frac{h}{x}\right)}{\frac{h}{x}} \to 1 \cdot x$$

$$= \frac{1}{x}$$

$$\frac{d}{dx} \sin x = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{2\cos\frac{x+h+x}{2} \cdot \sin\frac{x+h-x}{2}}{h}$$

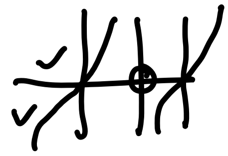
$$= \lim_{\substack{h\to 0 \\ \frac{h}{2}\to 0}} \frac{2\cos\left(x+h/2\right) \sin h/2}{2\, h/2} \to 1$$

$$\frac{\cancel{x}\cos x}{\cancel{x}} = \cos x.$$

$$\frac{d}{dx} \cos x = -\sin x \quad x \in R$$

$$\frac{d}{dx} \tan x \quad \Big| \quad y = \tan x \quad x \in R - (2n+1)\pi/2, n \in I.$$

$$\frac{d}{dx}\tan x = \underset{h\to 0}{\mathrm{lt}} \frac{\tan(x+h)-\tan x}{h}$$



$$= \underset{h\to 0}{\mathrm{lt}} \frac{\frac{\tan x+\tan h}{1-\tan x\tan h}-\tan x}{h}$$

$$= \underset{h\to 0}{\mathrm{lt}} \frac{\cancel{\tan x}+\tan h-\cancel{\tan x}+\tan^2 x\tan h}{h}$$

$$= \underset{h\to 0}{\mathrm{lt}} \underbrace{\frac{\tan h}{h}}_{\to 1}\{1+\tan^2 x\} = \underline{\underline{\sec^2 x}}.$$

lly $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x.$

$$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x\cot x.$$

$$\underset{h\to 0}{\mathrm{lt}} \frac{\operatorname{cosec}(x+h)-\operatorname{cosec} x}{h}$$

$$= \underset{h\to 0}{\mathrm{lt}} \frac{\frac{1}{\sin(x+h)}-\frac{1}{\sin x}}{h} = \underset{h\to 0}{\mathrm{lt}} \frac{\sin x-\sin(x+h)}{\sin x\sin(x+h)h}$$

$$= \underset{h\to 0}{\mathrm{lt}} \frac{\sin x-\sin(x+h)}{\sin x\sin(x+h)h}$$

$$= \underset{h\to 0}{\mathrm{lt}} \frac{2\cos\frac{x+x+h}{2}\cdot\sin\frac{-h}{2}}{\sin x\cdot\sin(x+h)\cdot h}$$

$$= \lim_{h \to 0} \frac{-2\cos\left(x+\frac{h}{2}\right) \cdot \sin\frac{h}{2}}{\sin x \sin(x+h) \cdot \frac{h}{2} \cdot 2}$$

$$= \frac{-2\cos x}{2\sin^2 x} = \underline{\underline{-\operatorname{cosec} x \cot x}}$$

Algebra of differentiation.

$y = u(x)$ | $y = u + v$ $\quad \frac{dy}{dx} = \frac{d}{dx}(u+v)$

$y = v(x)$ | $\frac{dy}{dx} = \frac{d}{dx}u + \frac{d}{dx}v$

$$\frac{dy}{dx} = \frac{\left(u(x+h) + v(x+h) - (u+v)\right)}{h}$$

$$= \frac{u(x+h) - u + v(x+h) - v}{h}$$

$$= \frac{u(x+h) - u(x)}{h} + \frac{v(x+h) - v_x}{h}$$

$$= u'_x + v'(x)$$

$$\Downarrow$$

$$\frac{d}{dx}u(x) + \frac{d}{dx}v(x) \; //$$

$$\frac{d}{dx}(u_1 + u_2 + \dots + u_n) = \frac{du_1}{dx} + \frac{du_2}{dx} + \dots + \frac{du_n}{dx}$$

||^{ly} $\frac{d}{dx}(U-V) = \frac{dU}{dx} - \frac{dV}{dx}$

$\frac{d}{dx} k f(x) = k \frac{d}{dx} f(x)$

$\lim_{h\to 0} \frac{k f(x+h) - k f(x)}{h}$

$= \lim_{h\to 0} k \left\{ \frac{f(x+h) - f(x)}{h} \right\}$

$= k \lim_{h\to 0} \underline{\underline{f'(x)}}. = k\frac{dy}{dx}.$

$\frac{d}{dx}(k_1 f(x) + k_2 g(x))$

$= \underline{\underline{k_1 \frac{d}{dx} f(x) + k_2 \frac{d}{dx} g(x)}}$

Find the derivative of

1) $y = \sqrt{x} + 2x^{5/3} + 3x^{2/3}$

$\frac{dy}{dx} = \frac{1}{2} x^{\frac{1}{2}-1} + 2 \cdot \frac{5}{3} x^{5/3-1} + 3 \cdot \frac{2}{3} x^{2/3-1}$

$= \frac{1}{2} x^{-1/2} + \underline{\underline{\frac{10}{3} x^{2/3}}} + 2x^{-1/3}$

2) $f(x) = \sin x + 2\cos x + e^x - \ln x$

$f'(x) = \cos x - 2\sin x + e^x - \frac{1}{x}$ //

$\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

$\frac{d}{dx}\cos^{-1}x = \frac{-1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$

$y = \sin^{-1}x \quad [-\frac{\pi}{2}, \pi/2] \rightarrow \cos y > 0.$

$x = \sin y$

$\frac{dx}{dy} = \cos y \qquad \therefore \frac{dy}{dx} = \frac{1}{\cos y} = \sec y.$

$= \frac{1}{\sqrt{1-x^2}}$ //

$x = \sin y$
$x^2 = \sin^2 y$
$1-x^2 = \cos^2 y$

$y = \cos^{-1}x \quad [0, \pi] \; \sin y > 0.$

$x = \cos y$

$\frac{dx}{dy} = -\sin y$

$\frac{dy}{dx} = -\operatorname{cosec} y = \frac{-1}{\sin y} = \frac{-1}{\sqrt{1-x^2}}$

$\sqrt{1-x^2}$
$= \sqrt{1-\cos^2 y}$
$= \sqrt{\sin^2 y}$
$= \sin y$ $\quad | \sin y > 0$

$|||^{ly}$ $y = \tan^{-1}x \qquad \frac{dy}{dx} = \frac{1}{1+x^2}$

||ly $y = \cot^{-1}x \quad \dfrac{dy}{dx} = \dfrac{-1}{1+x^2}$

$y = \mathrm{cosec}^{-1}x \quad y \in [-\frac{\pi}{2}, 0) \cup (0, \pi/2]$

$x = \mathrm{cosec}\, y$

$\dfrac{dx}{dy} = -\mathrm{cosec}\, y \cot y$

$\dfrac{dy}{dx} = -\sin y \tan y$

$\sin y = \dfrac{1}{x}$

$x^2 - 1 = \cot^2 y$

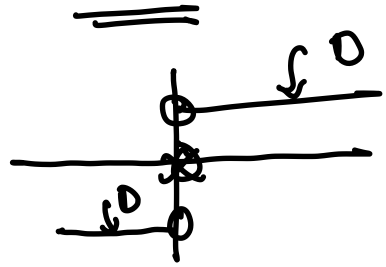
$\tan^2 y = \dfrac{1}{x^2-1}$

$\tan y = \dfrac{1}{\pm\sqrt{x^2-1}}$

$$= -\frac{1}{x}\frac{1}{\sqrt{x^2-1}} \quad x>0$$
$$= \frac{-1}{-x\sqrt{x^2-1}}, \quad x<0$$
$$\left.\right\} \frac{-1}{|x|\sqrt{x^2-1}} \text{ ''}$$

$y = \sec^{-1}x \quad \dfrac{dy}{dx} = \dfrac{1}{|x|\sqrt{x^2-1}}.$

$\dfrac{d}{dx}\mathrm{sgn}(x) = 0 \quad x \neq 0.$



$\dfrac{d}{dx} f(kx) = k f'(kx) \to \dfrac{d f(kx)}{d(kx)}$

$\dfrac{d}{dx}\sin 2x = 2\cos 2x$

$y = u_{(x)} \cdot v_{(x)}$

$$\frac{dy}{dx} = \frac{U_{(x+h)} \cdot V_{(x+h)} - U_{(x)} V_{(x)}}{h}$$

$$= \lim_{h \to 0} \frac{U_{(x+h)} \cdot V_{(x+h)} - U_{x+h} V_x + U_{x+h} V_x - U_x V_x}{h}$$

$$\lim_{h \to 0} U_{(x+h)} \left\{ \frac{V_{x+h} - V_x}{h} \right\} + \lim_{h \to 0} V_x \left\{ \frac{U_{x+h} - U_x}{h} \right\}$$

$$= U_{(x)} \cdot V'_{(x)} + V_x \cdot U'_{(x)}$$

$$= UV' + VU'.$$ → Product Rule.

$$\frac{d}{dx} x \sin x = x \cdot \cos x + \sin x \cdot 1$$

$$= x \cos x + \sin x.$$

$$\frac{d}{dx} \cdot x a^x = x \frac{d}{dx}(a^x) + a^x \times 1$$

$$= x \cdot a^x \ln a + a^x$$

$$= a^x (x \ln a + 1)$$

$$y = a^x$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h}$$

$$= \lim_{h \to 0} a^x \left(\frac{a^h - 1}{h} \right)$$

$$= a^x \underline{\underline{\ln a}}.$$

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx} \log_a x = \frac{d}{dx} \log_e x \cdot \log_a e$$

$$= \log_a e \cdot \frac{1}{x} = \frac{1}{x \log_e a} //$$

$$\frac{d}{dx} e^x \sin x = e^x \frac{d}{dx} \sin x + \sin x \cdot \frac{d}{dx} e^x$$

$$= e^x \cdot \cos x + \sin x \cdot e^x$$

$$= e^x \{ \cos x + \sin x \} //$$

$$\frac{d}{dx} \cdot e^x \cos x = e^x \frac{d}{dx} \cos x + \cos x \cdot \frac{d}{dx} e^x$$

$$= -e^x \sin x + \cos x \cdot e^x$$

$$= e^x (\cos x - \sin x) //$$

$$\frac{d}{dx} a^x \operatorname{cosec} x = a^x \frac{d}{dx} \operatorname{cosec} x + \operatorname{cosec} x \cdot \frac{d}{dx} a^x$$

$$= a^x \cdot - \operatorname{cosec} x \underline{\underline{\cot x + \operatorname{cosec} x \cdot a^x \ln a}}.$$

$$\frac{d}{dx}(\sec x - \tan x)(x^3 - \cot^{-1}x)$$

$$= (\sec x - \tan x)\left\{3x^2 + \frac{1}{1+x^2}\right\}$$

$$+ (x^3 - \cot^{-1}x)(\sec x \tan x - \sec^2 x)$$

$$=$$

fn	derivative.
x^n	nx^{n-1}
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \tan x$.

Quotient Rule.

$$y = \frac{P(x)}{Q(x)} \qquad \frac{dy}{dx} = \frac{Q(x) \cdot P'(x) - P(x) \cdot Q'(x)}{Q^2(x)}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{\frac{P(x+h)}{Q(x+h)} - \frac{P(x)}{Q(x)}}{h}$$

$$= \lim_{h \to 0} \frac{P(x+h)\,Q(x) - P(x)\,Q(x+h)}{Q(x+h)\,Q(x)\,h}$$

$$= \lim_{h \to 0} \frac{P(x+h)\,Q(x) - P(x)\,Q(x) + P(x)\,Q(x) - P(x)\,Q(x+h)}{Q(x+h)\,Q(x)\,h}$$

$$= \lim_{h \to 0} \frac{Q(x)\cdot\{P(x+h) - P(x)\} + P(x)\cdot\{Q(x) - Q(x+h)\}}{h\,Q(x+h)\,Q(x)}$$

$$= \lim_{h \to 0} \frac{Q(x)\cdot\left\{\frac{P(x+h)-P(x)}{h}\right\} - \left\{\frac{Q(x+h)-Q(x)}{h}\right\}P(x)}{Q(x+h)\,Q(x)}$$

$$\frac{d}{dx}\frac{P(x)}{Q(x)} = \frac{Q(x)\,P'(x) - Q'(x)\cdot P(x)}{Q^2(x)}$$ //

$$\frac{d}{dx}\left(\frac{x}{\log x}\right) = \frac{\log x \frac{d}{dx}x - x\frac{d}{dx}\log x}{(\log x)^2}$$

$$= \frac{(\log x)\times 1 - x\cdot\frac{1}{x\log_e a}}{(\log x)^2}$$

$$= \frac{(\log x) - \frac{1}{\log_e a}}{(\log x)^2}$$ //

$$\frac{d}{dx}\left(\frac{\sin x}{e^x}\right) = \frac{e^x \cdot \frac{d}{dx}\sin x - \sin x \cdot \frac{d}{dx}e^x}{(e^x)^2}$$

$$= \frac{e^x \cdot \cos x - \sin x \cdot e^x}{e^{2x}} \quad //$$

$y = |\cos x| + |\sin x|$

Find $\frac{dy}{dx}$ at $x = \frac{2\pi}{3}$.

$y = -\cos x + \sin x$ in $\left(\frac{2\pi}{3} - h, \frac{2\pi}{3} + h\right)$

$$\frac{dy}{dx} = -(-\sin x) + \cos x$$

$$= \underline{\underline{\cos x + \sin x}}.$$

If $\frac{d}{dx}\left(\frac{1+x^2+x^4}{1+x+x^2}\right) = ax+b$, $(a,b) =$ ___

$$\frac{(1+x+x^2)\frac{d}{dx}(1+x^2+x^4) - (1+x^2+x^4)\frac{d}{dx}(1+x+x^2)}{(1+x+x^2)^2}$$

check the calculation.

$$= \frac{(1+x+x^2)(2x+4x^3) - (1+x^2+x^4)(1+2x)}{(1+x+x^2)^2}$$

$x^2 \quad 3x^4 \quad 2x^5$

$$= (2x+4x^3) + \cancel{2x^2} + \cancel{4x^4} + \cancel{2x^3} + \cancel{4x^5}$$

$$- \{ 1+\cancel{2x}+\cancel{x^2}+\cancel{2x^3}+\cancel{x^4}+\cancel{2x^5} \}$$

$$\overline{\quad (1+x+x^2)^2 \quad}$$

$$= \boxed{\frac{4x^3 + x^2 + 3x^4 + 2x^5 - 1}{x^4+x^2+1+2x+2x^3+2x^2}}$$

$$\begin{array}{r|l} & 2x - 1 \\ \hline x^4+2x^3+3x^2+2x+1 & 2x^5+3x^4+4x^3+x^2-1 \\ & 2x^5+4x^4+6x^3+4x^2+2x \\ \hline & 0 + -x^4-2x^3-3x^2-2x-1 \\ & -x^4-2x^3-3x^2-2x-1 \\ \hline & 0 \end{array}$$

$2x-1 = ax+b$

$\therefore a=2$ $(a,b) = (2,-1)$
$b=-1$

$y = x|x|$ find $f'(x)$.

$x \geq 0$

$y = x^2$, $\frac{dy}{dx} = 2x = 2(x)$ $x > 0$

$x < 0$ $y = -x^2$, $\frac{dy}{dx} = -2x = 2(-x)$ $x < 0$

$= 2|x|$

$\frac{d}{dx} x|x| = 2|x|.$

$f(x) = |\cos x - \sin x|$

$f'(\pi/2) = $ ____ .

$f(x) = \sin x - \cos x$

$(\pi/2 - h, \frac{\pi}{2} + h)$

$f'(x) = \cos x + \sin x$

$x = \pi/2$

$= 1$