```python
import math
import numpy as np
import cv2
from google.colab.patches import cv2_imshow
```

Question 1: Compute the Euclidean distance between vector V_1 and V_2. Store the answer in variable d. Show screenshot\photograph of your calculation

$V_1 = \{2, -3, 5\}$ and $V_2 = \{6, 2, 1\}$

```python
x1,y1,z1,x2,y2,z2 = 2,-3,5,6,2,1
d = math.sqrt((x2-x1)**2 + (y2-y1)**2 + (z2-z1)**2)
print(f'The Euclidean distance is {d:.02f} .')
```

```
The Euclidean distance is 7.55 .
```

```python
def grader1(d, tol=1e-3):
    return math.isclose(d, math.sqrt(57), rel_tol=tol)
grader1(d)
```

```
True
```

Question 2: Find a Unit vector in the direction of $x = \{6, -8, 0\}$ and store it in variable 'x' as list. Show screenshot\photograph of your calculation

```python
x1,y1,z1 = 6,-8,0
p = abs(math.sqrt(x1**2 + y1**2 + z1**2)) # magnitude
print(f"The magnitude is {p} .")
x = [x1/p,y1/p,z1/p]
print(f'The unit vector is {x}.')
```

```
The magnitude is 10.0 .
The unit vector is [0.6, -0.8, 0.0].
```

```python
def grader2(v, tol=1e-3):
    cuv = (0.6, -0.8, 0)
    return all(math.isclose(v[i], cuv[i], rel_tol=tol) for i in range(3))
grader2(x)
```

```
True
```

Question 3: Find the determinant of the matrix and store the value in variable a A =

$$\begin{bmatrix} 3 & 4 & 2 \\ 2 & 1 & 5 \\ 6 & 0 & -1 \end{bmatrix}$$

Show screenshot\photograph of your calculation

```python
A = [[3,4,2],
    [2,1,5],
    [6,0,-1]]
det = (A[0][0] * (A[1][1] * A[2][2] - A[1][2] * A[2][1]) -
     A[0][1] * (A[1][0] * A[2][2] - A[1][2] * A[2][0]) +
     A[0][2] * (A[1][0] * A[2][1] - A[1][1] * A[2][0]))
print(f'The determinant is {det}.')
a = det
```

```
The determinant is 113.
```

```python
def grader3(ans, tol=1e-3):
    correct_det = 113
    return math.isclose(ans, correct_det, rel_tol=tol)

grader3(a)
```

```
True
```

Question 4: Compute the inverse (B^{-1}) of matrix B and store it in variable b

B =

$$\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$$

Show screenshot\photograph of your calculation

```
B = [[2,-1],[1,3]]
adjB = [[3,1],[-1,2]]
detB  = B[0][0]*B[1][1] - (B[0][1]*B[1][0])
b = []
for row in adjB:
    temp = []
    for element in row:
        temp.append(element/detB)
    b.append(temp)
print(f'Inverse of B is {b}')
```

```
Inverse of B is [[0.42857142857142855, 0.14285714285714285], [-0.14285714285714285, 0.2857142857142857]]
```

```
def grader4(B_inv, tol=1e-3):
    correct_B_inv = np.array([[3/7, 1/7], [-1/7, 2/7]])
    return np.allclose(B_inv, correct_B_inv, rtol=tol)
grader4(b)
```

```
True
```

Question 5: Compute the dot product value for $V_1 = \{1, 2, -1\}$ and $V_2 = \{3, -6, 2\}$ and store it in variable dot_prod. Show screenshot\photograph of your calculation

```
v1 = [1,2,-1]
v2 = [3,-6,2]
dotprod = 0
```

```
for i ,j in zip(v1,v2):
    dotprod += i*j
dot_prod = dotprod
print(f'The dot product is {dot_prod} .')
```

```
The dot product is -11 .
```

```
def grader5(dot_prod, tol=1e-3):
  is_correct = math.isclose(dot_prod, -11, rel_tol=tol)
  return is_correct
grader5(dot_prod)
```

```
True
```

Question 6: Compute the angle between V_1 = {1, 2} and $V_2 = \{3, 4\}$ using the dot product formula and store it in variable theta. Show screenshot\photograph of your calculation

```
v1,v2 = [1,2],[3,4]
dotpro = 0
for i ,j in zip(v1,v2):
    dotpro += i*j
magv1 = abs(math.sqrt(1**2 + 2**2))
magv2 = abs(math.sqrt(3**2 + 4**2))
theta = math.acos(dotpro/(magv1*magv2))
```

```
def grader6(theta, tol=1e-3):
    correct_theta = math.acos(11 / (math.sqrt(5) * 5))
    return math.isclose(theta, correct_theta, rel_tol=tol)

grader6(theta)
```

```
True
```

Question 7: Find the eigenvalues of C =

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

and store it in variables lamda1 and lambda2.

Show screenshot\photograph of your calculation

```
lambda1= 1 #write your answer
lambda2 = 3 #write your answer


def grader7(lamda1, lamda2):
    c1 = 3
    c2 = 1
    if (lamda1 == c1 and lamda2 == c2) or \
       (lamda1 == c2 and lamda2 == c1):
        return True
    else:
        return False
grader7(lambda1, lambda2)
```

```
True
```

```
# Open the image.
img = cv2.imread("/content/Q7.png")
cv2_imshow(img)
```

$$C = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$C - \lambda I = \begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}$$

$$(2-\lambda)(2-\lambda) - 1 = 0.$$

$$4 - 2\lambda - 2\lambda + \lambda^2 - 1 = 0.$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\lambda = 1, 3$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 - 4 \times 1 \times 3}}{2 \times 1}$$

$$\frac{4 \pm \sqrt{4}}{2}.$$

$$\frac{4 \pm 2}{2}. = \cancel{6}$$

roots are $\frac{2}{2}, \frac{6}{2}$

$= 1, 3$

Question 8: A box contains 4 red, 3 blue, and 2 green balls. What is the probability of drawing a red or blue ball? store the result in variable p. Show screenshot\photograph of your calculation

```
red_balls = 4
blue_balls = 3
green_balls = 2
total_outcomes = red_balls + blue_balls + green_balls
favorable_outcomes = red_balls + blue_balls

p = (favorable_outcomes/total_outcomes) #write your answer
print(f"probablity of drawing a red ball or blue ball is (4 + 3)/ 9 = {p}.")
```

```
probablity of drawing a red ball or blue ball is (4 + 3)/ 9 = 0.7777777777777778.
```

```
def grader8(prob):
    if abs(prob - 0.7777777777777777777) < 1e-6:
        return True
    else:
        return False
grader8(p)
```

```
True
```

Question 9: A test for a disease is 95% accurate, and 1% of people have the disease. If a person tests positive, what is the probability that they actually have the disease? Store the answer in variable p. Show screenshot\photograph of your calculation

```
p = 0.161
```

```
def grader9(p):
    if p==0.161:
        return True
```

```
    else:
        return False
grader9(p)
```

```
True
```

```
# Open the image.
img = cv2.imread("/content/Q9.jpg")
cv2_imshow(img)
```

9.

A - person has disease

B = Person test positive

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

$P(A) = 0.01$

$P(B|A) = 0.95$

$P(B|A') = 0.05$

$P(B) = P(B|A) \cdot P(A) + P(B|A') \cdot P(A')$

$= 0.0095 + 0.0495 = 0.059$

$$P(A|B) = \frac{0.95 \times 0.01}{0.059} = 0.161$$

Probablity that person ~~actully~~ that actually has disease, tests ~~positive~~ positive is 16.1%

Question 10: In a class, 70% of students pass math, and 50% pass physics. If 30% pass both, what is the probability that a student who passes math also passes physics?. Store your answer in variable p. Show screenshot\photograph of your calculation

```
p = round(0.30/0.70,4) #write your answer
```

```
def grader10(p):
    if p==0.4286:
        return True
    else:
        return False
grader10(p)
```

```
True
```

```
# Open the image.
img = cv2.imread("/content/Q10.png")
cv2_imshow(img)
```

10.

$P(\text{Math}) = 70\% = 0.70$

$P(\text{Physics}) = 50\% = 0.50.$

$P(\text{Physics and Math}) = 30\% = 0.30.$

$$P(\text{Physics} \mid \text{Math}) = \frac{P(\text{Physics and Math})}{P(\text{Math})} = \frac{0.30}{0.70} = \underline{\underline{0.486}}$$

Question 11: Compute the entropy of a biased coin with P(Heads) = 0.8 and P(Tails) = 0.2. Store your answer in variable e given below. Comment your oberservation about the uncertainity about the situation. Show screenshot\photograph of your calculation

```
e = 0.72193 #write your answer
```

```
def grader11(e):
    if e==0.72193:
        return True
    else:
```

```
        return False
grader11(e)
```

```
True
```

Entropy measures the uncertainty or randomness of a random variable. In this case, the entropy of the biased coin (0.721928) is less than the entropy of a fair coin (which would be 1), indicating that there is less uncertainty or randomness in the outcome of the biased coin. Since the probability of heads is higher (0.8), the outcome is more predictable, leading to lower entropy.

```
# Open the image.
img = cv2.imread("/content/Q11.JPG")
cv2_imshow(img)
```

11.

$$H = -(P(Heads) \cdot \log_2(P(Heads)) + P(Tails) \cdot \log_2(P(Tails)))$$

$$H = (0.8 \cdot \log_2(0.8) + 0.2 \cdot \log_2(0.2))$$

$$\log_2(0.8) = -0.3219$$

$$\log_2(0.2) = -2.32193$$

$$H = -(0.8(-0.3219) + 0.2(-2.32193))$$

$$= 0.72193.$$

Question 12: Given the random variable X that takes the values $\{1, 2, 3, 4\}$ with probabilities $\{0.1, 0.2, 0.3, 0.4\}$, compute the expected value of X. Store the answer in variable expected_value. Show screenshot\photograph of your calculation

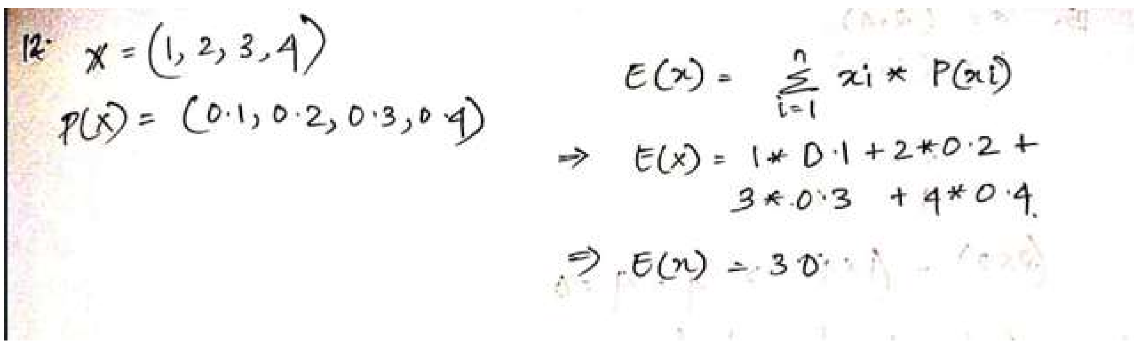
```
expected_value = 3.0
```

```
def grader12(expected_value, tol=1e-3):
    return math.isclose(expected_value, 3.0, rel_tol=tol)

grader12(expected_value)
```

```
True
```

```
# Open the image.
img = cv2.imread("/content/Q12.png")
cv2_imshow(img)
```



Question 13: Two dice are rolled. Find the probability that their sum is greater than 8. Store the answer in variable prob. Show screenshot\photograph of your calculation

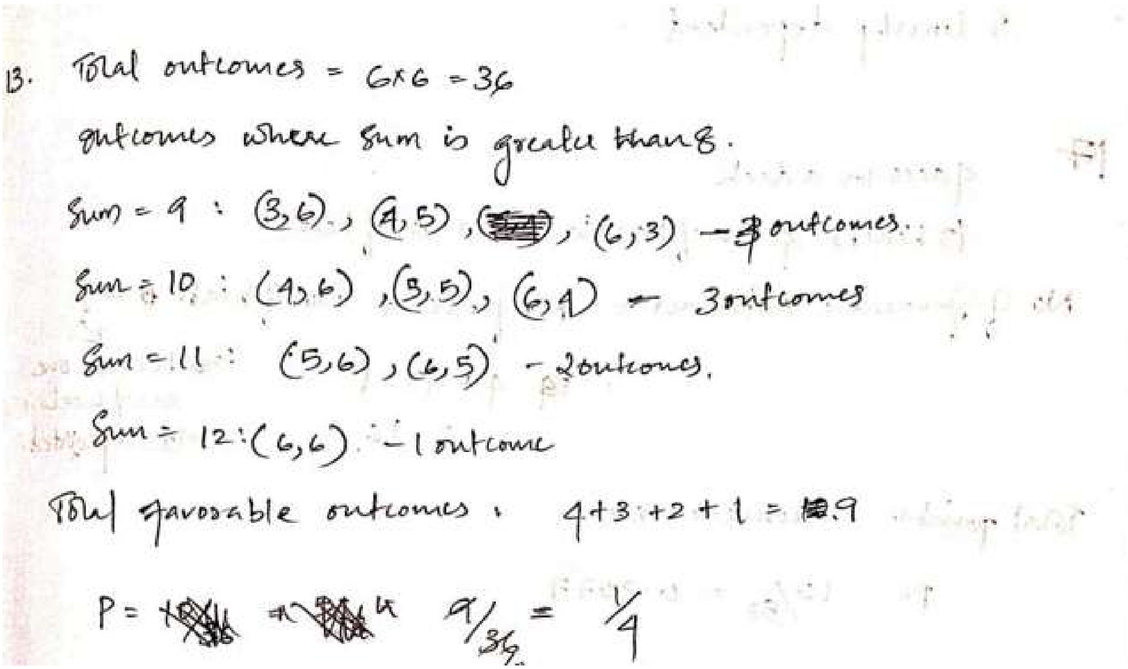
```
prob = 1/4 #write your answer
```

```
def grader13(prob, tol=1e-3):
    return math.isclose(prob, 0.25, rel_tol=tol)

grader13(prob)
```

```
True
```

```
# Open the image.
img = cv2.imread("/content/Q13.png")
cv2_imshow(img)
```



13. Total outcomes = 6×6 = 36

outcomes where sum is greater than 8.

Sum = 9 : (3,6), (4,5), ~~(5,4)~~, (6,3) — 4 outcomes.

Sum = 10 : (4,6), (5,5), (6,4) — 3 outcomes

Sum = 11 : (5,6), (6,5) — 2 outcomes.

Sum = 12 : (6,6) — 1 outcome

Total favorable outcomes : 4+3+2+1 = 9

P = $9/36$ = $1/4$

Question 14: A biased coin has P(Heads)=0.6. If it is flipped 10 times, what is the probability of getting exactly 7 heads? Store the answer in variable prob2. Show screenshot\photograph of your calculation

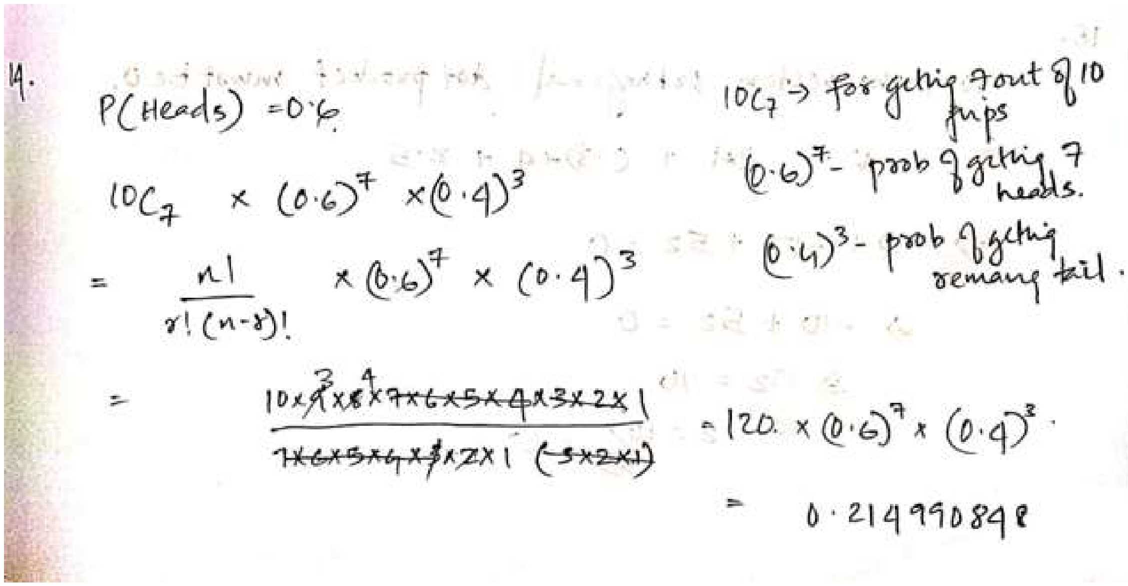
```
prob2 = 0.2149#write your answer
```

```
def grader(prob2, tol=1e-3):
    return math.isclose(prob2, 0.2013, rel_tol=tol)

grader(prob2)
```

```
False
```

```
# Open the image.
img = cv2.imread("/content/Q14.png")
cv2_imshow(img)
```



Question 15: Determine whether the vectors (2,4) and (1,2) are linearly independent. Store the boolean value in variable ans. Show screenshot\photograph of your calculation

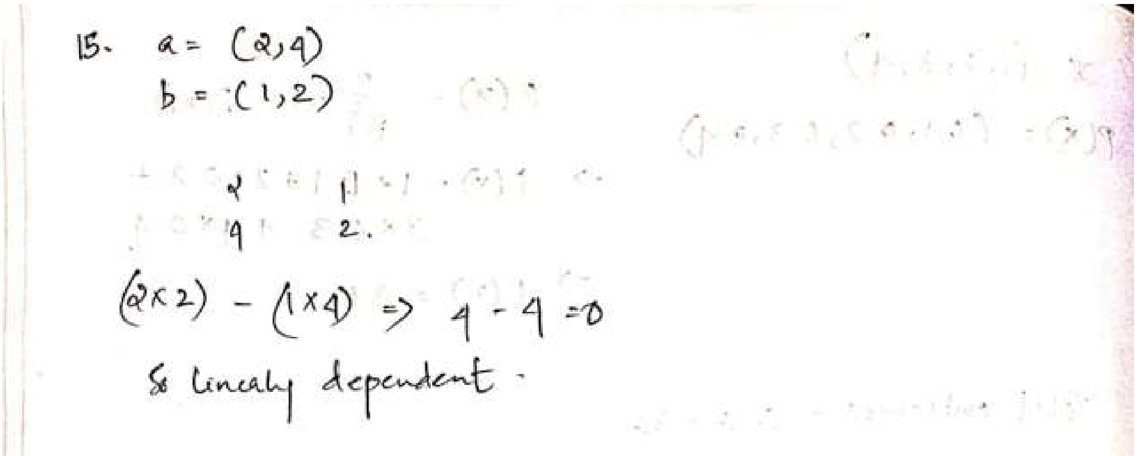
```
ans = False #write your answer(True/False)
```

```
def grader15(is_independent):
    return is_independent == False
```

```
grader(ans)
```

```
False
```

```
# Open the image.
img = cv2.imread("/content/Q15.png")
cv2_imshow(img)
```



Question 16: Find the dot product of $a = (1, -2, 3)$ and $b = (4, 0, -1)$. Store your answer in dot_prod variable. Show screenshot\photograph of your calculation

```
v1 = [1,-2,3]
v2 = [4,0,-1]
dotprod = 0
for i ,j in zip(v1,v2):
    dotprod += i*j
dot_prod = dotprod
print(f'The dot product is {dot_prod} .')
```

```
The dot product is 1 .
```

```
def grader16(dot_product):
    return dot_product == 4
```

```
grader16(dot_prod)
```

```
False
```

Question 17: In a deck of cards, there are 4 aces. What is the probability of drawing an ace or a heart from a deck of 52 cards? Store your answer in variable prob3. Show screenshot\photograph of your calculation

```
prob3 = round(4/13,6) #write your answer
```

```
def grader17(prob3):
    return math.isclose(prob3,0.307692)

grader17(prob3)
```

```
True
```

```
# Open the image.
img = cv2.imread("/content/Q17.png")
cv2_imshow(img)
```

17

4 aces in a deck

13 hearts & one of hearts is an ace of hearts.

No of favorable outcomes = No of Aces + No of hearts − 1

$= 4 + 13 - 1$

$= 16.$

because one ace of hearts is already counted

Total possible outcomes = 52.

$P = 16/52 = 0.3077.$

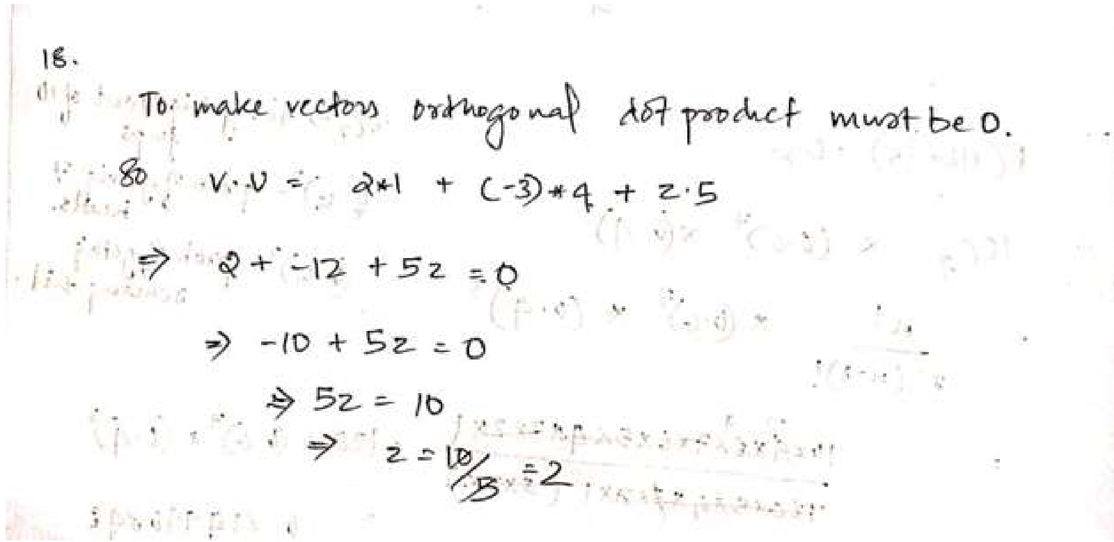
Question 18: Given the vector $v = \{2, -3, z\}$, find the value of z such that the vector v is orthogonal to the vector $u = \{1, 4, 5\}$. Store the value in variable z. Show screenshot\photograph of your calculation

```
z = 2 #write your answer
```

```
def grader_18(z_value):
    return np.isclose(z_value, 2)
grader_18(z)
```

```
True
```

```
# Open the image.
img = cv2.imread("/content/Q18.png")
cv2_imshow(img)
```


18.

To make vectors orthogonal dot product must be 0.

So $v \cdot v = 2*1 + (-3)*4 + z \cdot 5$

$\Rightarrow 2 + -12 + 5z = 0$

$\Rightarrow -10 + 5z = 0$

$\Rightarrow 5z = 10$

$\Rightarrow z = 10/5 = 2$

Question 19: The probability that it will rain tomorrow is 0.3, and the probability that a person will carry an umbrella is 0.6. If the probability of both events happening (rain and umbrella) is 0.2, what is the probability that it will rain given that the person carries an umbrella? Store your answer in variable prob4. Show screenshot\photograph of your calculation

```
prob4 = 0.333#write your answer
```

```
def grader19(prob4):
    return math.isclose(prob4, 0.333)

grader19(prob4)
```

```
True
```

```
# Open the image.
img = cv2.imread("/content/Q19.png"
```

19. $P(\text{Rain} \mid \text{Umbrella}) = \dfrac{P(\text{Umbrella} \mid \text{Rain}) \cdot P(\text{Rain})}{P(\text{Umbrella})}$

$P(\text{Rain}) = 0.3$

$P(\text{Umbrella}) = 0.6$

$P(\text{Rain and Umbrella}) = 0.2$.