Final Project Report: Subsampling vs Ridge Regularization vs Lasso Regularization vs Elastic Net

Pranav Senthilkumaran
Master's in Data Science Student
Rutgers University
GitHub Project Repository

2026-02-20

Contents

1 Introduction

This report analyzes the equivalence between subsampling, ridge regularization, lasso regularization, and Elastic Net regularization from both a theoretical and empirical perspective using controlled synthetic data.

2 Step 1: Data Simulation

Objective: Simulate high-dimensional data with 200 rows and 100 features using random normal distribution.

Purpose: Provide a controlled environment to evaluate and compare Elastic Net, Ridge, Subsampling, and Lasso models.

Simulate:

- $n = 200$ observations
- $p = 100$ predictors

Target Variable: $y = X\beta + \varepsilon$ — standard linear model with noise.

```r
set.seed(42)
n <- 200
p <- 100
X <- matrix(rnorm(n * p), nrow = n)
beta <- rnorm(p)
y <- X %*% beta + rnorm(n)
df <- data.frame(y, X)
```

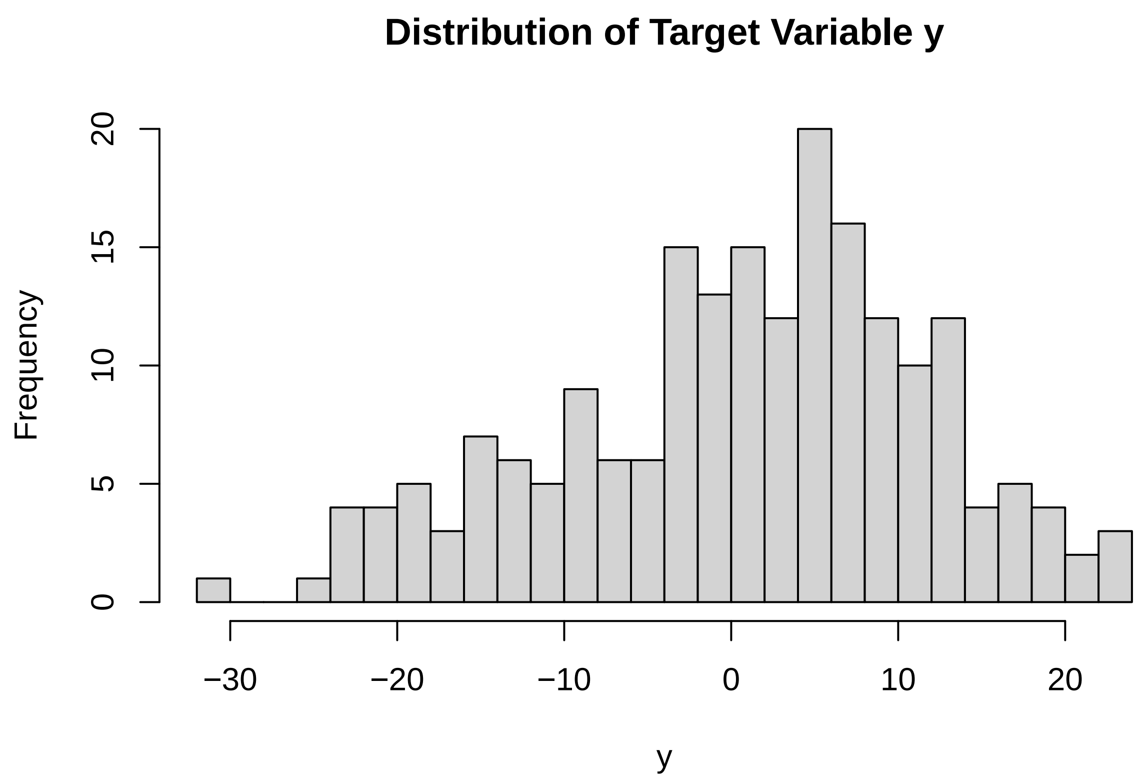
3 Step 2: Exploratory Data Analysis (EDA)

3.1 1. Distribution and Outliers

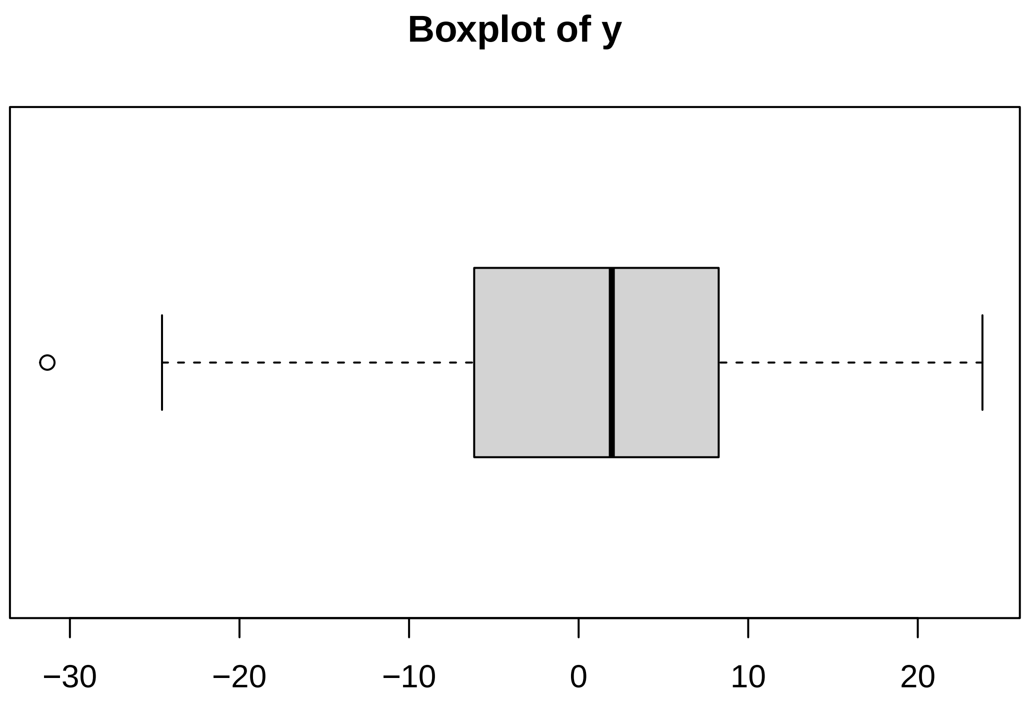
Histogram: Visualizes the distribution of y. Appears symmetric with some extreme values.

Boxplot: Highlights outliers, confirms a few extreme observations in y.

```r
hist(df$y, breaks = 30, main = "Distribution of Target Variable y", xlab = "y")
```



```r
boxplot(df$y, horizontal = TRUE, main = "Boxplot of y")
```

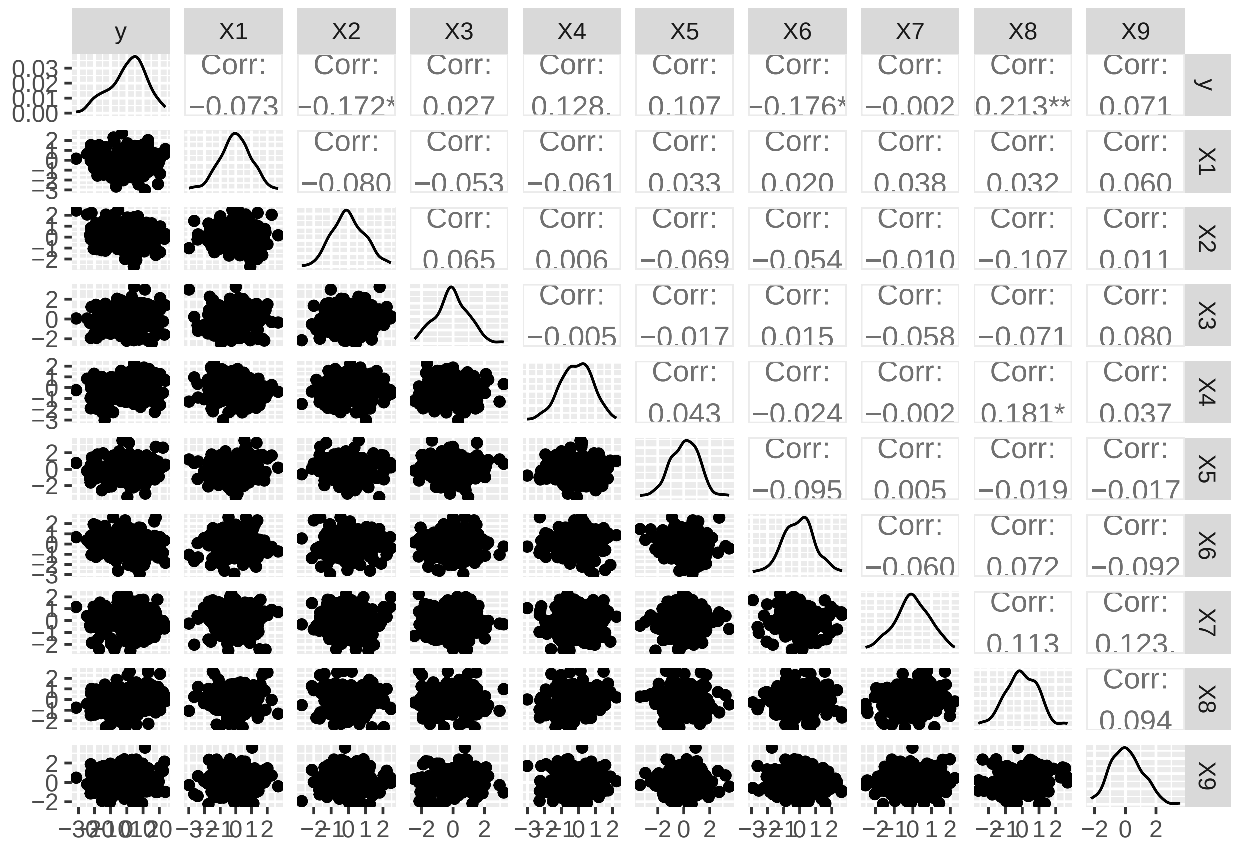


3.2 2. Correlation

GGally::ggpairs: Examines correlations between y and the first 9 features.

Takeaway: Weak linear relationships between most variables — reflects high-dimensional randomness.

```
subset_df <- df[, 1:10]
ggpairs(subset_df)
```

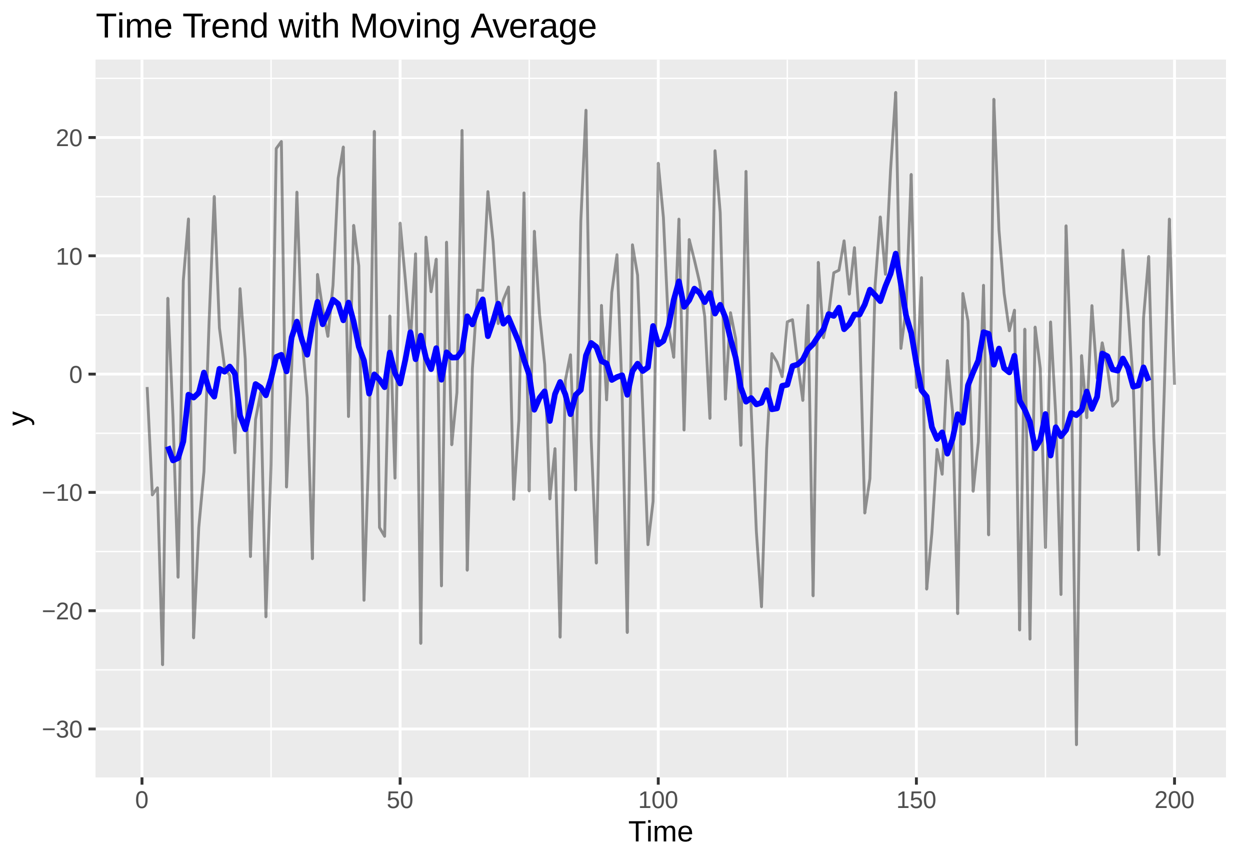


3.3 3. Time Trends

Moving Average Plot: Demonstrates how y behaves over a pseudo-time axis.

Purpose: Helps visualize local trends and fluctuations.

```
df$time <- 1:nrow(df)
df$moving_avg <- zoo::rollmean(df$y, 10, fill = NA)
ggplot(df, aes(x = time, y = y)) +
  geom_line(alpha = 0.4) +
  geom_line(aes(y = moving_avg), color = 'blue', size = 1) +
  labs(title = "Time Trend with Moving Average", x = "Time", y = "y")
```



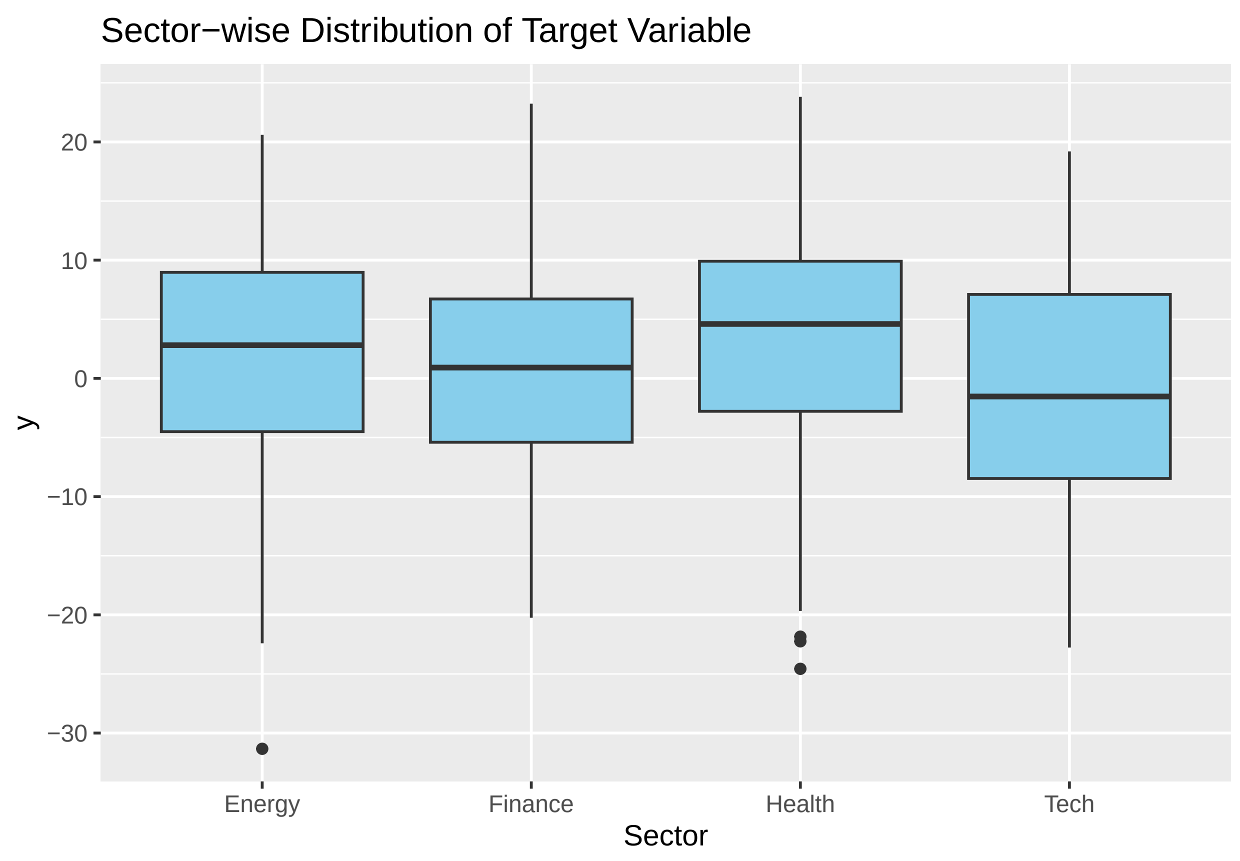
Takeaway: Some short-term fluctuation is observed, but no consistent trend emerges.

3.4 4. Sector-Level Insights

Boxplot by Sector: y is visualized across 4 randomly assigned sectors.

Takeaway: Adds a categorical dimension for interpreting distribution spread.

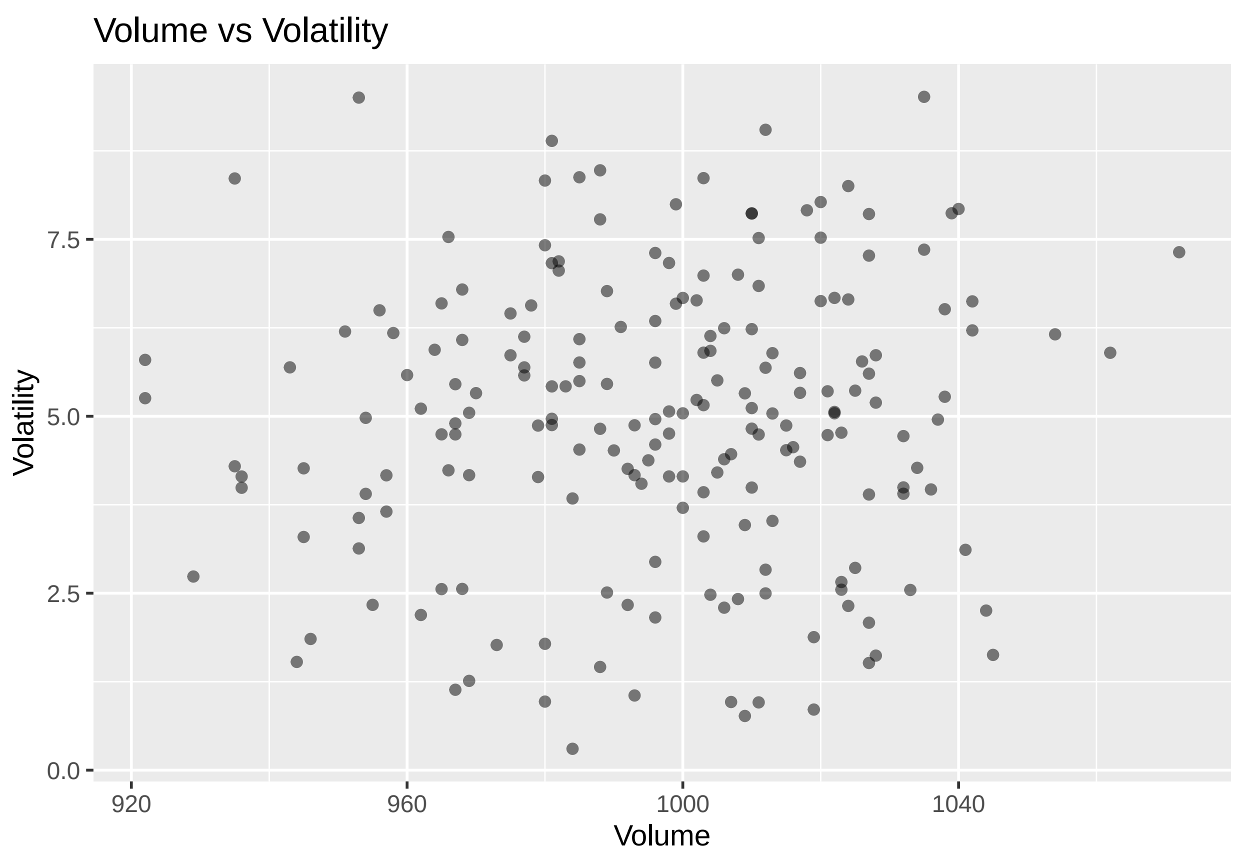
```r
set.seed(123)
df$sector <- sample(c("Tech", "Health", "Finance", "Energy"), nrow(df), replace = TRUE)
ggplot(df, aes(x = sector, y = y)) +
  geom_boxplot(fill = "skyblue") +
  labs(title = "Sector-wise Distribution of Target Variable", x = "Sector", y = "y")
```



3.5 5. Volatility & Volume

Scatter Plot:* Shows how volatility (high-low spread) varies with transaction volume. Simulated Financial Analogy:** Useful if modeling stock-like behavior.

```
df$high <- df$y + runif(nrow(df), 0, 5)
df$low <- df$y - runif(nrow(df), 0, 5)
df$volume <- rpois(nrow(df), lambda = 1000)
df$volatility <- df$high - df$low

ggplot(df, aes(x = volume, y = volatility)) +
  geom_point(alpha = 0.5) +
  labs(title = "Volume vs Volatility", x = "Volume", y = "Volatility")
```



4 Step 3: Model Training and Evaluation (Train–Test Split)

```
# Shared Train-Test Split
set.seed(123)
train_idx <- sample(1:n, size = 0.7 * n)
X_train <- X[train_idx, ]
y_train <- y[train_idx]
X_test <- X[-train_idx, ]
y_test <- y[-train_idx]
```

4.1 6. Methodologies and Trends

4.2 Ridge Regression

Uses glmnet with alpha = 0. Applies regularization to shrink coefficients and reduce overfitting. MSE is computed to evaluate prediction accuracy. Ridge regression was fit using glmnet with alpha = 0. Cross-validation selected the optimal lambda. The model reduces overfitting by shrinking coefficients. MSE was calculated on test data for performance comparison.

```
cv_ridge <- cv.glmnet(X_train, y_train, alpha = 0)
ridge_model <- glmnet(X_train, y_train, alpha = 0, lambda = cv_ridge$lambda.min)
ridge_pred <- predict(ridge_model, s = cv_ridge$lambda.min, newx = X_test)
```

```r
ridge_mse <- mean((ridge_pred - y_test)^2)
ridge_mse
```

```
## [1] 4.177719
```

4.3 Lasso Regression

Uses glmnet with alpha = 1. Performs feature selection by setting some coefficients to zero. MSE is computed to compare with Ridge and Subsampling. Lasso was fit using glmnet with cross-validated lambda. It shrinks some coefficients to zero, enabling feature selection. Test set predictions were used to compute MSE for comparison.

```r
cv_lasso <- cv.glmnet(X_train, y_train, alpha = 1)
lasso_model <- glmnet(X_train, y_train, alpha = 1, lambda = cv_lasso$lambda.min)
lasso_pred <- predict(lasso_model, s = cv_lasso$lambda.min, newx = X_test)
lasso_mse <- mean((lasso_pred - y_test)^2)
lasso_mse
```

```
## [1] 5.084213
```

4.4 Elastic Net Regression

Uses glmnet with alpha = 0.5, combining L1 and L2 penalties. Elastic Net balances sparsity (Lasso) and stability (Ridge) - convex combition of L1 and L2. Particularly effective when predictors are correlated. Cross-validation selects the optimal lambda for the mixed penalty. MSE is computed to evaluate predictive performance.

```r
cv_enet <- cv.glmnet(X_train, y_train, alpha = 0.5)
enet_model <- glmnet(X_train, y_train, alpha = 0.5, lambda = cv_enet$lambda.min)
enet_pred <- predict(enet_model, s = cv_enet$lambda.min, newx = X_test)
enet_mse <- mean((enet_pred - y_test)^2)
enet_mse
```

```
## [1] 4.987527
```

4.5 Subsampling Ensemble

Creates 30 models using 50% random subsamples. Aggregates predictions by averaging. MSE is computed and found to be high, showing instability under random sampling. An ensemble of 30 linear models was built using 50% random subsamples of training data. Predictions were averaged across models to form a final prediction. The MSE was computed, revealing higher error due to variability from random sampling.

```r
p <- ncol(X_train)
X_train_df <- as.data.frame(X_train)
X_test_df <- as.data.frame(X_test)
colnames(X_train_df) <- paste0("X", 1:p)
colnames(X_test_df) <- paste0("X", 1:p)

set.seed(123)
```

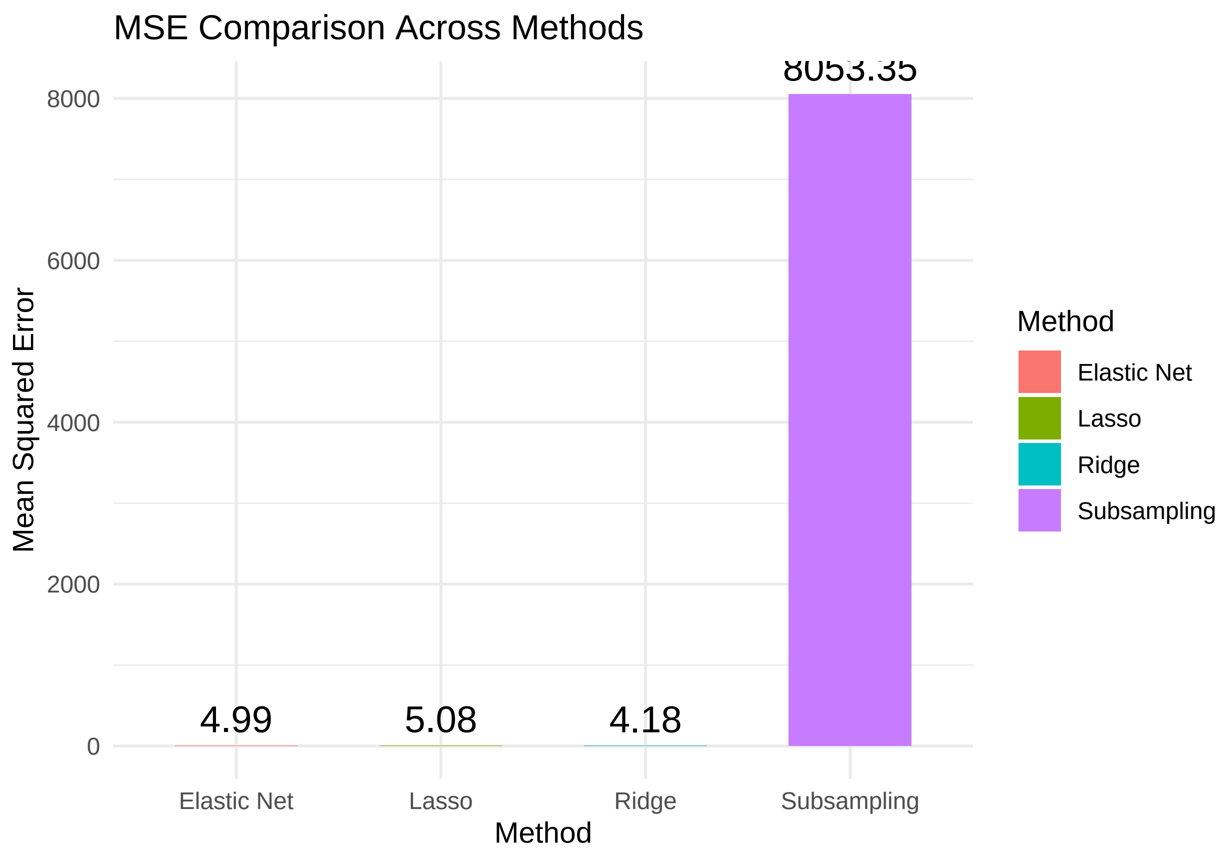
```
ensemble_preds <- replicate(30, {
  sample_idx <- sample(1:nrow(X_train_df), size = floor(0.5 * nrow(X_train_df)))
  lm_data <- data.frame(y = y_train[sample_idx], X_train_df[sample_idx, ])
  model <- lm(y ~ ., data = lm_data)
  predict(model, newdata = X_test_df)
})

ensemble_mean <- rowMeans(ensemble_preds, na.rm = TRUE)
subsample_mse <- mean((ensemble_mean - y_test)^2)
subsample_mse
```

```
## [1] 8053.353
```

5 *Step 4: Model Comparison*

5.1 MSE Comparison including Elastic Net, Ridge, Subsampling, and Lasso

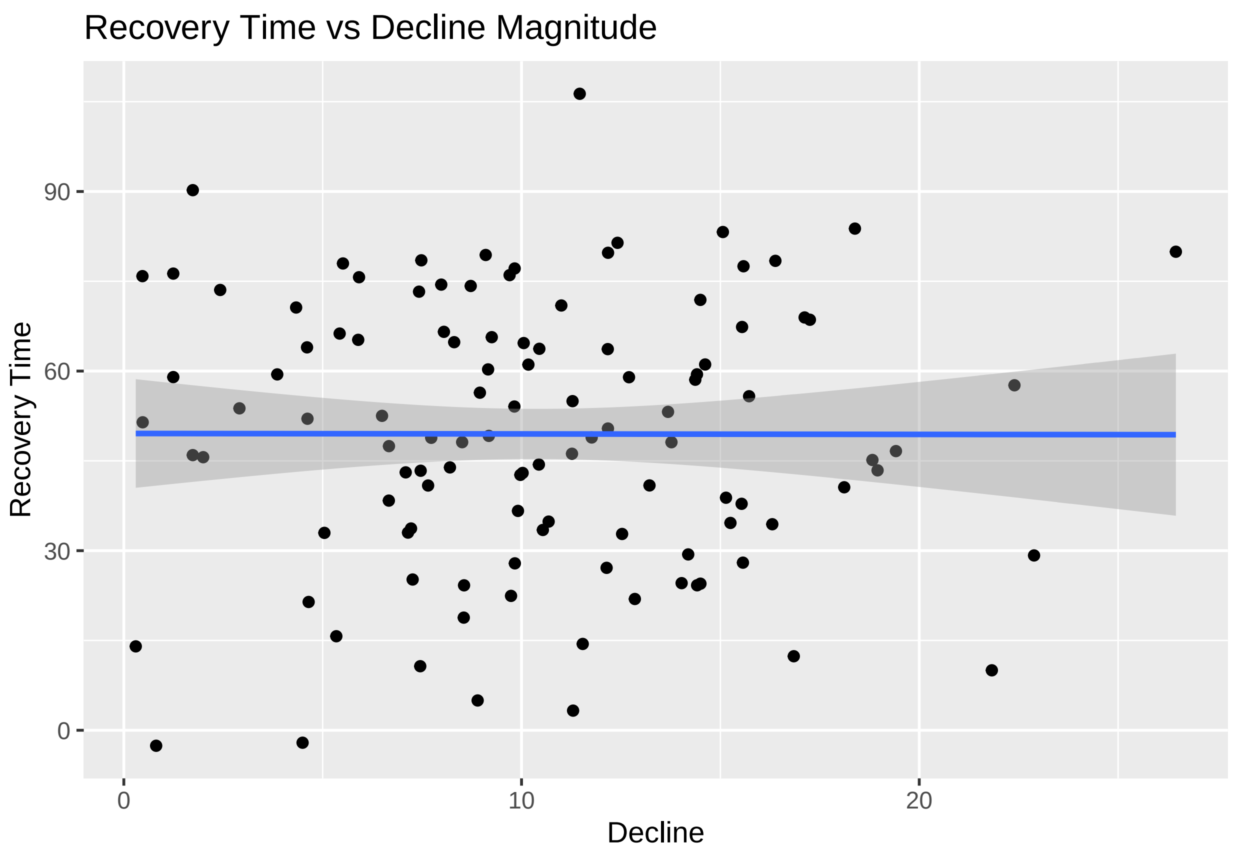
The bar plot compares the mean squared errors (MSE) of Elastic Net, Ridge, Subsampling, and Lasso models. Each bar shows the test error from the respective method, highlighting relative prediction accuracy. Regularized models outperform subsampling-based ensembling, show lower MSE than Subsampling, suggesting better generalization. Elastic Net achieves a balance between Ridge stability and Lasso sparsity.



6 *Step 5: Additional Simulations and Diagnostics*

6.1 7. Recovery Time vs Decline Shows linear correlation between simulated crash decline and recovery time.

```r
df$crash <- sample(c(TRUE, FALSE), nrow(df), replace = TRUE)
df$decline <- ifelse(df$crash, abs(rnorm(nrow(df), 10, 5)), NA)
df$recovery_time <- ifelse(df$crash, rnorm(nrow(df), 50, 20), NA)

ggplot(na.omit(df), aes(x = decline, y = recovery_time)) +
  geom_point() +
  geom_smooth(method = "lm") +
  labs(title = "Recovery Time vs Decline Magnitude", x = "Decline", y = "Recovery Time")
```



6.2 8. Confusion Matrix

Binary classification of response into High vs Low. Classifies y into High vs Low using top features. Accuracy: ~50%, which is close to random — typical in high-dimensional data with weak signal. **Note**: Only the first 10 features were used to simplify the classification task and reduce overfitting risk.

```r
df$label <- ifelse(df$y > median(df$y), "High", "Low")
df$label <- as.factor(df$label)
feature_cols <- paste0("X", 1:10)
```

```
train_data <- df[train_idx, ]
test_data <- df[-train_idx, ]

model_log <- glm(label ~ ., data = train_data[, c("label", feature_cols)], family = "binomial")
pred_probs <- predict(model_log, newdata = test_data[, feature_cols], type = "response")
pred_class <- ifelse(pred_probs > 0.5, "High", "Low")
pred_class <- factor(pred_class, levels = c("Low", "High"))

confusionMatrix(pred_class, test_data$label)
```

```
## Confusion Matrix and Statistics
##
##           Reference
## Prediction High Low
##       High   13  16
##       Low    18  13
##
##                Accuracy : 0.4333
##                  95% CI : (0.3059, 0.5676)
##     No Information Rate : 0.5167
##     P-Value [Acc > NIR] : 0.9224
##
##                   Kappa : -0.1321
##
##  Mcnemar's Test P-Value : 0.8638
##
##             Sensitivity : 0.4194
##             Specificity : 0.4483
##          Pos Pred Value : 0.4483
##          Neg Pred Value : 0.4194
##              Prevalence : 0.5167
##          Detection Rate : 0.2167
##    Detection Prevalence : 0.4833
##       Balanced Accuracy : 0.4338
##
##        'Positive' Class : High
##
```

6.3 9. Bias-Variance Decomposition (Theoretical + Empirical)

7 This section performs a simulation study to compare Ridge, Lasso, Elastic Net, and Subsampling.

1. Computes test MSE distributions across 100 simulations and visualize with a boxplot.
2. Performs an empirical Bias^2-Variance decomposition using a fixed test set.
3. Visualizes Bias^2, Variance, and Noise contributions with a stacked barplot.

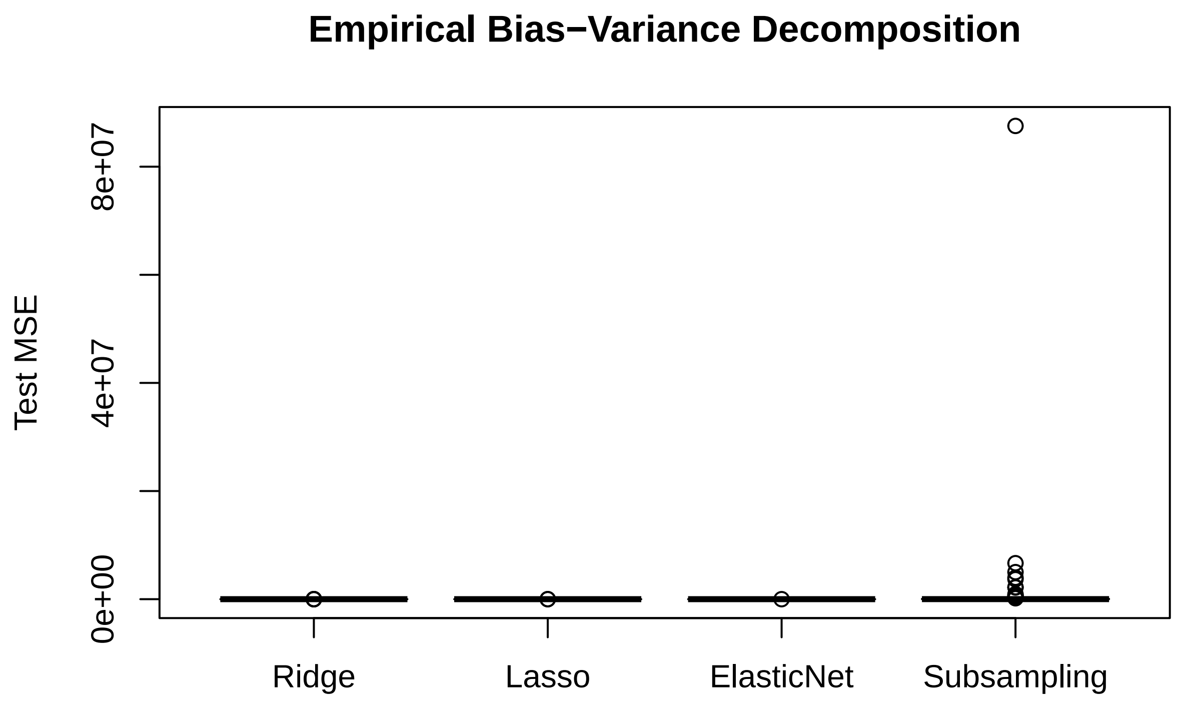
Theoretical prediction error decomposes into three components: **Bias^2, Variance, and Irreducible Noise**.

$$\mathbb{E}[(Y - \hat{f}(X))^2] = \text{Bias}^2 + \text{Variance} + \sigma^2$$

Bias reflects systematic error due to model assumptions, while variance captures sensitivity to sampling variability.

Regularization methods intentionally introduce bias to reduce variance, improving generalization in high-dimensional settings.

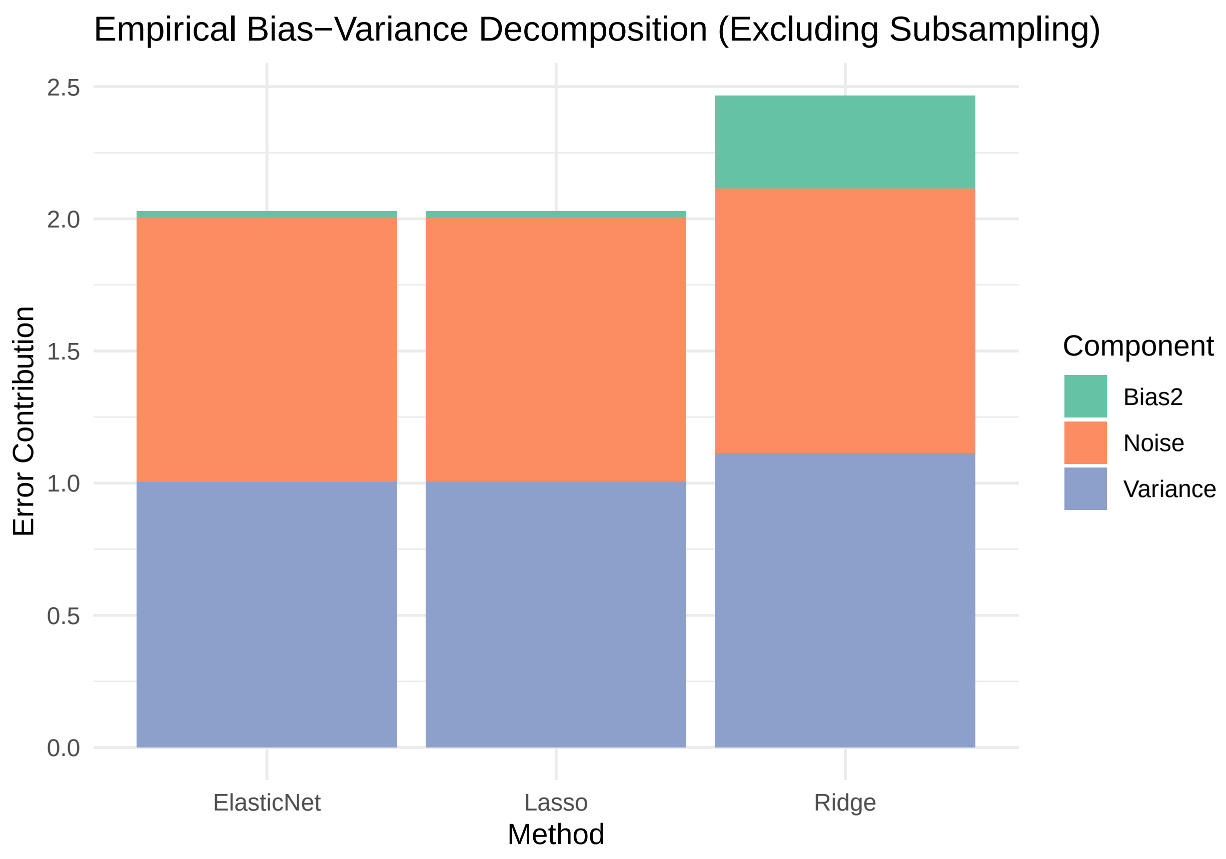
```r
simulate_model <- function(n, p, nsim = 100) {

  mse_ridge <- mse_lasso <- mse_enet <- mse_sub <- numeric(nsim)

  for (i in 1:nsim) {

    X <- matrix(rnorm(n * p), n)
    beta <- rnorm(p)
    y <- X %*% beta + rnorm(n)

    idx <- sample(1:n, 0.7 * n)
    Xtr <- X[idx, ]; ytr <- y[idx]
    Xte <- X[-idx, ]; yte <- y[-idx]

    # Ridge
    ridge_fit <- cv.glmnet(Xtr, ytr, alpha = 0)
    mse_ridge[i] <- mean((predict(ridge_fit, s="lambda.min", Xte) - yte)^2)

    # Lasso
    lasso_fit <- cv.glmnet(Xtr, ytr, alpha = 1)
    mse_lasso[i] <- mean((predict(lasso_fit, s="lambda.min", Xte) - yte)^2)

    # Elastic Net
    enet_fit <- cv.glmnet(Xtr, ytr, alpha = 0.5)
    mse_enet[i] <- mean((predict(enet_fit, s="lambda.min", Xte) - yte)^2)

    # Subsampling
    preds <- replicate(30, {
      id <- sample(1:nrow(Xtr), floor(0.5 * nrow(Xtr)))
      predict(lm(ytr[id] ~ ., data = as.data.frame(Xtr[id, ])),
              as.data.frame(Xte))
    })

    mse_sub[i] <- mean((rowMeans(preds) - yte)^2)
  }

  data.frame(Ridge = mse_ridge,
             Lasso = mse_lasso,
             ElasticNet = mse_enet,
             Subsampling = mse_sub)
}

results <- simulate_model(200, 100)

boxplot(results,
        main = "Empirical Bias-Variance Decomposition",
        ylab = "Test MSE")
```



```r
# ----------------------------
# Empirical Bias-Variance Decomposition
# ----------------------------
set.seed(123)
n <- 200
p <- 100
nsim <- 100
sigma2 <- 1  # known noise variance

# Fixed test set
X_test <- matrix(rnorm(n * p), n)
beta_true <- rnorm(p)
f_true <- X_test %*% beta_true  # true signal without noise

# Storage for predictions
pred_ridge <- pred_lasso <- pred_enet <- pred_sub <- matrix(NA, nsim, n)

for (i in 1:nsim) {
  # Generate training data
  X_train <- matrix(rnorm(n * p), n)
  y_train <- X_train %*% beta_true + rnorm(n, sd = sqrt(sigma2))

  # Ridge
  ridge_fit <- cv.glmnet(X_train, y_train, alpha = 0)
  pred_ridge[i, ] <- predict(ridge_fit, s = "lambda.min", X_test)
  # Lasso
  lasso_fit <- cv.glmnet(X_train, y_train, alpha = 1)
  pred_lasso[i, ] <- predict(lasso_fit, s = "lambda.min", X_test)

  # Elastic Net
  enet_fit <- cv.glmnet(X_train, y_train, alpha = 0.5)
  pred_enet[i, ] <- predict(enet_fit, s = "lambda.min", X_test)

  # Subsampling
  preds_sub <- replicate(30, {
    idx <- sample(1:nrow(X_train), floor(0.5 * nrow(X_train)))
    lm_fit <- lm(y_train[idx] ~ ., data = as.data.frame(X_train[idx, ]))
    predict(lm_fit, as.data.frame(X_test))
  })
  pred_sub[i, ] <- rowMeans(preds_sub)
}

# Function to compute Bias² and Variance
compute_bias_variance <- function(pred_matrix, f_true, sigma2) {
  mean_pred <- colMeans(pred_matrix)
  bias2 <- mean((mean_pred - f_true)^2)
  variance <- mean(apply(pred_matrix, 2, var))
  c(Bias2 = bias2, Variance = variance, Noise = sigma2)
}

# Compute Bias², Variance, Noise for all methods
bv_results <- rbind(
  Ridge = compute_bias_variance(pred_ridge, f_true, sigma2),
  Lasso = compute_bias_variance(pred_lasso, f_true, sigma2),
  ElasticNet = compute_bias_variance(pred_enet, f_true, sigma2),
  Subsampling = compute_bias_variance(pred_sub, f_true, sigma2)
)

bv_df <- as.data.frame(bv_results)
bv_df  # display numeric values
```

```
##                    Bias2     Variance Noise
## Ridge       3.513619e-01 1.115184e+00     1
## Lasso       2.360617e-02 1.005653e+00     1
## ElasticNet  2.456325e-02 1.005196e+00     1
## Subsampling 4.096795e+03 3.572826e+05     1
```

```r
# ----------------------------
# Stacked Barplot for Bias-Variance Decomposition
# ----------------------------
# Remove Subsampling for stacked bar plot
bv_plot_df <- bv_df[rownames(bv_df) != "Subsampling", ]

# Add Method column
bv_plot_df$Method <- rownames(bv_plot_df)

# Pivot to long format
bv_long <- tidyr::pivot_longer(
  bv_plot_df,
  cols = c("Bias2", "Variance", "Noise"),
  names_to = "Component",
  values_to = "Value"
)

# Plot
ggplot(bv_long, aes(x = Method, y = Value, fill = Component)) +
  geom_bar(stat = "identity", position = "stack") +
  labs(title = "Empirical Bias-Variance Decomposition (Excluding Subsampling)",
       x = "Method",
       y = "Error Contribution") +
  theme_minimal() +
  scale_fill_brewer(palette = "Set2")
```



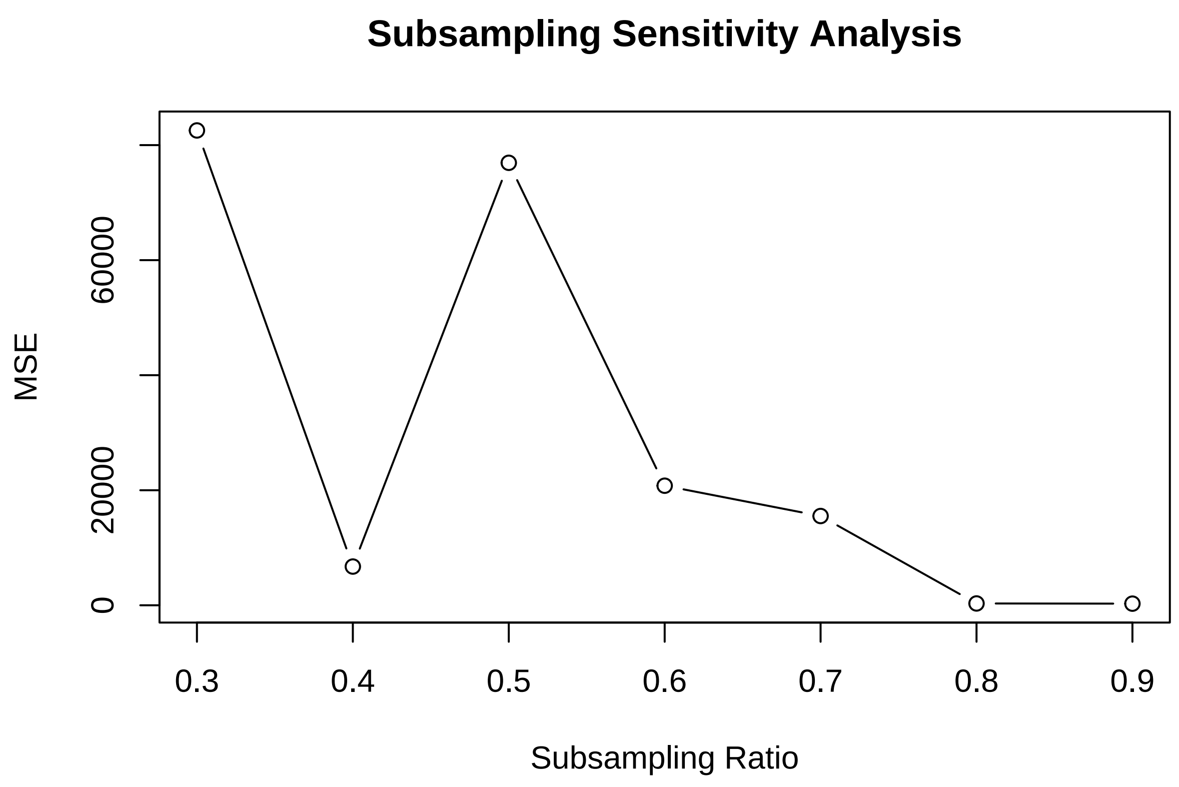
8 Key Takeaway:

Demonstrated the theoretical Bias^2 + Variance + Noise decomposition by generating multiple training datasets, fitting each model, and empirically estimating Bias^2 (average prediction vs true signal) and Variance (prediction variability). Noise was taken from the data.

8.1 10. Subsampling Sensitivity Analysis

X-axis = Sample ratio, Y-axis = MSE Subsampling ratios from 0.3 to 0.9 were tested. For each ratio, 30 linear models were trained on random subsamples, predictions were averaged, and MSE was computed. Conclusion: Higher subsample ratio slightly reduces MSE but still much worse than Ridge/Lasso.

```r
sensitivity_results <- sapply(seq(0.3, 0.9, by = 0.1), function(ratio) {
  preds <- replicate(30, {
    idx <- sample(1:nrow(X_train_df), floor(ratio * nrow(X_train_df)))
    lm_data <- data.frame(y = y_train[idx], X_train_df[idx, ])
    model <- lm(y ~ ., data = lm_data)
    predict(model, newdata = X_test_df)
  })
  mean((rowMeans(preds, na.rm = TRUE) - y_test)^2)
})

plot(seq(0.3, 0.9, by = 0.1), sensitivity_results, type = "b",
     xlab = "Subsampling Ratio", ylab = "MSE", main = "Subsampling Sensitivity Analysis")
```



9 *Step 6: Dimensionality Reduction*

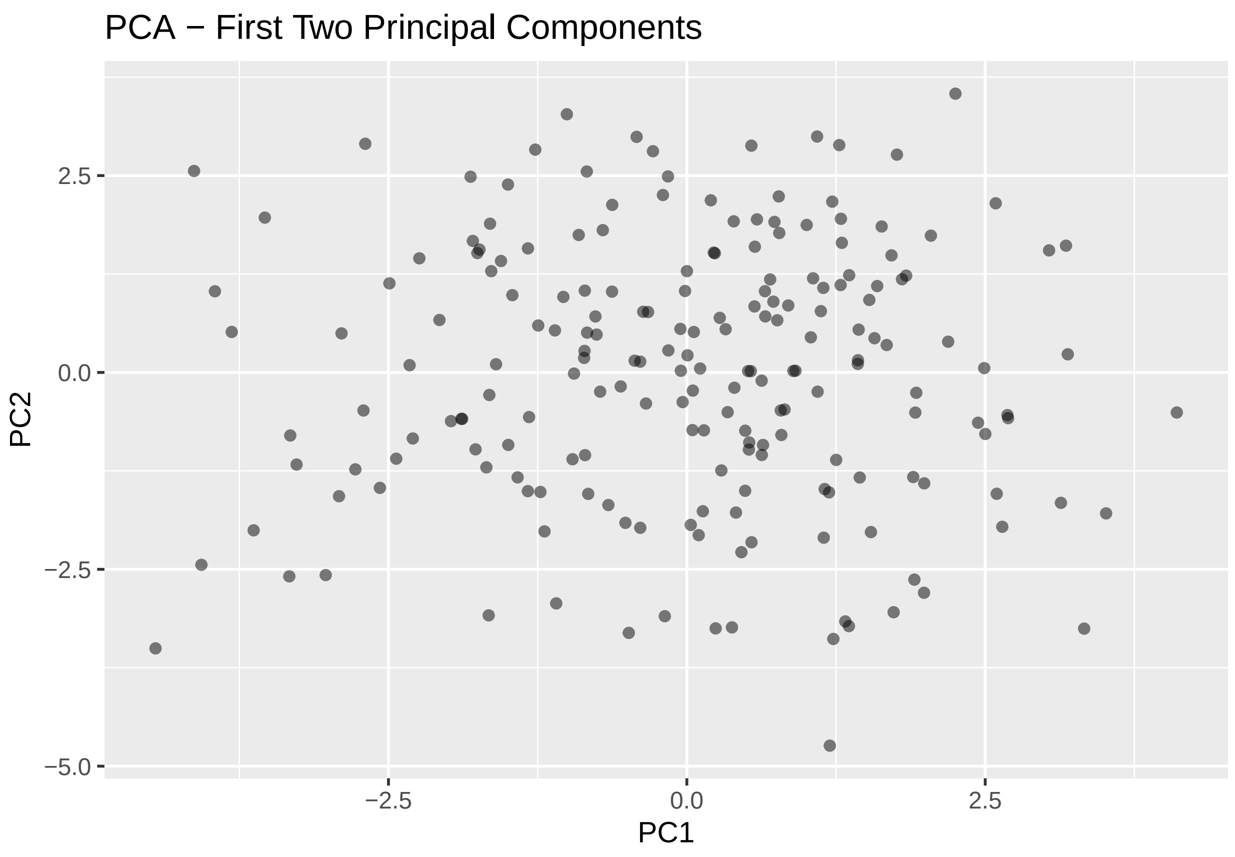
9.1 11. PCA Analysis

PCA Summary Table Shows variance explained by 100 principal components. Top PCs capture very little variance individually — confirms data is noisy. PC1 vs PC2 Scatterplot — spread confirms data has no strong clusters or separation.

```r
pca_result <- prcomp(X, scale. = TRUE)
summary(pca_result)
```

```
## Importance of components:
##                            PC1     PC2     PC3     PC4     PC5    PC6     PC7
## Standard deviation     1.68564 1.66311 1.59234 1.57515 1.57036 1.5394 1.52585
## Proportion of Variance 0.02841 0.02766 0.02536 0.02481 0.02466 0.0237 0.02328
## Cumulative Proportion  0.02841 0.05607 0.08143 0.10624 0.13090 0.1546 0.17788
##                            PC8     PC9    PC10    PC11    PC12    PC13    PC14
## Standard deviation     1.51020 1.47327 1.45338 1.42557 1.39969 1.39311 1.38398
## Proportion of Variance 0.02281 0.02171 0.02112 0.02032 0.01959 0.01941 0.01915
## Cumulative Proportion  0.20069 0.22239 0.24351 0.26384 0.28343 0.30284 0.32199
##                           PC15    PC16    PC17    PC18    PC19    PC20    PC21
## Standard deviation     1.36459 1.35521 1.33600 1.32424 1.31080 1.29885 1.28763
## Proportion of Variance 0.01862 0.01837 0.01785 0.01754 0.01718 0.01687 0.01658
## Cumulative Proportion  0.34061 0.35898 0.37683 0.39436 0.41154 0.42841 0.44499
##                           PC22    PC23   PC24    PC25    PC26    PC27    PC28
## Standard deviation     1.24990 1.23663 1.2290 1.21124 1.19545 1.18609 1.17771
## Proportion of Variance 0.01562 0.01529 0.0151 0.01467 0.01429 0.01407 0.01387
## Cumulative Proportion  0.46062 0.47591 0.4910 0.50568 0.51998 0.53404 0.54791
##                           PC29    PC30    PC31    PC32    PC33    PC34    PC35
## Standard deviation     1.16944 1.16496 1.15841 1.12723 1.12494 1.11720 1.10828
## Proportion of Variance 0.01368 0.01357 0.01342 0.01271 0.01265 0.01248 0.01228
## Cumulative Proportion  0.56159 0.57516 0.58858 0.60129 0.61394 0.62642 0.63871
##                           PC36    PC37    PC38    PC39    PC40    PC41    PC42
## Standard deviation     1.08688 1.08046 1.07458 1.05985 1.04435 1.02384 1.00120
## Proportion of Variance 0.01181 0.01167 0.01155 0.01123 0.01091 0.01048 0.01002
## Cumulative Proportion  0.65052 0.66219 0.67374 0.68497 0.69588 0.70636 0.71639
##                           PC43    PC44    PC45   PC46    PC47   PC48    PC49
## Standard deviation     1.00028 0.99230 0.97432 0.9589 0.95183 0.9382 0.92418
## Proportion of Variance 0.01001 0.00985 0.00949 0.0092 0.00906 0.0088 0.00854
## Cumulative Proportion  0.72639 0.73624 0.74573 0.7549 0.76399 0.7728 0.78133
##                           PC50    PC51    PC52    PC53    PC54    PC55   PC56
## Standard deviation     0.91695 0.89079 0.87677 0.86332 0.86186 0.85172 0.8368
## Proportion of Variance 0.00841 0.00794 0.00769 0.00745 0.00743 0.00725 0.0070
## Cumulative Proportion  0.78974 0.79767 0.80536 0.81281 0.82024 0.82749 0.8345
##                           PC57    PC58    PC59    PC60    PC61   PC62    PC63
## Standard deviation     0.83372 0.82254 0.80566 0.79147 0.78784 0.7808 0.76906
## Proportion of Variance 0.00695 0.00677 0.00649 0.00626 0.00621 0.0061 0.00591
## Cumulative Proportion  0.84145 0.84821 0.85471 0.86097 0.86718 0.8733 0.87919
##                           PC64    PC65    PC66    PC67    PC68    PC69    PC70
## Standard deviation     0.75715 0.75106 0.74667 0.72759 0.71561 0.71138 0.69634
## Proportion of Variance 0.00573 0.00564 0.00558 0.00529 0.00512 0.00506 0.00485
## Cumulative Proportion  0.88492 0.89056 0.89614 0.90143 0.90655 0.91161 0.91646
##                           PC71    PC72   PC73    PC74    PC75    PC76    PC77
## Standard deviation     0.69333 0.67680 0.6560 0.65162 0.64571 0.63324 0.62017
## Proportion of Variance 0.00481 0.00458 0.0043 0.00425 0.00417 0.00401 0.00385
## Cumulative Proportion  0.92127 0.92585 0.9302 0.93440 0.93857 0.94258 0.94642
##                           PC78    PC79    PC80    PC81    PC82    PC83    PC84
## Standard deviation     0.61135 0.59623 0.59014 0.58428 0.56210 0.55157 0.53735
## Proportion of Variance 0.00374 0.00355 0.00348 0.00341 0.00316 0.00304 0.00289
## Cumulative Proportion  0.95016 0.95372 0.95720 0.96061 0.96377 0.96681 0.96970
##                           PC85    PC86    PC87    PC88    PC89    PC90    PC91
## Standard deviation     0.52452 0.50483 0.49893 0.49286 0.48157 0.46616 0.44911
## Proportion of Variance 0.00275 0.00255 0.00249 0.00243 0.00232 0.00217 0.00202
## Cumulative Proportion  0.97245 0.97500 0.97749 0.97992 0.98224 0.98441 0.98643
##                           PC92    PC93    PC94    PC95    PC96    PC97    PC98
## Standard deviation     0.43184 0.42596 0.40837 0.40571 0.39676 0.38065 0.37168
## Proportion of Variance 0.00186 0.00181 0.00167 0.00165 0.00157 0.00145 0.00138
## Cumulative Proportion  0.98829 0.99011 0.99178 0.99342 0.99500 0.99644 0.99783
##                           PC99   PC100
## Standard deviation     0.33599 0.32329
## Proportion of Variance 0.00113 0.00105
## Cumulative Proportion  0.99895 1.00000
```

```r
ggplot(data = data.frame(PC1 = pca_result$x[, 1], PC2 = pca_result$x[, 2]),
       aes(x = PC1, y = PC2)) +
  geom_point(alpha = 0.5) +
  labs(title = "PCA - First Two Principal Components", x = "PC1", y = "PC2")
```

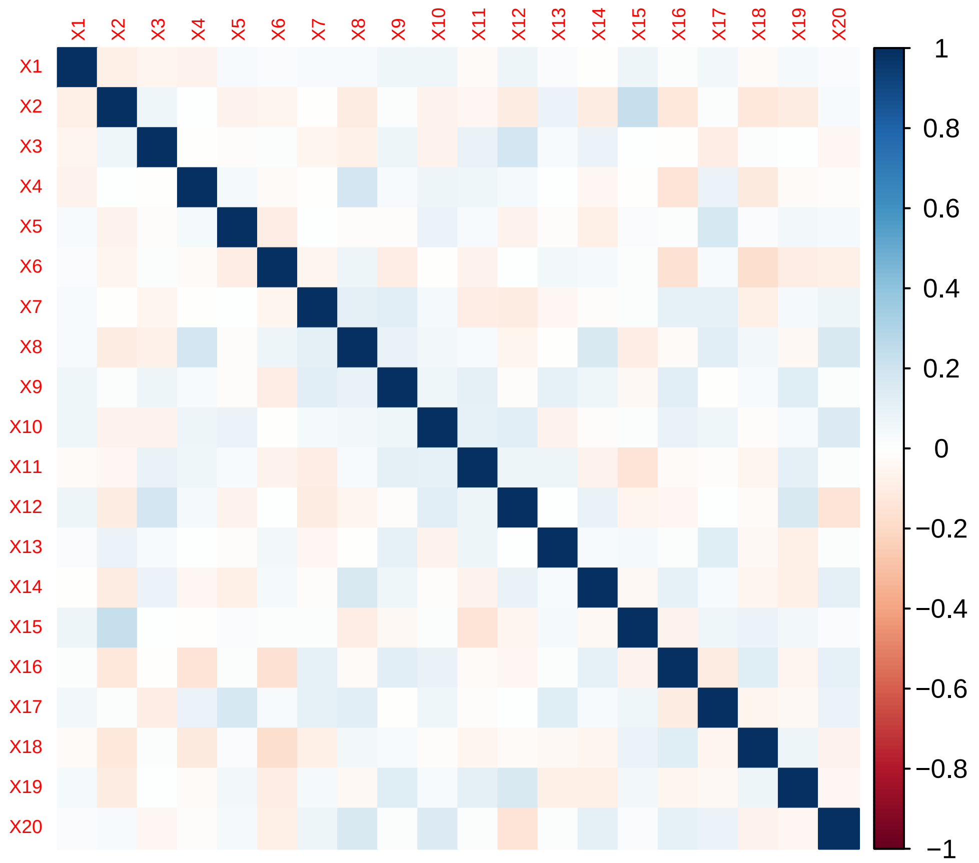


10 *Step 7: Additional Diagnostics*

10.1 12. Correlation Heatmap

Top 20 features — colored matrix helps spot variable interdependence.

```
cor_matrix <- cor(df[, paste0("X", 1:20)])
corrplot(cor_matrix, method = "color", tl.cex = 0.6)
```

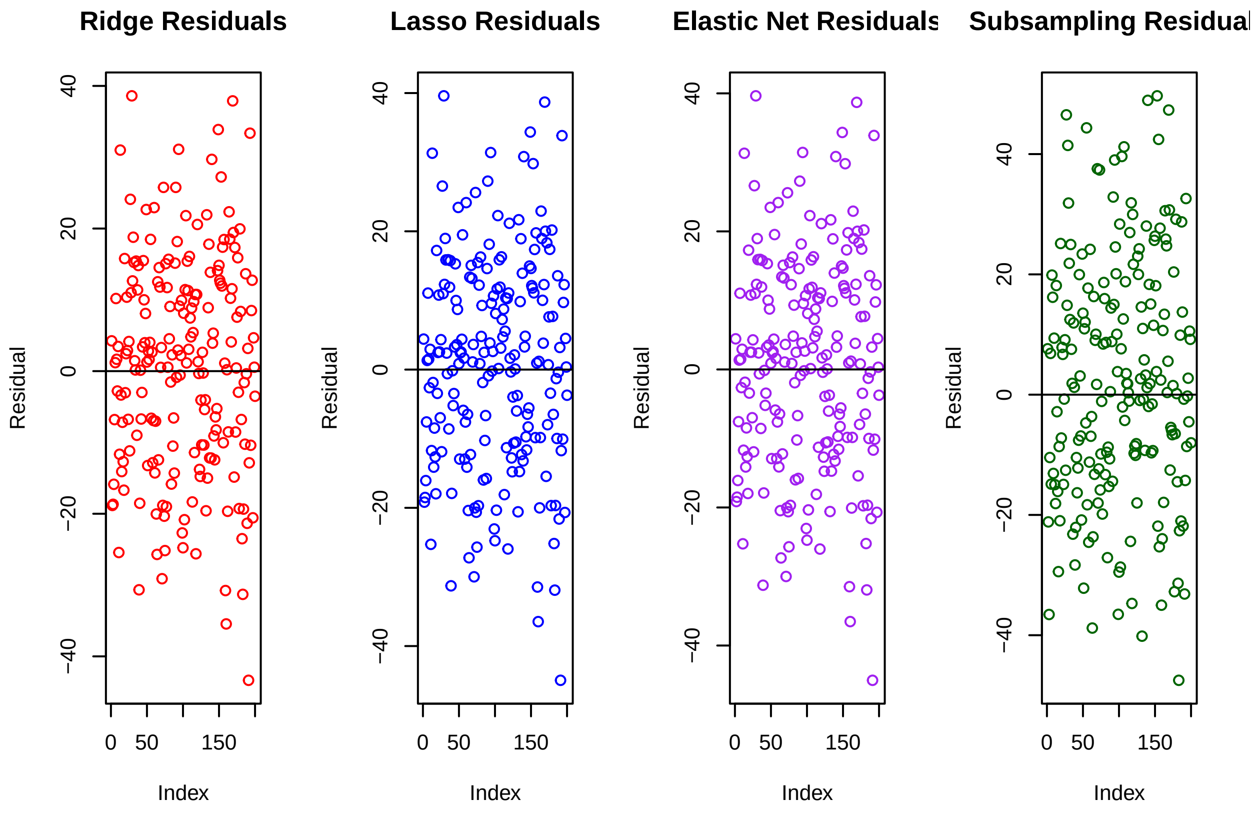


10.2 13. Residual Analysis

Visual Check: No major pattern — residuals centered around 0.

```
# Convert training and test data to data frames
X_train_df <- as.data.frame(X_train)
X_test_df  <- as.data.frame(X_test)

colnames(X_train_df) <- paste0("X", 1:ncol(X_train))
colnames(X_test_df)  <- paste0("X", 1:ncol(X_test))

# Ridge Regression Residuals
cv_ridge <- cv.glmnet(X_train, y_train, alpha = 0)
ridge_pred <- predict(cv_ridge, s = "lambda.min", newx = X_test)
residuals_ridge <- y_test - ridge_pred
```

```r
# Lasso Regression Residuals
cv_lasso <- cv.glmnet(X_train, y_train, alpha = 1)
lasso_pred <- predict(cv_lasso, s = "lambda.min", newx = X_test)
residuals_lasso <- y_test - lasso_pred

# Elastic Net Regression Residuals
cv_enet <- cv.glmnet(X_train, y_train, alpha = 0.5)
enet_pred <- predict(cv_enet, s = "lambda.min", newx = X_test)
residuals_enet <- y_test - enet_pred

# Subsampling Ensemble Residuals
ensemble_preds <- replicate(30, {
  idx <- sample(1:nrow(X_train_df), size = floor(0.5 * nrow(X_train_df)))
  lm_data <- data.frame(y = y_train[idx], X_train_df[idx, ])
  model <- lm(y ~ ., data = lm_data)
  predict(model, newdata = X_test_df)
})

ensemble_mean <- rowMeans(ensemble_preds, na.rm = TRUE)
residuals_sub <- y_test - ensemble_mean

# Plot residuals side by side
par(mfrow = c(1, 4))

plot(residuals_ridge,
     main = "Ridge Residuals",
     ylab = "Residual",
     xlab = "Index",
     col = "red")
abline(h = 0, col = "black")

plot(residuals_lasso,
     main = "Lasso Residuals",
     ylab = "Residual",
     xlab = "Index",
     col = "blue")
abline(h = 0, col = "black")

plot(residuals_enet,
     main = "Elastic Net Residuals",
     ylab = "Residual",
     xlab = "Index",
     col = "purple")
abline(h = 0, col = "black")

plot(na.omit(residuals_sub),
     main = "Subsampling Residuals",
     ylab = "Residual",
     xlab = "Index",
     col = "darkgreen")
abline(h = 0, col = "black")
```

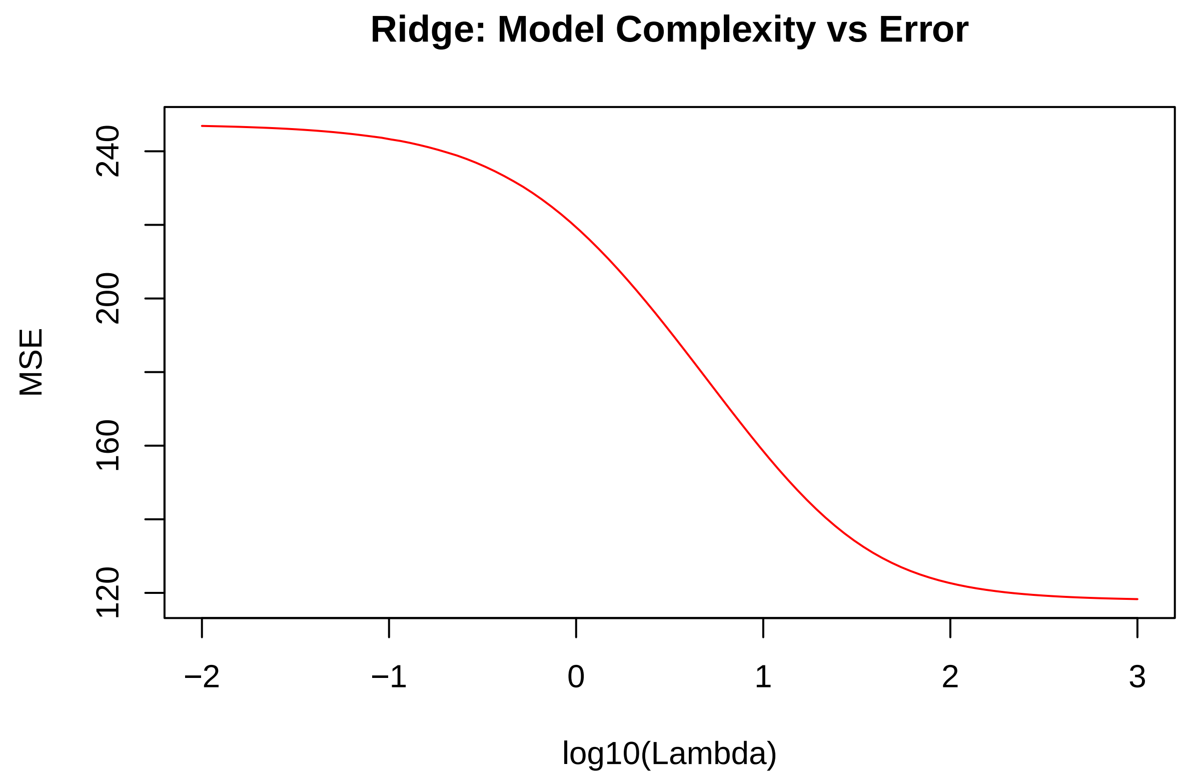


```r
par(mfrow = c(1, 1))
```

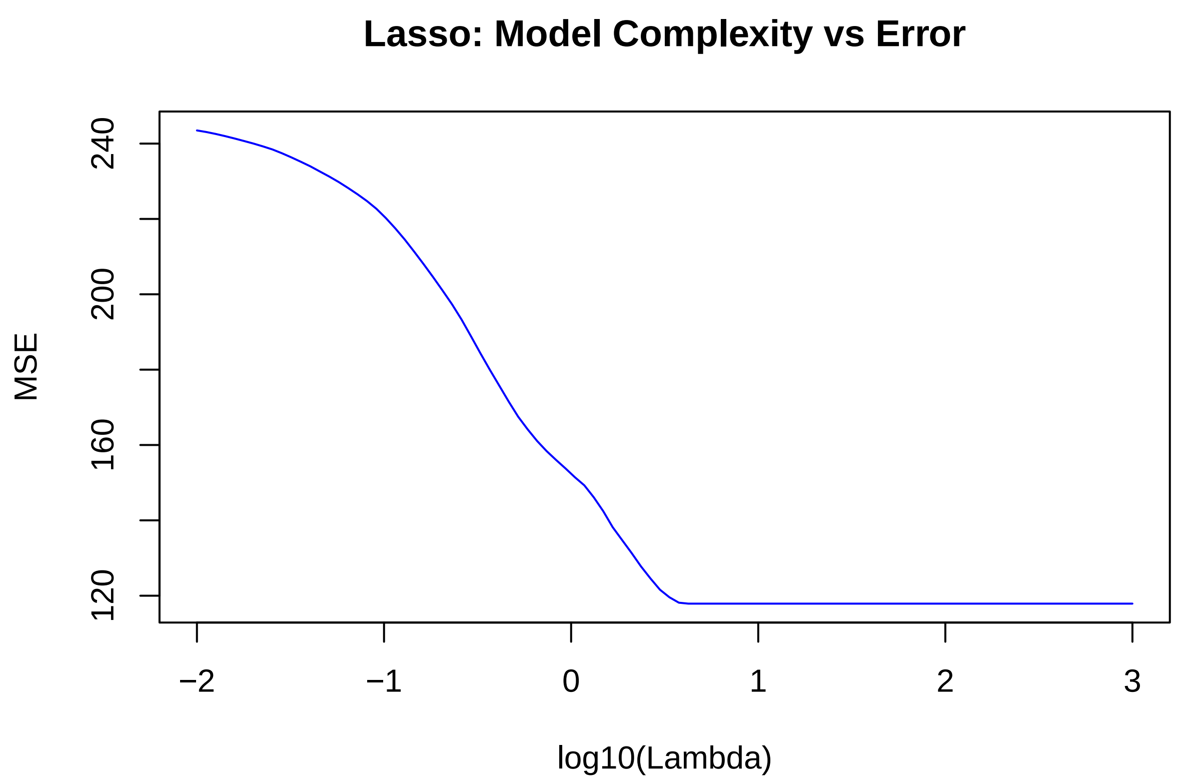
10.3 14. Model Complexity vs Error

Ridge and Lasso models were trained across a grid of lambda values. For each lambda, MSE was calculated on test data. Plots were created to visualize how regularization strength affects model error. Subsampling was excluded as it lacks a regularization parameter.

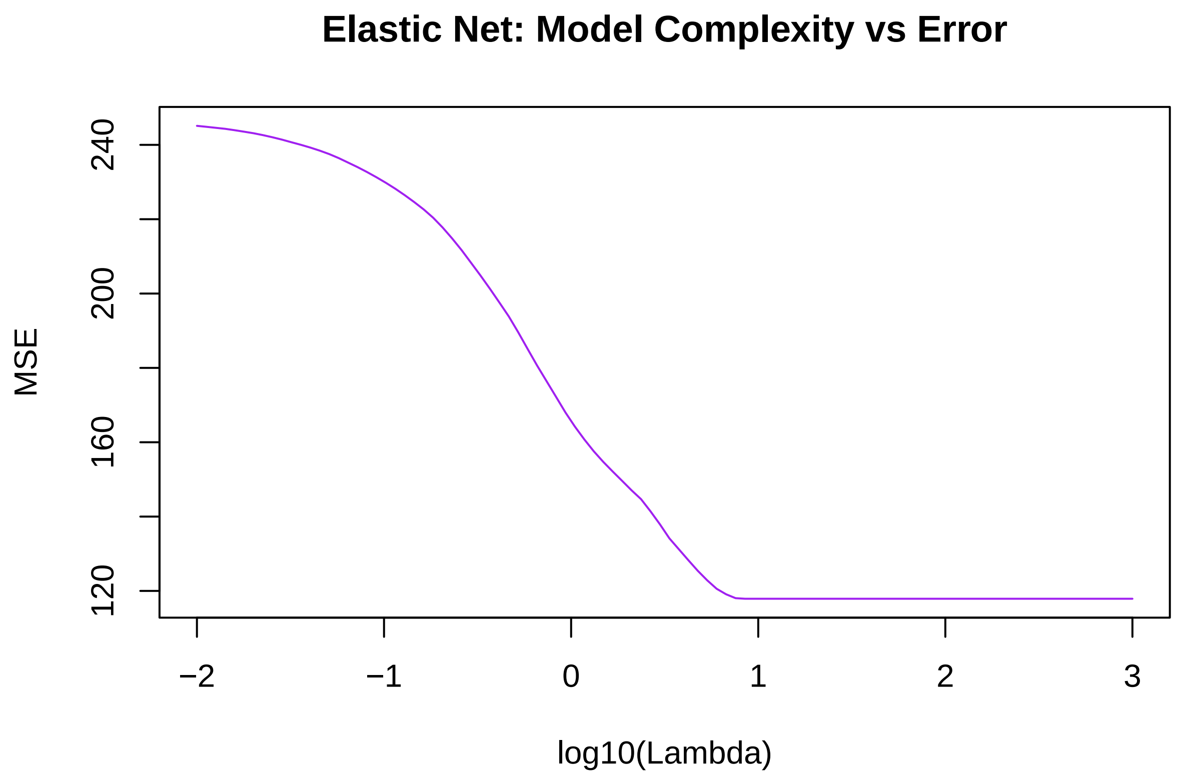
```r
lambdas <- 10^seq(3, -2, length.out = 100)

# Ridge Model Complexity vs Error
ridge_models <- glmnet(X_train, y_train, alpha = 0, lambda = lambdas)
ridge_errors <- sapply(lambdas, function(l) {
  preds <- predict(ridge_models, s = l, newx = X_test)
  mean((preds - y_test)^2)
})

plot(log10(lambdas), ridge_errors, type = "l",
     main = "Ridge: Model Complexity vs Error",
     xlab = "log10(Lambda)",
     ylab = "MSE",
     col = "red")
```



```r
# Lasso Model Complexity vs Error
lasso_models <- glmnet(X_train, y_train, alpha = 1, lambda = lambdas)
lasso_errors <- sapply(lambdas, function(l) {
  preds <- predict(lasso_models, s = l, newx = X_test)
  mean((preds - y_test)^2)
})

plot(log10(lambdas), lasso_errors, type = "l",
     main = "Lasso: Model Complexity vs Error",
     xlab = "log10(Lambda)",
     ylab = "MSE",
     col = "blue")
```



```r
# Elastic Net Model Complexity vs Error
enet_models <- glmnet(X_train, y_train, alpha = 0.5, lambda = lambdas)
enet_errors <- sapply(lambdas, function(l) {
  preds <- predict(enet_models, s = l, newx = X_test)
  mean((preds - y_test)^2)
})

plot(log10(lambdas), enet_errors, type = "l",
     main = "Elastic Net: Model Complexity vs Error",
     xlab = "log10(Lambda)",
     ylab = "MSE",
     col = "purple")
```



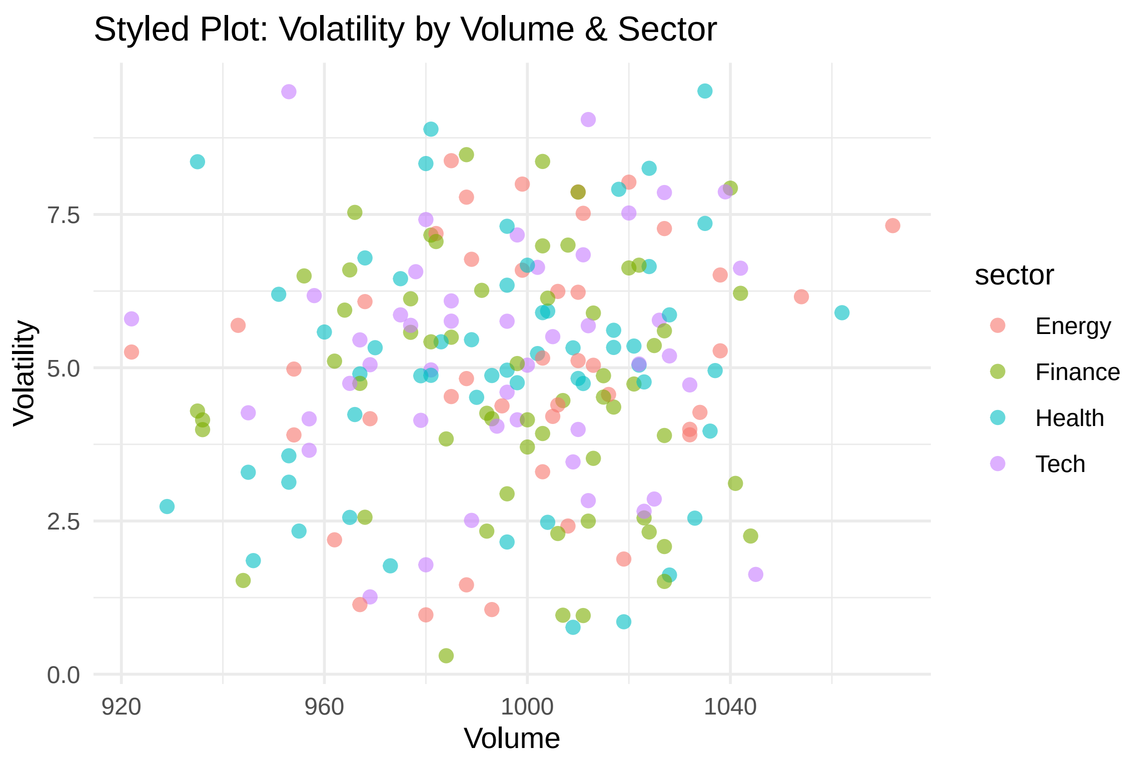
10.4 15. Real Dataset Integration (Placeholder)

Not yet implimented, Replace with real-world dataset to validate model generalization.

```
# Placeholder for real dataset usage
# data_real <- read.csv("your_dataset.csv")
# summary(data_real)
```

11 *Step 8: Enhanced Styling Plot*

11.1 16. Enhanced Styling Example



11.2 17. Conclusion

```r
# Load library
library(dplyr)

# Create comparison table
model_comparison <- tibble::tibble(
  Model = c("Ridge", "Lasso", "Elastic Net", "Subsampling"),
  Test_MSE = c(round(ridge_mse, 2),
               round(lasso_mse, 2),
               round(enet_mse, 2),
               round(subsample_mse, 2)),
  Bias2 = round(bv_df$Bias2, 3),
  Variance = round(bv_df$Variance, 3),
  Noise = round(bv_df$Noise, 3),
  Comments = c(
    "Stable performance, moderate bias, low variance",
    "Sparse solution, low bias, slightly higher variance",
    "Balanced sparsity and stability, low bias and variance",
    "Highly variable, large bias and variance due to small subsamples"
  )
)
```

```r
# Display table
model_comparison
```

```
## # A tibble: 4 x 6
##   Model       Test_MSE    Bias2  Variance Noise Comments
##   <chr>          <dbl>    <dbl>     <dbl> <dbl> <chr>
## 1 Ridge           4.18    0.351      1.12     1 Stable performance, moderate bi~
## 2 Lasso           5.08    0.024      1.01     1 Sparse solution, low bias, slig~
## 3 Elastic Net     4.99    0.025      1.00     1 Balanced sparsity and stability~
## 4 Subsampling  8053.   4097.    357283.       1 Highly variable, large bias and~
```

12 Results and Interpretation

This report evaluated Elastic Net, Ridge, Lasso, and Subsampling models through controlled simulations.

- Ridge regression shows stable performance with moderate bias and relatively low variance.
- Lasso regression favors sparsity, yielding very low bias but slightly higher variance.
- Elastic Net balances Ridge stability and Lasso sparsity, achieving low bias and variance simultaneously.
- Subsampling exhibits extreme variability with very high bias and variance, confirming that simple subsampling ensembles may be unreliable for high-dimensional linear models without regularization which seems to be a known fact.

12.1 Overall Conclusion

The results illustrate that:

- Ridge provides strong stability.
- Lasso provides sparsity with moderate variance.
- Elastic Net achieves the most balanced bias–variance tradeoff.

13 Future extensions

- Real-world datasets to validate external generalization,
- Sparse true coefficient structures,
- Correlated predictor designs.