MTH – 522: Advanced Mathematical Statistics
Homework 2
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Hat Matrix

1. In the toy data example:

```
# Price vs Age of used car
library(matrixcalc)

# Price of car - y
y <- matrix(c(6,9,8,10,11,12,11,13),8,1)
y

# Age of car - x
x <- matrix(0,8,2)
x[,1] <- 1
x[,2] <- c(6,5,4,3,2,2,1,1)
x
```

a. What is the hat matrix?

Solution:

```
# Hat Matrix for the data
hat <- x %*% solve(t(x) %*% x) %*% t(x)
hat
```

b. Verify that hat matrix is symmetric and idempotent.

Solution:

```
# Symmetric hat matrix
hat_trans <- t(hat)
hat_trans
all.equal(hat_trans,hat)
```

```
> # Symmetric hat matrix
> hat_trans <- t(hat)
> hat_trans
              [,1]        [,2]       [,3]  [,4]          [,5]          [,6]        [,7]        [,8]
[1,]  5.000000e-01  0.37500000 0.25000000 0.125 -5.551115e-17 -5.551115e-17 -0.12500000 -0.12500000
[2,]  3.750000e-01  0.29166667 0.20833333 0.125  4.166667e-02  4.166667e-02 -0.04166667 -0.04166667
[3,]  2.500000e-01  0.20833333 0.16666667 0.125  8.333333e-02  8.333333e-02  0.04166667  0.04166667
[4,]  1.250000e-01  0.12500000 0.12500000 0.125  1.250000e-01  1.250000e-01  0.12500000  0.12500000
[5,] -5.551115e-17  0.04166667 0.08333333 0.125  1.666667e-01  1.666667e-01  0.20833333  0.20833333
[6,] -5.551115e-17  0.04166667 0.08333333 0.125  1.666667e-01  1.666667e-01  0.20833333  0.20833333
[7,] -1.250000e-01 -0.04166667 0.04166667 0.125  2.083333e-01  2.083333e-01  0.29166667  0.29166667
[8,] -1.250000e-01 -0.04166667 0.04166667 0.125  2.083333e-01  2.083333e-01  0.29166667  0.29166667
> all.equal(hat_trans,hat)
[1] TRUE
>
```

```
# Idempotent hat matrix
hat_2 <- hat %*% hat
hat_2
all.equal(hat_2,hat)
```

```
> # Idempotent hat matrix
> hat_2 <- hat %*% hat
> hat_2
              [,1]        [,2]       [,3]  [,4]          [,5]          [,6]        [,7]        [,8]
[1,]  5.000000e-01  0.37500000 0.25000000 0.125 -1.121788e-16 -1.121788e-16 -0.12500000 -0.12500000
[2,]  3.750000e-01  0.29166667 0.20833333 0.125  4.166667e-02  4.166667e-02 -0.04166667 -0.04166667
[3,]  2.500000e-01  0.20833333 0.16666667 0.125  8.333333e-02  8.333333e-02  0.04166667  0.04166667
[4,]  1.250000e-01  0.12500000 0.12500000 0.125  1.250000e-01  1.250000e-01  0.12500000  0.12500000
[5,] -9.598803e-17  0.04166667 0.08333333 0.125  1.666667e-01  1.666667e-01  0.20833333  0.20833333
[6,] -9.598803e-17  0.04166667 0.08333333 0.125  1.666667e-01  1.666667e-01  0.20833333  0.20833333
[7,] -1.250000e-01 -0.04166667 0.04166667 0.125  2.083333e-01  2.083333e-01  0.29166667  0.29166667
[8,] -1.250000e-01 -0.04166667 0.04166667 0.125  2.083333e-01  2.083333e-01  0.29166667  0.29166667
> all.equal(hat_2,hat)
[1] TRUE
```

c. Verify that the diagonal elements of the hat matrix are in the range of [0,1].

Solution.

```
# Limits of diagonal elements
for (i in 1:8) {
  if(hat[i,i]<=1 | hat[i,i]>=0){
    print("TRUE")
  }else{
    print("FALSE")
  }
}
```

```
[1] "TRUE"
[1] "TRUE"
[1] "TRUE"
[1] "TRUE"
[1] "TRUE"
[1] "TRUE"
[1] "TRUE"
[1] "TRUE"
```

d. What is the trace of diagonal elements?

Solution.

```r
# Trace of hat matrix
trace <- 0
for (i in 1:8) {
  trace <- trace + hat[i,i]
}
trace
```

```
> trace
[1] 2
```

The trace of the hat matrix should be equal to p.
We see that the trace is equal to 2 for this example, which is true as p = 2 for this example.

2. Verify the following property of matrix trace

   trace(AB)=trace(BA)

Solution.

```r
# Trace of product of 2 matrices
for(i in 1:3){
  A<-matrix(sample(1:5,16, replace=TRUE),4,4);
  B<-matrix(sample(1:5,16, replace=TRUE),4,4);
  a <- sum(diag(A %*% B))
  b <- sum(diag(B %*% A))
  print(paste("Trace of AB is ", a))
  print(paste("Trace of BA is ", b))
}
```

```
[1] "Trace of AB is  141"
[1] "Trace of BA is  141"
[1] "Trace of AB is  161"
[1] "Trace of BA is  161"
[1] "Trace of AB is  139"
[1] "Trace of BA is  139"
```