

Monetary Policy with Heterogeneous Agents: Insights from TANK models *

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Abstract

How does heterogeneity affect the effectiveness of monetary policy and the properties of economic fluctuations? Using the distinction between constrained and unconstrained households at each point in time, we identify three channels at work in Heterogeneous Agent New Keynesian (HANK) models: (i) changes in the average consumption *gap between* constrained and unconstrained households, (ii) variations in consumption *dispersion within* unconstrained households, and (iii) changes in the *share* of constrained households. We analyze the quantitative importance of each of those factors for output fluctuations in a baseline HANK model. We show that a simple Two-Agent New Keynesian (TANK) model, with a constant share of constrained households and no heterogeneity within either type, approximates reasonably well the implications of a HANK model regarding the effects of aggregate shocks on aggregate output..

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1 Introduction

A growing literature has emerged in recent years that aims at re-examining some important macro questions through the lens of monetary models with heterogenous agents. Models in this literature commonly assume the presence of idiosyncratic shocks to individuals' income, together with the existence of incomplete markets and borrowing constraints. Those features are combined with the kind of nominal rigidities and monetary non-neutralities that are the hallmark of New Keynesian models. Following Kaplan et al. (2016), we refer to those models as HANK models (for "Heterogenous Agent New Keynesian" models).

Several lessons have been drawn from this literature. Thus, for instance, taking into account agents' heterogeneity has been shown to be important in order to understand the transmission of monetary policy, including the relative contribution of direct and indirect effects (Kaplan et al. (2016)) or its redistributive effects across income groups (Aucleart (2016)). In addition, several authors have emphasized how the transmission of monetary policy and its aggregate effects may vary significantly depending on the prevailing fiscal policy, as the latter determines how the implementation of monetary policy affects the distribution of individual income and wealth among agents with different marginal propensities to consume.

As is well known, solving for the equilibrium of HANK economies requires the use of nontrivial computational techniques, given the need to keep track of the wealth distribution, and the difficulties arising from the presence of occasionally binding borrowing constraints. The reliance on numerical techniques for the analysis of those models often presents a challenge when it comes to understanding the mechanisms underlying some of the findings, and may thus limit their usefulness in the classroom or as an input in policy institutions.

The purpose of the present paper is twofold. First, we provide a simple framework that helps understand and quantify the implications of heterogeneity for aggregate fluctuations. Our framework distinguishes between two types of households at each point in time, which we label as

"unconstrained" or "constrained", depending on whether their consumption satisfies or not a consumption Euler equation. Having made that distinction, we identify three dimensions of heterogeneity that explain differences in aggregate fluctuations between a HANK economy and its representative agent counterpart (RANK, for short): (i) changes in the average consumption gap *between* constrained and unconstrained households, (ii) variations in consumption dispersion *within* unconstrained households, and (iii) changes in the share of constrained households . We show that the previous three factors are captured through additive "wedges" showing up in a log-linearized Euler equation for aggregate consumption, and which determine the differential behavior of a HANK economy relative to its RANK counterpart. Furthermore, by tracing their responses to aggregate shocks, we can assess the quantitative significance of each of those heterogeneity factors in shaping aggregate output fluctuations.

A second objective of the present paper is to assess the ability of Two Agent New Keynesian (TANK) models to approximate the role of heterogeneity in richer HANK models. TANK models assume the existence of two types of consumers—"constrained" and "unconstrained"—with constant shares in the population, while allowing only for aggregate shocks (i.e. disregarding idiosyncratic shocks). A subset of households (the "unconstrained") are assumed to have full access to financial markets (including markets for stocks and bonds), while "constrained" households are assumed to behave in a "hand-to-mouth" fashion, consuming their current income at all times. This will be the case if they do not have access to financial markets, find themselves continuously against a binding borrowing constraint, or display a pure myopic behavior.

HANK and TANK models share a key feature that is missing in RANK models, namely, the fact that at any point in time a fraction of agents face a binding borrowing constraint (or behave as if they did), and thus do not adjust their consumption in response to changes in interest rates or any variable other than their current income. As a result, average consumption of constrained and unconstrained households will generally differ, with the gap between them changing over time

in response to aggregate shocks. This corresponds to the first heterogeneity factor in our general framework, and one that can be found in both HANK and TANK models.

On the other hand, HANK and TANK models differ in two important ways, related to the two remaining heterogeneity factors introduced above. Thus, in HANK models –but not in TANK– households face idiosyncratic shocks that cannot be fully insured against. As a result, there exists a non-degenerate wealth distribution that evolves over time, constituting an additional (infinite dimensional) state variable, and leading to a *dispersion* of consumption within the subset of unconstrained households. Secondly, the subset of households who are subject (or act "as if" subject) to a binding borrowing constraint does not change over time in TANK models, neither in terms of their identity nor their weight in the population. By contrast, in HANK models that fraction is endogenous and will generally vary over time, as a result of the interaction of aggregate shocks, the distribution and composition of wealth at any point in time, and the presence of borrowing limits.

From a more practical perspective, the analysis of TANK models is highly simplified relative to their HANK counterparts for there is no need to keep track of the wealth distribution and its changes over time. In fact, as we show below, the implied equilibrium conditions of a baseline TANK model can be reduced to a system of difference equations isomorphic to that of a RANK model.

A key finding of our analysis is that a simple TANK model approximates well, both from a qualitative and a quantitative viewpoint, the aggregate output dynamics of a canonical HANK model in response to aggregate shocks, monetary and non-monetary. This is because of two reasons. On the one hand, for plausible specifications of a HANK model, consumption heterogeneity *between* constrained and unconstrained households fluctuates significantly in response to aggregate shocks, and its fluctuations are well captured by a TANK model. On the other hand, we show that the two remaining heterogeneity factors, while significant, they tend to mutually offset each other.

The previous finding suggests that a TANK model may be used to obtain analytical results that provide useful insights on the role of heterogeneity in more general HANK models. In particular, our analysis below emphasizes the role of heterogeneity in amplifying or attenuating the impact on output of changes in monetary policy and other sources of fluctuations.

1.1 Related Literature

The paper is related to two main strands of the literature. On the one hand, the emerging literature introducing New Keynesian features into heterogeneous agent models with idiosyncratic risk and incomplete markets. Some examples are the works of Guerrieri and Lorenzoni (2017), McKay et al. (2016), Farhi and Werning (2017b), Gornemann et al. (2016), Kaplan et al. (2016), McKay and Reis (2016), Werning (2015), Auclert (2017), Luetticke (2017), and Ravn and Sterk (2014), among many others. The main difference with respect to that literature is the development of a simple TANK model with no idiosyncratic shocks, which admits an analytical solution, and the emphasis on the distinction between constrained and unconstrained households. On the other hand, the paper builds on the earlier literature on two-agent models, such as Campbell and Mankiw (1989), Galí et. al. (2007), Bilbiie (2008, 2018), Bilbiie and Straub (2013), Broer et. al. (2016) and Walsh (2017).¹ The main difference with respect to that literature is the comparison, both from a theoretical and a quantitative viewpoint, with more general heterogeneous agent models.

The remainder of the paper is organized as follows. In section 1 we introduce our "organizing framework." In section 2 we lay out a baseline HANK model and in section 3 we analyze the role of the different heterogeneity components in shaping aggregate fluctuations. In section 4 we introduce

¹Similarly, Bilbiie (2018) uses a TANK model to illustrate the "direct" and "indirect" effects of monetary policy shocks emphasized by Kaplan et al. (2016) in a more general HANK model. Farhi and Werning (2017a) use a variety of TANK models to analyze the size of fiscal multipliers in a liquidity trap and in currency unions. Ravn and Sterk (2017) build a tractable heterogeneous agent model with nominal rigidities and labor market frictions, giving rise to endogenous unemployment risk.

a baseline TANK model and compare its predictions with those of the HANK counterpart. Section 5 summarizes our main findings and concludes.

2 Heterogeneity and Aggregate Consumption: An Organizing Framework

In this section we derive an equilibrium condition for aggregate consumption that we use as a simple organizing device to think about the implications of heterogeneity, relative to a representative household model. Our approach is related to Werning (2015), but differs from the latter in the emphasis we attach to the distinction between constrained and unconstrained households –a distinction emphasized by TANK models– in our interpretation of the deviations from the standard Euler equation of a representative household model.

Consider an economy with a continuum of heterogeneous households, indexed by $s \in [0, 1]$. Each household seeks to maximize utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t(s), X_t(s); Z_t)$, where $C_t(s)$ denotes consumption, $X_t(s)$ includes other household-specific endogenous variables, and Z_t is an exogenous preference shifter following a stationary process. We assume $U(C, X; Z) \equiv \frac{C^{1-\sigma}-1}{1-\sigma}Z + V(X; Z)$. Households have identical preferences, but may differ in terms of their wealth, earnings, and/or the transfers they receive. Most importantly, we assume that in any given period t , a fraction $\lambda_t \in [0, 1]$ of households do not have access to (or simply do not make use of) financial markets in order to smooth consumption over time in the face of shocks. As a result, the standard consumption Euler equation will not hold for these households. Possible reasons for this lack of participation include the presence of a binding borrowing constraint or myopic behavior. The remaining households, representing a fraction $1 - \lambda_t$ of the population, are assumed to satisfy their consumption Euler equation. Such an environment is characteristic of most heterogenous agent models, tracing back to Bewley (1983), Huggett (1993) and Aiyagari (1994).

Next we derive a generalized Euler equation for aggregate consumption for such an economy

with heterogeneous households. We proceed in two steps. First we derive an Euler equation in terms of average consumption for unconstrained households. Then we rewrite that Euler equation in terms of aggregate consumption.

2.1 A Generalized Euler Equation for Unconstrained Households' Consumption

Let $\mathcal{U}_t \subset [0, 1]$ denote the set of households that in period t have *unconstrained* access to (and make effective use of) a market for one-period bonds yielding a (gross) riskless real return R_t .² The measure of \mathcal{U}_t is given by $1 - \lambda_t$. For any household $s \in \mathcal{U}_t$ the following Euler equation is satisfied:

$$Z_t C_t(s)^{-\sigma} = \beta R_t \mathbb{E}_t \{ Z_{t+1} C_{t+1}(s)^{-\sigma} \} \quad (1)$$

Integrating both sides of (1) over $s \in \mathcal{U}_t$, and letting $C_t^U \equiv \frac{1}{1-\lambda_t} \int_{s \in \mathcal{U}_t} C_t(s) ds$ denote average consumption among households that are unconstrained in period t , we can write

$$Z_t (C_t^U)^{-\sigma} = \beta R_t \mathbb{E}_t \{ Z_{t+1} (C_{t+1}^U)^{-\sigma} \Theta_{t+1} \} \quad (2)$$

where

$$\Theta_{t+1} \equiv \left(\frac{C_{t+1|t}^U}{C_{t+1}^U} \right)^{-\sigma} \frac{\int_{s \in \mathcal{U}_t} (C_{t+1}(s)/C_{t+1|t}^U)^{-\sigma} ds}{\int_{s \in \mathcal{U}_t} (C_t(s)/C_t^U)^{-\sigma} ds}$$

where $C_{t+1|t}^U \equiv \frac{1}{1-\lambda_t} \int_{s \in \mathcal{U}_t} C_{t+1}(s) ds$ denotes average consumption in period $t+1$ of households that were unconstrained in period t .

Note that (2) can be viewed as an Euler equation describing the average consumption of unconstrained households. It differs from the standard Euler equation for the representative household due to the presence of the Θ_{t+1} term. The latter term captures the wedge between, on the one hand, the average intertemporal marginal rate substitution across unconstrained households and,

²In the present section, we assume a riskless real bond for notational convenience. The analysis carries over to the case of a nominally riskless bond, as assumed below.

on the other, the intertemporal marginal rate of substitution of a "stand-in" household whose consumption is, period by period, equal to the average consumption among unconstrained households, C_t^U .

In order to get some intuition about the factors behind that wedge, consider the following second order approximation (derived in the appendix):

$$\Theta_{t+1} \simeq \left(\frac{C_{t+1|t}^U}{C_{t+1}^U} \right)^{-\sigma} \left[\frac{2 + \sigma(1 + \sigma) \text{var}_{s|t}\{c_{t+1}(s)\}}{2 + \sigma(1 + \sigma) \text{var}_{s|t}\{c_t(s)\}} \right] \quad (3)$$

where $\text{var}_{s|t}\{c_{t+k}(s)\} \equiv (1 - \lambda_t)^{-1} \int_{s \in \mathcal{U}_t} (c_{t+k}(s) - c_{t+k|t}^U)^2$ with $c_t(s) \equiv \log C_t(s)$ and $c_{t+k|t}^U \equiv \log C_{t+k|t}^U$. To understand the factors behind variations in Θ_{t+1} , consider an economy in which the set of unconstrained households doesn't change over time, i.e. $\mathcal{U}_t = \mathcal{U}$ for all t .³ In that case, $C_{t+1}^U = C_{t+1|t}^U$ and for all t . As a result, deviations of Θ_{t+1} from unity will be exclusively due to anticipated changes in the cross-sectional variance of consumption across the (fixed set of) unconstrained households, i.e. if $\text{var}_s\{c_{t+1}(s)\}$ differs from $\text{var}_s\{c_t(s)\}$. In particular, if that variance is expected to increase, then $\Theta_{t+1} > 1$, which lowers C_t^U , given C_{t+1}^U , an effect analogous to that of precautionary savings.

On the other hand, even if the cross-sectional variance of consumption among unconstrained households were constant, Θ_{t+1} may differ from unity and vary over time as a result of some households switching from an unconstrained status to a constrained one and viceversa. Such switches may occur in connection with changes in λ_t over time. To the extent that households who become constrained at $t + 1$ have on average lower consumption in that period than those who are unconstrained, then $C_{t+1|t}^U < C_{t+1}^U$. Furthermore, in the presence of switching, the variance of consumption at $t + 1$ across households who were unconstrained at t is likely to be larger than their current (i.e. period t) consumption variance, i.e. $\text{var}_{s|t}\{c_{t+1}(s)\} > \text{var}_{s|t}\{c_t(s)\}$. Both phenomena

³A particular instance of such an environment is given by an economy with heterogeneous households but in which none are constrained, i.e. $\mathcal{U} = [0, 1]$.

tend to raise the anticipated average future marginal utility of currently unconstrained households, leading to a decrease in their current consumption for any given interest rate and expected average consumption C_{t+1}^U .

Given the previous considerations, we can interpret Θ_t as a wedge that captures the impact of consumption heterogeneity *within* the set of unconstrained households on the dynamics of their average consumption. The size of that wedge will generally vary over time in response to shocks that trigger the mechanisms discussed above.

For future reference, we can log-linearize (2) around a steady state to yield

$$\hat{c}_t^U = \mathbb{E}_t\{\hat{c}_{t+1}^U\} - \frac{1}{\sigma}\hat{r}_t - \frac{1}{\sigma}\mathbb{E}_t\{\Delta z_{t+1}\} - \frac{1}{\sigma}\mathbb{E}_t\{\hat{\theta}_{t+1}\} \quad (4)$$

where lower case letters denote the natural log of a variable and a " $\hat{\cdot}$ " indicates deviation from steady state. Equation (4) makes clear that an anticipated increase in consumption dispersion within unconstrained households, reflected in an increase in $\mathbb{E}_t\{\hat{\theta}_{t+1}\}$, will lower current average unconstrained consumption, *ceteris paribus*.

2.2 A Generalized Euler Equation for Aggregate Consumption

Next we proceed to relate average consumption among the unconstrained, C_t^U , with aggregate consumption $C_t \equiv \int_0^1 C_t(s)ds$, in order to derive an Euler equation for the latter.

Let $\mathcal{K}_t \subset [0, 1]$ denote the set of constrained households in period t , i.e. households that do not satisfy (2) in that period. Recall that such households represent a fraction λ_t of all households. We denote their average consumption by $C_t^K \equiv \frac{1}{\lambda_t} \int_{s \in \mathcal{K}_t} C_t(s)ds$. Note that aggregate consumption C_t satisfies $C_t = (1 - \lambda_t)C_t^U + \lambda_t C_t^K$.

We define $\gamma_t \equiv (C_t^U - C_t^K)/C_t^U$, an index of the average consumption *gap* between constrained and unconstrained households, which thus captures consumption heterogeneity *between* the two

type of households.⁴ In much of the discussion above we assume that steady state average consumption among unconstrained households is above its counterpart among constrained households. Accordingly, $\gamma \equiv (C^U - C^K)/C^U \in [0, 1]$. That property is satisfied in all the models considered below.

Note that we can write:

$$C_t = C_t^U(1 - \lambda_t \gamma_t)$$

Thus, the extent to which aggregate consumption differs from average unconstrained consumption will be the result of two factors: (i) changes in the gap measure γ_t , and (ii) variations in the share of constrained households, $\hat{\lambda}_t$. In particular, and for any *given* level of average unconstrained consumption, aggregate consumption will be larger the smaller is the gap between the two household types and the smaller is the share of unconstrained households.

In a neighborhood of the steady state, and up to a first order:

$$\hat{c}_t = \hat{c}_t^U - \frac{\lambda}{1 - \lambda\gamma} \hat{\gamma}_t - \frac{\gamma}{1 - \lambda\gamma} \hat{\lambda}_t \quad (5)$$

where $\hat{\lambda}_t \equiv \lambda_t - \lambda$ and $\hat{\gamma}_t \equiv \gamma_t - \gamma$ denote deviations from steady state values. Combining (4) and (5) yields the following Euler equation for aggregate consumption:

$$\hat{c}_t = \mathbb{E}_t\{\hat{c}_{t+1}\} - \frac{1}{\sigma} \hat{r}_t - \frac{1}{\sigma} \mathbb{E}_t\{\Delta z_{t+1}\} - \mathbb{E}_t\{\Delta \hat{h}_{t+1}\}$$

where \hat{h}_t is a heterogeneity index defined by

$$\hat{h}_t \equiv \hat{h}_t^\gamma + \hat{h}_t^\theta + \hat{h}_t^\lambda$$

with

$$\hat{h}_t^\gamma \equiv -\frac{\lambda}{1 - \lambda\gamma} \hat{\gamma}_t$$

⁴Note that, since the consumption Euler equation does not hold for constrained households, heterogeneity *within* those households is not relevant for the purposes of deriving an Euler equation for aggregate consumption. The latter can instead be obtained combining the average consumption of unconstrained households, and the consumption *gap* between constrained and unconstrained households.

$$\widehat{h}_t^\theta \equiv -\frac{1}{\sigma} \sum_{k=1}^{\infty} \mathbb{E}_t \{\widehat{\theta}_{t+k}\}$$

and

$$\widehat{h}_t^\lambda \equiv -\frac{\gamma}{1-\lambda\gamma} \widehat{\lambda}_t$$

Assuming all variables are expected to revert asymptotically to their steady state values, we can solve the previous difference equation forward to obtain the following expression for aggregate consumption:

$$\widehat{c}_t = -\frac{1}{\sigma} \widehat{r}_t^L + \frac{1}{\sigma} z_t + \widehat{h}_t \quad (6)$$

where $\widehat{r}_t^L \equiv \sum_{k=0}^{\infty} \mathbb{E}_t \{\widehat{r}_{t+k}\}$.

The first two factors behind aggregate consumption fluctuations, namely, current and expected real interest rates (as summarized by \widehat{r}_t^L) and the exogenous demand shifter z_t , are already found in the representative household model. The third factor, \widehat{h}_t , summarizes the impact of heterogeneity on aggregate consumption. The heterogeneity index \widehat{h}_t has, in turn, three different components, respectively associated with variations in $\widehat{\gamma}_t$, $\widehat{\theta}_t$, and $\widehat{\lambda}_t$. Henceforth we refer to these three components as the *gap*, *dispersion* and *share* components, respectively.

Note that in the representative household model, $\widehat{h}_t = 0$ for all t . Thus, the extent to which aggregate consumption (and, in equilibrium, aggregate output and employment) in a HANK model behaves differently from its RANK counterpart, conditional on the monetary policy stance (as summarized by \widehat{r}_t^L) will depend on response of \widehat{h}_t to different aggregate shocks. That response will in turn depend on the joint endogenous response of $\widehat{\gamma}_t$, $\widehat{\theta}_t$, and $\widehat{\lambda}_t$. Thus, the response of aggregate consumption to an expansionary shock will be amplified if it is accompanied, *ceteris paribus*, by:

(i) an increase in \widehat{h}_t^γ , resulting from a reduction in the consumption gap between constrained and unconstrained households resulting from a redistribution of resources towards the latter as a result of the shock;

- (ii) an increase in \hat{h}_t^θ , i.e. caused by a lower anticipated cross-sectional dispersion of consumption among currently unconstrained households;
- (iii) an increase in \hat{h}_t^λ i.e. triggered by a reduction in the share of constrained households for, ceteris paribus, this represents an increase in average consumption of those switching to an unconstrained status.

It is important to note that the responses of these three factors to an aggregate shock will generally not be independent from each other, as the analysis below will illustrate.

Instead, a standard TANK model with a constant share of hand-to-mouth households and no heterogeneity among unconstrained households, $\hat{\lambda}_t = \hat{\theta}_t = 0$ for all t . Accordingly, $\hat{h}_t = \hat{h}_t^\gamma$, with variations in the gap variable $\hat{\gamma}_t$ being the only source of deviations from the predictions of a RANK model.

It is an open question, both empirical and theoretical, whether fluctuations in \hat{h}_t , as well as its underlying components, \hat{h}_t^γ , \hat{h}_t^θ , and \hat{h}_t^λ , contribute significantly to fluctuations in aggregate consumption. The analysis of the dynamics of those factors, i.e. the size of their fluctuations and their response to aggregate shocks, should contribute to our understanding of the implications of heterogeneity for aggregate fluctuations. Unfortunately, it is not clear that currently available data (at least for the U.S. economy) would allow one to determine the identity of constrained and unconstrained agents at business cycle frequencies and to match this information with individual consumption.⁵ Instead, below we use a baseline HANK model as a laboratory economy, where we analyze quantitatively the behavior of the heterogeneity index \hat{h}_t and its underlying components (\hat{h}_t^γ , \hat{h}_t^θ , and \hat{h}_t^λ), and their role in shaping aggregate fluctuations.

⁵Ideally one would need a long balanced panel with quarterly consumption and wealth data for individual households.

3 A Baseline HANK Model

Next we lay out a baseline HANK model which captures in a stylized way many key features of the existing models in the literature.

3.1 Households

As in the general framework above, we assume a continuum of households, indexed by $s \in [0, 1]$, with identical preferences given by $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t(s), \mathcal{N}_t(s); Z_t)$, where $Z_t \equiv \exp\{z_t\}$ is an exogenous preference shifter, $C_t(s) \equiv \left(\int_0^1 C_t(s, i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ is a consumption index with $C_t(s, i)$ denoting the quantity of good i consumed by the household s in period t , and $\mathcal{N}_t(s)$ denote work hours. As above we specialize the utility function to be of the form $U(C, \mathcal{N}; Z) \equiv \left(\frac{C^{1-\sigma}-1}{1-\sigma} - \frac{\mathcal{N}^{1+\varphi}}{1+\varphi}\right) Z$. The household's period budget constraint is given by:

$$\begin{aligned} \frac{1}{P_t} \int_0^1 P_t(i) C_t(s, i) di + Q_t F_t(s) + \frac{B_t(s)}{P_t} &\leq \frac{B_{t-1}(s)(1+i_{t-1})}{P_t} + [Q_t + (1-\delta)D_t] F_{t-1}(s) \\ &\quad + W_t \mathcal{N}_t(s) \exp\{e_t(s)\} + T_t(s) \end{aligned}$$

for $t = 0, 1, 2, \dots$, where $P_t(i)$ is the price of good i . W_t is the nominal wage per efficiency unit of labor. $B_t(s)$ represents purchases of one-period discount bonds (yielding an interest rate i_t). $F_t(s)$ are the holdings of shares in an equity fund (described below). Q_t is the price of those shares. $T_t(s)$ denotes net transfers. $e_t(s)$ is an idiosyncratic productivity shock, satisfying $\int_0^1 \exp\{e_t(s)\} ds = 1$ for all t .

An equity fund hold claims to a fraction $1 - \delta$ of firms' profits. Shares in the equity fund are tradable and fully liquid, and can be used for consumption smoothing purposes. The remaining fraction δ of firms' profits, which we refer to as *illiquid*, have no tradable claims associated with it. Instead they are allocated across households according to a distribution/transfer rule described below.

We assume a borrowing constraint of the form

$$\frac{B_t(s)}{P_t} \geq -\psi Y \quad (7)$$

for all t , where Y denotes steady state output and $\psi \geq 0$. In addition, we assume short-selling of shares in the equity fund is not allowed, i.e. $F_t(s) \geq 0$, for all t and s .

As in most of the HANK literature, we do not model explicitly households' portfolio decisions. Instead we assume that all households with positive net wealth allocate an identical share $v_t \in [0, 1]$ of that wealth to the equity fund, i.e. $Q_t F_t(s) = \max[0, v_t \mathcal{A}_t(s)]$ for all $s \in [0, 1]$, where $\mathcal{A}_t(s) \equiv Q_t F_t(s) + B_t(s)/P_t$ denotes household s financial wealth.

Optimal allocation of expenditures implies $C_t(s, i) = (P_t(i)/P_t)^{-\epsilon} C_t(s)$. We assume employment is demand determined, and uniformly distributed across households, i.e. $\mathcal{N}_t(s) = N_t$ for all $s \in [0, 1]$. Below we make assumptions that guarantee that it is optimal for each households to meet that labor demand at the prevailing wage.

As in the general framework of section 1, households who are unconstrained (i.e. for whom (7) is not binding) will satisfy a standard consumption Euler equation. Formally

$$Z_t C_t(s)^{-\sigma} = \beta(1 + i_t) \mathbb{E}_t \{ Z_{t+1} C_{t+1}(s)^{-\sigma} (P_t/P_{t+1}) \}$$

for all $s \in \mathcal{U}_t$ where $\mathcal{U}_t \equiv \{s | B_t(s) > -\psi P_t Y\}$. On the other hand, the price of shares in the equity fund is assumed to satisfy the difference equation

$$Q_t = \mathbb{E}_t \{ \Lambda_{t,t+1}^+ [Q_{t+1} + (1 - \delta) D_{t+1}] (P_t/P_{t+1}) \}$$

where $\Lambda_{t,t+1}^+ \equiv \beta(C_{t+1|t}^+ / C_t^+)^{-\sigma} (Z_{t+1}/Z_t)$ is the stochastic discount factor used by the fund, with C_t^+ and $C_{t+1|t}^+$ denoting consumption in period t and $t + 1$ of households with positive net wealth in period t , weighted by their share holdings (or, equivalently, by their wealth).

We need to specify how the illiquid component of profits is allocated among households. We assume the following distribution rule:

$$T_t(s) = \left[1 + \tau_t^a \left(\frac{\mathcal{A}_t^+(s)}{\mathcal{A}_t^+} - 1 \right) + \tau_t^e (\exp\{e_t(s)\} - 1) \right] \delta D_t \quad (8)$$

where $\mathcal{A}_t^+(s) \equiv \max[0, \mathcal{A}_t(s)]$ and $\mathcal{A}_t^+ \equiv \int_0^1 \mathcal{A}_t^+(s) ds$. Note that $\int_0^1 T(s) ds = \delta D_t$. The previous rule is governed by two parameters, $\tau_t^a \in [0, 1]$ and $\tau_t^e \in [0, 1]$, which capture, respectively, the weight attached to net financial wealth and labor income as a determinant of the share of illiquid profits allocated to each household. Note that such a rule may capture both institutional and fiscal mechanisms. We assume households take their allocated transfers as lump-sum.

Rule (8) embeds several special cases of interest found in the literature. A first case, which we refer to as "wealth-based rule" (or W-rule, for short) corresponds to $\tau_t^a = 1$ and $\tau_t^e = 0$, implying that illiquid profits are distributed only to current shareholders, in proportion to their holdings. This is similar to the case analyzed in Gornemann et al. (2017) and the one that we view as the most realistic (among the three considered). At the other extreme we have by $\tau_t^a = 0$ and $\tau_t^e = 1$ in which case illiquid profits are transferred to households in proportion to their productivity (or, equivalently, their labor income). This corresponds to the case considered by Kaplan et al. (2018), which they interpret as capturing profit sharing in the form of bonuses. Below we refer to this rule as "productivity-based" or P-rule. Another special case of interest is the rule assumed in McKay et al. (2016) which corresponds to $\tau_t^a = \tau_t^e = 0$, which implies that illiquid profits are distributed uniformly among households, independently of their wealth or productivity. Below we refer to this transfer rule as "uniform" or U-rule.

Note that our baseline HANK model satisfies all the assumptions of the general framework in section 1. Thus we can write an Euler equation for aggregate consumption as

$$\hat{c}_t = \mathbb{E}_t\{\hat{c}_{t+1}\} - \frac{1}{\sigma}(\hat{i}_t - \mathbb{E}_t\{\pi_{t+1}\}) - \frac{1}{\sigma}\mathbb{E}_t\{\Delta z_{t+1}\} - \mathbb{E}_t\{\Delta \hat{h}_{t+1}\} \quad (9)$$

where $\hat{h}_t = \hat{h}_t^\gamma + \hat{h}_t^\theta + \hat{h}_t^\lambda$ summarizes the influence of the relevant heterogeneity factors, as discussed in section 1.

3.2 Supply Side

Our focus is on aggregate demand dynamics, so we keep the supply side of the model as simple as possible. In particular we assume a wage schedule

$$W_t = \mathcal{M}^w C_t^\sigma N_t^\varphi \quad (10)$$

where $C_t \equiv \int_0^1 C_t(s)ds$ denotes average consumption and where $\mathcal{M}^w > 1$ is a constant (gross) average wage markup. Throughout we assume $W_t \geq C_t(s)^\sigma N_t^\varphi$ for all $s \in [0, 1]$ and all t , so that all households will be willing to supply the work hours demanded by firms at wage W_t .

On the supply side, a continuum of firms, indexed by $i \in [0, 1]$ is assumed. Each firm produces a differentiated good with a linear production function

$$Y_t(i) = A_t N_t(i) \quad (11)$$

where $A_t \equiv \exp\{a_t\}$ is an exogenous technology parameter common to all firms. Each firm sets the price of its good optimally each period, subject to a quadratic adjustment cost $\mathcal{C}(\cdot) \equiv \frac{\xi}{2} P_t Y_t \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2$ where $\xi > 0$ and a sequence of demand constraints $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$, where Y_t denotes aggregate output. Profit maximization, combined with the symmetric equilibrium conditions $P_t(i) = P_t$ and $Y_t(i) = Y_t$ for all $i \in [0, 1]$, implies:

$$\Pi_t (\Pi_t - 1) = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left(\frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1} (\Pi_{t+1} - 1) \right\} + \frac{\epsilon}{\xi} \left(\frac{1}{\mathcal{M}_t^p} - \frac{1}{\mathcal{M}^p} \right) \quad (12)$$

where $\Pi_t \equiv P_t/P_{t-1}$ is the (gross) price inflation rate, and $\mathcal{M}_t^p \equiv A_t/W_t$ is the average gross markup, with the latter's optimal value in the absence of price adjustment costs given by $\mathcal{M}^p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$. Aggregate profits are given by $D_t = Y_t \Delta^p(\Pi_t) - W_t N_t$ where $\Delta^p(\Pi_t) \equiv 1 - (\xi/2)(\Pi_t - 1)^2$.

Log-linearization of (12) around the zero inflation steady state yields the inflation equation

$$\pi_t = \beta \mathbb{E}_t \{ \pi_{t+1} \} - \omega \hat{\mu}_t^p \quad (13)$$

where $\hat{\mu}_t^p \equiv \log(\mathcal{M}_t^p / \mathcal{M}^p)$ and $\omega \equiv \epsilon_p / (\xi \mathcal{M}^p)$. Combining (10) and (11) (after taking logs) we can express the deviations of the (log) price markup from steady state can be written as:

$$\hat{\mu}_t^p = (1 + \varphi) a_t - (\sigma + \varphi) \hat{y}_t \quad (14)$$

Note that by setting $\hat{\mu}_t^p = 0$ all t we can solve for the natural (i.e. flexible price) level of output, \hat{y}_t^n (expressed in log deviations from steady state). In the baseline model analyzed here, the latter variable is as a function of technology only, given by

$$\hat{y}_t^n = \frac{1 + \varphi}{\sigma + \varphi} a_t \equiv \psi_a a_t.$$

More generally, however, \hat{y}_t^n may also depend on other exogenous shocks that may shift the markup-output schedule (14), including labor supply shocks and shocks to the desired markup, among others. Importantly, under our assumptions, heterogeneity factors do not have an impact on the natural level of output, which is determined by the supply side. Accordingly, any effect of heterogeneity on aggregate output will be result from its impact on aggregate demand combined with the presence of nominal rigidities.

Independently of the number and nature of the shocks affecting \hat{y}_t^n , the following relation will generally hold:

$$\hat{\mu}_t^p = -(\sigma + \varphi) \tilde{y}_t \quad (15)$$

where $\tilde{y}_t \equiv y_t - y_t^n$ is the output gap, i.e. the log deviation of output from its flexible price counterpart. Substituting (15) into (13) we obtain a version of the New Keynesian Phillips curve:

$$\pi_t = \beta \mathbb{E}_t \{\pi_{t+1}\} + \kappa \tilde{y}_t \quad (16)$$

where $\kappa \equiv \omega (\sigma + \varphi)$.

The fact that the supply side of our baseline HANK model is not affected by the presence of heterogeneity allows us to focus the impact of the latter on aggregate demand (which coincides with aggregate consumption in our simple model), in the spirit of Werning (2015).

3.3 Monetary Policy

The central bank is assumed to follow a Taylor-type rule given by

$$\hat{i}_t = \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t \quad (17)$$

where v_t is an exogenous monetary policy shock, which follows an $AR(1)$ process. The previous rule, often assumed in the New Keynesian literature, is meant to capture in a parsimonious way the behavior of central banks' in "normal" times.

3.4 Equilibrium

We complete the description of our model by listing several market clearing conditions that will need to be satisfied in equilibrium. Goods market clearing requires

$$Y_t(i) = C_t(i) + X_t(i)$$

for all $i \in [0, 1]$, where $X_t(i) = (P_t(i)/P_t)^{-\epsilon_p}(\xi/2)Y_t(\Pi_t - 1)^2$ captures the demand for good i to meet price adjustment costs. Noting that in equilibrium all firms set the same prices, thus implying $Y_t(i) = Y_t$ and $C_t(i) = C_t$ for all $i \in [0, 1]$, we can write

$$C_t = Y_t \Delta^p(\Pi_t) \quad (18)$$

where $\Delta_t^p(\Pi_t) \equiv 1 - (\xi/2)(\Pi_t - 1)^2$. Furthermore, market clearing in the bonds and stock markets implies that $\int_0^1 B_t(s)ds = 0$ and $\int_0^1 F_t(s)ds = 1$ for all t .

Aggregate employment is given by

$$\begin{aligned} N_t &= \int_0^1 N_t(i)di \\ &= Y_t/A_t \end{aligned} \quad (19)$$

and is assumed to be distributed uniformly among household types, so that $N_t(s) = N_t$ for all $s \in [0, 1]$ all t .

Note that, up to a first-order approximation and in a neighborhood of the zero inflation steady state (18) can be written as

$$\hat{c}_t = \hat{y}_t$$

Combining the previous condition with Euler equation (9) we obtain a version of the dynamic IS equation for the HANK model:

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma} \left(\hat{i}_t - \mathbb{E}_t\{\pi_{t+1}\} - \hat{r}_t^n \right) - \mathbb{E}_t\{\Delta \hat{h}_{t+1}\} \quad (20)$$

where

$$\begin{aligned} \hat{r}_t^n &\equiv -\mathbb{E}_t\{\Delta z_{t+1}\} + \sigma \mathbb{E}_t\{\Delta y_{t+1}^n\} \\ &= (1 - \rho_z)z_t - \sigma(1 - \rho_a)\psi_a a_t \end{aligned} \quad (21)$$

is the natural rate of interest in the associated RANK economy, which is independent of heterogeneity factors. Solving (20) forward we obtain:

$$\tilde{y}_t = -\frac{1}{\sigma} \tilde{r}_t^L + \hat{h}_t \quad (22)$$

where $\tilde{r}_t^L \equiv \sum_{k=0}^{\infty} \mathbb{E}_t\{\hat{i}_{t+k} - \pi_{t+k} - \hat{r}_{t+k}^n\}$.

Knowledge of the response of \hat{h}_t to aggregate shocks would be sufficient to solve for the equilibrium of the model above. Unfortunately, there is no simple relation between \hat{h}_t and aggregate shocks, due to its dependence on the wealth distribution at each point in time. Keeping track of the latter requires the use of nontrivial non-linear methods.⁶

3.5 Calibration and Solution Method

The baseline calibration of the model closely follows recent studies in the HANK literature, and is summarized in Table 1. In particular, each period is assumed to be a quarter, and we set the discount factor β such that the steady-state real risk-free rate is 3 percent per year.⁷

⁶See, e.g. Algan et al. (2014) for a recent survey and comparison of alternative solution methods.

⁷This implies values for β of 0.9745, 0.9743 and 0.9679, for the wealth-based, labor-based and uniform specification, respectively.

Regarding preferences, we set the coefficient of risk aversion $\sigma = 1$ (log-utility), the (inverse) Frisch elasticity of substitution $\phi = 1$, the elasticity of substitution among good varieties $\epsilon = 10$, which implies that profits are 10 percent of GDP. Also, we set the price adjustment cost parameter $\xi = 105$ implying a slope of the Phillips curve $\kappa = 0.17$, which corresponds to the value that would arise in a model with sticky prices à la Calvo with average price duration of four quarters.

Regarding the idiosyncratic productivity shock, and following McKay et al. (2016) and Auclert (2016), we assume that $e_t(s)$ follows an AR(1) with persistence parameter $\rho_e = 0.9777$ and standard deviation $\sigma_e = 0.1928$. This parameterization implies that at an annual frequency individual wages display an autocorrelation of 0.92 and a standard deviation of 0.7, which are consistent with the estimates of Floden and Lindé (2001) and the calibration of Kaplan et. al. (2018).

Furthermore, following the taxonomy of U.S. assets contained in Kaplan et. al. (2018, Table 2), the fraction of illiquid assets δ is assumed to equal to 0.92, which implies a value of liquid net worth of 25 percent of annual GDP.⁸ Finally, we set the borrowing limit $\psi = 0.5$, which implies that between 21% and 27% of the households are borrowing constrained (depending on the specification of the transfer rule), which is in the middle of the range of values used in the literature. Section 5 considers alternative values of ψ and δ . Regarding the distribution of illiquid profits, as discussed above, we consider three different transfer rules which are nested in our framework, namely, "wealth-based" or W-rule (with $\tau_a = 1, \tau_e = 0$), the "productivity-based" or P-rule ($\tau_a = 0, \tau_e = 1$), and the "uniform" or U-rule ($\tau_a = 0, \tau_e = 0$).

As to the interest rate rule coefficients, it is assumed that $\phi_\pi = 1.5$ and $\phi_y = 0.5/4$.⁹ Finally, it is assumed that each aggregate shock follows an AR(1) process, where the persistence parameters are set to $\rho_v = \rho_z = 0.5$ for the monetary and preference shocks, and to $\rho_a = 0.9$ for the technology shock. These are standard values in the New Keynesian literature.

⁸Total net worth, calculated as the present discount value of firm's profits, equals 3.3 times annual GDP, which is also consistent with the evidence reported in Kaplan et. al. (2018).

⁹These values were proposed by Taylor (1993) as providing a good approximation to U.S. monetary policy in the Greenspan years.

The numerical solution algorithm is based on the projection and perturbation method developed by Reiter (2010). In particular, the individual consumption choices and the implied wealth distribution are approximated on a coarse grid for the two individual state variables, i.e. "cash-on-hand" (the sum of income, transfers and liquid wealth) and the idiosyncratic shock.¹⁰ The steady state is calculated with a fixed point iteration for the discount factor β and the value of \mathcal{A}_t^+ , while the dynamic responses to aggregate shocks are computed using a (linear) perturbation method around the steady state.

4 Aggregate Fluctuations in HANK: The Role of Heterogeneity

In this section we report our findings regarding the role of heterogeneity in the HANK model described above. In doing so, we take the Representative Agent New Keynesian (RANK) model as a natural benchmark. Note that the RANK model may be viewed as a limiting case of our baseline HANK model with $e_t(s) = 0$ for all $s \in [0, 1]$ and t , together with the assumptions of identical initial conditions (i.e. $B_{t-1}(s) = 0$, for all $s \in [0, 1]$) and absence of borrowing constraint (7). Note that in that case $\hat{h}_t = 0$ for all $s \in [0, 1]$, with (20) collapsing into the familiar dynamic IS equation

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma} \left(\hat{i}_t - \mathbb{E}_t\{\pi_{t+1}\} - \hat{r}_t^n \right) \quad (23)$$

or, equivalently,

$$\tilde{y}_t = -\frac{1}{\sigma} \tilde{r}_t^L$$

Note that (23), together with the New Keynesian Phillips curve (16) and the interest rate rule (17) fully describe the equilibrium dynamics of the RANK economy. In the analysis below all the parameters defining the RANK model are calibrated as their counterparts in HANK.¹¹

¹⁰The exogenous idiosyncratic shock is discretized on a grid of 11 points, while cash-on-hand is discretized using 80 points. The policy functions are approximated using a piecewise cubic spline.

¹¹In the case of β , as in the HANK model, it is set to generate an annualized steady state real rate of 3%.

Figure 1 displays a scatterplot of the output gap \tilde{y}_t and the heterogeneity factor \hat{h}_t generated by a simulation (of 10,000 periods) of our calibrated HANK model for each of the transfer rules considered, under the assumption of an exogenous $AR(1)$ process for the real interest rate \hat{r}_t (or, equivalently, a monetary policy shock v_t , under fully rigid prices). The previous assumption guarantees that any differential behavior between HANK and RANK is not influenced by the assumed monetary policy rule, and is instead only due to the heterogeneity factors, since $\tilde{r}_t^L = (1 - \rho_r)^{-1} \hat{r}_t$ is the same in both models in that case.

For each rule we report (i) the standard deviation of the output gap \tilde{y}_t relative to RANK, (ii) the elasticity of \hat{h}_t with respect to \tilde{y}_t , denoted by Φ , and which we obtain as the estimated coefficient of an OLS regression of \hat{h}_t on \tilde{y}_t , and (iii) the first-order autocorrelation of \tilde{y}_t . The same sequence of shocks is fed into the model across the three transfer rule specifications. Note that the three graphs have an identical scale, for ease of comparison.

A number of observations, uncovered by Figure 1, are worth making. First, the size of fluctuations in \tilde{y}_t and \hat{h}_t , as well as the sign of the correlation between those two variables, depend strongly on the transfer rule in place. In particular, we see that output gap fluctuations relative to RANK are amplified under the W-rule, whereas they are damped under the U-rule, with respective relative standard deviations being 1.84 and 0.77. In the case of the P-rule, \hat{h}_t is largely acyclical, suggesting a very limited contribution of heterogeneity to aggregate fluctuations (the relative standard deviation is 1.07). The extent of amplification or damping of output gap volatility is related to the elasticity Φ . Thus, when the elasticity is positive (negative) the response of \hat{h}_t amplifies (dampens) the effects of real rate shocks on the output gap. This is consistent with the analysis in Werning (2015), who emphasized the role of the cyclicity of heterogeneity for amplification or damping of demand shocks.

Secondly, the "thinness" of the scatterplots points to a limited role of the changing wealth distribution as a state variable (in addition to \hat{r}_t). This is particularly true for the P- and U-rules.

That feature is also reflected in an autocorrelation of the output gap which, ranging from 0.53 to 0.63 is only slightly higher than the autocorrelation of the driving force itself (0.5). Note that gap between the two can be viewed as a measure of the importance of endogenous persistence mechanisms associated with heterogeneity.¹²

In Figure 2 we try to dig into the factors underlying the HANK-generated variations in \hat{h}_t , by plotting also its three components against the output gap. Note that the gap component, \hat{h}_t^γ , appears to display the closest connection with the overall heterogeneity index \hat{h}_t , and to account for much of the latter's elasticity with respect to the output gap. On the other hand, and especially for the W- and P-rules, the dispersion and share components, \hat{h}_t^θ and \hat{h}_t^λ , display much larger volatility than \hat{h}_t , but are far less tightly connected to the output gap. This is not so much the case for the U-rule. Most importantly, in all cases, \hat{h}_t^θ and \hat{h}_t^λ show a nearly-perfect negative correlation, largely offsetting each other (as they lie close to the -45 line), as shown in the bottom panel of the Figure. That feature largely neutralizes their combined effect on the output gap.

Figure 3 displays the same information as Figures 1 and 2, but conditional on each of the three shocks introduced in the model: monetary, preference and technology. With few exceptions, the observations made above under the assumption of an exogenous real rate shock carry over to these additional shocks. In particular, the gap component, \hat{h}_t^γ , tracks reasonably well the overall heterogeneity index \hat{h}_t , while the dispersion and share components, \hat{h}_t^θ and \hat{h}_t^λ , though they display substantial variation, are highly negatively correlated and tend to neutralize each other.

4.1 Discussion

The cyclical properties of the heterogeneity index, \hat{h}_t , which in turn determine the extent of amplification or attenuation of shocks relative to RANK, have been seen to depend critically on the nature of the transfer rule considered. It is possible to explain, at least qualitatively, the cyclical

¹²In the RANK model, which has no endogenous state variables, the autocorrelation of the output gap is the same as that of the driving forces.

behavior of \hat{h}_t through the lens of the gap component, \hat{h}_t^γ . Thus, in the case of the W-rule, where all profits are allocated to unconstrained agents, the reduction in the markup (and profit share) brought about by an increase in the output gap implies a reallocation resources from unconstrained to constrained households. That leads to a decrease in the consumption gap between the two types (as measured by γ_t) with consequent increase in the gap component, \hat{h}_t^γ , and an amplification of the effects of the shock. By contrast, in the case of a U-rule, the reduction in the profit share affects all households uniformly, which in turn implies a larger relative impact for households with lower labor income, a larger fraction of whom are constrained. As a result, the gap measure increases, bringing down the gap component \hat{h}_t^γ and attenuating the impact of the shock. Finally, in the case of the P-rule, (illiquid) profits are distributed in proportion to labor income, and hence affect all households' income in nearly the same proportion.¹³

What is the source of the systematic negative correlation between the dispersion and the share components, \hat{h}_t^θ and \hat{h}_t^λ ? Consider, for sake of concreteness, an expansionary aggregate shock that raises overall wealth and consumption. In response to that shock we would expect a decrease in λ_t , the share of constrained households and, as a result, a procyclical response of the share component \hat{h}_t^λ , as observed in Figure 3. In that environment, more households with relatively low idiosyncratic shocks join the set of unconstrained households, increasing the consumption dispersion within that set in a persistent fashion. That raises the anticipated values of $\hat{\theta}_{t+k}$, for $k = 1, 2, \dots$ lowering the dispersion component \hat{h}_t^θ , which has a dampening effect on output. Accordingly a negative relation between \hat{h}_t^λ and \hat{h}_t^θ arises, which leads to a partial neutralization of their joint impact on fluctuations.

The findings of the present section suggest that much of the observed cyclicity of \hat{h}_t is inher-

¹³Note that only a fraction $1 - \delta$ of profits is distributed in proportion to wealth and, hence, accrues disproportionately to unconstrained households. That should generate a negative elasticity of the gap component with respect to the output gap. On the other hand there are factors other than the markup and the profit share that influence the gap component in response to changes in real interest rates (such as the Fisher effect and the interest rate exposure effect, as discussed in Auclert (2017))

ited from the cyclical properties of the *gap* component, \hat{h}_t^γ . In other words, much of the role of heterogeneity in shaping aggregate fluctuations has to do with the reallocation of resources between constrained and unconstrained households in the wake of aggregate shocks. Interestingly, that is the dimension of heterogeneity that Two-agent New Keynesian models (TANK) may potentially capture. This motivates the next section, which lays out a version of a TANK model and compares its properties to the HANK model analyzed above.

5 HANK vs TANK

The purpose of this section is to compare the predictions of the baseline HANK model analyzed in the previous section with those of a Two-Agent New Keynesian (TANK) model, which we describe briefly next. Again, the supply side of TANK is assumed to be identical to that of our baseline HANK model, so we restrict our description below to the determinants of aggregate consumption.

5.1 The TANK model

Next we describe a version of a Two-Agent New Keynesian (TANK) model. A key advantage of TANK models, relative to their richer HANK counterparts, is that one can generally derive an analytical expression for the heterogeneity index \hat{h}_t and solve for the equilibrium in closed form. Such a characterization will then be used to understand the role of (a particular dimension of) heterogeneity for aggregate fluctuations.

The only source of heterogeneity in the class of TANK models considered here is the assumption that a time-invariant subset of households do not participate in financial markets and just consume their current income, in a hand-to-mouth fashion. Such TANK models have a long tradition in macroeconomics, and have been used to account for a number of empirical observations which are at odds with the predictions of the representative agent model.¹⁴ Our version of the TANK

¹⁴Campbell and Mankiw (1989) introduced the two-agent framework to account for empirical deviations from

model is close to that in Bilbiie (2008), the main difference lying in our distinction between liquid and illiquid components of firms' profits, and the introduction of an explicit transfer rule, both of which facilitate comparability with our HANK model.

We assume a continuum of households represented by the unit interval, with preferences as in the HANK model above. There are two types of households. A fraction $1 - \lambda$ of households are assumed to have unconstrained access to financial markets. We refer to them as "unconstrained" households, and are assumed to constitute a time-invariant subset of all households. In particular, unconstrained households can trade two types of assets: one-period nominally riskless bonds and shares in an equity fund that owns claims to a fraction $1 - \delta$ of aggregate profits D_t . Their period budget constraint is given by

$$\frac{1}{P_t} \int_0^1 P_t(i) C_t^U(i) di + \frac{B_t^U}{P_t} + Q_t F_t^U = \frac{B_{t-1}^U(1 + i_{t-1})}{P_t} + W_t N_t^U + [Q_t + (1 - \delta) D_t] F_{t-1}^U + T_t^U \quad (24)$$

where the notation is analogous to that in the HANK model of the previous section, with the superscript U referring to variables specific to unconstrained households.

The remaining fraction λ of households are assumed to consume their current labor income plus transfers each period, possibly (but not necessarily) because they do not have access to financial markets. We refer to those households as "constrained" or "Keynesian" and use a superscript K to denote variables specific to their type. Formally their period budget constraint is given by

$$\frac{1}{P_t} \int_0^1 P_t(i) C_t^K(i) di = W_t N_t^K + T_t^K \quad (25)$$

Importantly, and in contrast with the HANK model above, households in the TANK model do not face any form of idiosyncratic uncertainty. We assume that they take the wage as given

the permanent-income hypothesis. Galí et al. (2005, 2007) embedded that framework into a New Keynesian model in order to re-examine the conditions for equilibrium uniqueness, and to account for the effects of government purchases on consumption. Bilbiie (2008) analyzed the properties of a version of TANK closer to the one considered here (i.e. without physical capital).

and are willing to supply as much labor as demanded by firms. Labor demand is assumed to be distributed uniformly among all households, thus implying $N_t^U = N_t^K = N_t$.

The intertemporal optimality condition for unconstrained households takes the form

$$Z_t(C_t^U)^{-\sigma} = \beta(1 + i_t)\mathbb{E}_t \{Z_{t+1}(C_{t+1}^U)^{-\sigma}(P_t/P_{t+1})\}$$

while the price of shares in the equity fund must satisfy:

$$Q_t = \mathbb{E}_t \{\Lambda_{t,t+1}^U(Q_{t+1} + (1 - \delta)D_{t+1})\} \quad (26)$$

where $\Lambda_{t,t+1}^U \equiv \beta(Z_{t+1}/Z_t)(C_{t+1}^U/C_t^U)^{-\sigma}$ is the relevant stochastic discount factor, since only unconstrained households own shares in the fund.

The following distribution/transfer rule determines how the illiquid component of profits, δD_t , is allocated:

$$T_t^U = \left(1 + \frac{\tau\lambda}{1 - \lambda}\right) \delta D_t$$

$$T_t^K = (1 - \tau)\delta D_t$$

Note that $(1 - \lambda)T_t^U + \lambda T_t^K = \delta D_t$. Thus, when $\tau = 1$ all profits, associated with both the liquid and illiquid components of equity, end up in the hands of unconstrained households, a case which should be viewed as corresponding to the "wealth-based" rule in the HANK model above. On the other hand, the case of $\tau = 0$ is associated with a uniform distribution of illiquid profits across all households, and can be associated naturally to the "uniform" distribution case considered in the HANK model, as well as the "productivity-based," given that there are no differences in productivity across households in the TANK model.

In the TANK model, the index of the consumption gap between unconstrained and constrained households, $\gamma_t \equiv 1 - \frac{C_t^K}{C_t^U}$, is given by,

$$\begin{aligned}\gamma_t &= 1 - \frac{W_t N_t + T_t^K}{W_t N_t + \frac{1-\delta}{1-\lambda} D_t + T_t^U} \\ &= \frac{(1 - \delta(1 - \tau)) D_t}{(1 - \lambda) W_t N_t + (1 - \delta(1 - \tau) \lambda) D_t} \\ &= \frac{(\Delta^p(\Pi_t) \mathcal{M}_t^p - 1)(1 - \delta(1 - \tau))}{1 - \lambda + (\Delta^p(\Pi_t) \mathcal{M}_t^p - 1)(1 - \delta(1 - \tau) \lambda)}\end{aligned}$$

Note that as long as $\delta(1 - \tau) < 1$ the gap variable γ_t is increasing in the price markup \mathcal{M}_t^p , reflecting the fact that a disproportionate amount of profits is allocated to unconstrained households.

Log-linearizing the above relation around a zero inflation steady state we obtain:

$$\begin{aligned}\hat{\gamma}_t &= \Psi \hat{\mu}_t^p \\ &= -(\sigma + \varphi) \Psi \tilde{y}_t\end{aligned}$$

where $\Psi \equiv \frac{(1-\lambda)(1-\delta(1-\tau))}{[1-\lambda+(\mathcal{M}^p-1)(1-\delta(1-\tau)\lambda)]^2}$. Using the fact that $C_t = C_t^U(1 - \lambda\gamma_t)$ and taking a first order approximation we can write:

$$\begin{aligned}\hat{c}_t &= \hat{c}_t^U - \frac{\lambda}{1 - \lambda\gamma} \hat{\gamma}_t \\ &= \hat{c}_t^U + \Phi \tilde{y}_t\end{aligned}$$

where $\Phi \equiv \frac{\lambda(\sigma+\varphi)\Psi}{1-\lambda\gamma}$ is assumed to be less than one.¹⁵

The previous relation can be combined with the log-linearized Euler equation for unconstrained households, given by

$$\hat{c}_t^U = \mathbb{E}_t\{\hat{c}_{t+1}^U\} - \frac{1}{\sigma} \left(\hat{i}_t - \mathbb{E}_t\{\pi_{t+1}\} \right) - \frac{1}{\sigma} \mathbb{E}_t\{\Delta z_{t+1}\}$$

¹⁵The $\Phi > 1$ is analyzed in Bilbiie (2008), who refers to it as "inverted aggregate demand logic".

to obtain an Euler equation for aggregate consumption of the form:

$$\widehat{c}_t = \mathbb{E}_t\{\widehat{c}_{t+1}\} - \frac{1}{\sigma} \left(\widehat{i}_t - \mathbb{E}_t\{\pi_{t+1}\} \right) - \frac{1}{\sigma} \mathbb{E}_t\{\Delta z_{t+1}\} - \mathbb{E}_t\{\Delta \widehat{h}_{t+1}\}$$

where

$$\begin{aligned} \widehat{h}_t &= -\frac{\lambda}{1 - \lambda\gamma} \widehat{\gamma}_t \\ &= \Phi \widetilde{y}_t \end{aligned}$$

Note that in the present model $\widehat{h}_t = \widehat{h}_t^\gamma$, i.e. the impact of heterogeneity on the Euler equation for aggregate consumption is now restricted to the presence of a *gap* component, which is proportional to the output gap.

Combining the previous relations with the goods market clearing condition $\widehat{c}_t = \widehat{y}_t$ one can derive a "modified" dynamic IS equation:

$$\widetilde{y}_t = \mathbb{E}_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma(1 - \Phi)} \left(\widehat{i}_t - \mathbb{E}_t\{\pi_{t+1}\} - r_t^n \right) \quad (27)$$

where r_t^n is given by (21). Together with (16) and (17), equation (27) describes the equilibrium dynamics of the TANK model. Note that such equilibrium dynamics are isomorphic to those of a RANK model with the inverse elasticity of intertemporal substitution modified to be $\sigma(1 - \Phi)$ instead of σ . Accordingly, under the (plausible) assumption that $\delta(1 - \tau) < 1$ (implying $0 < \Phi < 1$), the presence of constrained households ($\lambda > 0$) in the TANK model tends to make the output gap more responsive to changes in real interest rates, as well as to shocks affecting the *natural* rate of interest (for any given real rate). The reason for this is that an increase in the output gap leads to a shift of resources towards constrained households, as a consequence of the resulting increase in the labor income share and the fact that labor income accounts for a larger fraction of constrained households' income. Their higher marginal propensity to consume leads to an amplification of the effects on the output gap.¹⁶

¹⁶See Bilbiie (2018) for a detailed discussion of that multiplier mechanism, as well as the decomposition between

5.2 HANK vs TANK: Main Findings

Next we report the main findings from the comparison of the properties of our TANK and HANK models. For the sake of concreteness we restrict our analysis to the W-rule, since it is the one for which the role of heterogeneity in the HANK model has been shown to be more important quantitatively (and, arguably, the most realistic one as well). We calibrate the TANK model as follows. Parameters that are common across the two models, including the coefficients of the New Keynesian Phillips curve (16) and the interest rate rule (17), the autoregressive coefficients of the exogenous driving forces, as well as σ are equated to their counterparts in HANK. As in our RANK model, β is set to imply a 3 percent (annualized) interest rate in the steady state. Parameter λ is set equal to the fraction of constrained households in the steady state of the HANK model. Parameter τ is set to 1, in a way consistent with the W-rule.

The left panel of Figure 4 displays a scatterplot of the output gap \tilde{y}_t and the heterogeneity factor \hat{h}_t generated by a simulation of both HANK and TANK models, under the assumption of an exogenous AR(1) process for the real interest rate \hat{r}_t , and over 10,000 periods. As the Figure makes clear the TANK model is quite successful in replicating the cyclical patterns generated by HANK. The right panel of Figure 4 displays the corresponding simulated times series for output generated by HANK, TANK and RANK equilibria, for an arbitrary subsample of 100 periods. Note that the TANK model tracks very closely the output fluctuations generated by HANK, though significant differences with respect to RANK are observable.

Figure 5 shows identical information for our three alternative sources of fluctuations (monetary policy, preference and technology shocks). Independently of the nature of the shock, the output generated by the TANK model tracks very well its HANK counterpart.¹⁷ As shown in the first column of Table 2 the standard deviations of output (relative to RANK) for both HANK and

direct and indirect effects of monetary policy, in TANK models.

¹⁷See the Appendix for the impulse responses of selected variables to the different shocks.

TANK are very similar, and the correlation between the time series for output generated by the two models is close to one. This finding remains true under the alternative transfer rules (P- and U-rules), as reported in the second and third columns of the same Table. The larger differences arise in the case of demand shocks (both monetary policy and preferences) under the U-rule, for which the standard deviation of output implied by TANK is about 15 percent larger than its HANK counterpart, while the correlation between the HANK and TANK model is close to one in all cases.

As a final exercise, we examine the ability of the TANK and RANK models to capture the predictions of HANK with regard to some changes in the environment. Figure 6 shows the cumulative responses over 16 quarters of output to monetary, preference and technology shocks under alternative settings for the interest rate rule coefficients ϕ_π and ϕ_y (again, under a W-rule). We see that the TANK model, and to a lesser extent the RANK model, generate similar predictions about the effects of changes in the monetary policy rule. In particular, as the strength of the anti-inflation stance of the central bank (as measured by ϕ_π) is increased, the smaller (larger) is the impact of demand (technology) shocks on output. On the other hand, increases in the output coefficient ϕ_y are predicted to have a stabilizing influence on output by the three models, and independently of the source of fluctuations.

Figure 7 displays analogous evidence, but focusing now on the effects of changes in the borrowing constraint limit, ψ , the fraction of illiquid profits, δ , and the transfer rule parameter, τ^a . Under HANK, an increase in the borrowing limit has a small effect on the cumulative responses of output to the three shocks, despite the implied changes in the fraction of constrained households (which ranges from 0.15 in the case of $\psi = 1$ to 0.28 when $\psi = 0$). The effects are similarly small, though slightly more pronounced under TANK. In the case of changes in the fraction of illiquid profits the TANK model is shown to capture pretty well the predictions of HANK. Note in particular that as the fraction of illiquid profits becomes small, the possibilities for self-insurance

in HANK increase, lowering the fraction of constrained households in the steady state, and making its predictions closer to the RANK model. A similar phenomenon occurs in the TANK model, as parameter λ is adjusted accordingly. Finally, both in HANK and TANK an increase in τ^a , which implies that a larger fraction of profits is distributed to wealth-rich households, is shown to amplify (dampen) the effects of demand (technology) shocks. This suggests that, in both models, the amplification/dampening of aggregate shocks depends critically on the cyclical properties of markups (or equivalently the labor share), and on the fraction of illiquid profits distributed to households with high marginal propensity to consume.

6 Conclusions

We have identified three dimensions of heterogeneity dynamics that explain the differential behavior of a HANK economy relative to its RANK counterpart: (i) changes in average consumption gap *between* constrained and unconstrained households, (ii) changes in consumption dispersion *within* the subset of unconstrained households, and (iii) changes in the share of constrained households. Using a baseline HANK model we characterize the behavior of the three heterogeneity components. Among other results, we show that the cyclical properties of those components and, as a result, the extent to which heterogeneity amplifies or dampens aggregate fluctuations, depends substantially on how illiquid profits are assumed to be distributed among households. While our analysis has been restricted to an artificial HANK economy, we believe that shedding light on the empirical properties of the different heterogeneity components should be useful for the development of future HANK models.

We also show that a tractable two-agent (TANK) model, which only captures one dimension of heterogeneity (the one which we refer to as the *gap* component), approximates reasonably well the predictions of a baseline HANK model regarding the effects of aggregate shocks on aggregate variables, as well as its predictions regarding the consequences of changes in the environment, once

the fraction of constrained (and the transfer rule) are calibrated accordingly. The previous findings notwithstanding, it should be clear that a simple TANK model will never be able to address many other questions involving heterogeneity (such as the effects of monetary policy on income and wealth distribution and, possibly, welfare) for which a richer HANK model is needed.

The TANK model used above can be extended along several dimensions while preserving its relative tractability. Thus, one can introduce some form of idiosyncratic risk without departing from the two-agent structure. One possibility is to consider that in each period a fraction of Ricardian agent might become Keynesian, and viceversa, as e.g. in Nisticó (2016) and Bilbiie (2017). That extension of the model could be useful to address the so-called “forward-guidance” puzzle inherent in representative agent models, and that are also present in the current version of our TANK model. Second, the comparison between TANK and HANK models could be extended to alternative frameworks—e.g. models with capital, government debt and other assets (liquid and illiquid)—to understand to what extent TANK models might be able to capture some of the defining features of richer HANK models.

References

- Aiyagari, Rao (1994): "Uninsured Idiosyncratic Risk and Aggregate Savings," *Quarterly Journal of Economics*, Volume 109, Issue 3, 1 August 1994, Pages 659–684,
- Algan, Yann, Olivier Allais, Wouter den Haan, and Pontus Rendahl (2014): "Solving and Simulating Models with Heterogeneous Agents and Aggregate Uncertainty," in Schmedders and Judd (eds). *Handbook of Computational Economics*, vol. 3.
- Auclert, Adrien (2017): "Monetary Policy and the Redistribution Channel," mimeo.
- Bayer, Christian, Ralph Luetticke, Lien Pham-Do and Volker Tjaden (2015): "Precautionary Savings, Illiquid Assets, and the Aggregate Consequences of Shocks to Household Income Risk," *Econometrica*, forthcoming.
- Bewley, Truman (1983): "A Difficulty with the Optimum Quantity of Money", *Econometrica*, 51(5), 1485-1504.
- Bilbiie, Florin (2008): "Limited Asset Market Participation, Monetary Policy and (Inverted) Aggregate Demand Logic," *Journal of Economic Theory*, 140(1), 162-196.
- Bilbiie, Florin (2017): "A Catch-22 for HANK Models: No Puzzles, No Amplification," miemo.
- Bilbiie, Florin and Ragot, Xavier (2017): "Optimal Monetary Policy and Liquidity with Heterogeneous Households,"CEPR Discussion Paper No. DP11814.
- Bilbiie, Florin (2018): "The New Keynesian Cross,"mimeo.
- Broer, Tobias, Per Krusell, Niels-Jakob Hansen, and Erik Oberg (2016): "The New Keynesian Transmission Channel: A Heterogeneous-Agent Perspective," NBER working paper no. 22418.
- Campbell, John Y. and N. Gregory Mankiw (1989), "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence," in O.J. Blanchard and S. Fischer (eds.), *NBER Macroeconomics Annual 1989*, 185-216, MIT Press.
- Farhi, Emmanuel and Iván Werning (2017a): "Fiscal Multipliers: Liquidity Traps and Currency

Unions," *Handbook of Macroeconomics* 2: 2417-2492.

Farhi, Emmanuel and Iván Werning (2017b): "Monetary Policy, Bounded Rationality and Incomplete Markets," mimeo.

Floden, Martin and Jesper Lindé (2001): "Idiosyncratic Risk in the United States and Sweden: Is there a Role for Government Insurance?," *Review of Economic Dynamics* 4(2), 406-437.

Galí, Jordi, David López-Salido, and Javier Vallés (2007): "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, 5(1), 227-270.

Gornemann, Nils, Keith Kuester and Makoto Nakajima, (2016): "Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy", mimeo.

Guerrieri, Veronica and Guido Lorenzoni (2017): "Credit Crises, Precautionary Savings, and the Liquidity Trap," *Quarterly Journal of Economics* 132(3), 1427-1467.

Huggett, Mark (1993): "The Risk Free Rate in Heterogeneous Agent Incomplete Insurance Economies," *Journal of Economic Dynamics and Control* 17 (5-6), 953-969.

Kaplan, Greg, Giovanni L. Violante, and Justin Weidner (2014): "The Wealthy Hand-to-Mouth," Brookings Papers on Economic Activity 2014 (Spring), 77-138.

Kaplan, Greg, Benjamin Moll and Giovanni L. Violante (2016), "Monetary Policy According to HANK", NBER Working paper No. 21897.

Luetticke, Ralph (2017): "Transmission of Monetary Policy with Heterogeneity in Household Portfolios", mimeo.

McKay, Alisdair and Ricardo Reis (2016) "The Role of Automatic Stabilizers in the U.S. Business Cycle, *Econometrica* 84(1) 141-94.

McKay, Alisdair, Emi Nakamura and Jon Steinsson (2016): "The Power of Forward Guidance Revisited," *American Economic Review*, 106(10), 3133-3158.

Nisticó, Salvatore (2016): "Optimal Monetary Policy and Financial Stability in a non-Ricardian

Economy," *Journal of the European Economic Association*, 14(5), 1225-1252.

Ravn, Morten and Vincent Sterk (2014): "Job Uncertainty and Deep Recessions," Discussion Papers 1501, Centre for Macroeconomics (CFM).

Ravn, Morten and Vincent Sterk (2017): "Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach," mimeo.

Reiter, Michael (2010): "Solving Heterogeneous Agent Models by Projection and Perturbation," *Journal of Economic Dynamics and Control* 33, 649-665.

Taylor, John (1993): "Discretion vs Policy Rules in Practice," *Carnegie Rochester Conference Series on Public Policy*, 39, 195-214.

Walsh, Carl (2017): "Workers, Capitalists, Wage Flexibility and Welfare," mimeo.

Werning, Iván (2015): "Incomplete Markets and Aggregate Demand," mimeo.

Table 1: Parameter Values

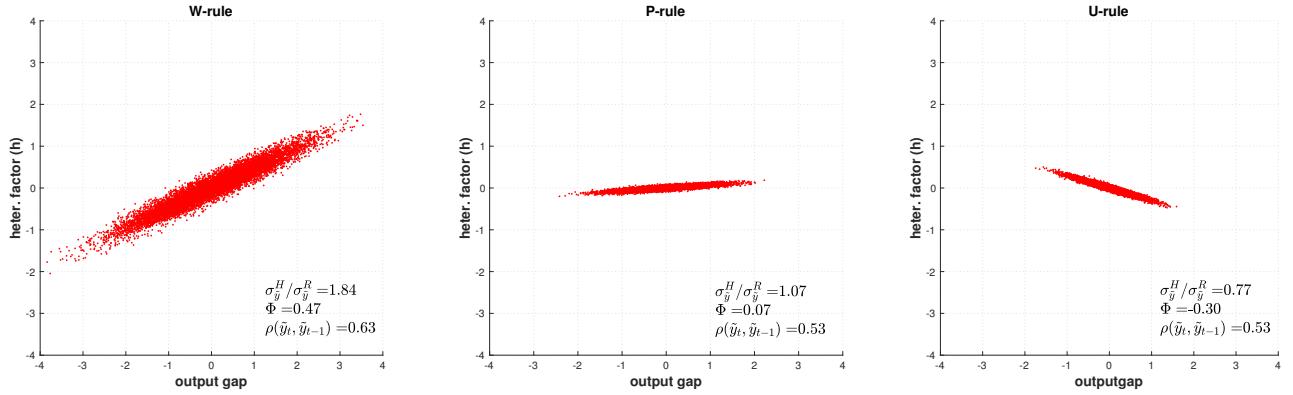
Parameter	Description	Target/Source
<i>Preferences and technology</i>		
$\sigma = 1$	Risk aversion	standard
$\varphi = 1$	Frisch elasticity of labor supply	standard
$\beta = \begin{cases} 0.9745 & W - rule \\ 0.9743 & P - rule \\ 0.9679 & U - rule \end{cases}$	Discount factor	avg. real interest rate $\bar{r} = 3\%$
$\epsilon = 10$	Elasticity of substitution among goods	profits share of 10%
$\xi = 105.63$	Price adjustment cost	avg. price duration of 1 year (Calvo)
<i>Asset markets</i>		
$\delta = 0.92$	Fraction of illiquid assets / Total Assets	Kaplan et. al. (2018)
$\psi = 0.5$	Borrowing limit	Share of constr. households $\sim 21\%$
<i>Exogenous Processes</i>		
$\rho_e = 0.9777$	Persistence of idiosyn. shock	persist. annual wage = 0.92
$\sigma_e = 0.1928$	Std. if innovation of idiosyn. shock	std. of annual wage = 0.7
$\rho_z = \rho_v = 0.5$	Persist. of pref. and mon. pol. shocks	standard
$\rho_a = 0.9$	Persist. of technology shock	standard
<i>Monetary Policy</i>		
$\phi_\pi = 1.5, \phi_y = 0.5/4$	Interest rate rule coefficients	Taylor (1993)
<i>Transfer rule of illiquid profits</i>		
(i) $\tau_a = 0, \tau_e = 1$	labor-based	Kaplan et. al. (2018)
(ii) $\tau_a = 1, \tau_e = 0$	wealth-based	Gornemann et. al. (2016)
(iii) $\tau_a = 0, \tau_e = 0$	uniform	McKay et. al. (2016)

Table 2: Patterns of Fluctuations: HANK vs TANK

	<i>W-rule</i>	<i>P-rule</i>	<i>U-rule</i>
<i>Monetary Policy</i>			
σ_y^H/σ_y^R	1.17	1.04	0.88
σ_y^T/σ_y^R	1.31	1.02	1.03
$\rho(y_t^T, y_t^H)$	0.99	0.99	0.99
<i>Preference</i>			
σ_y^H/σ_y^R	1.17	1.03	0.88
σ_y^T/σ_y^R	1.31	1.02	1.03
$\rho(y_t^T, y_t^H)$	0.99	0.99	0.99
<i>Technology</i>			
σ_y^H/σ_y^R	0.99	0.99	1.00
σ_y^T/σ_y^R	0.99	1.00	1.00
$\rho(y_t^T, y_t^H)$	0.99	0.99	0.99

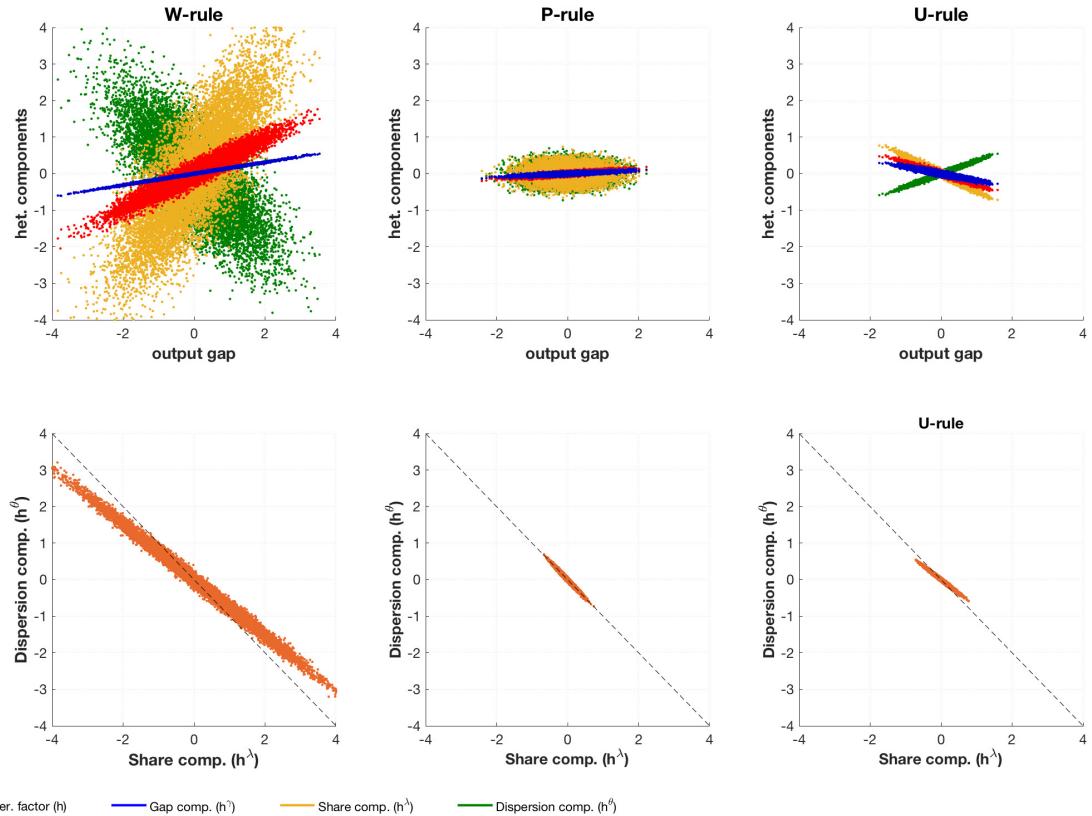
Note: The table contains summary statistics corresponding to simulations of 10,000 periods of the RANK, TANK and HANK model, in response to monetary policy, preference and technology shocks, under the three transfer rule considered (W-, P- and U-rule, in columns).

Figure 1: Output-gap and Heterogeneity Factor in HANK - Real Rate Shocks



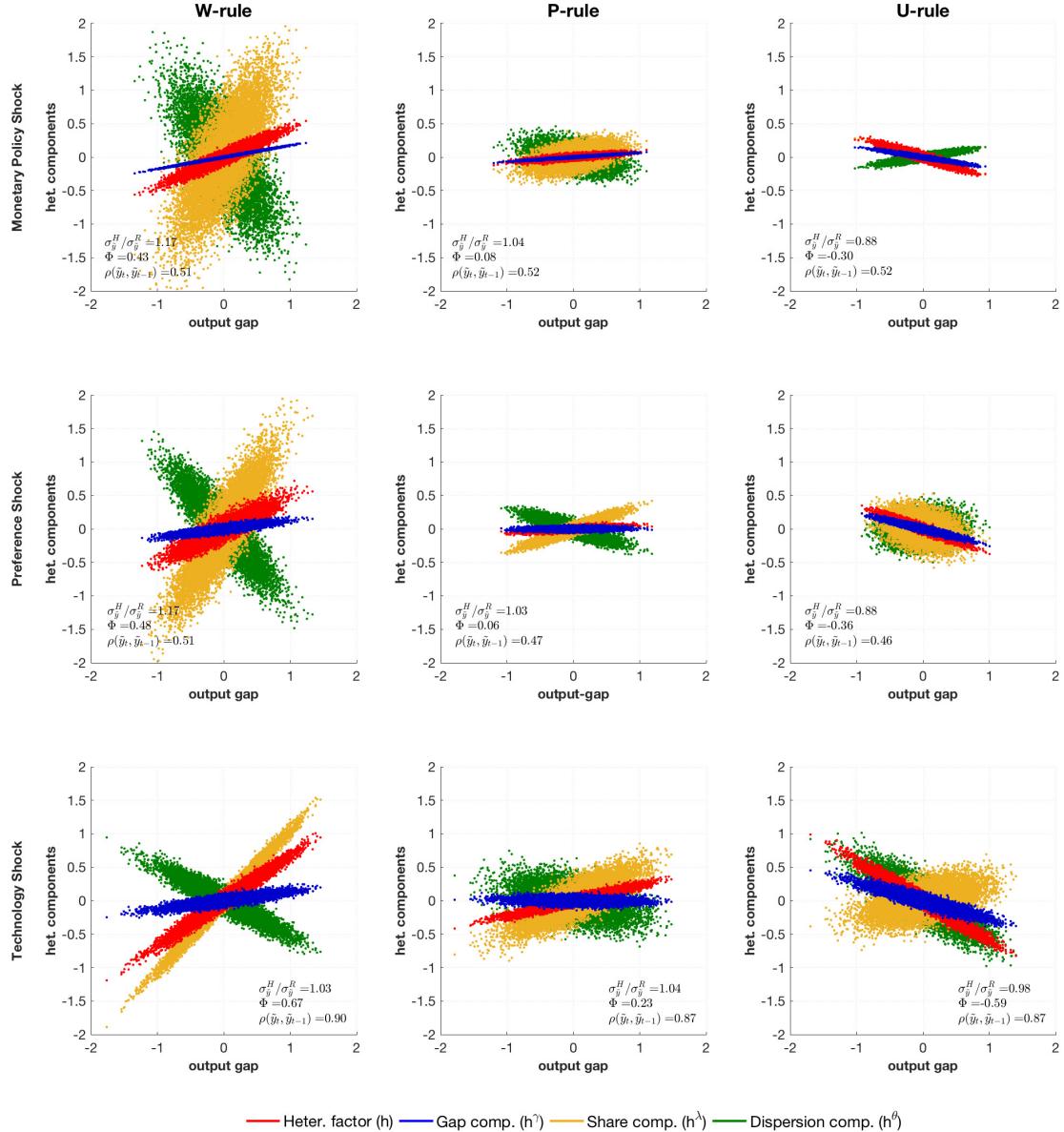
Note: The figure shows scatterplots of the output gap (horizontal axis) and the heterogeneity factor \hat{h} (vertical axis) for the HANK models under the W-rule (first column), the P-rule (second column) and the U-rule (third column), generated from a random simulation (of 10,000 periods) of the model in response to real rate shocks.

Figure 2: Output-gap and Heterogeneity Components in HANK- Real Rate Shocks



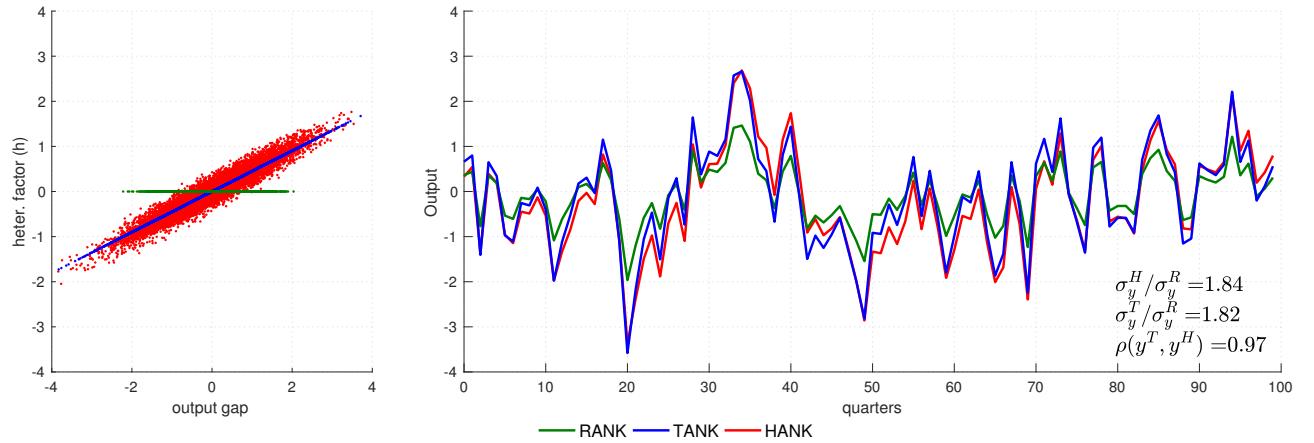
Note: The first row shows scatterplots of the output gap (horizontal axis) and the heterogeneity components (vertical axis) for the HANK models under the W-rule (first column), the P-rule (second column) and the U-rule (third column), generated from a random simulation (of 10,000 periods) of the model in response to real rate shocks. The second row shows the associated scatterplots for the share component \hat{h}^λ (horizontal axis) and the dispersions component \hat{h}^θ (vertical axis).

Figure 3: Output-gap and Heterogeneity Components in HANK - Demand and Supply Shocks



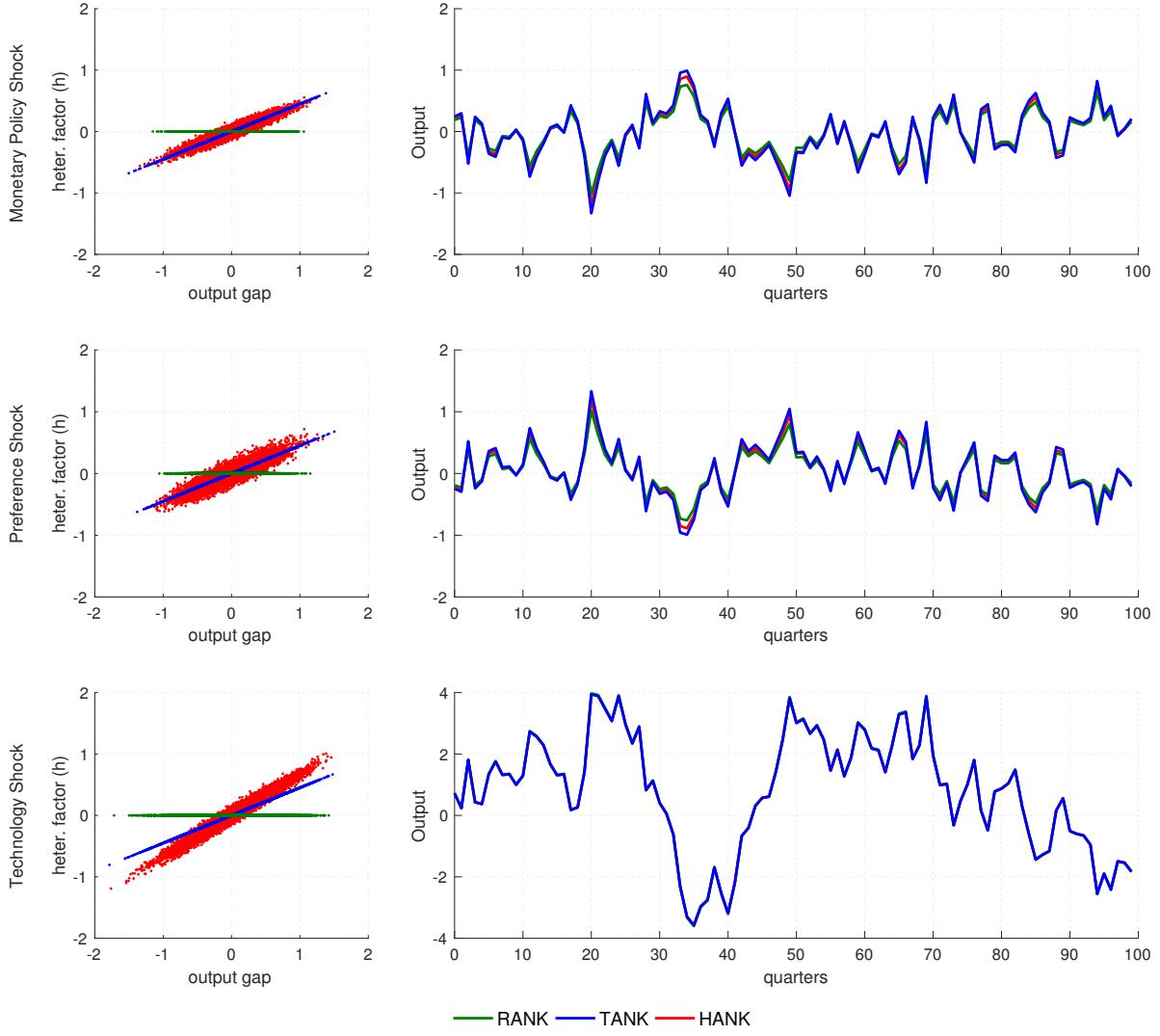
Note: The figure shows scatterplots of the output gap (horizontal axis) and the heterogeneity factor \hat{h} (vertical axis) for the HANK models under the W-rule (first column), the P-rule (second columns) and the U-rule (third column), generated from random simulations (of 10,000 periods) of the model in response to monetary policy (first row), preference (second row) and technology (third row) shocks.

Figure 4: HANK vs TANK - Real Rate Shock



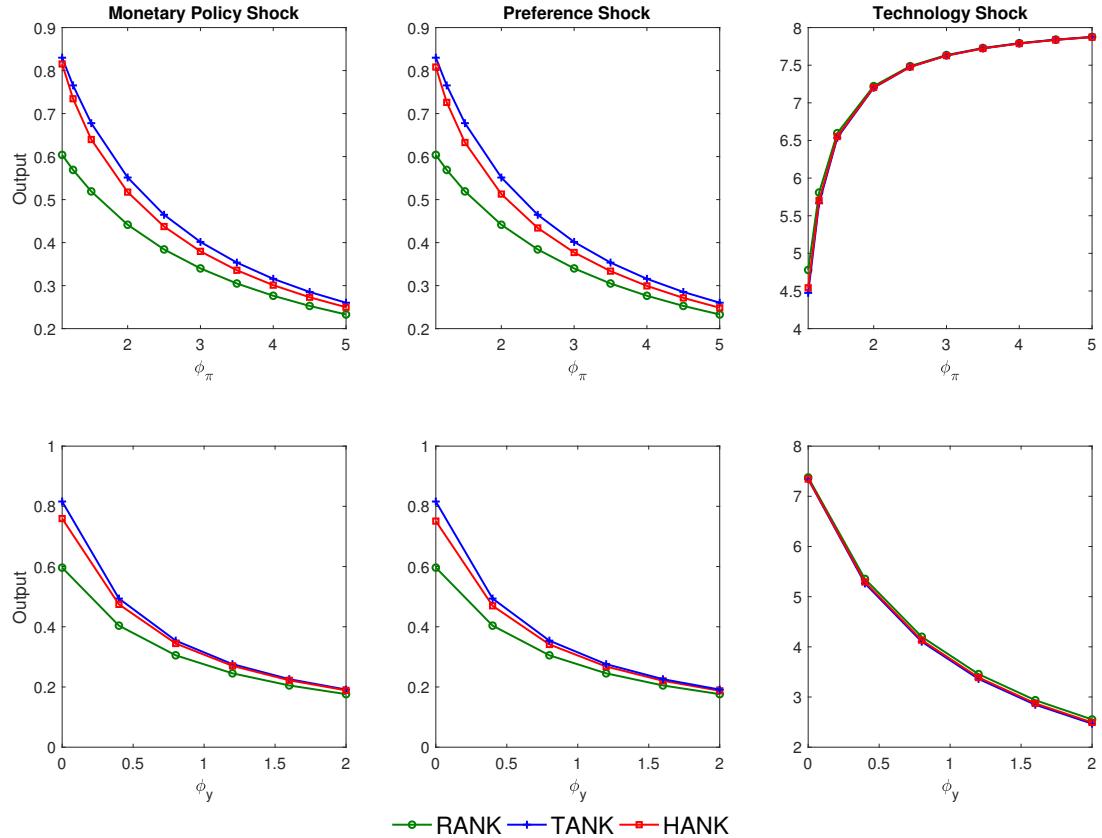
Note: The left panel shows a scatterplot of the output gap (horizontal axis) and the heterogeneity factor \hat{h} (vertical axis) for the HANK, TANK and RANK models under the W-rule, generated from a random simulation (of 10,000 periods) of the models in response to real rate shocks. The right panel shows the corresponding path of output for a subsample of 100 periods.

Figure 5: HANK vs TANK - Demand and Supply Shocks



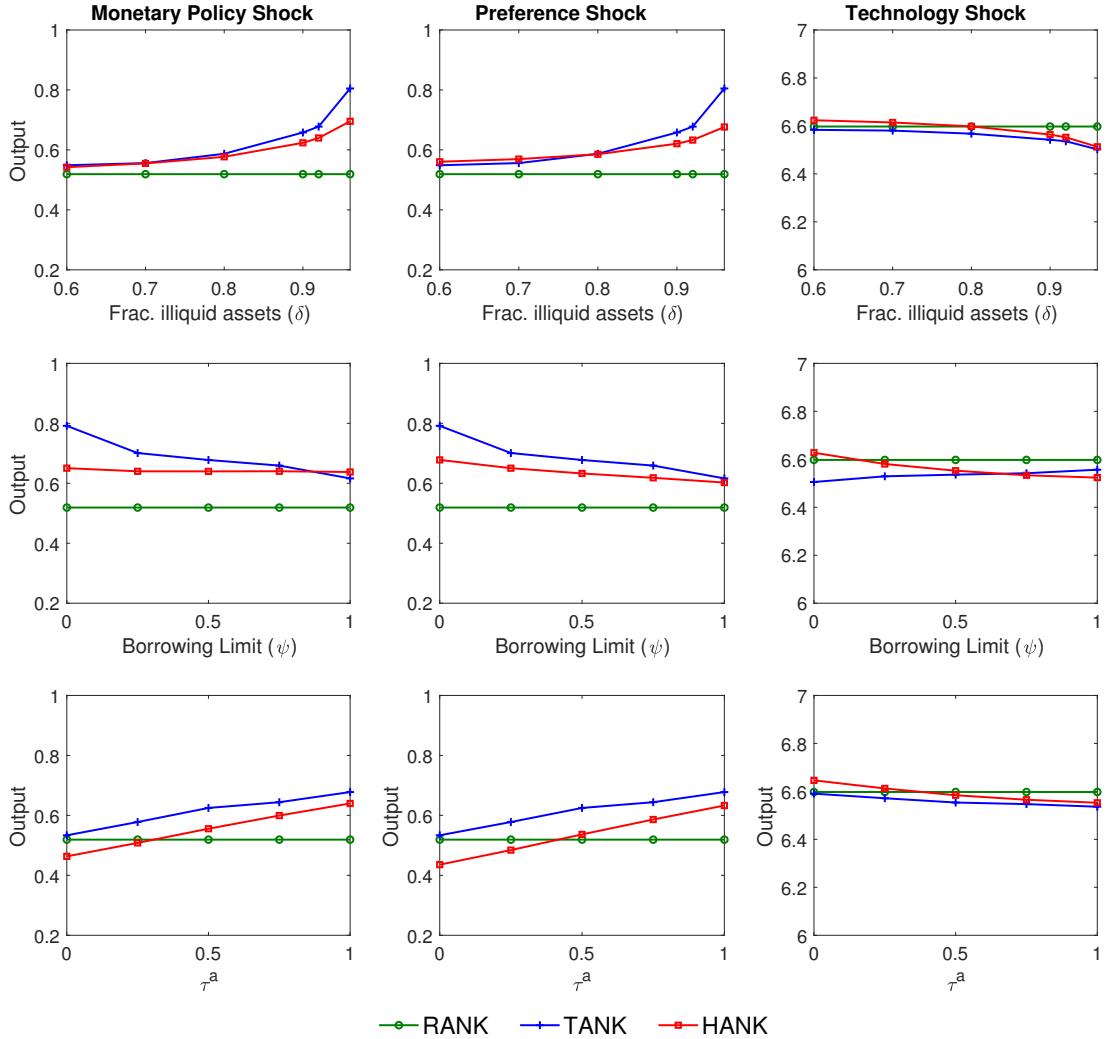
Note: The first column shows scatterplots of the output gap (horizontal axis) and the heterogeneity factor \hat{h} (vertical axis) for the HANK, TANK and RANK models generated from random simulations (of 10,000 periods) of the model in response to monetary policy (first row), preference (second row) and technology (third row) shocks. The second columns shows, for each shock, the corresponding path of output for a subsample of 100 periods.

Figure 6: Changes in Environment - Monetary Policy Rule



Note: The figure compares the cumulative response of output over 16 quarters under alternative values for the interest rule parameters ϕ_π (first row) and ϕ_y (second row), for the RANK, TANK and HANK models under the W-rule, conditional on monetary policy (first column), preference (second column) and technology (third column) shocks.

Figure 7: Changes in Environment - Fraction of Illiquid Assets, Borrowing Limit and Transfer Rule



Note: The figure compares the cumulative response of output over 16 quarters under alternative values for the fraction of illiquid assets δ (first row), the borrowing limit ψ (second row) and the transfer policy τ^a (third row), for the RANK, TANK and HANK model, conditional on monetary policy (first column), preference (second column) and technology (third column) shocks.

Appendix

A. An Organizing Framework: Derivations

A second-order approximation $\int_{s \in \mathcal{U}_t} (C_{t+1}(s)/C_{t+1|t}^U)^{-\sigma} ds$ around a symmetric allocation where $C_{t+1}(s) = C_{t+1|t}^U$ for all $s \in \mathcal{U}_t$ is given by

$$\begin{aligned} \int_{s \in \mathcal{U}_t} (C_{t+1}(s)/C_{t+1|t}^U)^{-\sigma} ds &\simeq \int_{s \in \mathcal{U}_t} \left[1 - \sigma \left(\frac{C_{t+1}(s)}{C_{t+1|t}^U} - 1 \right) + \frac{\sigma(1+\sigma)}{2} \left(\frac{C_{t+1}(s)}{C_{t+1|t}^U} - 1 \right)^2 \right] ds \\ &= (1 - \lambda_t) \left[1 + \frac{\sigma(1+\sigma)}{2} \frac{1}{1 - \lambda_t} \int_{s \in \Theta_t} \left(\frac{C_{t+1}(s)}{C_{t+1|t}^U} - 1 \right)^2 ds \right] \\ &\simeq (1 - \lambda_t) \left[1 + \frac{\sigma(1+\sigma)}{2} var_{s|t}\{c_{t+1}(s)\} \right]. \end{aligned}$$

where $var_{s|t}\{c_{t+1}(s)\} \equiv \frac{1}{1-\lambda_t} \int_{s \in \mathcal{U}_t} (c_t(s) - c_t^U)^2 ds$ and where in the last step we have used the fact that up to a second order approximation $\left(\frac{C_{t+1}(s)}{C_{t+1|t}^U} - 1 \right)^2 \simeq (c_t(s) - c_{t+1|t}^U)^2 \simeq (c_t(s) - \bar{c}_t^U)^2$ where $\bar{c}_t^U \equiv \frac{1}{1-\lambda_t} \int_{s \in \mathcal{U}_t} c_t(s) ds$.

Similarly, we can show that up to a second order approximation,

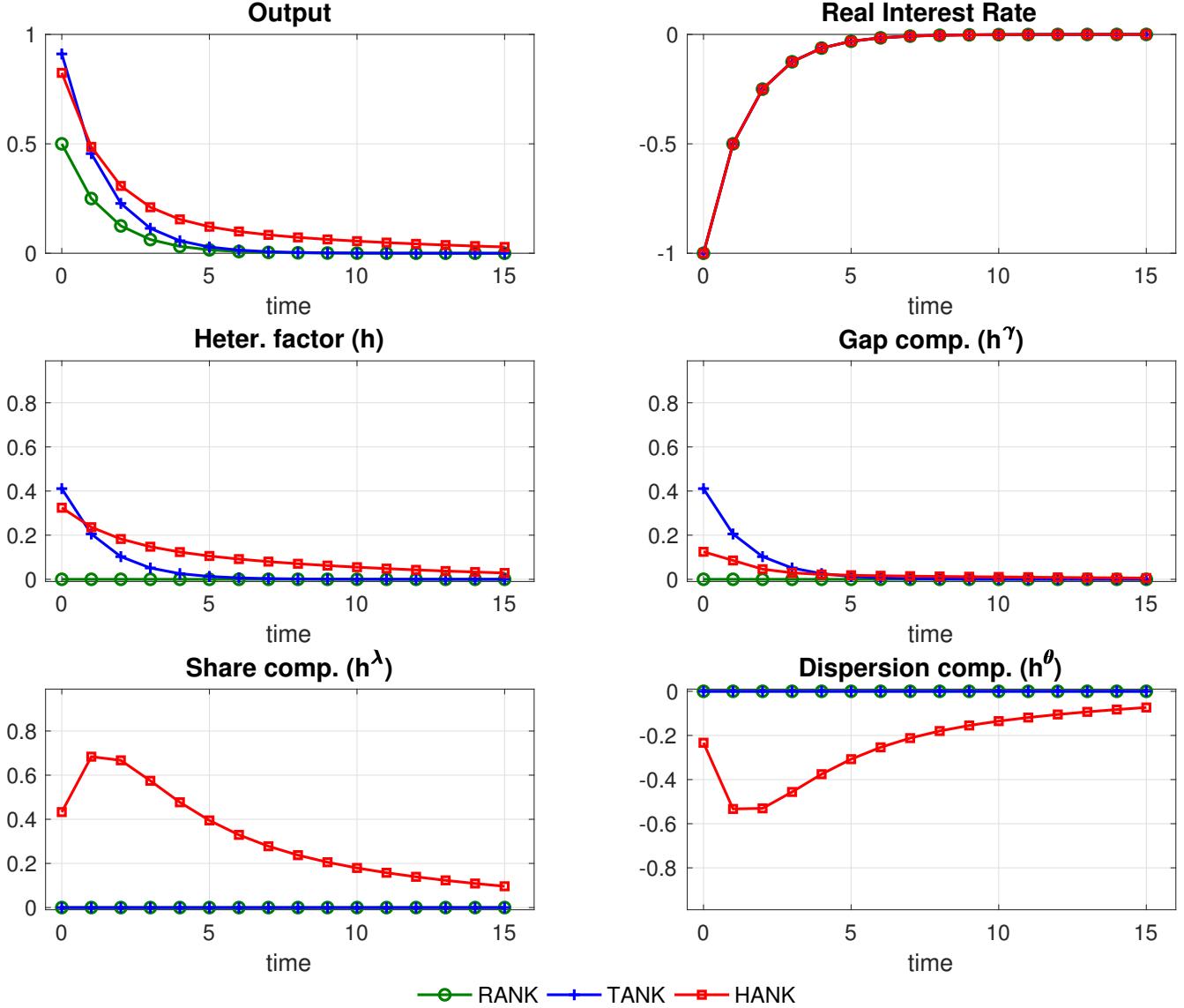
$$\int_{s \in \mathcal{U}_t} (C_t(s)/C_t^U)^{-\sigma} ds \simeq (1 - \lambda_t) \left[1 + \frac{\sigma(1+\sigma)}{2} var_{s|t}\{c_t(s)\} \right]$$

Hence,

$$\frac{\int_{s \in \mathcal{U}_t} (C_{t+1}(s)/C_{t+1|t}^U)^{-\sigma} ds}{\int_{s \in \mathcal{U}_t} (C_t(s)/C_t^U)^{-\sigma} ds} = \frac{2 + \sigma(1+\sigma) var_{s|t}\{c_{t+1}(s)\}}{2 + \sigma(1+\sigma) var_{s|t}\{c_t(s)\}}$$

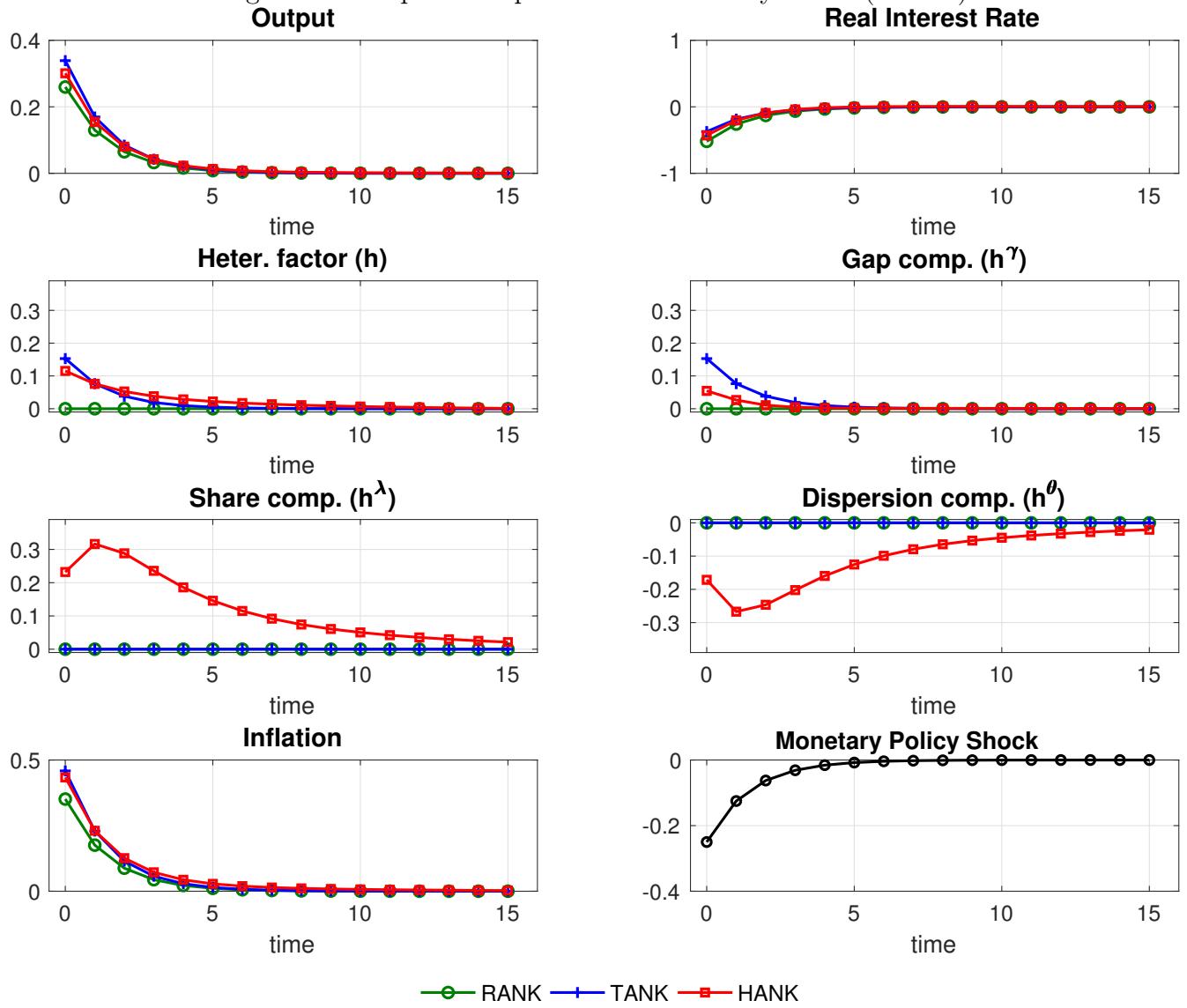
B. Impulse Responses to Aggregate Shocks

Figure B.1: Impulse Responses to a Real Rate Shock (W-rule)



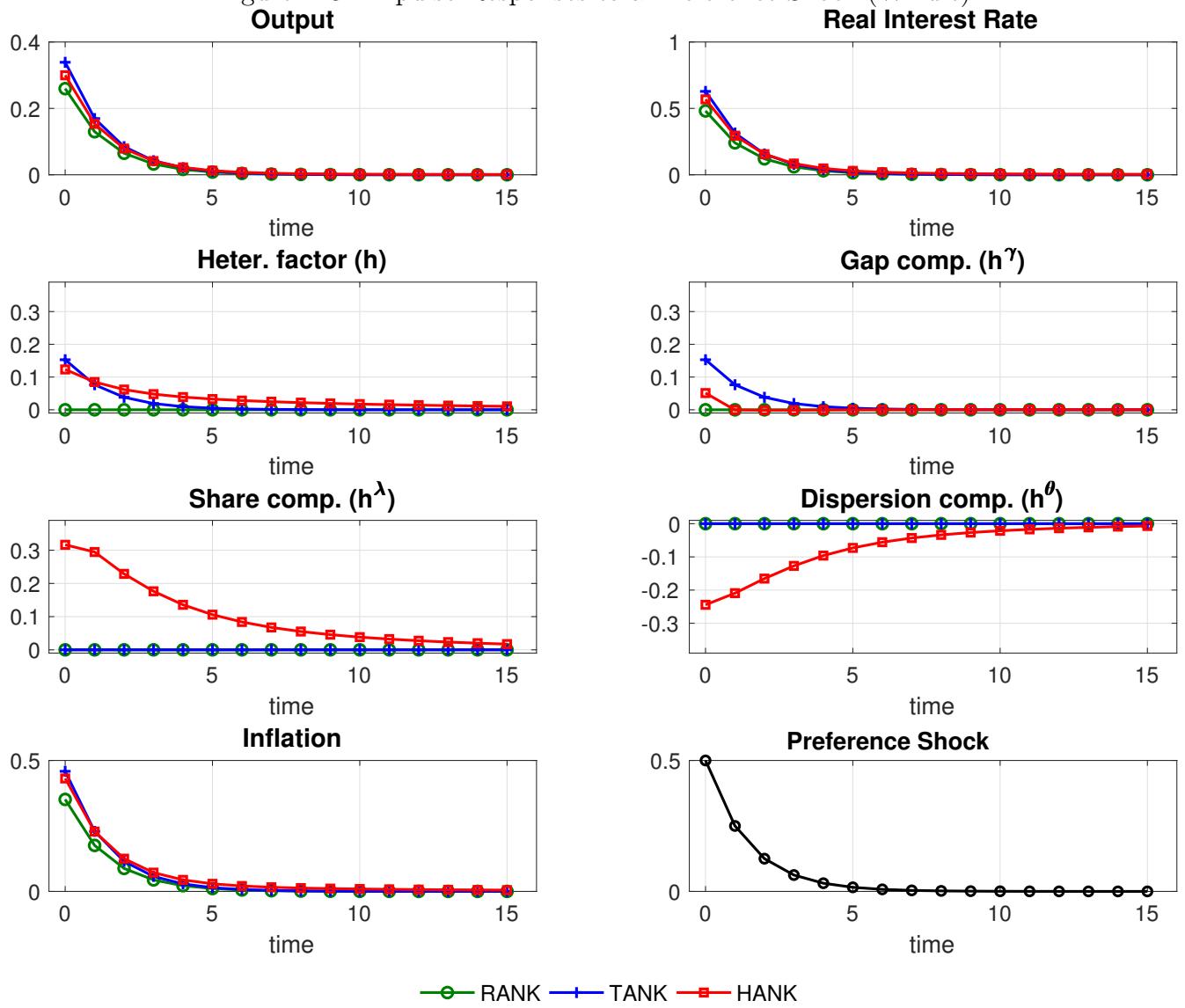
Note: The figure compares the cumulative response of output over 16 quarters under alternative values for the fraction of illiquid assets δ (first row), the borrowing limit ψ (second row) and the transfer policy (τ^e), for the RANK, TANK and HANK model, conditional on monetary policy (first column), preference (second column) and technology (third column) shocks.

Figure B.2: Impulse Responses to a Monetary Shock (W-rule)



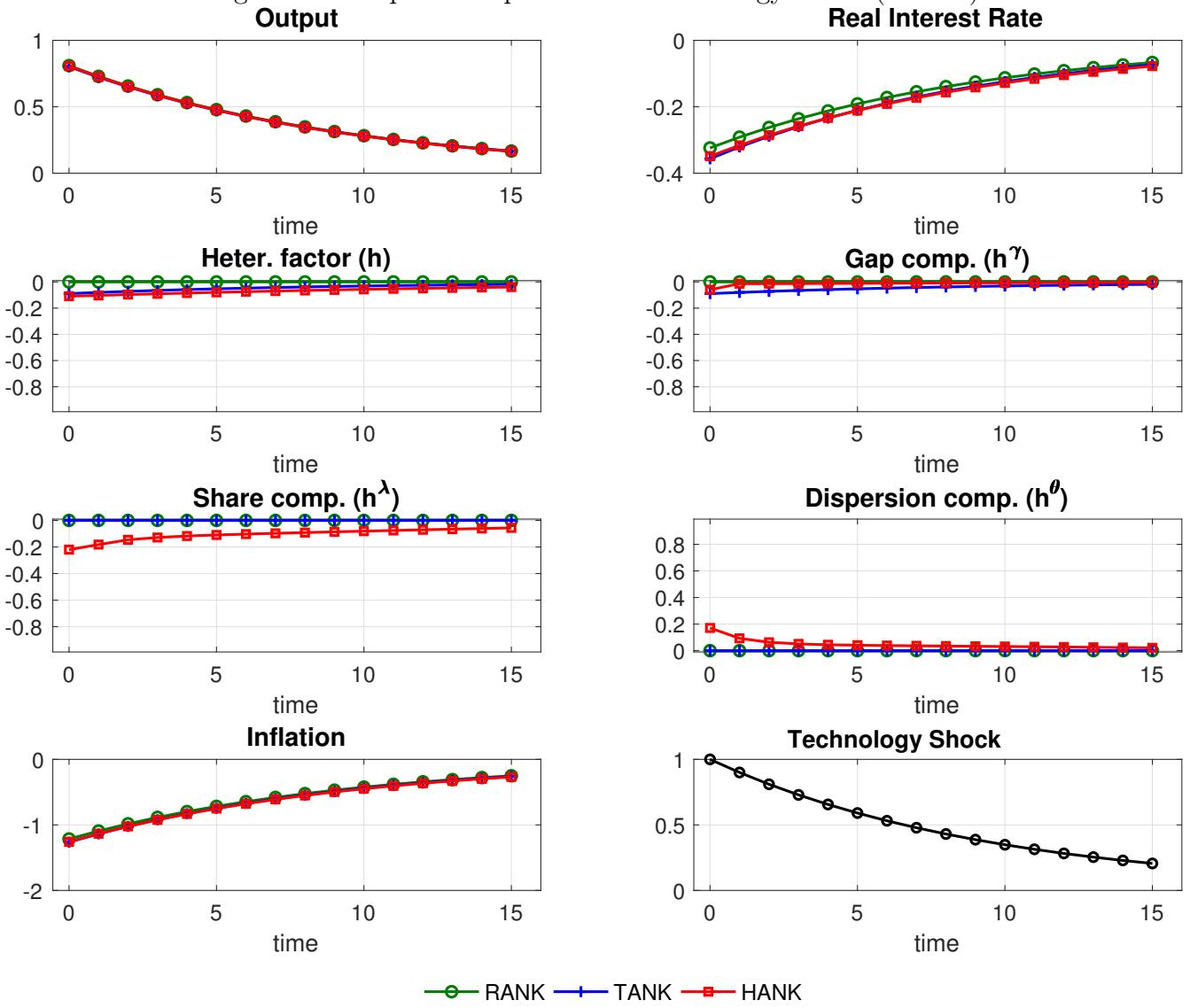
Note: The figure compares the cumulative response of output over 16 quarters under alternative values for the fraction of illiquid assets δ (first row), the borrowing limit ψ (second row) and the transfer policy (τ^e), for the RANK, TANK and HANK model, conditional on monetary policy (first column), preference (second column) and technology (third column) shocks.

Figure B.3: Impulse Responses to a Preference Shock (W-rule)



Note: The figure compares the cumulative response of output over 16 quarters under alternative values for the fraction of illiquid assets δ (first row), the borrowing limit ψ (second row) and the transfer policy (τ^e), for the RANK, TANK and HANK model, conditional on monetary policy (first column), preference (second column) and technology (third column) shocks.

Figure B.4: Impulse Responses to a Technology Shock (W-rule)



Note: The figure compares the cumulative response of output over 16 quarters under alternative values for the fraction of illiquid assets δ (first row), the borrowing limit ψ (second row) and the transfer policy (τ^e), for the RANK, TANK and HANK model, conditional on monetary policy (first column), preference (second column) and technology (third column) shocks.