

# Thoughts on OLGs

Pranay Gundam

January 23, 2024

## Introduction

As some context beforehand, overlapping generations models (OLGs) at a very basic sense with only groups of consumers looks at the trends in savings and consumption for households who live for only a finite number of time periods. From this standpoint we can then incorporate firms with production functions that determine the amount of goods available for households to consume who offer labor to firms in exchange, and also governments who can tax and offer other consumption services. Let's put the basic OLG model into math language before we discuss how we can extend the problem.

Specifically, let  $N$  many households be born in each time period  $t = 1, 2, 3, \dots$ . Each household lives for two periods; the period in which they were born and the next consecutive period in which they consume a single type of a non-storable good. Let there also be  $N$  many "initially old" households at time  $t = 1$ : this means that at time  $t = 1$  we also populate it with a group of households that act as if they were born in  $t = 0$  but are not able to consume anything in that period. The utility for an household born in period  $t$  who values consumption in a current period with the function  $u(c)$  is

$$U(c_t^t, c_{t+1}^t) = u(c_t^t) + \beta u(c_{t+1}^t).$$

Here  $\beta \in (0, 1]$  is a discount factor, and  $c_i^j$  is the consumption for a household in period  $i$  who was born in period  $j$ . Households are meant to optimize their utility with respect to the conditions that

$$\begin{aligned} c_t^t &\leq w_t^t, \\ c_{t+1}^t &\leq w_{t+1}^t. \end{aligned}$$

Here, similarly to the indexing on consumption,  $w_i^j$  is the endowment of the consumption good for a household in period  $i$  who was born in period  $j$ . In a market in which each of these households interact with each other we allow the existence of consumption bonds, or agreements between households that in exchange for one household giving another household units of the consumption good in the current period, in the next period the household who received the goods

must then give some number of consumption goods in the future period to the household that originally gave the goods. Note that this restricts households to trading bonds only with other households born in the same time period since if they were to trade with households in different periods then in the next period the previously old generation of households would no longer be alive and would not be able to fulfill their debt obligations. Furthermore, it is also logically consistent that if all the households born in a certain time period are homogenous in both utility functions and endowment then it would make no sense for them to trade bonds with each other since there is no scenario in which a trade between two households would benefit both parties.

With this setup in mind in order to solve one such scenario we can setup the following lagrangian maximization problem for a consumer at time  $t$

$$\mathcal{L}(c_t^t, c_{t+1}^t, \mu_1, \mu_2) = u(c_t^t) + \beta u(c_{t+1}^t) + \mu_1 (w_t^t - c_t^t) + \mu_2 (w_{t+1}^t - c_{t+1}^t),$$

and the following first order conditions:

$$\begin{aligned} c_t^t : u'(c_t^t) &= \mu_1, \\ c_{t+1}^t : u'(c_{t+1}^t) &= \mu_2, \\ \mu_1 : c_t^t &= w_t^t, \\ \mu_2 : c_{t+1}^t &= w_{t+1}^t. \end{aligned}$$

Which would then very clearly imply that  $c_t^t = w_t^t$  and  $c_{t+1}^t = w_{t+1}^t$  meaning each household consumes exactly their in period endowment. This is not a very exciting outcome but introducing even a little bit of complexity can lead to more interesting scenarios.

In class we discussed a few variations of the setup above:

1. Added in-period heterogeneity that incentivizes consumption bond trading.
2. Added in the concept of fiat money.
3. Added in governments that tax households and also consume some amount goods as well.
4. Added in varying population sizes over time.
5. We discussed a very basic version of adding firms with production functions

There are some things that I want to explore that we didn't explicitly discuss in class:

1. Households that live for  $N$  many periods
2. Add in stochasticity
3. Discuss a generalized sense of heterogeneity

4. Develop a more involved model with firms
5. Consider a continuum of firms and/or firms
6. What other financial instruments can we consider other than fiat money and consumption bonds
7. Add a more complicated government (basically a government that does more than just taxes and consumes goods)
8. Multiple goods

I'm sure that each of these concepts have been picked apart and discussed in fine detail since whenever the first theory on OLGs was developed but I want to work through and model some of them myself without having been influenced by any of the papers or textbooks that discuss these topics (which also leaves a lot of room for errors on my end so be aware of that) as a means to work out my - econ related - creative muscles. In addition, to preface, with the foresight of having already done a bit of work on each of these questions, in order to come to some level of a meaningful answer that relates a trend of savings or an actual equilibrium we have to impose more restrictions on the structure of the endowments (if not actual quantities then at least relationships between values in the form of inequalities). All that being said, here are my thoughts on a few of these items.

## **Multiple Goods**

## **$N$ Period Lives**

## **Generalized Heterogeneity**

## **Stochasticity**

## **Fleshed out Firm Sector**