Linear Independence in Echelon

Here's a detailed chart to understand and remember **linear independence** and **dependence** based on conditions in an echelon matrix, along with examples:

Condition	Linear Independence/Dependence	Explanation	Example
Number of pivot columns = r	Linearly **Independent**	If the rank r equals the number of columns, all vectors are independent.	$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (All vectors are independent)
$r < n$ (where n = columns)	Linearly **Dependent**	If the number of pivot columns (rank) is less than the total columns, some vectors are linear combinations of others.	$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$ (3rd column is dependent)
Zero row(s) in RREF	Linearly **Dependent** if extra columns exist	A zero row implies a missing constraint, meaning there are infinite solutions or a dependency.	$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$
Free variables exist	Linearly **Dependent**	If there are free variables, the vectors are not linearly independent.	$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix}$
Vectors form a **basis**	Linearly **Independent**	A basis is a set of vectors where all are linearly independent, spanning the vector space.	Vectors $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
Augmented matrix has no solution	Linearly **Independent** (of the augmented column)	If the augmented column doesn't create inconsistency, it means the original set is independent.	$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, augmented by $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ (Consistent and independent)

Condition	Linear Independence/Dependence	Explanation	Example
Augmented matrix is inconsistent	Linearly **Dependent** (with the augmented column)	If adding a column makes the system inconsistent, the original set was dependent.	$A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$, augmented by $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ (Inconsistent)

Key Tips:

- Check the **rank** of the matrix (number of pivot columns in REF or RREF).
- Linearly **Independent**: Rank = number of columns (no free variables, no zero rows except in augmented matrix).
- Linearly **Dependent**: Rank < number of columns (free variables exist, or zero rows imply dependency).

Would you like visualizations for better understanding?