# JEE MAIN 2024
## Paper with Solution

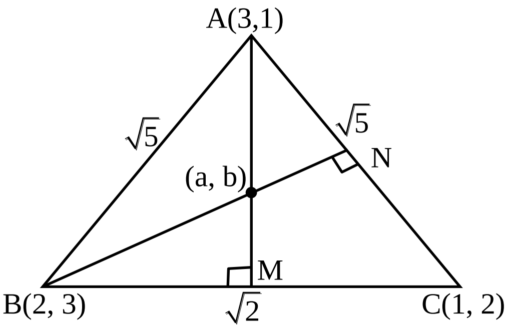
**MATHS | 27th January 2024 _ Shift-1**

## SECTION – A

**1.** If $(a,b)$ be the orthocentre of the triangle whose vertices are $(1,2),(2,3)$ and $(3,1)$, and $I_1=\int_a^b x\sin\left(4x-x^2\right)dx, I_2=\int_a^b \sin\left(4x-x^2\right)dx$, then $36\frac{I_1}{I_2}$ is equal to:

(1) 80 (2) 66 (3) 72 (4) 88

**Sol.** **(3)**




$$BC: y-3=\frac{2-3}{1-2}(x-2)$$

$$BC: y-x=1$$

$$\left.\begin{array}{l} AM: y+x=4 \\ BN: y-2x=-1 \end{array}\right\} x:\left(\frac{5}{3},\frac{7}{3}\right)$$

$$I_1=\int_a^b x\,\mathrm{Sin}\left(4x-x^2\right)dx$$

$$\downarrow K\&A$$

$$2I_1=4\int_a^b \sin\left(4x-x^2\right)dx$$

$$2I_1=4I_2$$

$$36\frac{I_1}{I_2}=36.2=72$$

**2.** Let $a_1,a_2,\ldots a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k=50$ and $\sum_{\forall k<j} a_k\cdot a_j=1100$. Then the standard deviation of $a_1,a_2,\ldots,a_{10}$ is equal to :

(1) 10 (2) $\sqrt{5}$ (3) $\sqrt{115}$ (4) 5

**Sol.** **(2)**

$$\sum_{k=1}^{10} a_k=50 \quad \Rightarrow \mu=5$$

variance = $\dfrac{\sum(a_i)^2}{n}-(\mu)^2$ ...(1)

$(a_1+a_2+\ldots+a_{10})^2 = \sum_{\substack{i\neq j\\ i,j=1,2\ldots,10}}\left(a_i^2+2(a_ia_j)\right)$

$\Rightarrow (50)^2 - 2(1100) = \Sigma a_i^{\,2}$

$\Rightarrow \Sigma a_i^{\,2} = 2500 - 2200$

$\Rightarrow \Sigma a_i^{\,2} = 300$

variance = $\dfrac{300}{10}-25$

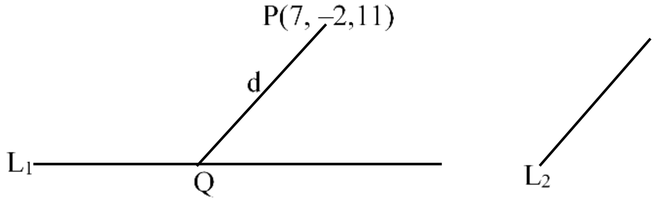
variance = $\dfrac{50}{10}=5$

S.D. = $\sqrt{5}$

**3.** The distance, of the point $(7,-2,11)$ from the line $\dfrac{x-6}{1}=\dfrac{y-4}{0}=\dfrac{z-8}{3}$ along the line $\dfrac{x-5}{2}=\dfrac{y-1}{-3}=\dfrac{z-5}{6}$, is :

(1) 12 (2) 18 (3) 21 (4) 14

**Sol.** **(4)**




$L_{PQ}:\dfrac{x-7}{2}=\dfrac{y+2}{-3}=\dfrac{z-11}{6}=\lambda$

$Q:(2\lambda+7,-2-3\lambda,11+6\lambda)$

lies on $L_1$

$\dfrac{2\lambda+7-6}{1}=\dfrac{-2-3\lambda-4}{0}=\dfrac{11+6\lambda-8}{3}$

$\Rightarrow 3\lambda + 6 = 0 \Rightarrow \lambda = -2$

Hence

Q = (3, 4, –1)

$d(PQ)=\sqrt{16+36+144}=\sqrt{196}=14$

**4.** Let $x = x(t)$ and $y = y(t)$ be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, $a, b \in \mathbf{R}$. Given that $x(0) = 2; y(0) = 1$ and $3y(1) = 2x(1)$, the value of $t$, for which $x(t) = y(t)$, is :

**(1)** $\log_{\frac{2}{3}} 2$ (2) $\log_4 3$ (3) $\log_{\frac{4}{3}} 2$ (4) $\log_3 4$

**Sol.** **(3)**

$\frac{dx}{dt} + ax = 0$ & $\frac{dy}{dt} + by = 0$

| | |
|---|---|
| $\ln x = -at + \lambda$ | $\ln y = -bt + m$ |
| $x = k_1 e^{-at}$ | $y = K_2 e^{-bt}$ |
| $x(0) = 2$ | $y(0) = 1$ |
| $\Rightarrow K_1 = 2$ | $\Rightarrow K_2 = 1$ |

$3y(1) = 2x(1)$

$3e^{-b} = 2.2e^{-a}$

$e^{a-b} = \frac{4}{3}$

$x(t) = y(t)$

$2e^{-at} = e^{-bt}$

$e^{(a-b)t} = 2$

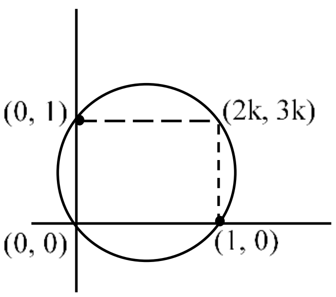
$\left(\frac{4}{3}\right)^t = 2$

$t = \log_{\frac{4}{3}} 2$

**5.** Four distinct points $(2k, 3k), (1, 0), (0, 1)$ and $(0, 0)$ lie on a circle for $k$ equal to : Options

(1) $\frac{1}{13}$ (2) $\frac{2}{13}$ (3) $\frac{5}{13}$ (4) $\frac{3}{13}$

**Sol.** **(3)**




$(x - 1)x + y(y - 1) = 0$

$x^2 + y^2 - x - y = 0$

Now (2K, 3K) lies

$4k^2 + 9k^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

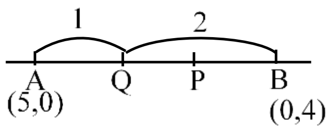
$k = 0 \mid k = \dfrac{5}{13}$

**6.** The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines $L_1$ and $L_2$ passing through the origin. The tangent of an angle between the lines $L_1$ and $L_2$ is :

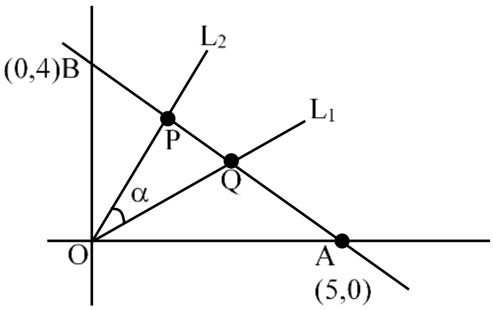
(1) $\dfrac{8}{5}$ (2) $\dfrac{25}{41}$ (3) $\dfrac{2}{5}$ (4) $\dfrac{30}{41}$

**Sol.** **(4)**

Q: $\left(\dfrac{10}{3}, \dfrac{4}{3}\right)$




P : $\left(\dfrac{5}{3}, \dfrac{8}{3}\right)$




$m_{OP} = \dfrac{8}{5}, M_{OQ} = \dfrac{2}{5}$

$\tan\alpha = \left|\dfrac{\frac{8}{5} - \frac{2}{5}}{1 + \frac{16}{25}}\right|$

$= \left|\dfrac{6.5}{41}\right|$

$\tan\alpha = \dfrac{30}{41}$

**7.** If $a = \lim\limits_{x\to 0}\dfrac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$ and $b = \lim\limits_{x\to 0}\dfrac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$, then the value of $ab^3$ is :

(1) 30 (2) 36 (3) 25 (4) 32

**Sol.** **(4)**

$a = \lim\limits_{x\to 0}\dfrac{1+\sqrt{1+x^4} - 2}{x^4} \cdot \dfrac{1}{\sqrt{1+\sqrt{1+x_4}} + \sqrt{2}}$

$a = \lim\limits_{x\to 0}\dfrac{x^4}{x^4} \cdot \dfrac{1}{\sqrt{1+x^4}+1} \cdot \dfrac{1}{\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}} = \dfrac{1}{2} \cdot \dfrac{1}{2\sqrt{2}} \Rightarrow a = \dfrac{1}{4\sqrt{2}}$

$$b=\lim_{x\to 0}\frac{\left(\frac{S^2x}{x^2}\right)\cdot x^2}{\frac{(2-1-cx)\cdot x^2}{x^2}}\cdot(\sqrt{2}+\sqrt{1+\cos x})$$

$$b=\frac{1}{\frac{1}{2}}\cdot 2\sqrt{2}\Rightarrow b=4\sqrt{2}$$

$$\text{Now } ab^3=\frac{1}{(4\sqrt{2}}(4\sqrt{2})^3=32$$

**8.** If the shortest distance between the lines $\frac{x-4}{1}=\frac{y+1}{2}=\frac{z}{-3}$ and $\frac{x-\lambda}{2}=\frac{y+1}{4}=\frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of $\lambda$ is :

(1) 10 (2) 8 (3) 5 (4) 7

**Sol.** **(2)**

$$SD=\left|\frac{(\bar{b}-\bar{a})\cdot(\bar{p}\times\bar{q})}{|\bar{p}\times\bar{q}|}\right|=\frac{6}{\sqrt{5}}$$

$$\Rightarrow\left|\frac{((\lambda-4)\hat{i}+0\hat{j}+2\hat{k})\cdot(2\hat{i}-\hat{j}+0\hat{k})}{\sqrt{4+1}}\right|=\frac{6}{\sqrt{5}}$$

$$\bar{p}\times\bar{q}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&2&-3\\2&4&-5\end{vmatrix}=\langle 2,-1,0\rangle$$

$$\left|\frac{2\lambda-8}{5^2}\right|=\frac{6}{88}\Rightarrow\left|\frac{2\lambda-8}{\sqrt{5}}\right|=\frac{6}{\sqrt{5}}$$

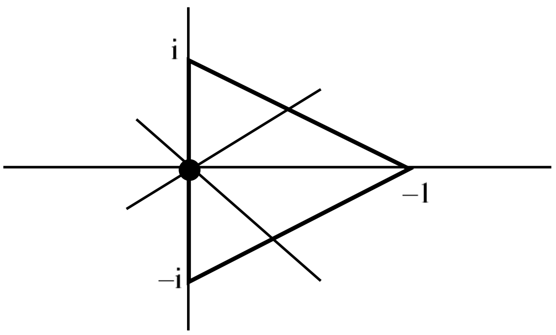
$|\lambda-4|=1 \qquad \Rightarrow l = 7$ or $1$

sum of $\lambda = 7 + 1 = 8$

**9.** If $S=\{z\in C:|z-i|=|z+i|=|z-1|\}$, then, $n(S)$ is :

(1) 2 (2) 0 (3) 3 (4) 1

**Sol.** **(4)**



**10.** Consider the function.

$$f(x)=\begin{cases}\dfrac{a\left(7x-12-x^2\right)}{b\left|x^2-7x+12\right|} & ,x<3\\ 2^{\frac{\sin(x-3)}{x-[x]}} & ,x>3\\ b & ,x=3\end{cases}$$

where $[x]$ denotes the greatest integer less than or equal to $x$. If $S$ denotes the set of all ordered pairs $(a,b)$ such that $f(x)$ is continuous at $x=3$, then the number of elements in $S$ is :

(1) 1 (2) 4 (3) 2 (4) Infinitely many

**Sol.** **(1)**

$$f\left(3^+\right)=\lim_{x\to3^+}2\frac{\sin(x-3)}{x-\left(3^+\right)}=2^1=2$$

f(3) = b

$$f\left(3^-\right)=\lim_{x\to3^-}\frac{-a(x-3)(x-4)}{b\,|(x-3)(x-4)|}$$

$$=\lim_{x\to3^-}-\frac{a}{b}\frac{(x-5)(x-4)}{(x-1)(x-4)}=\frac{-a}{b}$$

f(x) is can't at x = 3

$f(3^-) = f(3) = f(3^+)$

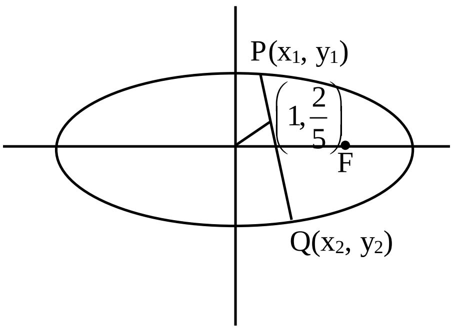
$2 = b = \dfrac{-a}{b} \Rightarrow b = 2,\ a = -4$

only one ordered pair

**11.** The length of the chord of the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$, whose mid point is $\left(1,\dfrac{2}{5}\right)$, is equal to :

(1) $\dfrac{\sqrt{1541}}{5}$ (2) $\dfrac{\sqrt{1691}}{5}$ (3) $\dfrac{\sqrt{1741}}{5}$ (4) $\dfrac{\sqrt{2009}}{5}$

**Sol.** **(2)**



$\therefore$ Mid point of chord is given as $\left(1, \frac{2}{5}\right)$

$\Rightarrow$ equation of chord is

$T = S_1$

$$\Rightarrow \frac{x.1}{25} + \frac{y.\frac{2}{5}}{16} = \frac{1}{25} + \frac{4}{16 \times 25}$$

$$\Rightarrow \frac{x}{5} + \frac{y}{4} = \frac{1}{4} \text{ (equation of chord)}$$

Now solving this with equation of ellipse

$\Rightarrow$ Coordinate of point $P = \left(x_1, 2 - \frac{8x_1}{5}\right)$

and Coordinate of point $\frac{x^2}{25} + \frac{1}{16}\left(2 - \frac{8x}{5}\right)^2 = 1$

$$\therefore \quad PQ = \sqrt{(x_1 - x_2)^2 + \frac{64}{25}(x_1 - x_2)^2} = |x_1 - x_2|\sqrt{\frac{89}{25}}$$

Now for $|x_1 - x_2|$

$$\frac{x^2}{25} + \frac{1}{16}\left(2 - \frac{8x}{5}\right)^2 = 1$$

$\Rightarrow \quad 4x^2 - 8x - 15 = 0$

$\Rightarrow \quad |x_1 - x_2| = \sqrt{19}$

There fore, $PQ = \sqrt{19} \times \sqrt{\frac{89}{25}} = \frac{\sqrt{1691}}{5}$

**12.** If A denotes the sum of all the coefficients in the expansion of $\left(1 - 3x + 10x^2\right)^n$ and B denotes the sum of all the coefficients in the expansion of $\left(1 + x^2\right)^n$, then :

(1) $A = B^3$ (2) $3A = B$ (3) $A = 3B$ (4) $B = A^3$

**Sol.** **(1)**

$y = (1 - 3x + 10x^2)^n$

$A = (1 - 3 + 10)^n = 8^n$

S"y $B = 2^n$

then $A = B^3$

**13.** The number of common terms in the progressions $4,9,14,19,\ldots\ldots$, up to $25^{th}$ term and $3,6,9,12,\ldots\ldots$, up to $37^{th}$ term is :

(1) 5 (2) 8 (3) 7 (4) 9

**Sol.** **(3)**

4, 9, 14, 19, ...,, 124 $\left.\begin{array}{l}\rightarrow CD=5\\ \rightarrow CD=3\end{array}\right\} LCM=15$

3, 6, 9, 12,..., 111,

9, 24, .... ,

$9(n-1).15 \le 111$

$(n-1)15 \le 102$

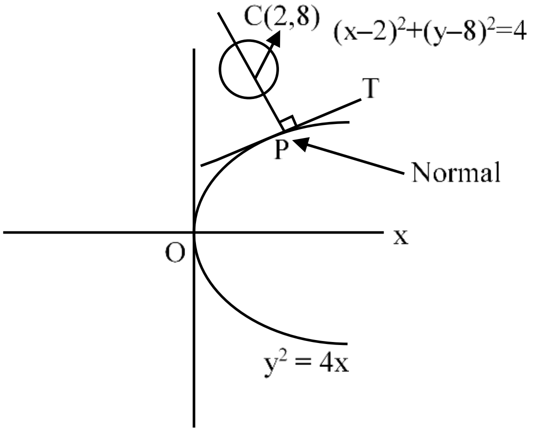
$(n-1)15 = 90$

$n-1 \Rightarrow n = 7$

**14.** If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is $d$, then $d^2$ is equal to :

(1) 16 (2) 20 (3) 24 (4) 36

**Sol.** **(2)**




r = 2

(minimum distance will be along the common normal)

N and to parabola

$y = mx - 2am - am^3$

$y = mx - 2am - m^3$

$\Rightarrow 8 = 2m - 2m - m^3$

$\Rightarrow m = -2$

$\Rightarrow N : y = -2x + 4 + 8$

$\Rightarrow y = -2x + 12$ for 'p' Solve 'N' with parabola

$y = 2x + 12$

$4x = (-2x + 12)^2$

$\Rightarrow 4x = 4x^2 + 144 - 48x$

$\Rightarrow 4x = 52x + 144 = 0$

$\Rightarrow x^2 - 13x + 36 = 0$

$\Rightarrow x = \dfrac{13 \pm \sqrt{169-144}}{2}$

$\Rightarrow x = \dfrac{13 \pm 5}{2}, 9$

x = 9 , 4

y = – 6, 4 in Ist quadrant

P(4, 4) C(2, 8)

$d = |PC| = \sqrt{4+16} = \sqrt{20}$

$d^2 = 20$

**15.** Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Given below are two statements :

Statement I : $f(-x)$ is the inverse of the matrix $f(x)$.

Statement II : $f(x)f(y) = f(x+y)$.

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

**Sol.** **(3)**

$$f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

S-I $$f(-x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$f^{-1}(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$$

$$f^{-1}(x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} = f(-x)$$

S-I is True

S-II $\quad f(x)f(y)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$

$$=\begin{bmatrix}\cos(x+y) & -\sin(x+y) & 0\\ \sin(x+y) & \cos(x+y) & 0\\ 0 & 0 & 1\end{bmatrix}=f(x+y)$$

S-II is True

**16.** Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k},\vec{b}=3(\hat{i}-\hat{j}+\hat{k})$. Let $\vec{c}$ be the vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$ Then $\vec{a}\cdot((\vec{c}\times\vec{b})-\vec{b}-\vec{c})$ is equal to :

(1) 20 (2) 24 (3) 36 (4) 32

**Sol.** **(2)**

$\vec{a}=\hat{i}+\hat{j}+\hat{k}$

$\vec{b}=3(\hat{i}-\hat{j}+\hat{k})$

then $=\vec{a}\cdot(\vec{c}\times\vec{b})-\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}$

$=[\vec{a}\ \vec{c}\ \vec{b}]-0-3=\vec{b}\cdot\vec{b}-3$

$=(\vec{b})^2-3=27-3=24$

**17.** The function $f:\mathbf{N}-\{1\}\to\mathbf{N}$; defined by $f(n)=$ the highest prime factor of $n$, is :

(1) neither one-one nor onto (2) one-one only
(3) both one-one and onto (4) onto only

**Sol.** **(1)**

$f: N-\{1\}\to N$

f(n) = the highest prime factor of n

| | |
|---|---|
| f(2) = 2 | f(2) = f(4) $(2\neq 4)$ |
| f(3) = 3 | $\Rightarrow$ Not one-one |
| f(4) = 2 | and a $\nexists$ any $n\in$ co-domain |
| | such that f(n) = 1(not onto) |

$\Rightarrow$ Not onto

$\Rightarrow$ Neither one – one nor onto

**18.** Let $S=\{1,2,3,\ldots,10\}$. Suppose M is the set of all the subsets of S, then the relation $R=\{(A,B):A\cap B\neq\phi;A,B\in M\}$ is :

(1) reflexive only (2) symmetric and reflexive only
(3) symmetric and transitive only (4) symmetric only

**Sol.** **(4)**

S = { 1, 2, 3, ……………10}

M = P(S)

A, B $\in$ M

$R = \{(A, B) : A \cap B \neq \phi \; A, B \in M\}$

if $\quad$ A = $\phi$ = B then $A \cap B = \phi$

$\phi \not{R} \phi \Rightarrow$ not Reflexive

Symmetric $\quad$ Yes

Not Transitive

A = {1, 2 } $\quad$ B = {2, 3, 4} $\quad$ C = {4, 5, 6}

$\Rightarrow$ ARB, BRC

But $A \not{R} C$

**19.** If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}} dx = a + b\sqrt{2} + c\sqrt{3}$, where $a, b, c$ are rational numbers, then $2a + 3b - 4c$ is equal to :

(1) 4 $\quad$ (2) 7 $\quad$ (3) 10 $\quad$ (4) 8

**Sol.** **(4)**

$$\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}} dx = a + b\sqrt{x} + c\sqrt{3}$$

Rationalize

$$\int_0^1 \left(\frac{\sqrt{3+x}-\sqrt{1+x}}{2}\right) dx$$

$$= \frac{1}{2}\left[\frac{2}{3}(3+x)^{\frac{3}{2}} - \frac{2}{3}(1+x)^{3/2}\right]_0^1$$

$$= \frac{1}{3}\left[(8-2\sqrt{2}) - (3\sqrt{3}-1)\right]$$

$$= \frac{1}{3}[8 - 2\sqrt{2} - 3\sqrt{2} + 1]$$

$$= \frac{1}{3}[9 - 2\sqrt{2} - 3\sqrt{3}]$$

$$= 3 - \frac{2}{3}\sqrt{2} - \sqrt{3}$$

$$= a + b\sqrt{2} + c\sqrt{3}$$

$$a = 3 \;\left|\; b = -\frac{2}{3} \;\right|\; c = -1$$

$$2(3) + 3\left(-\frac{2}{3}\right) - 4(-1)$$

$\Rightarrow \quad 6 - 2 + 4$

$\Rightarrow \quad 8$

**20.** ${}^{n-1}C_r = \left(k^2-8\right){}^nC_{r+1}$ if and only if :

(1) $2\sqrt{3} < k \le 3\sqrt{2}$ (2) $2\sqrt{2} < k \le 3$ (3) $2\sqrt{3} < k < 3\sqrt{3}$ (4) $2\sqrt{2} < k < 2\sqrt{3}$

**Sol.** **(2)**

$${}^{n-1}C_r = \left(k^2-8\right){}^nC_{r+1}$$

$$\Rightarrow {}^{n-1}C_r = \left(k^2-8\right)\cdot\frac{n}{r+1}\cdot C_r$$

$$\Rightarrow k^2-8 = \frac{r+1}{n}$$

$$0 < \frac{r+1}{n} \le 1$$

$$0 < k^2-8 \le 1$$

$$\Rightarrow 8 < k^2 \le 9$$

$$\Rightarrow \left[-3,-2\sqrt{2}\right)\cup\left(2\sqrt{2},3\right]$$

$$\Rightarrow 2\sqrt{2} < k \le 3$$

## SECTION – B

**21.** The least positive integral value of $\alpha$, for which the angle between the vectors $\alpha\hat{i}-2\hat{j}+2\hat{k}$ and $\alpha\hat{i}+2\alpha\hat{j}-2\hat{k}$ is acute, is

**Sol.** **5**

$$\vec{a} = \alpha\hat{i}-2\hat{j}+2k$$

$$\vec{b} = \alpha\hat{i}+2\alpha\hat{j}-2k$$

cos $\theta$ should positive for acute

$$\cos\theta = \left|\frac{\alpha^2-4\alpha-4}{\sqrt{\alpha^2+8}\sqrt{5\alpha^2+4}}\right| \ge 0$$

$$= \left|\alpha^2-4\alpha-4\right| \ge 0$$

$$\Rightarrow (\alpha-2)^2-8 \ge 0$$

$$\Rightarrow (\alpha-2)^2 \ge 8$$

$$\Rightarrow \left|(\alpha-2)\right| \ge 2\sqrt{2}$$

$$(\alpha-2) \in (-\infty-2\sqrt{2}]\cup[2\sqrt{2},\infty]$$

$$\Rightarrow (-\infty,-2\sqrt{2}+2]\cup[2+2\sqrt{2},\infty)$$

$$2(1+\sqrt{2})$$

$\alpha$ = 2(1 + 1.141) = 2(2.414)

= 4.828

$\Rightarrow$ least positive $\alpha$ is = 5

**22.** A fair die is tossed repeatedly until a six is obtained. Let $X$ denote the number of tosses required and let $a = P(X=3), b = P(X \ge 3)$ and $c = P(X \ge 6 | X > 3)$. Then $\frac{b+c}{a}$ is equal to

**Sol.** **12**

$P(A) = 1/6$

$P(x=3) = 5/6 \times 5/6 \times 1/6 \Rightarrow \frac{25}{216} = a$

$P(x \ge 3) = \left(\frac{5}{6}\right)^2 \frac{1}{6} + \left(\frac{5}{6}\right)^3 (1/6) + \cdots$

$= \frac{\frac{25}{216}}{1-5/6} \Rightarrow 25/36 = b$

$P(x \ge 6 / x > 3) = \frac{P(x \ge 6)}{P(x > 3)}$

$= \frac{\left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^6 \cdot \left(\frac{1}{6}\right) + \cdots}{\left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right) + \cdots}$

$= \frac{\left(\frac{5}{6}\right)^5 \left(\frac{1}{6}\right)}{\left(1-\frac{5}{6}\right)} \times \frac{\left(1-\frac{5}{6}\right)}{\left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right)}$

$= (5/6)^2 = \frac{25}{36} = C$

$\frac{\left(\frac{50}{36}\right)}{25/216} = 12$

**23.** If the solution of the differential equation $(2x+3y-2)dx + (4x+6y-7)dy = 0, y(0) = 3$, is $\alpha x + \beta y + 3\log_e |2x+3y-\gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to

**Sol.** **29**

$(2x + 3y - 2)dx + (4x + 6y - 7)dy = 0$

$\frac{dy}{dx} = -\frac{(2x+3y-2)}{(4x+6y-7)}$

$\frac{dy}{dx} = \frac{-(2x+3y-2)}{(2)(2x+3y)-7}$

$$\left|\begin{array}{l} 2x+3y=v \\ 2+3\cdot\frac{dy}{dx}=\frac{dv}{dx} \end{array}\right.$$

$$\Rightarrow\left(\frac{dv}{dx}-2\right)\frac{1}{3}=\frac{-(v-2)}{2v-7}$$

$$\Rightarrow\frac{dv}{dx}-2=\frac{-3v+6}{2v-7}$$

$$\Rightarrow\quad\frac{dv}{dx}=\frac{-3v+6}{2v-7}+2=\frac{-3v+6+4v-14}{2v-7}$$

$$\Rightarrow\frac{dv}{dx}=\frac{v-8}{2v-7}$$

$$\Rightarrow\left(\frac{2v-7}{v-8}\right)dv=dx$$

$$\Rightarrow\left(\frac{(v-8)+v+1}{(v-8)}\right)dv=dx$$

$$\Rightarrow\left(1+\frac{v+1}{v-8}\right)dv=dx$$

$$\Rightarrow dv+\left(\frac{v-8+9}{v-8}\right)dv=dx$$

$$\Rightarrow dv+dv+\left(\frac{9}{v-8}\right)dv=dx$$

$$\Rightarrow\int 2dx+\int\frac{9}{v-8}dx=\int dx$$

$\Rightarrow 2v + 9\log|v-8| = x + c$

$\Rightarrow 2(2x + 3y) + 9\log|2x + 3y - 8| = x + c$

$y(0) = 3$

$2(0 + 9) + 9\log|9 - 8| = 0 + C \Rightarrow C = 18$

$\Rightarrow 9(2x + 3y) + 9\log|2x - 3y - 8| = x + 18$

$\Rightarrow 4x + 6y + 9\log|2x + 3y - 8| = x + 18$

$\Rightarrow 3x + 6y + 9\log|2x + 3y - 8| = 18$

$\Rightarrow x + 2y + 3\log|2x + 3y - 8| = 6$

$\Rightarrow \alpha x + \beta y + 3\log|2x + 3y - \gamma| = 6$

$\alpha = 1, \beta = 2, \gamma = 8$

$1 + 4 + 24 = 29$

**24.** If $\alpha$ satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, A, B, C $\geq 0$, then $5(3A - 2B - C)$ is equal to ______.

**Sol.** **5**

$\alpha^2 + \alpha + 1 = 0 \langle^{\omega}_{\omega^2} |\ \omega^3 = 1$ $(x^2 + x + 1 = 0)$ $0\langle^{\omega}_{\omega^2} |\ \omega^3 = 1$ let $\alpha = \omega$

$(1+\alpha)^7 = 1 + {}^7c_1\alpha + {}^7c_2\alpha^2 + {}^7c_3\alpha^3 + {}^7c_4\alpha^4 + {}^7c_5\alpha^5 + {}^7c_6\alpha^6 + {}^7c_7\alpha^7$

$= 1 + 7\alpha + 21\alpha^2 + 35\alpha^3 + 35\alpha^4 + 24\alpha^5 + 7\alpha^6 + \alpha^9$

$\because \alpha = \omega$

$= 1 + 7\omega + 21\omega^2 + 35\omega^3 + 35\omega^4 + 21\omega^5 + 7\omega^6 + \omega^7$

$(\because \omega^3 = 1)$

$= 1 + 7\omega + 21\omega^2 + 35 + 35\omega + 21\omega + 7 + \omega$

$= 43 + 43\omega + 42\omega^2$

$= 43 + 43\alpha + 42\alpha^2 = A + B\alpha + C\alpha^2$

$\Rightarrow A = 43 \mid B = 43 \mid C = 42$

$5(129 - 86 - 42)$

$= 5(129 - 128) = 5$

**25.** If $8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + \cdots\cdots\infty$, then the value of $p$ is

**Sol.** **9**

$\left(3 + \frac{3}{4} + \frac{3}{4^2} + \frac{3}{4^3} + \ldots\right)$

$+\left(\frac{P}{4} + \frac{2P}{4^2} + \frac{3P}{4^3} + \ldots\infty\right) = 8$

$\Rightarrow \frac{3}{1 - \frac{1}{4}} + \frac{p}{4} + \frac{2p}{4^2} + \ldots\infty = 8$

$\Rightarrow \frac{p}{4} + \frac{2p}{4^2} + \frac{3P}{4^3} + \ldots\infty = 4 \qquad \ldots(1)$

Let $\quad \Rightarrow S = \frac{p}{4} + \frac{2p}{4^2} + \frac{3p}{4^3} \ldots\infty$

$\frac{S}{4} = \qquad \frac{p}{4^2} + \frac{2p}{4^3} + \ldots\infty$

on Substraction

$$\frac{3S}{4}=\frac{p}{4}+\frac{p}{4^2}+\frac{p}{4^3}+\ldots\infty$$

$$\frac{3S}{4}=P\left(\frac{\frac{1}{4}}{1-\frac{1}{4}}\right)=\frac{p}{3}$$

Now from (1)

$$\frac{4p}{9}=4\Rightarrow P=9$$

**26.** Let $A=\begin{bmatrix}2&0&1\\1&1&0\\1&0&1\end{bmatrix}, B=[B_1,B_2,B_3]$, where $B_1,B_2,B_3$ are column matrics, and

$$AB_1=\begin{bmatrix}1\\0\\0\end{bmatrix}, AB_2=\begin{bmatrix}2\\3\\0\end{bmatrix}, AB_3=\begin{bmatrix}3\\2\\1\end{bmatrix}$$

If $\alpha=|B|$ and $\beta$ is the sum of all the diagonal elements of $B$, then $\alpha^3+\beta^3$ is equal to

**Sol.** **28**

$$\begin{bmatrix}2&0&1\\1&1&0\\1&0&1\end{bmatrix} \quad B=\begin{array}{ccc}B_1&B_2&B_3\\ \multicolumn{3}{c}{\begin{bmatrix}a&d&g\\b&e&h\\c&f&i\end{bmatrix}}\end{array}$$

$$AB_1=\begin{bmatrix}2a+c\\a+b\\a+c\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}\Rightarrow \begin{matrix}a=-b\\a=-c\end{matrix} \qquad a=1,\ b=-1,\ c=-1$$

$$AB_2=\begin{bmatrix}2d+f\\d+e\\d+f\end{bmatrix}=\begin{bmatrix}2\\3\\0\end{bmatrix}$$

$d=-f$

$d=2,\ f=-2$

$e=1$

$$AB_3=\begin{bmatrix}2g+i\\g+h\\g+i\end{bmatrix}=\begin{bmatrix}3\\2\\1\end{bmatrix}\Rightarrow g=1-i$$

$2-2i+i=3$

$i=-1$

$g = 2, h = 0$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$\alpha = |B| = 1(-1)-2(1) + 2(2+1) = 3$

$\alpha = 3, \beta = \text{trace}(B) = 1$

$\alpha^3 + \beta^3 = 3^3 + 1^3 = 28$

**27.** Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in \mathbf{R}$. Then $f'(10)$ is equal to

**Sol.** **202**

$f(x) = x^3 + x^2 f'(1) + x f''(2) + f''(3)$

$f'(x) = 3x^2 + 2xf'(1) + f''(2)$ ...(1)

Put $x = 1$

$f'(1) = 3 + 2f'(1) + f''(2)$

$\Rightarrow f'(1) + f''(2) + 3 = 0$ ...(2)

again differentiation (1)

$f''(x) = 6x + 2f'(1)$ Put $x = 2$

$f''(2) = 12 + 2f'(1)$ ...(3)

Solving (2) & (3)

$2f'(1) - f''(2) + 12 = 0$

$2f'(1) + 2f''(2) + 6 = 0$

$3f''(2) - 6 = 0$

$f''(2) = 2$

$f'(1) = -5$

$f'(10) = 300 + 20(-5) + 2$

$= 300 - 100 + 2$

$= 202$

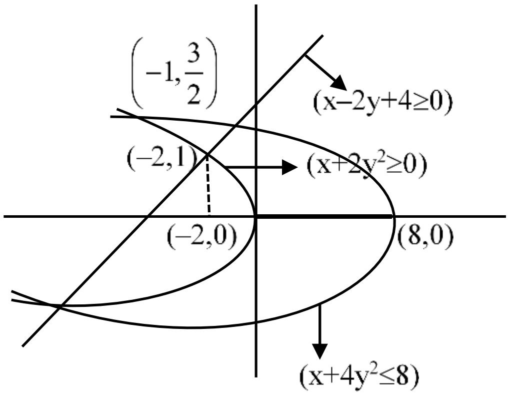
**28.** Let the area of the region $\{(x, y) : x - 2y + 4 \geq 0, x + 2y^2 \geq 0, x + 4y^2 \leq 8, y \geq 0\}$ be $\frac{m}{n}$, where $m$ and $n$ are coprime numbers. Then $m + n$ is equal to

**Sol.** **119**

$x - 2y + 4 \geq 0$

$x + 2y^2 \geq 0$

$x + 4y^2 \leq 8,\ y \geq 0$




$2y^2 = -x$

$$y = \frac{\sqrt{-x}}{\sqrt{2}}$$

$$\int_{-2}^{-1} \left(\frac{x+4}{2}\right) + \int_{-1}^{8} \frac{\sqrt{8-x}}{2} - \frac{1}{\sqrt{2}} \int_{-2}^{0} \sqrt{-x}$$

$$\Rightarrow \frac{1}{2}\left[\frac{x^2}{2} + 4x\right]_{-2}^{-1} + \frac{1}{2}\left[\left(-\frac{2}{3}\right)(8-x)^{3/2}\right]_{-1}^{8} + \frac{1}{\sqrt{2}} \times \frac{2}{3}\left[(-x)^{\frac{3}{2}}\right]_{-2}^{0}$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{2} - 4 - (2-8)\right] - \frac{1}{3}[0-27] + \frac{\sqrt{2}}{3}[-2\sqrt{2}]$$

$$\frac{5}{4} + 9 - \frac{4}{3} = \frac{321}{36} = \frac{107}{12}$$

$$\frac{45+324-48}{36} = \frac{107}{12} = \frac{m}{n} \Rightarrow m + n = 107+12 = 119$$

**29.** Let for a differentiable function $f:(0,\infty) \to \mathbf{R}, f(x) - f(y) \geq \log_e\left(\frac{x}{y}\right) + x - y, \forall x, y \in (0,\infty)$.

Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to

**Sol.** **2890**

$f;(0, \infty) \to R$

$f(x) - f(y) \geq \log(x) - \log y + x - y$

$f(x) - \log x - x \geq f(y) - \ln y - y$

$x \leftrightarrow y$

$f(y) - \log y - y \geq f(x) - \ln x - x$

$\Rightarrow f(x) = \ln x + x + \lambda$

$f'(x) = \frac{1}{x} + 1$

$\sum_{n=1}^{20} \left(n^2 + 1\right)$

$20 + \frac{20 \times 21 \times 41}{6} = 20 + 2870 = 2890$

**30.** Let the set of all $a \in \mathbf{R}$ such that the equation $\cos 2x + a\sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ$, then pqr is equal to ____ .

**Sol.** **48**

$\cos 2x + a \sin x = 2a - 7$

$\Rightarrow 1 - 2\sin^2 x + 7 = a(2 - \sin x)$

$\Rightarrow \frac{2\left(4 - \sin^2 x\right)}{2 - \sin x} = a$

$\Rightarrow \frac{2(2 - \sin x)(2 + \sin x)}{2 - \sin x} = a$

$\Rightarrow a = 4 + 2\sin x$

$\Rightarrow a \in [2, 6] \cong [p, q]$

$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$

$= \tan 9^\circ - \tan 27^\circ - \cot 27^\circ + \cot 9^\circ$

$= \frac{\sin 9^\circ}{\cos 9^\circ} + \frac{\cos 9^\circ}{\sin 9^\circ} - \left(\frac{\cos 27^\circ}{\sin 27^\circ} + \frac{\sin 27^\circ}{\cos 27^\circ}\right)$

$= \frac{2}{2} \times \frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{2}{2} \frac{1}{\sin 27^\circ \cos 27^\circ}$

$= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$

$\frac{2}{\sqrt{5} - 1} \times 4 - \frac{2}{\sqrt{5} + 1} \times 4 = 8\left[\frac{\sqrt{5} + 1}{4} - \frac{\sqrt{5} - 1}{4}\right]$

$= 2[2] = 4 \Rightarrow pqr = 48$

Continuing to keep the pledge

## of **imparting education** for the **last 17 Years**

| | | | | |
|---|---|---|---|---|
| **65136+** SELECTIONS SINCE 2007 | JEE (Advanced) **12142** | JEE (Main) **32584** | NEET/AIIMS **17875** (Under 50000 Rank) | NTSE/OLYMPIADS **2535** (6th to 10th class) |

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- **AIR-1 to 10** 25 Times
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2747/5182 = **53.01%**
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1756/4818 = **36.45%**

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