Assignment 7

Recommendations System

① Given,

	M	N	P	Q	R
A	1	2	3	4	5
B	2	3	2	5	3
C	5	5	5	3	2

Subtracting the average for each row

$$Avg\,(A) = \frac{15}{5} = 3$$

$$Avg\,(B) = \frac{15}{5} = 3$$

$$Avg\,(C) = \frac{20}{5} = 4$$

	M	N	P	Q	R
A	-2	-1	0	1	2
B	-1	0	-1	2	0
C	1	1	1	-1	-2

Subtracting the average for each column

$$Avg\,(M) = \frac{-2}{3} = -0.66$$

$$Avg\,(N) = 0$$

$$Avg\,(P) = 0$$

$$Avg\,(Q) = \frac{2}{3} = 0.66$$

$$Avg\,(R) = 0$$

	M	N	P	Q	R
A	$-4/3$	-1	0	$1/3$	2
B	$-1/3$	0	-1	$4/3$	0
C	$5/3$	1	1	$-5/3$	-2

∴ Largest element = 2

$(C, P) = 1$

② Given,

$$
\begin{array}{c|cccccc}
A & 1 & 0 & 1 & 0 & 1 & 2\alpha \\
B & 1 & 1 & 0 & 0 & 1 & 6\alpha \\
C & 0 & 1 & 0 & 1 & 0 & 2\alpha
\end{array}
$$

cos A, B

for $\alpha = 0 \Rightarrow$

$$
\cos(A, B) = \frac{1 + 1 + 12\alpha^2}{\sqrt{3 + 4\alpha^2}\ \sqrt{3 + 36\alpha^2}} = \frac{2 + 12\alpha^2}{\sqrt{9 + 120\alpha^2 + 144\alpha^4}}
$$

$$
\cos(B, c) = \frac{1 + 12\alpha^2}{\sqrt{3 + 36\alpha^2}\ \sqrt{2 + 4\alpha^2}} = \frac{1 + 12\alpha^2}{\sqrt{6 + 84\alpha^2 + 144\alpha^4}}
$$

$$
\cos(C, A) = \frac{4\alpha^2}{\sqrt{2 + 4\alpha^2}\ \sqrt{3 + 4\alpha^2}} = \frac{4\alpha^2}{\sqrt{6 + 20\alpha^2 + 16\alpha^4}}
$$

for $\alpha = 0 \Rightarrow \cos(A, B) = \frac{2}{3} = 0.66$

$\cos(B, c) = 0.408$

$\cos(C, A) = 0$

for $\alpha = 0.5 \Rightarrow \cos(A, B) = 0.7216$

$\cos(B, c) = 0.6667$

$\cos(C, A) = 0.28868$

for $\alpha = 1 \Rightarrow \cos(A, B) = 0.8473$

$\cos(B, c) = 0.8498$

$\cos(C, A) = 0.6172$

A is closer to C
than B

$\Rightarrow \theta = 18.89$

for $\alpha = 2 \Rightarrow \cos(A, B) = 0.9461$

$\cos(B, c) = 0.9926$

$\cos(C, A) = 0.8652$

$\Rightarrow \theta = 30.6$

③ Given,

	a	b	c	d	e	f	g	h
A	4	5			5	1		3	2
B			3	4	3	1	2	1
C	2		1	3		4	5	3

Updating 3, 4, 5 — to 1

1, 2, empty to 0

	a	b	c	d	e	f	g	h
A	1	1	0	1	0	0	1	0
B	0	1	1	1	0	0	0	0
C	0	0	0	1	0	1	1	1

$JD(a, b) = 1 - JS(a, b) = 1 - \frac{1}{2} = \frac{1}{2}$

$JD(a, c) = 1 - 0 = 1$

$JD(a, d) = 1 - \frac{1}{3} = \frac{2}{3}$

$JD(a, e) = 1 - 0 = 1$

$JD(a, f) = 1 - 0 = 1$

$JD(a, g) = 1 - \frac{1}{2} = \frac{1}{2}$

$JD(a, h) = 1 - 0 = 1$

$JD(b, c) = 1 - \frac{1}{2} = \frac{1}{2}$

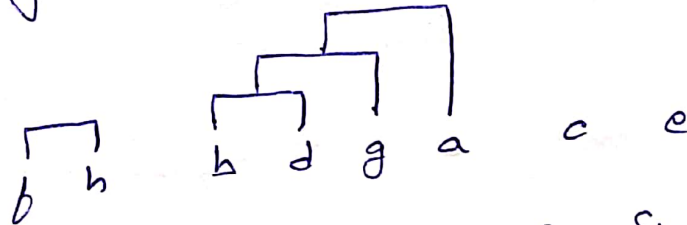$JD(b, d) = 1 - \frac{2}{3} = \frac{1}{3}$

$JD(b, e) = 1 - 0 = 1$

$JD(b, f) = 1 - 0 = 1$

$JD(b, g) = 1 - \frac{1}{3} = \frac{2}{3}$

$JD(b, h) = 1 - 0 = 1$

$JD(c,d) = 1 - \frac{1}{3} = \frac{2}{3}$

$JD(e,e) = 1 - 0 = 1$

$JD(c,b) = 1 - 0 = 1$

$JD(c,g) = 1 - 0 = 1$

$JD(c,h) = 1 - 0 = 1$

$JD(d,e) = 1 - 0 = 1$

$JD(d,b) = 1 - \frac{1}{3} = \frac{2}{3}$

$JD(d,g) = 1 - \frac{2}{3} = \frac{1}{3}$

$JD(d,h) = 1 - \frac{1}{3} = \frac{2}{3}$

$JD(e,b) = 1 - 0 = 1$

$JD(e,g) = 1 - 0 = 1$

$JD(e,h) = 1 - 0 = 1$

$JD(b,g) = 1 - \frac{1}{2} = \frac{1}{2}$

$JD(b,h) = 1 - 1 = 0$

$JD(g,h) = 1 - \frac{1}{2} = \frac{1}{2}$

Clustering the items hierarchially :-

b h   h d g a   c   e

∴ The final clusters are $\{b, h\}$, $\{h, d, g, a\}$,

$\{c\}$, $\{e\}$

④ Given, $H = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ → ①

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 5 & 5 & 5 \end{bmatrix} = \begin{bmatrix} 5x & 5x & 5x \\ 5y & 5y & 5y \\ 5z & 5z & 5z \end{bmatrix}$ → ②

The RMSE of ① and ② is,

$(5x-1)^2 + (5x-2)^2 + (5x-3)^2 + (5y-4)^2 + (5y-5)^2$

$+ (5y-6)^2 + (5z-7)^2 + (5z-8)^2 + (5z-9)^2$

→ ③

differentiate eqn ③ with x,

$$150x - 60 = 0$$

$$\boxed{x = \frac{2}{5}}$$

differentiating eqn ③ with y,

$$150y - 150 = 0$$

$$\boxed{y = 1}$$

differentiating eqn ③ with z,

$$150z - 240 = 0$$

$$\boxed{z = \frac{8}{5}}$$