Assignment 8

Dimentionality Reduction

(1) Given three columns are

$$C_1 = \left[\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right]$$

$$C_2 = \left[\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}\right]$$

$$C_3 = [x, y, z]$$

the dot product of any two columns must be 0

$$C_1 \cdot C_2 = (2/7 \times 6/7) + (3/7 \times 2/7) + (6/7 \times -3/7)$$

$$= \frac{12}{49} + \frac{6}{49} - \frac{18}{49} = \frac{0}{49} = 0$$

$$C_2 \cdot C_3 = (6/7 \times x) + (2/7 \times y) + (-3/7 \times z) = 0$$

$$6x + 2y - 3z = 0 \rightarrow (1)$$

$$C_3 \cdot C_1 = (x \times 2/7) + (y \times 3/7) + (z \times 6/7) = 0$$

$$2x + 3y + 6z = 0 \rightarrow (2)$$

$$(1) \times 2 \Rightarrow 12x + 4y - \cancel{6z} = 0$$

$$(2) \Rightarrow 2x + 3y + \cancel{6z} = 0$$

$$14x + 7y = 0$$

$$2x + y = 0$$

$$\boxed{y = -2x}$$

② × 3 ⇒ $\cancel{6x} + 9y + 18z = 0$

① ⇒ $-\cancel{6x} \pm 2y \mp 3z = 0$

$7y + 21z = 0$

$y + 3z = 0$

$\boxed{y = -3z}$ $\boxed{-2 : 1 : -3}$

② Given matrix $A = \begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix}$

Let the eigen vector be of the form $\begin{bmatrix} 1 \\ e \end{bmatrix}$

To find the eigen values $Ax = \lambda x$

$$\begin{bmatrix} 2 & 3 \\ 3 & 10 \end{bmatrix} \begin{bmatrix} 1 \\ e \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ e \end{bmatrix}$$

$2 + 3e = \lambda \rightarrow$ ①

$3 + 10e = \lambda e \rightarrow$ ②

substitute ① in ②

$3 + 10e = (2 + 3e)e$

$3 + 10e = 2e + 3e^2$

$3e^2 - 8e - 3 = 0$

$3e^2 - 9e + e - 3 = 0$

$3e(e-3) + 1(e-3) = 0$

$(3e+1)(e-3) = 0 \Rightarrow e = 3, \frac{-1}{3}$

So the eigen vectors are $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ -1/3 \end{bmatrix}$

the corresponding eigen values are

$\lambda = 2 + 3e = 2 + 3(3) = 11$

$\lambda = 2 + 3e = 2 + 3\left(\frac{-1}{3}\right) = 1$

(3) Given eigen vector is $A = \begin{bmatrix} 1 \\ 3 \\ 4 \\ 5 \\ 7 \end{bmatrix}$

To get the unit eigen vector of the given matrix we need to divide each component by the squareroot of sum of squares in the same direction.

i.e

$$\text{sum of squares} = 1^2 + 3^2 + 4^2 + 5^2 + 7^2$$
$$= 1 + 9 + 16 + 25 + 49 = 100$$

$$\text{Squareroot of sum of squares} = \sqrt{100} = 10$$

$$\therefore \text{Unit Eigen vector} = \begin{bmatrix} 1/10 \\ 3/10 \\ 4/10 \\ 5/10 \\ 7/10 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.7 \end{bmatrix}$$

(4) The given three points in a two dimensional space are

(1, 1), (2, 2), (3, 4)

Construct a matrix whose rows correspond to ~~the~~ point and columns correspond to dimensions of the space.

$$M = \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{bmatrix}$$

Perform $M^T M \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 17 \\ 17 & 21 \end{bmatrix}$

(5) Given diagonal matrix $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

The pseudo inverse has 0's off the diagonals.
The diagonal elements of new matrix is 1 divided by the corresponding element of given matrix M.
For diagonal element 0 we leave it as it is, rather than infinity

$\therefore$ pseudo inverse of $M = \begin{bmatrix} 1/1 & 0 & 0 \\ 0 & 1/2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(6) Given matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$

Probability with which we choose row = $\dfrac{\text{Sum of squares of element in the row}}{\text{Sum of squares of elements in the matrix}}$

$$P(R_1) = \frac{1^2+2^2+3^2}{1^2+2^2+\dots+12^2} = \frac{14}{650} = 0.02$$

$$P(R_2) = \frac{4^2+5^2+6^2}{650} = \frac{77}{650} = 0.12$$

$$P(R_3) = \frac{7^2+8^2+9^2}{650} = \frac{194}{650} = 0.298$$

$$P(R_4) = \frac{10^2+11^2+12^2}{650} = \frac{365}{650} = 0.56$$