# Case Study 1

# Multi Room Heat Balance (Conduction only)

## ❖ Introduction

This assignment studies the heat balance in a multi-room system considering conduction as the only mode of heat transfer. Steady-state energy balance equations are used to determine the temperature distribution of each room.

## ❖ Assumptions

1. Steady-state heat transfer
2. One-dimensional conduction
3. Uniform temperature within each room
4. Constant thermal resistance
5. No convection or radiation
6. Perfect thermal contact between rooms

## ❖ Mathematical Modelling

Case Study - 1

Multi room Heat Balance (Conduction only)

★ Mathematical modelling

Acc. to Fourier's law

$$Q = \frac{T_i - T_j}{R}$$

Q = heat transfer rate (W)

$T_i, T_j$ = room temperature (°C)

R = thermal resistance (K/W)

Conductance, $G = \frac{1}{R}$

$$Q = G(T_i - T_j)$$

→ Energy Balance for room

at steady state

$$\Sigma Q_{in} - \Sigma Q_{out} + Q_{gen} = 0$$

$$\therefore \Sigma G(T_i - T_j) + G(T_i - T_{amb}) = Q_{gen\,i}$$

Considering 4 rooms

$R_1$ = resistance b/w adjacent rooms

$R_2$ = resistance b/w room and ambient

$T_{amb}$ = ambient temperature

$Q_{gen}$ = heat generated in top room

$$\therefore G_1 = \frac{1}{R_1}, \quad G_2 = \frac{1}{R_2}$$

→ For room 0 [Ground room]

$$G_1(T_0 - T_1) + G_2(T_0 - T_{amb}) = 0$$

$$(G_1 + G_2)T_0 - G_1T_1 = G_2 T_{amb}$$

→ For room 1

$$G_1(T_1 - T_0) + G_1(T_1 - T_2) + G_2(T_1 - T_{amb}) = 0$$

$$(G_2 + 2G_1)T_1 - G_1 T_0 - G_1 T_2 = G_2 T_{amb}$$

→ For room 2

$$G_1(T_2 - T_1) + G_1(T_2 - T_3) + G_2(T_2 - T_{amb}) = 0$$

$$(G_2 + 2G_1)T_2 - G_1 T_1 - G_1 T_3 = G_2 T_{amb}$$

→ For room 3 (Top room with heat generation)

$$G_1(T_3 - T_2) + G_2(T_3 - T_{amb}) = Q_{gen}$$

$$(G_1 + G_2)T_3 - G_1 T_2 = G_2 T_{amb} + Q_{gen}$$

→ Matrix

$$[A] = \begin{bmatrix} G_2+G_1 & -G_1 & 0 & 0 \\ -G_1 & G_2+2G_1 & -G_1 & 0 \\ 0 & -G_1 & G_2+2G_1 & -G_1 \\ 0 & 0 & -G_1 & G_2+G_1 \end{bmatrix}$$

Temp vector

$$[T] = \begin{bmatrix} T_0 \\ T_1 \\ T_2 \\ T_3 \end{bmatrix}$$

$$[B] = \begin{bmatrix} G_2 T_{amb} \\ G_2 T_{amb} \\ G_2 T_{amb} \\ G_2 T_{amb} + Q_{gen} \end{bmatrix}$$

$$\therefore [A][T] = [B]$$

## ❖ CODE

```
clc;
clear;

% Thermal resistances (K/W)
R1 = 0.1;     % Between rooms
R2 = 0.48;    % To ambient

% Given data
Tamb = 35;    % Ambient temperature (°C)
Qgen = 50;    % Heat generation in top room (W)

% Conductances
G1 = 1/R1;
G2 = 1/R2;

% Coefficient matrix
A = [ (G1+G2)    -G1          0           0;
      -G1     (2*G1+G2)     -G1          0;
       0        -G1     (2*G1+G2)       -G1;
       0         0         -G1        (G1+G2) ];

% Right-hand side vector
B = [ G2*Tamb;
      G2*Tamb;
      G2*Tamb;
      G2*Tamb + Qgen ];

% Solve linear system
T = A\B;

% Display results
```

```
fprintf('Room 0 Temperature = %.2f °C\n', T(1));

fprintf('Room 1 Temperature = %.2f °C\n', T(2));

fprintf('Room 2 Temperature = %.2f °C\n', T(3));

fprintf('Room 3 Temperature = %.2f °C\n', T(4));


% Plot results

rooms = 0:3;

figure;

plot(rooms, T, '-o', 'LineWidth', 1.5);

grid on;

xlabel('Room Number');

ylabel('Temperature (°C)');

title('Multi-Room Temperature Distribution (Conduction Only)');
```

```
Command Window
Room 0 Temperature = 38.78 °C
Room 1 Temperature = 39.56 °C
Room 2 Temperature = 41.30 °C
Room 3 Temperature = 44.35 °C
```

**Figure 1: Output of code**

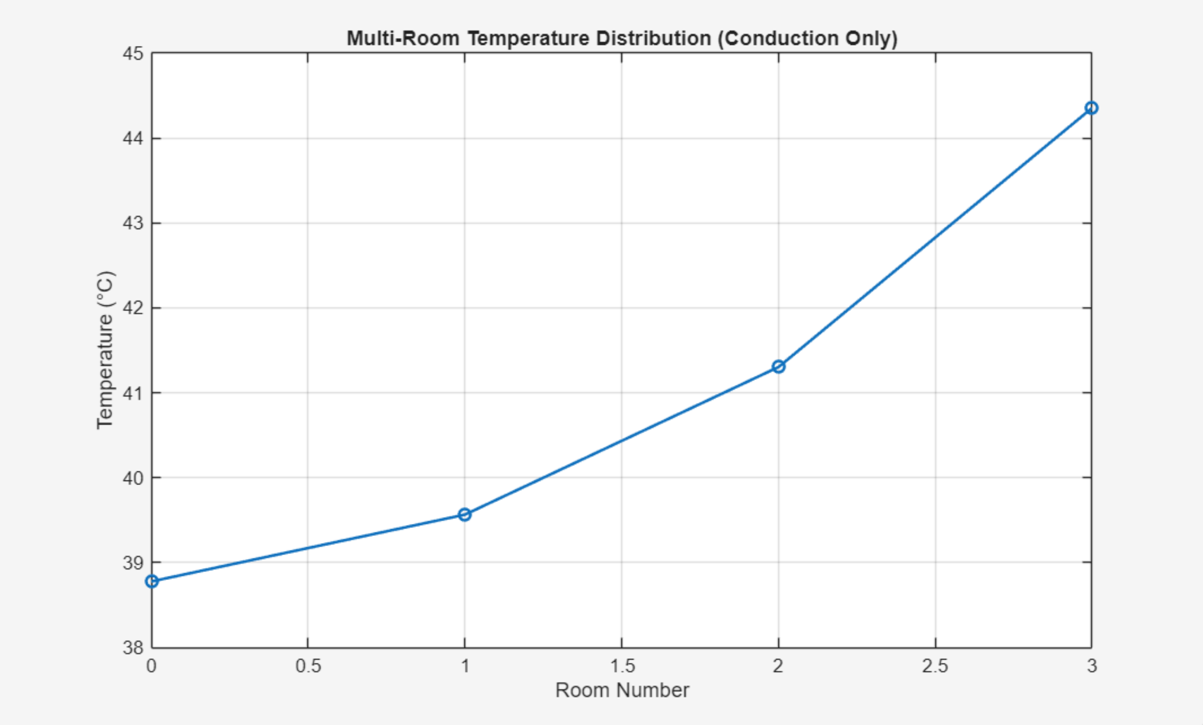


**Figure 2: Graph between Temperature and Room Number**

## ❖ Explanation of the Graph

Axes description

- X-axis (Room Number):
  Indicates the room index (Room 0 to Room 3).
- Y-axis (Temperature in °C):
  Shows the computed steady-state temperature of each room.

Observed Temperature Trend

- The temperature increases progressively from Room 0 to Room 3.
- Room 0 (Ground room) has the lowest temperature.
- Room 3 (Top room) has the highest temperature.

Physical Reason Behind the Curve

1. Heat conduction between adjacent rooms
   - Heat flows from higher temperature rooms to lower temperature rooms through walls.
   - Each room exchanges heat only with its neighboring rooms and the ambient.
2. Heat loss to ambient

    - All rooms lose heat to the surrounding environment through conduction.
    - Lower rooms are closer to ambient influence, so their temperatures remain lower.
3. Internal heat generation
    - The top room contains internal heat generation, which raises its temperature.
    - This causes a temperature gradient from top to bottom.
4. Steady-state condition
    - Since the system is at steady state, temperatures do not change with time.
    - The plotted values represent the final equilibrium temperatures.

Why the curve is smooth and increasing

- The governing equations are linear energy balance equations.
- Thermal resistances are constant.
- Hence, temperature variation follows a smooth, monotonic trend rather than abrupt jumps.

## ❖ Conclusion

The graph shows a steady increase in room temperatures from bottom to top, indicating effective conduction-based heat transfer and the influence of internal heat generation in the upper room.