11.16.2.2.2

EE24BTECH11011 - Pranay Kumar

Question:
A die is thrown. Describe the following event: (i) A: A number less than 7.
(ii) B: A number greater than 7.

Also, find the following set operations: $A \cup B, A \cap B$

Solution:

Textual solution:
Event A represents outcomes less than 7. Since a fair die has outcomes {1, 2, 3, 4, 5, 6}, this event includes all possible outcomes:

$$A = \{1, 2, 3, 4, 5, 6\}. \tag{0.1}$$

The probability of event A occurring is:

$$P(A) = \frac{|A|}{6} = \frac{6}{6} = 1. \tag{0.2}$$

Event B represents outcomes greater than 7. Since a fair die has outcomes {1, 2, 3, 4, 5, 6}, this event is impossible:

$$B = \emptyset, \quad P(B) = 0. \tag{0.3}$$

Using standard probability rules and set operations:

$$A \cup B = A, \tag{0.4}$$

$$A \cap B = \emptyset. \tag{0.5}$$

Additional Analysis Using A and B:
Since B is an impossible event:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + 0 - 0 = P(A), \tag{0.6}$$

$$P(A \cap B) = 0. \tag{0.7}$$

Since A includes all possible outcomes of a fair die:

$$P(A) = 1. \tag{0.8}$$

Thus,

$$P(A \cup B) = 1, \tag{0.9}$$

$$P(A \cap B) = 0. \tag{0.10}$$

Computational solution:

To verify the theoretical results, we perform a simulation by rolling a die N times and tracking outcomes.

Definitions

Probability Mass Function (PMF): For a six-sided die:

$$P(X = k) = \frac{1}{6}, \quad k \in \{1, 2, 3, 4, 5, 6\}. \tag{0.11}$$

Cumulative Distribution Function (CDF):

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{x}{6}, & x \in \{1, 2, 3, 4, 5, 6\}, \\ 1, & x > 6. \end{cases} \tag{0.12}$$

The probability $P(B)$ is computed as:

$$P(B) = 1 - F(6) = 1 - 1 = 0. \tag{0.13}$$

Calculation of Set Operations

Using simulated data, we compute set probabilities:

$$P(A \cup B) = P(A) = 1, \tag{0.14}$$

$$P(A \cap B) = 0. \tag{0.15}$$

$$\tag{0.16}$$

Simulation Process

We roll a die N times and compute probabilities empirically. The following steps outline the process:

1) Simulating Outcomes: A random integer X is generated for each trial, where $X \in \{1, 2, 3, 4, 5, 6\}$. This is done by using a random number generator function.
2) Tracking Occurrences: For each simulated roll, the number of occurrences of each outcome is tracked. We count how many times each number from 1 to 6 appears.
3) Computing PMF: The PMF is computed by dividing the number of occurrences of each outcome by the total number of trials N. For example, the probability of rolling a "1" would be calculated as:
$$P(1) = \frac{\text{Occurrences of 1}}{N}.$$
4) Computing CDF: The CDF is derived from the PMF. For each outcome i, the cumulative probability is the sum of the PMF values for all outcomes less than or equal to i:
$$F(i) = \sum_{k=1}^{i} P(k).$$
5) Verifying Theoretical Probability: We verify the computed value of $P(B)$ to see if it matches the theoretical result, which should be 0.

Calculation of Set Operations

Using simulated data, we compute set probabilities:

$$P(A \cup B) = P(A), \tag{5.1}$$

$$P(A \cap B) = 0, \tag{5.2}$$

$$\tag{5.3}$$

The computation of set operations is based on the event data collected during the simulation. Since B is the empty set, we find that $A \cup B = A$ and $A \cap B = \emptyset$.

Output Representation

The output of the simulation and calculation process includes two main results:

- PMF: The probability distribution for each outcome $\{1, 2, 3, 4, 5, 6\}$, showing how likely each outcome is.
- CDF: The cumulative probability for each outcome, showing the probability of all outcomes less than or equal to a given value.

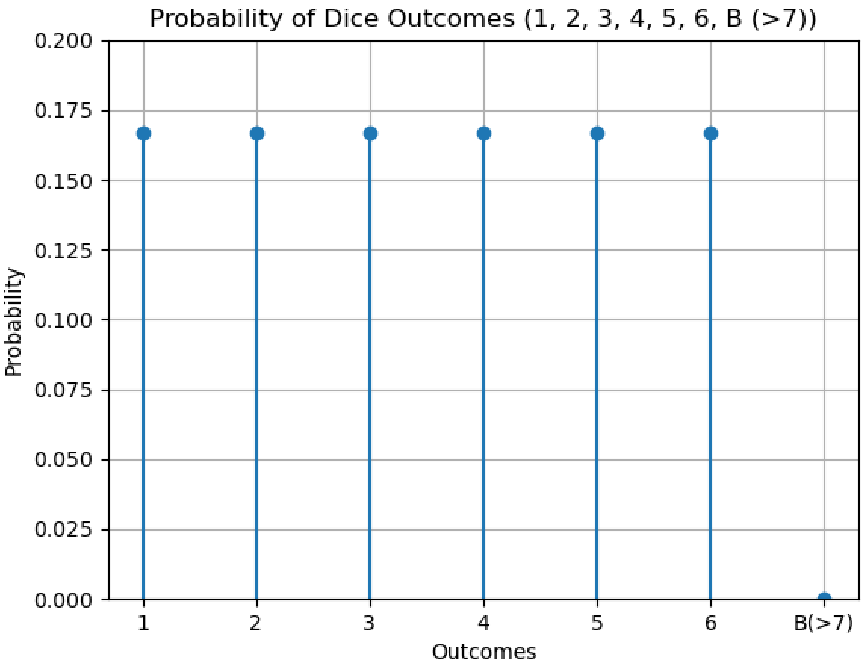

Fig. 5.1: Probability analysis of dice roll events