10.4.ex.2

EE24BTECH11011-PRANAY

Question:

Find the roots of the quadratic equation:

$$(x-2)^2 + 1 = 2x - 3 \tag{0.1}$$

Solution:

Rearranging terms,

$$(x-2)^2 + 1 - (2x-3) = 0 \tag{0.2}$$

$$x^2 - 4x + 4 + 1 - 2x + 3 = 0 \tag{0.3}$$

$$x^2 - 6x + 8 = 0 \tag{0.4}$$

Theoretical solution (Quadratic formula):

The roots are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{0.5}$$

$$x = \frac{6 \pm \sqrt{36 - 32}}{2} \tag{0.6}$$

$$x = \frac{6 \pm 2}{2} \tag{0.7}$$

$$x = 4 \text{ and } x = 2 \tag{0.8}$$

Computational solution:

(1) **Eigenvalues of Companion Matrix:**

The roots of a polynomial equation $x^n + b_{n-1}x^{n-1} + \cdots + b_1 x + b_0 = 0$ can be found by computing the eigenvalues of the companion matrix (C).

$$C = \begin{pmatrix} 0 & 1 \\ -b_0 & -b_1 \end{pmatrix} \tag{0.9}$$

For the equation $x^2 - 6x + 8 = 0$, we have:

$$b_0 = 8, \quad b_1 = -6 \tag{0.10}$$

$$C = \begin{pmatrix} 0 & 1 \\ -8 & 6 \end{pmatrix} \tag{0.11}$$

Using eigenvalue decomposition, the roots are:

$$x_1 = 4, \quad x_2 = 2 \tag{0.12}$$

(2) **Newton-Raphson iterative method:**

The function and its derivative are:

$$f(x) = x^2 - 6x + 8 \tag{0.13}$$

$$f'(x) = 2x - 6 \tag{0.14}$$

Difference equation:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.15}$$

$$x_{n+1} = x_n - \frac{x_n^2 - 6x_n + 8}{2x_n - 6} \tag{0.16}$$

Starting with initial guesses:

$$x_0 = 3 \text{ converges to } 2 \tag{0.17}$$

$$x_0 = 5 \text{ converges to } 4 \tag{0.18}$$

Verification:
Substituting $x = 4$:

$$(4 - 2)^2 + 1 = 2(4) - 3 \tag{0.19}$$

$$4 + 1 = 8 - 3 \tag{0.20}$$

$$5 = 5 \tag{0.21}$$

Substituting $x = 2$:

$$(2 - 2)^2 + 1 = 2(2) - 3 \tag{0.22}$$

$$0 + 1 = 4 - 3 \tag{0.23}$$

$$1 = 1 \tag{0.24}$$

Hence, the roots are verified.

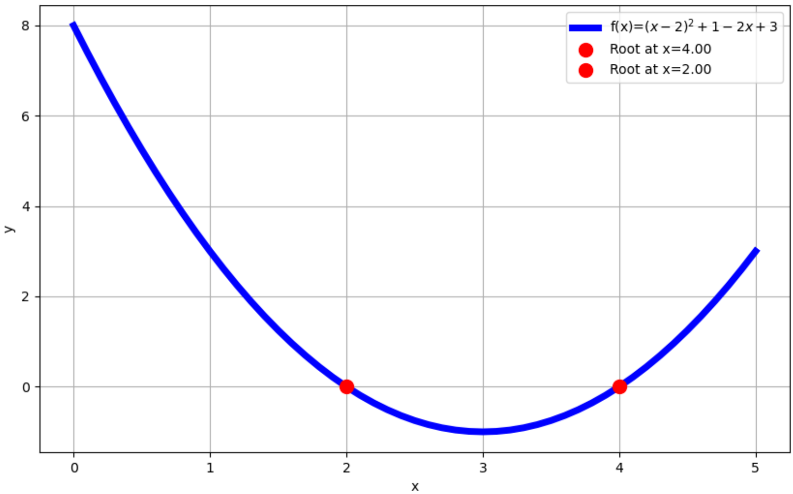

Fig. 0.1: Plot using Newton-Raphson method