11.16.2.2.2

EE24BTECH11011 - Pranay Kumar

Question:
A die is thrown. Describe the following event: (i) A: A number less than 7.
(ii) B: A number greater than 7.
Also, find the following Boolean operations: $A \vee B, A \wedge B$
Solution:

Textual solution:
Event A represents outcomes less than 7. Since a fair die has outcomes $\{1, 2, 3, 4, 5, 6\}$, this event includes all possible outcomes:

$$A = \{1, 2, 3, 4, 5, 6\}. \tag{0.1}$$

The probability of event A occurring is:

$$P(A) = \frac{|A|}{6} = \frac{6}{6} = 1. \tag{0.2}$$

Event B represents outcomes greater than 7. Since a fair die has outcomes $\{1, 2, 3, 4, 5, 6\}$, this event is impossible:

$$B = \emptyset, \quad P(B) = 0. \tag{0.3}$$

Using Boolean operations:

$$A \vee B = A, \tag{0.4}$$

$$A \wedge B = \emptyset. \tag{0.5}$$

Additional Analysis Using A and B:
Since B is an impossible event:

$$P(A \vee B) = P(A) + P(B) - P(A \wedge B) = P(A) + 0 - 0 = P(A), \tag{0.6}$$

$$P(A \wedge B) = 0. \tag{0.7}$$

Since A includes all possible outcomes of a fair die:

$$P(A) = 1. \tag{0.8}$$

Thus,

$$P(A \vee B) = 1, \tag{0.9}$$

$$P(A \wedge B) = 0. \tag{0.10}$$

Computational solution:

Computation of Probabilities for Rolling a Die

To verify the theoretical results, we perform a simulation by rolling a die N times and tracking outcomes.

Definitions

Probability Mass Function (PMF): For a six-sided die:

$$P(X = k) = \frac{1}{6}, \quad k \in \{1, 2, 3, 4, 5, 6\}. \tag{0.11}$$

Cumulative Distribution Function (CDF):

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{x}{6}, & x \in \{1, 2, 3, 4, 5, 6\}, \\ 1, & x > 6. \end{cases} \tag{0.12}$$

The probability $P(B)$ is computed as:

$$P(B) = 1 - F(6) = 1 - 1 = 0. \tag{0.13}$$

Calculation of Boolean Operations

Using simulated data, we compute probabilities:

$$P(A \vee B) = P(A) = 1, \tag{0.14}$$

$$P(A \wedge B) = 0. \tag{0.15}$$

$$\tag{0.16}$$

Simulation Process

We roll a die N times and compute probabilities empirically. The following steps outline the process:

1) Simulating Outcomes: A random integer X is generated for each trial, where $X \in \{1, 2, 3, 4, 5, 6\}$.
2) Tracking Occurrences: For each simulated roll, the number of occurrences of each outcome is tracked.
3) Computing PMF: The PMF is computed by dividing occurrences by N.
4) Computing CDF: The CDF is derived from the PMF.
5) Verifying Theoretical Probability.

Output Representation

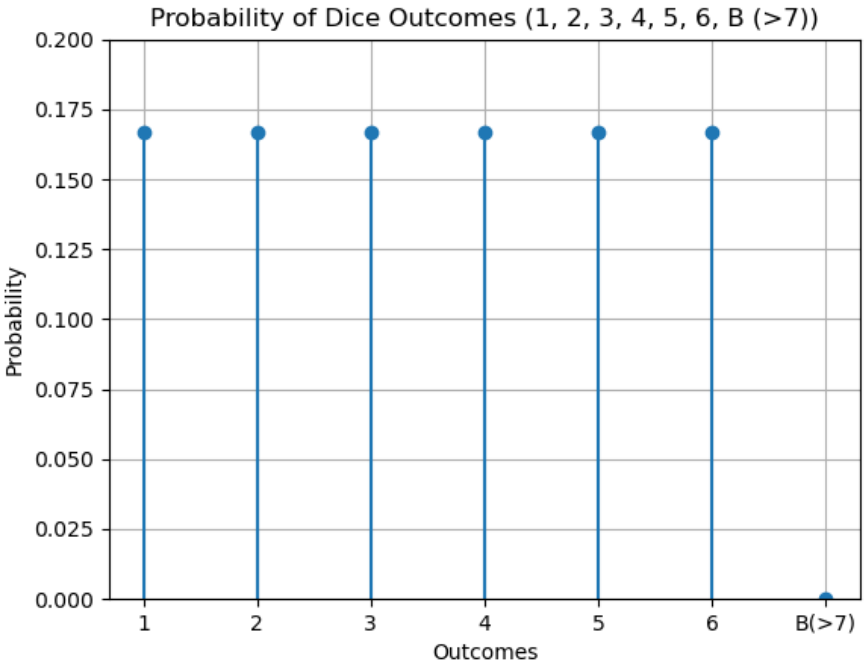

Fig. 5.1: Probability analysis of dice roll events