10.3.5.4.2

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QUESTION

A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator, and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

SOLUTION

Let the fraction be represented as $\frac{x}{y}$. From the given conditions:

$$\frac{x-1}{y} = \frac{1}{3} \tag{0.1}$$

$$\frac{x}{y+8} = \frac{1}{4} \tag{0.2}$$

Rewriting the equations in matrix form $A\mathbf{x} = \mathbf{b}$:

$$A = \begin{pmatrix} 1 & -\frac{1}{3} \\ 1 & -\frac{1}{4} \end{pmatrix} \tag{0.3}$$

$$b = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.4}$$

Performing LU decomposition:

$$A = L \cdot U, \tag{0.5}$$

where:

$$L = \begin{pmatrix} 1 & 0 \\ \frac{3}{4} & 1 \end{pmatrix}, \tag{0.6}$$

$$U = \begin{pmatrix} 1 & -\frac{1}{3} \\ 0 & -\frac{1}{12} \end{pmatrix}. \tag{0.7}$$

Factorization of LU: Given a matrix $\mathbf{A}$ of size $n \times n$, LU decomposition is performed row by row and column by column. The update equations are as follows:

1. Start by initializing $\mathbf{L}$ as the identity matrix $\mathbf{L} = \mathbf{I}$ and $\mathbf{U}$ as a copy of $\mathbf{A}$.
2. For each column $j \geq k$, the entries of U in the k-th row are updated as:

$$U_{k,j} = A_{k,j} - \sum_{m=1}^{k-1} L_{k,m} \cdot U_{m,j} \quad \forall \quad j \geq k \tag{0.8}$$

3. For each row $i > k$, the entries of L in the k-th column are updated as:

$$L_{i,k} = \frac{1}{U_{k,k}} \left(A_{i,k} - \sum_{m=1}^{k-1} L_{i,m} \cdot U_{m,k} \right) \quad \forall \quad i > k \tag{0.9}$$

Solving $L\mathbf{y} = \mathbf{b}$ using forward substitution:

$$y_1 = 1 \tag{0.10}$$

$$\frac{3}{4}y_1 + y_2 = 0 \tag{0.11}$$

$$y_2 = -\frac{3}{4} \tag{0.12}$$

Thus:

$$\mathbf{y} = \begin{pmatrix} 1 \\ -\frac{3}{4} \end{pmatrix}. \tag{0.13}$$

Next, solving $U\mathbf{x} = \mathbf{y}$ using backward substitution:

$$-\frac{1}{12}y = -\frac{3}{4} \tag{0.14}$$

$$y = 9 \tag{0.15}$$

$$x - \frac{y}{3} = 1 \tag{0.16}$$

$$x = 5 \tag{0.17}$$

Hence, the fraction is $\frac{5}{12}$.

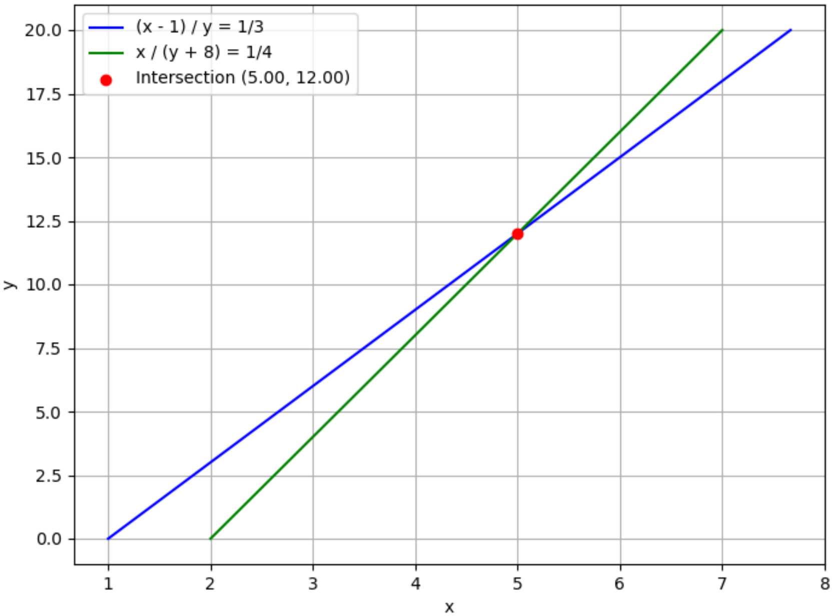

Fig. 0.1: Solution to fraction problem using LU decomposition