11.16.2.2.2

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Question: A die is thrown. Describe the following events:

- (i) AA: A number less than 7.
- (ii) BB: A number greater than 7.

Also, find the following $A \cup B$,$A \cap B$

Solution:

1) **Total Number of Possible Outcomes:** The sample space of rolling a fair six-sided die is:

$$S = 1, 2, 3, 4, 5, 6 \tag{1}$$

This means there are 6 possible outcomes when a die is rolled.

2) **Probability of Events:**

$$P(A) = P(X < 7) = 1 \quad \text{(since all outcomes are less than 7)} \tag{2}$$

$$P(B) = P(X > 7) = 0 \quad \text{(since no outcomes are greater than 7)} \tag{3}$$

Here, event A is a certain event because it includes all possible outcomes of the die roll. Event B is an impossible event since there are no outcomes greater than 6 on a standard die.

Boolean Algebra Derivations:

The Boolean algebra derivations provided are:

$$A + A' = 1 \quad \text{(Law of Complementarity)}, \tag{4}$$

$$AB + A'B = B \quad \text{(Distributive Law)}, \tag{5}$$

$$\implies P(AB) + P(A'B) = P(B) \quad \text{(Probability of the above)}, \tag{6}$$

$$B + B' = 1 \quad \text{(Law of Complementarity)}, \tag{7}$$

$$AB + AB' = A \quad \text{(Distributive Law)}, \tag{8}$$

$$\implies P(AB) + P(AB') = P(A) \quad \text{(Probability of the above)}, \tag{9}$$

$$A + B = AB + AB' + A'B \quad \text{(Expansion of Union)}, \tag{10}$$

$$\implies P(A + B) = P(AB) + P(AB') + P(A'B) \quad \text{(Probability of the above)}. \tag{11}$$

Step 1: Find $A \cup B$ (Union of A and B)

Using the expansion of the union:

$$A + B = AB + AB' + A'B$$

Taking probabilities on both sides:

$$P(A + B) = P(AB) + P(AB') + P(A'B)$$

From the problem:

$P(A) = 1, \quad P(B) = 0, \quad$ and $\quad P(AB) = 0 \quad$ (since A and B are mutually exclusive).

$$P(AB') = P(A) - P(AB) = 1 - 0 = 1,$$

$$P(A'B) = P(B) - P(AB) = 0 - 0 = 0.$$

Substituting these values:

$$P(A + B) = 0 + 1 + 0 = 1$$

Thus:

$$A \cup B = 1$$

Step 2: Find $A \cap B$ (Intersection of A and B)

The intersection $A \cap B$ is the event where both A and B occur simultaneously. From the problem:

$$A : \text{A number less than 7}, \quad B : \text{A number greater than 7}.$$

Since no number can be both less than 7 and greater than 7 at the same time, $A \cap B$ is an impossible event. Therefore:

$$P(AB) = 0$$

Thus:

$$A \cap B = 0$$

Final Results:

$A \cup B = 1$ (The union of A and B is certain because A is certain),

$A \cap B = 0$ (The intersection of A and B is impossible because A and B are mutually exclu

3) **Probability Mass Function (PMF):**

$$P_X(x) = \begin{cases} \frac{1}{6}, & x \in \{1, 2, 3, 4, 5, 6\} \\ 0, & \text{otherwise} \end{cases} \tag{12}$$

The PMF describes the probability distribution of the outcomes of the die roll. Each outcome from 1 to 6 has an equal probability of $\frac{1}{6}$.

4) **Cumulative Distribution Function (CDF):**

$$F_X(k) = \begin{cases} 0, & k < 1 \\ \frac{k}{6}, & 1 \leq k \leq 6 \\ 1, & k > 6 \end{cases} \tag{13}$$

The CDF gives the probability that the die roll will result in a value less than or equal to k. It increases linearly from 0 to 1 as k increases from 1 to 6.

5) **Monte Carlo Simulation:** We approximate probabilities by simulating a large number of die rolls and computing relative frequencies. This method is useful for verifying theoretical probabilities and understanding the behavior of random variables through empirical data.
6) **Additional Insights:**
   - The events A and B are mutually exclusive because they cannot occur simultaneously. This is evident from the fact that $P(AB) = 0$.
   - The probability of the union of two mutually exclusive events is the sum of their individual probabilities, as shown by $P(A + B) = P(A) + P(B)$.
   - The PMF and CDF are fundamental tools in probability theory for describing the distribution of discrete random variables.