

11.16.2.2.2

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Question:

A die is thrown. Describe the following event: (ii) B : A number greater than 7.

Also, find the following set operations: $A \cup B, A \cap B$

Solution:

Textual solution:

Event B represents outcomes greater than 7. Since a fair die has outcomes $\{1, 2, 3, 4, 5, 6\}$, this event is impossible:

$$B = \emptyset, \quad P(B) = 0. \quad (0.1)$$

Using standard probability rules and set operations:

Computational solution:

COMPUTATION OF PROBABILITIES FOR ROLLING A DIE

To verify the theoretical results, we perform a simulation by rolling a die N times and tracking outcomes.

Definitions

Probability Mass Function (PMF): For a six-sided die:

$$P(X = k) = \frac{1}{6}, \quad k \in \{1, 2, 3, 4, 5, 6\}. \quad (0.2)$$

Cumulative Distribution Function (CDF):

$$F(x) = \begin{cases} 0, & x < 1, \\ \frac{x}{6}, & x \in \{1, 2, 3, 4, 5, 6\}, \\ 1, & x > 6. \end{cases} \quad (0.3)$$

The probability $P(B)$ is computed as:

$$P(B) = 1 - F(6) = 1 - 1 = 0. \quad (0.4)$$

Simulation Process

We roll a die N times and compute probabilities empirically. The following steps outline the process:

- 1) Simulating Outcomes: A random integer X is generated for each trial, where $X \in \{1, 2, 3, 4, 5, 6\}$. This is done by using a random number generator function.
- 2) Tracking Occurrences: For each simulated roll, the number of occurrences of each outcome is tracked. We count how many times each number from 1 to 6 appears.
- 3) Computing PMF: The PMF is computed by dividing the number of occurrences of each outcome by the total number of trials N . For example, the probability of rolling a "1" would be calculated as:

$$P(1) = \frac{\text{Occurrences of 1}}{N}.$$

- 4) Computing CDF: The CDF is derived from the PMF. For each outcome i , the cumulative probability is the sum of the PMF values for all outcomes less than or equal to i :

$$F(i) = \sum_{k=1}^i P(k).$$

- 5) Verifying Theoretical Probability: We verify the computed value of $P(B)$ to see if it matches the theoretical result, which should be 0.

Calculation of Set Operations

Using simulated data, we compute set probabilities:

$$P(A \cup B) = P(A), \tag{5.1}$$

$$P(A \cap B) = 0, \tag{5.2}$$

$$\tag{5.3}$$

The computation of set operations is based on the event data collected during the simulation. Since B is the empty set, we find that $A \cup B = A$ and $A \cap B = \emptyset$.

Output Representation

The output of the simulation and calculation process includes two main results:

- PMF: The probability distribution for each outcome $\{1, 2, 3, 4, 5, 6\}$, showing how likely each outcome is.
- CDF: The cumulative probability for each outcome, showing the probability of all outcomes less than or equal to a given value.

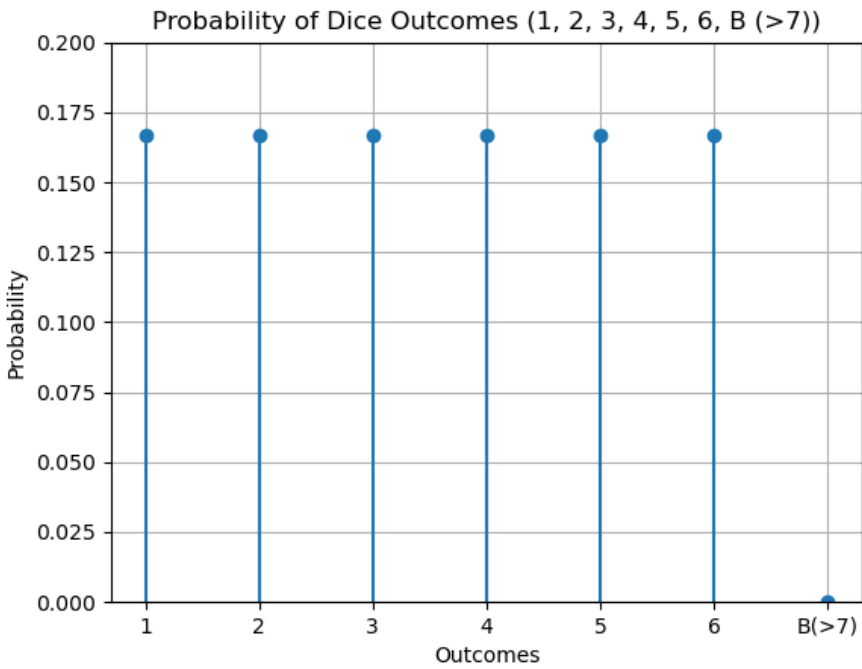


Fig. 5.1: Probability analysis of dice roll events