10.3.5.4.2

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QUESTION

A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator, and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

SOLUTION

Let the fraction be represented as $\frac{x}{y}$. From the given conditions:

$$\frac{x-1}{y} = \frac{1}{3} \tag{0.1}$$

$$\frac{x}{y+8} = \frac{1}{4} \tag{0.2}$$

Rewriting the equations in matrix form $A\mathbf{x} = \mathbf{b}$:

$$A = \begin{pmatrix} 1 & -\frac{1}{3} \\ 1 & -\frac{1}{4} \end{pmatrix} \tag{0.3}$$

$$b = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.4}$$

Performing LU decomposition:

$$A = L \cdot U, \tag{0.5}$$

where:

$$L = \begin{pmatrix} 1 & 0 \\ \frac{3}{4} & 1 \end{pmatrix}, \tag{0.6}$$

$$U = \begin{pmatrix} 1 & -\frac{1}{3} \\ 0 & -\frac{1}{12} \end{pmatrix}. \tag{0.7}$$

Expanding the LU decomposition step-by-step:

- Compute U:

$$U_{11} = A_{11} = 1, \tag{0.8}$$

$$U_{12} = A_{12} = -\frac{1}{3}, \tag{0.9}$$

$$U_{22} = A_{22} - L_{21}U_{12} = -\frac{1}{4} - \frac{3}{4} \times (-\frac{1}{3}) = -\frac{1}{12}. \tag{0.10}$$

- Compute L:

$$L_{21} = \frac{A_{21}}{U_{11}} = \frac{3}{4}. \tag{0.11}$$

Solving $L\mathbf{y} = \mathbf{b}$ using forward substitution:

$$y_1 = 1 \tag{0.12}$$

$$\frac{3}{4}y_1 + y_2 = 0 \tag{0.13}$$

$$y_2 = -\frac{3}{4} \tag{0.14}$$

Thus:

$$\mathbf{y} = \begin{pmatrix} 1 \\ -\frac{3}{4} \end{pmatrix}. \tag{0.15}$$

Next, solving $U\mathbf{x} = \mathbf{y}$ using backward substitution:

$$-\frac{1}{12}y = -\frac{3}{4} \tag{0.16}$$

$$y = 9 \tag{0.17}$$

$$x - \frac{y}{3} = 1 \tag{0.18}$$

$$x = 5 \tag{0.19}$$

Hence, the fraction is $\frac{5}{12}$.

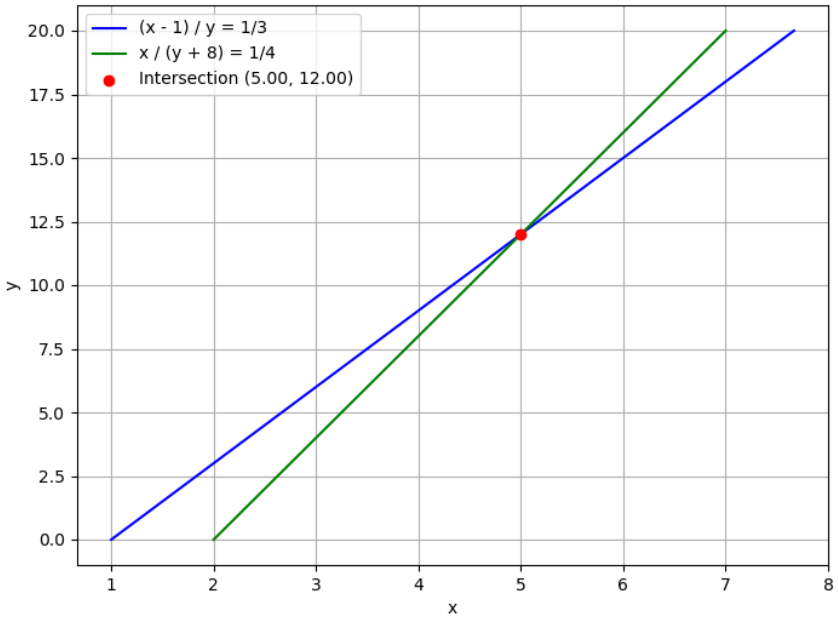

Fig. 0.1: Solution to fraction problem using LU decomposition