ASSIGNMENT 2

EE24BTECH11011 - PRANAY

27) Let $I = \int \frac{f(z)}{(z-1z-2)} dz$,where $f(x) = \sin\frac{\pi z}{2} + \cos\frac{\pi z}{2}$ and C is the curve $|z| = 3$ oriented anticlockwise.Then the value of I is
    a) $4\pi i$
    b) 0
    c) $-2\pi i$
    d) $-4\pi i$

28) Let $\sum_{-\infty}^{\infty} b_n z^n$ be the Laurent series expansion of the function $\frac{1}{z\sinh z}, 0 < |z| < \pi$. Then which one of the following is correct?

    a) $b_{-2} = 1, b_0 = -\frac{1}{6}, b_2 = \frac{7}{360}$
    c) $b_{-2} = 0, b_0 = -\frac{1}{6}, b_2 = \frac{7}{360}$
    b) $b_{-3} = 1, b_{-1} = -\frac{1}{6}, b_1 = \frac{7}{360}$
    d) $b_0 = 1, b_2 = -\frac{1}{6}, b_6 = \frac{7}{360}$

29) Under the transformation $w = \sqrt{\frac{1-iz}{z-i}}$, the region $D = \{z \in \mathbb{C} : |z| < 1\}$ is transformed to
    a) $\{z \in \mathbb{C} : 0 < argz < \pi\}$
    b) $\{z \in \mathbb{C} : -\pi < argz < 0\}$
    c) $\left\{z \in \mathbb{C} : 0 < argz < \frac{\pi}{2} \text{ or } \pi < argz < \frac{3\pi}{2}\right\}$
    d) $\left\{z \in \mathbb{C} : \frac{\pi}{2} < argz < \pi \text{ or } \frac{3\pi}{2} < argz < 2\pi\right\}$

30) Let $y(x)$ be the solution of the initial value problem

$$y''' - y'' + 4y' - 4y = 0 \quad , y(0) = y'(0) = 2 \quad , y''(0) = 0 \tag{1}$$

Then the value of $y\left(\frac{\pi}{2}\right)$

    a) $\frac{1}{5}\left(4e^{\frac{\pi}{2}} - 6\right)$
    b) $\frac{1}{5}\left(6e^{\frac{\pi}{2}} - 4\right)$
    c) $\frac{1}{5}\left(8e^{\frac{\pi}{2}} - 2\right)$
    d) $\frac{1}{5}\left(8e^{\frac{\pi}{2}} + 2\right)$

31) Let $y(x)$ be the solution of the initial value problem

$$x^2y'' + xy' + y = x, \quad y(1) = y'(1) = 1 \tag{2}$$

Then the value of $y\left(e^{\frac{\pi}{2}}\right)$ is

    a) $\frac{1}{2}\left(1 - e^{\frac{\pi}{2}}\right)$
    b) $\frac{1}{2}\left(1 + e^{\frac{\pi}{2}}\right)$
    c) $\frac{1}{2} + \frac{\pi}{4}$
    d) $\frac{1}{2} - \frac{\pi}{4}$

32) Let $T : P_3[0,1] \to P_2[0,1]$ be defined by $(Tp)(x) = p''(x) + p'(x)$.Then the matrix representation of T with respect to the bases $\left\{1, x, x^2, x^3\right\}$ and $\left\{1, x, x^2\right\}$ of $P_3[0,1]$ and $P_2[0,1]$ respectively is

    a) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 2 & 0 \\ 0 & 6 & 3 \end{pmatrix}$
    b) $\begin{pmatrix} 0 & 1 & 2 & 0 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 3 \end{pmatrix}$
    c) $\begin{pmatrix} 0 & 2 & 1 & 0 \\ 6 & 2 & 0 & 0 \\ 3 & 0 & 0 & 0 \end{pmatrix}$
    d) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 2 \\ 3 & 6 & 0 \end{pmatrix}$

33) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformations defined by $T(x, y, z) = (x + y, y + z, z + x)$ then the orthonormal basis for the range T is

a) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right\}$
c) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)\left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)\right\}$
b) $\left\{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)\right\}$
d) $\left\{\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)\right\}$

34) Consider the basis $\{u_1, u_2, u_3\}$ of $\mathbb{R}^3$,where $u_1 = (1, 0, 0)$ $u_2(1, 1, 0)$, $u_3(1, 1, 1)$.Let $\{f_1, f_2, f_3\}$ be the dual basis of $\{u_1, u_2, u_3\}$ ad f be a linear functional defined by $f(a, b, c) = a + b + c, (a, b, c) \in \mathbb{R}^3$.If $f = \alpha_1 f_1 + \alpha_2 f_2 + \alpha_3 + f_3$,then $(\alpha_1, \alpha_2, \alpha_3)$ is

a) $(1, 2, 3)$
b) $(1, 3, 2)$
c) $(2, 3, 1)$
d) $(3, 2, 1)$

35) The following table gives the cost matrix of a transportation problem The basic feasible solution

4	5	6
3	2	2
1	1	2

given by $x_{11} = 3, x_{13} = 1, x_{23} = 6, x_{31} = 2, x_{32} = 5$ is
a) degenerate and optimal
b) optimal but not degenerate
c) degenerate but not optimal
d) neither degenerate nor optimal

36) If z^* is the optimal value of the linear programming problem

$$\text{Maximize } z = 5x_1 + 9x_2 + 4x_3 \tag{3}$$

$$\text{subject to } x_1 + x_2 + x_3 = 5 \tag{4}$$

$$4x_1 + 3x_2 + 2x_3 = 12 \tag{5}$$

$$x_1, x_2, x_3 \geq 0, \tag{6}$$

then
a) $0 \leq z^* 10$
b) $10 \leq z^* 20$
c) $20 \leq z^* 30$
d) $30 \leq z^* 40$

37) Let G_1 be an abelian group of order 6 and $G_2 = S_3$.For $j = 1, 2$,let P_j be the statement "G_j has an unique order of 2".Then
a) both P_1 ad P_2 holds
b) neither P_1 nor P_2 holds
c) P_1 holds but not P_2
d) P_2 holds but not P_1

38) Let G be group of all symmetries of the square .Then the number of conjugate classes in G is

a) 4
b) 5
c) 6
d) 7

39) Consider the polynomial ring $\mathbf{Q}[x]$.The ideal of $\mathbf{Q}[x]$ generated by $x^2 - 3$ is

a) maximal but not prime
b) prime but not maximal
c) both maximal and prime
d) neither maximal nor prime