

3.3.20

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Question:

Draw a triangle PQR in which $QR = 3$ cm, $QP - PR = 6$ cm, and $\angle PQR = 45^\circ$.

Solution:

For triangle PQR with $QR = 3$ cm, $QP - PR = 6$ cm, and $\angle PQR = 45^\circ$.
Using law of cosines:

$$QP^2 = QR^2 + PR^2 - 2(QR)(PR)\cos PQR \quad (0.1)$$

$$K(QP + PR) = QR^2 - 2(QR)(PR)\cos PQR \quad (0.2)$$

Where,

$$K = QP - PR \quad (0.3)$$

$$\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} QP \\ PR \end{pmatrix} = \begin{pmatrix} QR^2 - 2(QR)(PR)\cos PQR \\ K \end{pmatrix} \quad (0.4)$$

$$\begin{pmatrix} QP \\ PR \end{pmatrix} = \frac{1}{K} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} QR^2 - 2(QR)(PR)\cos PQR \\ K \end{pmatrix} \quad (0.5)$$

$$\therefore \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} = 2I \quad (0.6)$$

$$PR = \frac{1}{2(1 + \frac{2(QR)\cos\angle(PQR)}{K})} \mathbf{e}^T \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{QR^2}{K} \\ K \end{pmatrix} \quad (0.7)$$

The coordinates of $\triangle PQR$ are

$$\mathbf{P} = PR \begin{pmatrix} \cos\angle PQR \\ \sin\angle PQR \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} QR \\ 0 \end{pmatrix} \quad (0.8)$$

substituting $QR = 3$ and $K = -6$ in (0.7)

$$QR = \frac{45}{6(2 - \sqrt{2})} \quad (0.9)$$

$$QR = 11.803300859 \quad (0.10)$$

Hence;

$$\mathbf{P} = \frac{11.803300859}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{0.11}$$

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{0.12}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}. \tag{0.13}$$

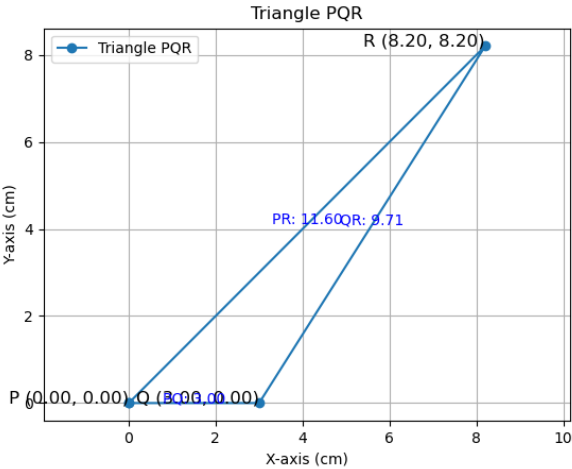


Fig. 0.1: Triangle PQR