ASSIGNMENT 5

EE24BTECH11011 - PRANAY

40) Using the Gauss-Seidel iteration method with the initial guess $\left\{x_1^{(0)} = 3.5, x_2^{(0)} = 2.25, x_3^{(0)} = 1.625\right\}$,the second approximation $\left\{x_1^{(2)}, x_2^{(2)}, x_3^{(2)}\right\}$ for the solution to the system of equations

$$2x_1 - x_2 = 7 \tag{1}$$
$$-x_1 + 2x_2 - x_3 = 1 \tag{2}$$
$$-x_2 + 2x_2 = 1 \tag{3}$$

is

a) $x_1^{(2)} = 5.3125, x_2^{(2)} = 4.4491, x_3^{(2)} = 2.1563$

b) $x_1^{(2)} = 5.3125, x_2^{(2)} = 4.3125, x_3^{(2)} = 2.6563$

c) $x_1^{(2)} = 5.3125, x_2^{(2)} = 4.4491, x_3^{(2)} = 2.6563$

d) $x_1^{(2)} = 5.4491, x_2^{(2)} = 4.4491, x_3^{(2)} = 2.1563$

41) The fourth order Runge-Kutta method given by

$$u_{j+1} = u_j + \frac{h}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right], j = 0, 1, 2, \ldots, \tag{4}$$

is used to solve the initial value problem $\frac{du}{dt} = u, u(0) = \alpha$ .If $u(1) = 1$ is obtained by taking the step size $h = 1$,then the value of K_4 is ______________

42) A particle P of mass m moves along the cycloid $x = (\theta - \sin\theta)$ and $y = (1 + \cos\theta), 0 \leq \theta \leq 2\pi$.Let g denote the accelaration due to gravity.Neglecting the fractional force,the Lagrangian associated with the motion of particle P is :

a) $m(1 - \cos\theta)\theta^2 - mg(1 + \cos\theta)$
b) $m(1 + \cos\theta)\theta^2 + mg(1 + \cos\theta)$
c) $m(1 + \cos\theta)\theta^2 + mg(1 - \cos\theta)$
d) $m(1 - \sin\theta)\theta^2 - mg(1 + \cos\theta)$

43) Suppose that X is a population random variable with probability density function

$$f(x;\theta) = \begin{cases} \theta x^{\theta-1}, & \text{if } 0 < x < 1 \\ 0 \text{ otherwise} \end{cases} \tag{5}$$

where θ is a parameter .In order to test the null hypothesis $H_0 : \theta = 2$,against the alternative hypothesis $H_1 : \theta = 3$,the following test is used:Reject the null hypothesis if $X_1 \geq \frac{1}{2}$ and accept otherwise,where X_1 is a random sample of size 1 drawn from the above population.Then the power of the test is ______________

44) Suppose that $X_1, X_2, \ldots, X_3$ is a random sample of size n from a population with probability density function

$$f(x;\theta) = \begin{cases} \frac{x}{\theta^2} e^{-\frac{x}{\theta}} \text{ if } x > 0 \\ 0 \text{ otherwise} \end{cases} \tag{6}$$

where θ is a parameter such that $\theta > 0$.The maximum likelihood estimator of θ is

a) $\frac{\sum_{i=1}^{n} X_i}{n}$

b) $\frac{\sum_{i=1}^{n} X_i}{n-1}$

c) $\frac{\sum_{i=1}^{n} X_i}{2n}$

d) $\frac{2\sum_{i=1}^{n} X_i}{n}$

45) Let $\mathbf{F}$ be a vector field on $\mathbb{R}^2/\{(0,0)\}$ by $\mathbf{F}(x,y) = \frac{y}{x^2+y^2}\hat{i} - \frac{x}{x^2+y^2}\hat{j}$. Let $\gamma, \alpha\colon [0,1] \to \mathbb{R}^2$ be defined by

$$\gamma(t) = (8\cos 2\pi t, 17\sin 2\pi t) \text{ and } \alpha(t) = (26\cos 2\pi t, -10\sin 2\pi t) \tag{7}$$

If $3\int_\alpha \mathbf{F}\cdot d\mathbf{r} - 4\int_\gamma \mathbf{F}\cdot d\mathbf{r} = 2m\pi$ then m is ____________

46) Let $g\colon \mathbb{R}^3 \to \mathbb{R}$ be defined by $g(x,y,z) = (3y+4z, 2x-3z, x+3y)$ and let

$$S = \left\{(x,y,z) \in \mathbb{R}^3 \colon 0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1\right\} \tag{8}$$

If

$$\iiint\limits_{g(s)} (2x+y-2z)\, dxdydz = \alpha \iiint\limits_{S} z dxdydz \tag{9}$$

Then α is ____________

47) Let $T_1, T_2\colon \mathbb{R}^5 \to \mathbb{R}^3$ be the linear transformations such that $rank(T_1) = 3$ and $nullity(T_2) = 3$. Let $T_3\colon \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that $T_3 \circ T_1 = T_2$ Then $rank(T_3)$ is ____________

48) Let $\mathbb{F}_3$ be the field of 3 elements and let $\mathbb{F}_3 \times \mathbb{F}_3$ be a vector space of $\mathbb{F}_3$. Then the number of distinct linearly dependent sets of the form $\{u,v\}$,where $u, v \in \mathbb{F}_3 \times \mathbb{F}_3/\{(0,0)\}$ and $u \ne v$ is ____________

49) Let $\mathbb{R}_{125}$ be a field of 125 elements. Then number of non-zero elements $\alpha \in \mathbb{F}_{125}$ such that $\alpha^5 = \alpha$ is ____________

50) The value of $\iint\limits_R xydxdy$, where R is the region in the first quadrant bounded by the curves $y = x^2, y + x = 2$ and $x = 0$ is ____________

51) Consider the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < \pi, t > 0 \tag{10}$$

with the boundary conditions $u(0,t) = 0, u(\pi,t) = 0$ for $t > 0$ and the intial condition $u(x,0) = \sin x$. Then $u\left(\frac{\pi}{2}, 1\right)$ is ____________

52) Consider the partial order in $\mathbb{R}^2$ given by the relation $(x_1,y_1) < (x_2,y_2)$ EITHER if $x_1 < x_2$ OR if $x_1 = x_2$ and $y_1 < y_2$. Then the order topology on $\mathbb{R}^2$ defined by the above order

a) $[0,1]\times\{1\}$ is compact but $[0,1]\times[0,1]$ is NOT compact

b) $[0,1]\times[0,1]$ is compact but $[0,1]\times\{1\}$ is NOT compact

c) both $[0,1]\times[0,1]$ and $[0,1]\times\{1\}$ are compact

d) both $[0,1]\times[0,1]$ and $[0,1]\times\{1\}$ are NOT compact