ASSIGNMENT 8

EE24BTECH11011 - PRANAY

27) The probability density function of the random vector (X, Y) is given by
$$f_{X,Y}(x, y) = \begin{cases} c, 0 < x < y < 1 \\ 0, \text{otherwise} \end{cases} \tag{1}$$
Then the value of c is equal to ...
28) Let $\{X_n\}_{n\geq 1}$ be a sequence of independent and identically distributed normal random variables with mean 4 and variance 1 . Then $\lim\limits_{n\to\infty} P\left(\frac{1}{n}\sum_{i=1}^{n} X_i > 4.006\right)$ is equal to ...
29) Let (X_1, X_2) be a random vector following bivariate normal distribution with mean vector $(0, 0)$,Variance (X_1) = Variance $(X_2) = 1$ and corelation coeffecient ρ , where $|\rho| < 1$.Then $P(X_1 + X_2 > 0)$ is equal to ...
30) Let $X_1, \ldots, X_n$ be a random sample from normal distribution with mean μ and variance 1. Let ϕ be thw cumulative distribution function of the standard normal distribution.Given $\phi(1.96) = 0.975$,the minimum sample size required such that the length of the 95% confidence interval for μ does NOT exceed 2 is ...
31) Let X be a random variable with probability density function $f(x; \theta) = \theta e^{-\theta x}$, where $x \geq 0$ and $\theta > 0$. To test $H_o: \theta = 1$ against $H_1: \theta > 1$,the following test is used :
$$\text{Reject } H_o \text{ if and only if } X > log_e 20 \tag{2}$$
Then the size of the test is ...
32) Let $\{X_n\}_{n\geq 0}$ be a discrete time Markov chain on the square space $\{1, 2, 3\}$ with one-step transisition probability matrix
$$\begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 0.4 & 0.3 & 0.3 \\ 0.5 & 0.2 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{pmatrix} \end{array} \tag{3}$$
and initial distribution $P(X_0 = 1) = 0.5$, $P(X_0 = 2) = 0.2$, $P(X_0 = 3) = 0.3$. Then $P(X_1 = 2, X_2 = 3, X_3 = 1)$ (rounded off to three decimal places) is equal to ...
33) Let f be a continuous and positive real-valued function on $[0, 1]$. Then
$$\int_0^1 f(\sin x)\cos x\, dx \tag{4}$$
is equal to ...
34) A random sample of size 100 is classified into 10 class intervals covering all the data points. To test whether the data comes from a normal population with unknown mean and unknown variance, the chi-squared goodness of fit test is used. The degrees of freedom of the test statistic is equal to ...
35) For $i = 1, 2, 3, 4$, let $Y_i = \alpha + \beta x_i + \varepsilon_i$ where x_i's are fixed covariates and ε_i's are uncorrelated random variables with mean 0 and variance 3. Here, α and β are unknown parameters. Given the following observations, the variance of the least squares estimator of β is equal to ...

Y_i	2	2.5	-0.5	1
x_i	3	2	-4	-1

36) Let $a_n = \frac{(-1)^{n+1}}{n!}, n \geq 0$ and $b_n = \sum_{k=0}^{n} a_k, n \geq 0$. Then,for $|x| < 1$, the series $\sum_{n=0}^{\infty} b_n x^n$ converges to

a) $\frac{-e^{-x}}{1+x}$ b) $\frac{-e^{-x}}{1+x^2}$ c) $\frac{-e^{-x}}{1-x}$ d) $-(1+x)\,e^{-x}$

37) Let $\{X_k\}_{k\geq 1}$ be a sequence of independent and indentically distributes Bernoulli random variables with success probability $p \in (0,1)$.Then as $n \to \infty$

$$\frac{1}{n}\sum_{k=1}^{n}(X_k)^k \tag{5}$$

converges almost surely to

a) p b) $\frac{1}{1-p}$ c) $\frac{1-p}{p}$ d) 1

38) Let X and Y be two independent random variables with χ^2_m and χ^2_n distributions, respectively, where m and n are positive integers. Then which of the following statements is true?

a) For $m < n, P(X > a) \geq P(Y > a)$ for all $a \in \mathbb{R}$.
b) For $m > n, P(X > a) \geq P(Y > a)$ for all $a \in \mathbb{R}$.
c) For $m < n, P(X > a) = P(Y > a)$ for all $a \in \mathbb{R}$.
d) None of the above.

39) The matrix

$$\begin{pmatrix} 1 & x & z \\ 0 & 2 & y \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

is diagonalizable when (x, y, z) equals

a) $(0,0,1)$
b) $(1,1,0)$
c) $\left(\sqrt{2},\ \sqrt{2}, 2\right)$
d) $\left(\sqrt{2},\ \sqrt{2},\ \sqrt{2}\right)$