Solution to System of Equations

EE24BTECH11011 - B. Pranay Kumar

Problem Statement

Find the solution of the system of equations:

$$\frac{3}{x} + \frac{8}{y} = -1, \tag{1}$$

$$\frac{1}{x} - \frac{2}{y} = 2, \quad x, y \neq 0. \tag{2}$$

Variable Substitution

To simplify, let:

$$a = \frac{1}{x},$$
$$b = \frac{1}{y}.$$

The equations then become:

$$3a + 8b = -1, \tag{3}$$
$$a - 2b = 2. \tag{4}$$

Matrix Representation of the Problem

We can represent the system in matrix form as:

$$\begin{pmatrix} 3 & 8 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Solving Using Row Operations

Performing row operations to solve for a and b:

$$\begin{pmatrix} 3 & 8 & -1 \\ 1 & -2 & 2 \end{pmatrix} \xrightarrow{R_2 \to 3R_2 - R_1} \begin{pmatrix} 3 & 8 & -1 \\ 0 & -14 & 7 \end{pmatrix}$$

Then,

$$\begin{pmatrix} 3 & 8 & -1 \\ 0 & -14 & 7 \end{pmatrix} \xrightarrow{R_2 \to \frac{R_2}{-7}} \begin{pmatrix} 3 & 8 & -1 \\ 0 & 2 & -1 \end{pmatrix}$$

$$\xrightarrow{R_1 \to R_1 - 4R_2} \begin{pmatrix} 3 & 0 & 3 \\ 0 & 2 & -1 \end{pmatrix}$$

Final Solution

The matrix simplifies to:

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -\frac{1}{2} \end{pmatrix}$$

This gives:

$$a = 1,$$
$$b = -\frac{1}{2}.$$

Since $a = \frac{1}{x}$ and $b = \frac{1}{y}$, we find:

$$x = 1, \quad y = -2.$$

C Code for Calculating Points

The following C code calculates points for both equations to plot:

```c
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include "libs/matfun.h"
#include "libs/geofun.h"

double calculate_y1(double x) {
    if (x == 0) return 0;
    return 8 / (-1 - (3 / x));
}

double calculate_y2(double x) {
    if (x == 0) return 0;
    return 2 / (1/x - 2);
}

void plotPointsToFile(const char *filename,
    double **points, int num_points) {
    FILE *file = fopen(filename, "w");
```

C Code for Generating Data Files

Main function to generate data files for plotting:

```c
int main() {
    double x_start = -10.0, x_end = 10.0,
        step = 0.1;
    int num_points = (int)((x_end - x_start)
        / step) + 1;
    double **points1 = createMat(num_points,
        2);
    double **points2 = createMat(num_points,
        2);

    if (points1 == NULL || points2 == NULL)
        {
        printf("Error: Memory allocation
            failed.\n");
        return 1;
        }

    int index = 0;
    for (double x = x_start; x <= x_end; x
```

Python Code for Plotting

This Python code reads the data files and plots the functions and intersection:

```python
import numpy as np
from scipy.optimize import fsolve
import matplotlib.pyplot as plt

def equations(vars):
    x, y = vars
    return [(3 / x) + (8 / y) + 1, (1 / x) -
        (2 / y) - 2]

guesses = [(1, 1), (-1, -1)]
intersection_points = [fsolve(equations,
    guess) for guess in guesses]

def read_points(filename):
    x_points, y_points = [], []
    with open(filename, 'r') as file:
        for line in file:
            x, y = map(float, line.split())
```
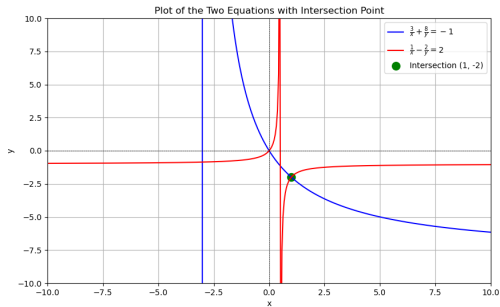
Graphical Solution

Figure: Graphical representation of the system of equations