29 JAN 2024-2

EE24BTECH11011-B.PRANAY KUMAR

1) Let $\mathbf{OA} = \mathbf{a}, \mathbf{OB} = 12\mathbf{a} + 4\mathbf{b}$ and $\mathbf{OC} = \mathbf{b}$, where $\mathbf{O}$ is the origin.If S is the parallelogram with adjacent sides $\mathbf{OA}$ and $\mathbf{OC}$, then $\frac{\text{area of the quadrilateral } OABC}{\text{area of } S}$ is equal to

   a) 8
   b) 7
   c) 6
   d) 10

2) Let a unit vector $\mathbf{u} = x\hat{i} + y\hat{j} + z\hat{k}$ makes angles $\frac{\pi}{2}\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}, \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ respectively.If $\mathbf{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ then $|\mathbf{u} - \mathbf{v}|^2$ is equal to

   a) 9
   b) $\frac{5}{2}$
   c) 7
   d) $\frac{11}{2}$

3) The function $f(x) = 2x + 3(x)^{\frac{2}{3}}, x \in \mathbb{R}$ has
   a) exactly one point of local minima and no point of local laxima
   b) exactly one point of local maxima and exactly one point of local minima
   c) exactly one point of local maxima and no point of local minima
   d) exactly two points of local maxima and exactly one point of local minima

4) If each term of a geometric peogression $a_1, a_2, a_3, \ldots$ with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$ is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \cdots + a_n$ then $S_{20} - S_{18}$ is equal to

   a) 2^{18}
   b) -2^{18}
   c) 2^{15}
   d) -2^{15}

5) If $\log_e a, \log_e b, log_e c$ are in A.P and $log_e a - \log_e 2b, \log_e 2b - \log_e 3c, \log_e 3c - \log_e a$ are also in an A.P then $a : b : c$ is equal to
   a) 16: 4: 1
   b) 6: 3: 2
   c) 25: 10: 4
   d) 9: 6: 4

6) Let $\mathbf{A}(3, 2, 3), \mathbf{Q}(4, 6, 2)$ and $\mathbf{R}(7, 3, 2)$ be the vertices of $\triangle PQR$.Then the angle $\angle QPR$ is
   a) $\cos^{-1}\left(\frac{1}{18}\right)$

b) $\frac{\pi}{3}$
c) $\frac{\pi}{6}$
d) $\cos^{-1}\left(\frac{7}{18}\right)$

7) Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

a) 16
b) 18
c) 15
d) 12

8) The distance of the point $(2, 3)$ from the line $2x - 3y + 28 = 0$ measured parallel to the line $\sqrt{3}x - y + 1 = 0$ is equal to

a) $4 + 6\sqrt{3}$
b) $3 + 4\sqrt{2}$
c) $6\sqrt{3}$
d) $4\sqrt{2}$

9) Let $y = log_e\left(\frac{1-x^2}{1+x^2}\right), -1 < x < 1$. Then at $x = \frac{1}{2}$, then the value off $225\left(y' - y''\right)$ is equal to

a) 736
b) 746
c) 732
d) 742

10) If

$$\int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x \sin(x - \theta)}} dx = A\sqrt{\cos\theta\tan x - \sin\theta} + B\sqrt{\cos\theta - \sin\theta\cot x} + C \tag{10.1}$$

where C is the integration constant, then AB is equal to

a) $4\operatorname{cosec}(2\theta)$
b) $4\sec\theta$
c) $2\sec\theta$
d) $8\operatorname{cosec}(2\theta)$

11) If $\mathbf{R}$ is the smallest equivalence relation on the set $\{1, 2, 3, 4\}$ such that $\{(1, 2), (1, 3)\} \subset \mathbf{R}$, then the number of elements in $\mathbf{R}$ is

a) 12
b) 15
c) 8
d) 10

12) Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos\left(2\sin^{-1} x\right) = \frac{1}{9}$ and let $\alpha, \beta\, (\alpha > \beta)$ be the roots of the equation $mx^2 - nx - m + n = 0$.Then the point (α, β) lies on the line

a) $5x + 8y = 9$
b) $3x - 3y = -2$
c) $5x - 8y = -9$

d) $3x + 2y = 2$

13) The sum of the solutions $x \in \mathbb{R}$ of the equation

$$\frac{3\cos 2x + \cos^3 2x}{\cos^{6x} - \sin^3 6x} = x^3 - x^2 + 6 \tag{13.1}$$

is

a) -1
b) 1
c) 3
d) 0

14) Let $\mathbf{A} = \begin{pmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{pmatrix}$ and $\mathbf{P} = \begin{pmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{pmatrix}$. The sum of the prime factors of $\left|\mathbf{P}^{-1}\mathbf{A}\mathbf{P} - 2\mathbf{I}\right|$ is equal to

a) 26
b) 66
c) 23
d) 27

15) An integer is chosen at random from the integers $1, 2, 3, \ldots, 50$. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is

a) $\frac{9}{50}$

b) $\frac{8}{25}$

c) $\frac{21}{50}$

d) $\frac{14}{25}$