ASSIGNMENT 1

EE24BTECH11011 - PRANAY

1) If the boolean expression $(p \wedge q) \odot (p \oplus q)$ is a tautology , then $\odot$ and $\oplus$ are respectively given by :

   a) $\wedge, \rightarrow$
   b) $\rightarrow, \rightarrow$
   c) $\vee, \rightarrow$
   d) $\vee, \wedge$

2) Let the tangent to the circle $x^2 + y^2 = 25$ at the point $\mathbf{R}(3, 4)$ meet $x-$axis and $y-$axis at points $\mathbf{P}$ and $\mathbf{Q}$, respectively. If r is the radius of the circle passing through the origin $\mathbf{O}$ and having the centre at the incentre of triangle OPQ, then r^2 is equal to :

   a) $\frac{625}{72}$
   b) $\frac{585}{66}$
   c) $\frac{125}{72}$
   d) $\frac{529}{64}$

3) Let a computer program generate only the digits 0 and 1 to form a string of numbers with probability of occurence of 0 at even places be $\frac{1}{2}$ and probability of occurence of 0 at the odd place be $\frac{1}{3}$. Then the probability that $'10'$ is followed by $'01'$ is equal to :

   a) $\frac{1}{6}$
   b) $\frac{1}{18}$
   c) $\frac{1}{9}$
   d) $\frac{1}{3}$

4) The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$

   a) 5
   b) 2
   c) 4
   d) 3

5) If the equation of plane passing through the mirror image of point $(2, 3, 1)$ with respect to the line $\frac{x+1}{2} = \frac{y-3}{2} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{3} = \frac{z+1}{2}$ is $\alpha x + \beta y + \gamma z = 24$ then $\alpha + \beta + \gamma$ is equal to :

   a) 21
   b) 19
   c) 18
   d) 20

6) Consider the function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = \begin{cases} \left(2 - \sin\left(\frac{1}{x}\right)\right)|x| & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$. Then f is

   a) monotonic on $(0,\infty)$ only
   b) Non monotonic on $(-\infty, 0)$ and $(0, \infty)$
   c) monotonic on $(-\infty, 0)$
   d) monotonic on $(-\infty, 0) \cup (0, \infty)$

7) Let $\mathbf{O}$ be the origin . Let $\mathbf{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\mathbf{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}, x, y \in \mathbf{R}, x > 0$ be such that $|\mathbf{PQ}| = \sqrt{20}$ and the vector $\mathbf{OP}$ is perpendicular to $\mathbf{OQ}$. If $\mathbf{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}, z \in \mathbf{R}$, is coplanar with $\mathbf{OP}$ and $\mathbf{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to :

a) 2
c) 1
b) 9
d) 7

8) Let $\mathbf{L}$ be a tangent line to the parabola $y^2 = 4x - 20$ at $(6, 2)$ If $\mathbf{L}$ is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$, then the value of b is equal to :

a) 20
d) 11
b) 14
c) 16

9) Let $f : \mathbf{R} \to \mathbf{R}$ be defined as $f(x) = e^{-x} \sin x$. If $F : [0,1] \to \mathbf{R}$ is a differentiable function such that $F(x) = \int_0^x f(t)\, dt$, Then the value of $\int_0^1 (F(x) + f(x))\, e^x dx$ lies in the interval :

a) $\left[\frac{330}{360}, \frac{331}{360}\right]$
c) $\left[\frac{327}{360}, \frac{329}{360}\right]$

b) $\left[\frac{331}{360}, \frac{334}{360}\right]$
d) $\left[\frac{335}{360}, \frac{336}{360}\right]$

10) If x, y, z are in arithmetic progression with the common difference $d, x \neq 3d$ and the determinent of the matrix $\begin{pmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{pmatrix}$ is zero , then the value of k^2 is :

a) 6
c) 72
b) 36
d) 12

11) If the integral $\int_0^{10} \frac{[\sin 2\pi x]}{e^{|x|}} dx = \alpha e^{-1} + \beta e^{\frac{-1}{2}} + \gamma$, where α, β, γ are integers and $[x]$ denotes the greatest integer less than or equal to x , then the value of $\alpha + \beta + \gamma$ is equal to :

a) 20
c) 25
b) 0
d) 10

12) Let $y = y(x)$ be the solution of the differential equation $(\cos(3 \sin x + \cos x + 3))\, dy = (1 + y \sin x (3 \sin x + \cos x + 3))\, dx$, $0 \leq x \leq \frac{\pi}{2}, y(0) = 0$. Then $y\left(\frac{\pi}{3}\right)$ is equal to:

a) $3 \log_e \left(\frac{2\sqrt{3}+10}{11}\right)$
c) $2 \log_e \left(\frac{3\sqrt{3}-8}{4}\right)$

b) $2 \log_e \left(\frac{\sqrt{3}+7}{2}\right)$
d) $3 \log_e \left(\frac{2\sqrt{3}+9}{6}\right)$

13) The value of the limit $\lim\limits_{x \to 0} \frac{\tan\left(\pi \cos^2 \theta\right)}{\sin\left(2\pi \sin^2 \theta\right)}$ is equal to :

a) $\frac{-1}{2}$
c) 0

b) $\frac{-1}{4}$
d) $\frac{1}{4}$

14) If the curve $y = y(x)$ is the solution of the differential equation $2(x^2 + x^{\frac{5}{4}})dy - y(x + x^{\frac{1}{4}})dx = 2x^{\frac{9}{4}}$, $x > 0$ which passes through the point $\left(1, 1 - \frac{4}{3} \log_e 2\right)$ then the value of $y(16)$ is equal to :

a) $\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

b) $4\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

c) $\left(\frac{31}{3} + \frac{8}{3}\log_e 3\right)$

d) $4\left(\frac{31}{3} - \frac{8}{3}\log_e 3\right)$

15) Let S_1,S_2andS_3 be three sets defined as

$$S_1 = \left\{z \in \mathbb{C} : |z - 1| \leq \sqrt{2}\right\}$$
$$S_2 = \{z \in \mathbb{C} : \mathrm{Re}\left((1 - i)\, z\right) \geq 1\}$$
$$S_3 = \{z \in \mathbb{C} : \mathrm{Im}\,(z) \leq 1\}$$

Then the set $S_1 \cap S_2 \cap S_3$

a) Has infinitely many elements
b) Has exactly 2 elements
c) has exactly 3 elements
d) is singleton