6 APRIL 2023-1

EE24BTECH11011 - PRANAY

1) The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line $\mathbf{L} : 9x+5y = 45$ between the axes. If m_1 and m_2 are slopes of the lines l_1 and l_2 then the point of intersection of line $y = (m_1 + m_2)\, x$ with $\mathbf{L}$ lies on :

   a) $6x + y = 10$
   b) $6x - y = 15$
   c) $y - 2x = 5$
   d) $y - x = 5$

2) Let the position vectors of the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ be $5\hat{i} + 5\hat{j} + 2\lambda\hat{k}, \hat{i} + 2\hat{j} + 3\hat{k}, -2\hat{i} + \lambda\hat{j} + 4\hat{k}$ and $-\hat{i} + 5\hat{j} + 6\hat{k}$. Let the set $\mathbf{S} = \{\lambda \in \mathbb{R}$: the points $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ are coplanar$\}$. Then $\sum_{\lambda \in \mathbf{S}} (\lambda + 2)^2$ is equal to :

   a) $\frac{37}{2}$
   b) 13
   c) 25
   d) 41

3) Let

$$\mathbf{I}(x) = \int \frac{x^2 \left(x \sec^2 x + \tan x\right)}{(x \tan x + 1)^2} dx \tag{1}$$

   If $\mathbf{I}(0) = 0$, then $\mathbf{I}\left(\frac{\pi}{4}\right)$ is equal to :

   a) $\log_e \frac{(\pi+4)^2}{16} + \frac{\pi^2}{4(\pi+4)}$
   b) $\log_e \frac{(\pi+4)^2}{32} - \frac{\pi^2}{4(\pi+4)}$
   c) $\log_e \frac{(\pi+4)^2}{16} - \frac{\pi^2}{4(\pi+4)}$
   d) $\log_e \frac{(\pi+4)^2}{32} + \frac{\pi^2}{4(\pi+4)}$

4) The sum of the first 20 terms of the series $5 + 11 + 19 + 29 + 41 + \dots$ is

   a) 3450
   b) 3420
   c) 3520
   d) 3250

5) A pair of dice is thrown 5 times. For each throw , a total of 5 is considered a success.If probability of a least 4 success is $\frac{k}{3^{11}}$, the k is equal to :

   a) 164
   b) 123
   c) 82
   d) 75

6) Let $\mathbf{A} = \left(a_{ij}\right)_{2\times2}$, where $a_{ij} \neq 0$ for all i, j and $\mathbf{A^2} = \mathbf{I}$. Let a be the sum of all diagonal elements of $\mathbf{A}$ and $b = \left|A\right|$ Then $3a^2 + 4b^2$ is equal to :

a) 14
b) 4
c) 3
d) 7

7) Let $a_1, a_2, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. If $d > 0$ is its common difference then

$$\lim_{a\to\infty} \sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}\cdots\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right) \tag{2}$$

is :

a) $\frac{1}{\sqrt{2}}$
b) 1
c) $\sqrt{d}$
d) 0

8) If ${}^{2n}C_3 : {}^{n}C_3 : 10 : 1$, then the ratio $\left(n^2+3n\right) : \left(n^2-3n+4\right)$ is :

a) 27: 11
b) 35: 16
c) 2: 1
d) 65: 37

9) Let $A = \{x \in \mathbb{R} : [x+3] + [x+4] \le 3\}$, $B = \left\{x \in \mathbb{R} : 3^x \left(\sum_{r=1}^{\infty} \frac{3}{10^r}\right) < 3^{-3x}\right\}$, where $[t]$ denotes greatest integer function. Then ,

a) $A \subset B, A \neq B$
b) $A \cap B = \emptyset$
c) $A = B$
d) $B \subset C, A \neq B$

10) One vertex of a rectangular parallelopiped is at the origin **O** and the length of its edges along x, y and z axes are $3, 4$ and 5 respectively.Let **P** be the vertex $(3, 4, 5)$ .Then the shortest distance between the diagonal **OP** and an edge parallel to z axis , not passisng through **O** or **P** is

a) $\frac{12}{5\sqrt{5}}$
b) $12\sqrt{5}$
c) $\frac{12}{5}$
d) $\frac{12}{\sqrt{5}}$

11) If the equation of the plane passing through the line of intersection of planes $2x - y + z = 3, 4x - 3y + 5z + 9 = 0$ and parallel to the line

$$\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5} \tag{3}$$

is $ax + by + cz + 6 = 0$ then $a + b + c$ is equal to

a) 15
b) 14
c) 13
d) 12

12) If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{x} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6} : 1$, then the third term from the beggining is

a) $30\sqrt{2}$
b) $60\sqrt{2}$
c) $30\sqrt{3}$
d) $60\sqrt{3}$

13) The sum of all the roots of the equation $\left|x^2 - 8x + 15\right| - 2x + 7 = 0$ is

a) $11 - \sqrt{3}$
b) $9 - \sqrt{3}$
c) $9 + \sqrt{3}$
d) $11 + \sqrt{3}$

14) From the top A of a vertical wall AB of height 30m, the angles of depression of the top P and bottom Q of a vertical tower PQ are $15°$ and $60°$ respectively , B and Q are on the same horizontal level . If C is a point on AB such that $CB = PQ$, then the area $\left(\text{in } m^2\right)$ of the quadrilateral $BCPQ$ is equal to

a) $200\left(3 - \sqrt{3}\right)$
b) $300\left(\sqrt{3} + 1\right)$
c) $300\left(\sqrt{3} - 1\right)$
d) $600\left(\sqrt{3} - 1\right)$

15) Let $\mathbf{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}, \mathbf{b} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\mathbf{c} = -\hat{i} + 4\hat{j} + 3\hat{k}$. If $\mathbf{d}$ is a vector perpendicular to both $\mathbf{b}$ and $\mathbf{c}$, and $\mathbf{a} \cdot \mathbf{d} = 18$ then $[\mathbf{a} \times \mathbf{d}]^2$ is equal to

a) 760
b) 640
c) 720
d) 680