3.3.20

EE24BTECH11011-B.PRANAY KUMAR

Question:
Draw a triangle PQR in which $QR = 3$ cm, $QP - PR = 6$ cm, and $\angle PQR = 45°$.

Solution:

For triangle PQR with $QR = 3$ cm, $QP - PR = 6$ cm, and $\angle PQR = 45°$.
Using law of cosines:

$$QP^2 = QR^2 + PR^2 - 2(QR)(PR)\angle PQR \tag{0.1}$$

$$K(QP + PR) = QR^2 - 2(QR)(PR)\angle PQR \tag{0.2}$$

Where,

$$K = QP - PR \tag{0.3}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} QP \\ PR \end{pmatrix} = \begin{pmatrix} QR^2 - 2(QR)(PR)\cos PQR \\ K \end{pmatrix} \tag{0.4}$$

$$\begin{pmatrix} QP \\ PR \end{pmatrix} = \frac{1}{K}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} QR^2 - 2(QR)(PR)\cos PQR \\ K \end{pmatrix} \tag{0.5}$$

$$\because \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = 2I \tag{0.6}$$

$$PR = \frac{1}{2(1 + \frac{2(QR)\ COS\angle(PQR)}{K})}\mathbf{e}^\top \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} \frac{QR^2}{K} \\ K \end{pmatrix} \tag{0.7}$$

The coordinates of $\triangle PQR$ are

$$\mathbf{P} = PR\begin{pmatrix} Cos\angle PQR \\ Sin\angle PQR \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{R} = \begin{pmatrix} QR \\ 0 \end{pmatrix} \tag{0.8}$$

substituting $QR = 3$ and $K = -6$ in (0.7)

$$QR = \frac{45}{6(2 - \sqrt{2})} \tag{0.9}$$

$$QR = 11.803300859 \tag{0.10}$$

Hence;

$$\mathbf{P} = \frac{11.803300859}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \tag{0.11}$$

$$\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{0.12}$$

$$\mathbf{R} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}. \tag{0.13}$$

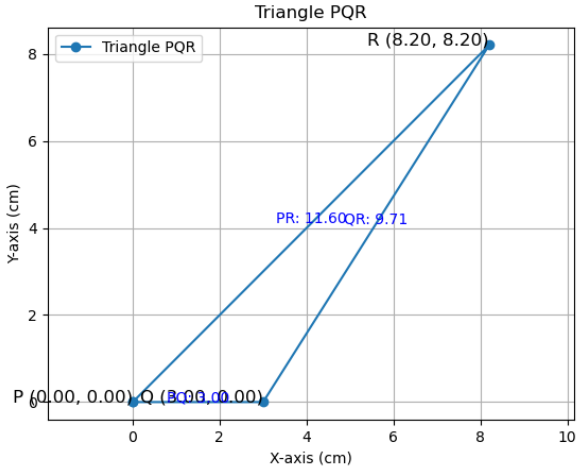

Fig. 0.1: Triangle *PQR*