

ASSIGNMENT 1

EE24BTECH11011 - PRANAY

1) If the boolean expression $(p \wedge q) \odot (p \oplus q)$ is a tautology, then \odot and \oplus are respectively given by :

- | | |
|-------------------------------|------------------------|
| a) \wedge, \rightarrow | c) \vee, \rightarrow |
| b) \rightarrow, \rightarrow | d) \vee, \wedge |

2) Let the tangent to the circle $x^2 + y^2 = 25$ at the point $\mathbf{R}(3,4)$ meet x – axis and y – axis at points \mathbf{P} and \mathbf{Q} , respectively. If r is the radius of the circle passing through the origin \mathbf{O} and having the centre at the incentre of triangle OPQ , then r^2 is equal to :

- | | |
|---------------------|---------------------|
| a) $\frac{625}{72}$ | c) $\frac{125}{72}$ |
| b) $\frac{585}{66}$ | d) $\frac{529}{64}$ |

3) Let a computer program generate only the digits 0 and 1 to form a string of numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to :

- | | |
|-------------------|------------------|
| a) $\frac{1}{6}$ | c) $\frac{1}{9}$ |
| b) $\frac{1}{18}$ | d) $\frac{1}{3}$ |

4) The number of solutions of the equation $x + 2 \tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$

- | | |
|------|------|
| a) 5 | c) 4 |
| b) 2 | d) 3 |

5) If the equation of plane passing through the mirror image of point $(2, 3, 1)$ with respect to the line

$$\frac{x+1}{2} = \frac{y-3}{2} = \frac{z+2}{-1} \quad (1)$$

and containing the line

$$\frac{x-2}{3} = \frac{1-y}{3} = \frac{z+1}{2} \quad (2)$$

is $\alpha x + \beta y + \gamma z = 24$ then $\alpha + \beta + \gamma$ is equal to :

- | | |
|-------|-------|
| a) 21 | c) 18 |
| b) 19 | d) 20 |

6) Consider the function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f(x) = \begin{cases} \left(2 - \sin\left(\frac{1}{x}\right)\right)|x| & , x \neq 0 \\ 0 & , x = 0 \end{cases}$. Then f is

- a) monotonic on $(0, \infty)$ only
 b) Non monotonic on $(-\infty, 0)$ and $(0, \infty)$

- c) monotonic on $(-\infty, 0)$
 d) monotonic on $(-\infty, 0) \cup (0, \infty)$

7) Let \mathbf{O} be the origin . Let $\mathbf{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\mathbf{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, $x, y \in \mathbf{R}$, $x > 0$ be such that $|\mathbf{PQ}| = \sqrt{20}$ and the vector \mathbf{OP} is perpendicular to \mathbf{OQ} . If $\mathbf{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, $z \in \mathbf{R}$, is coplanar with \mathbf{OP} and \mathbf{OQ} , then the value of $x^2 + y^2 + z^2$ is equal to :

- a) 2
 b) 9
 c) 1
 d) 7

8) Let \mathbf{L} be a tangent line to the parabola $y^2 = 4x - 20$ at $(6, 2)$ If \mathbf{L} is also a tangent to the ellipse $\frac{x^2}{2} + \frac{y^2}{b} = 1$, then the value of b is equal to :

- a) 20
 b) 14
 c) 16
 d) 11

9) Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be defined as $f(x) = e^{-x} \sin x$. If $F : [0, 1] \rightarrow \mathbf{R}$ is a differentiable function such that $F(x) = \int_0^x f(t) dt$, Then the value of $\int_0^1 (F(x) + f(x)) e^x dx$ lies in the interval :

- a) $\left[\frac{330}{360}, \frac{331}{360} \right]$
 b) $\left[\frac{331}{360}, \frac{334}{360} \right]$
 c) $\left[\frac{327}{360}, \frac{329}{360} \right]$
 d) $\left[\frac{335}{360}, \frac{336}{360} \right]$

10) If x, y, z are in arithmetic progression with the common difference d , $x \neq 3d$ and the determinant of the matrix $\begin{pmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{pmatrix}$ is zero , then the value of k^2 is :

- a) 6
 b) 36
 c) 72
 d) 12

11) If the integral

$$\int_0^{10} \frac{[\sin 2\pi x]}{e^{|x|}} dx = \alpha e^{-1} + \beta e^{\frac{-1}{2}} + \gamma \quad (3)$$

, where α, β, γ are integers and $[x]$ denotes the greatest integer less than or equal to x , then the value of $\alpha + \beta + \gamma$ is equal to :

- a) 20
 b) 0
 c) 25
 d) 10

12) Let $y = y(x)$ be the solution of the differential equation

$$(\cos 3 \sin x + \cos x + 3) dy = (1 + y \sin x (3 \sin x + \cos x + 3)) dx, 0 \leq x \leq \frac{\pi}{2}, y(0) = 0 \quad (4)$$

Then $y\left(\frac{\pi}{3}\right)$ is equal to:

a) $3 \log_e \left(\frac{2\sqrt{3}+10}{11} \right)$

c) $2 \log_e \left(\frac{3\sqrt{3}-8}{4} \right)$

b) $2 \log_e \left(\frac{\sqrt{3}+7}{2} \right)$

d) $3 \log_e \left(\frac{2\sqrt{3}+9}{6} \right)$

13) The value of the limit $\lim_{x \rightarrow 0} \frac{\tan(\pi \cos^2 \theta)}{\sin(2\pi \sin^2 \theta)}$ is equal to :

a) $\frac{-1}{2}$

c) 0

b) $\frac{-1}{4}$

d) $\frac{1}{4}$

14) If the curve $y = y(x)$ is the solution of the differential equation

$$2(x^2 + x^{\frac{5}{4}})dy - y(x + x^{\frac{1}{4}})dx = 2x^{\frac{9}{4}}, x > 0 \quad (5)$$

which passes through the point $(1, 1 - \frac{4}{3} \log_e 2)$ then the value of $y(16)$ is equal to :

a) $\left(\frac{31}{3} - \frac{8}{3} \log_e 3 \right)$

c) $\left(\frac{31}{3} + \frac{8}{3} \log_e 3 \right)$

b) $4 \left(\frac{31}{3} + \frac{8}{3} \log_e 3 \right)$

d) $4 \left(\frac{31}{3} - \frac{8}{3} \log_e 3 \right)$

15) Let S_1, S_2 and S_3 be three sets defined as

$$S_1 = \{z \in \mathbb{C} : |z - 1| \leq \sqrt{2}\}$$

$$S_2 = \{z \in \mathbb{C} : \operatorname{Re}((1 - i)z) \geq 1\}$$

$$S_3 = \{z \in \mathbb{C} : \operatorname{Im}(z) \leq 1\}$$

Then the set $S_1 \cap S_2 \cap S_3$

a) Has infinitely many elements

b) Has exactly 2 elements

c) has exactly 3 elements

d) is singleton