ASSIGNMENT 9

EE24BTECH11011 - PRANAY

40) Let $X_1, X_2, \ldots, X_{10}$ be a random sample size 10 from a population having $N\left(0, \theta^2\right)$ distribution where $\theta > 0$ is and unknowm parameter.Let $T = \frac{1}{10}\sum_{i=1}^{10} X_i{}^2$. If the mean square error of $cT\,(c > 0)$, as an estimator of θ^2, is minimized at $c = c_0$, then the value of c_0 equals

    a) $\frac{5}{6}$ b) $\frac{2}{3}$ c) $\frac{3}{5}$ d) $\frac{1}{2}$

41) Suppose that $X_1, X_2, \ldots, X_{10}$ are independent and identically istributed random vectors each having $N_2(\mu, \sum)$ distribution, where $\sum$ is non-singular. If

$$U = \frac{1}{1 + \left(\overline{X} - \mu\right)^{\top} \sum^{-1} \left(\overline{X} - \mu\right)} \tag{1}$$

where $\overline{X} = \frac{1}{10}\sum_{i=1}^{10} X_i$ then the value of $\log_e P\left(U \leq \frac{1}{2}\right)$ equals

    a) -5 b) -10 c) -2 d) -1

42) Suppose that (X, Y) has joint probability mass function

$$P(X = 0, Y = 0) = P(X = 1, Y = 1) = \theta, \tag{2}$$

$$P(X = 1, Y = 0) = P(X = 0, Y = 1) = \frac{1}{2} - \theta, \tag{3}$$

where $0 \leq \theta \leq \frac{1}{2}$ is an unknown parameter. Consider testing $H_0 : \theta = \frac{1}{4}$ against $H_1 : \theta = \frac{1}{3}$ based on a random sample $\{(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)\}$ from the above probability mass function. Let M be the cardinality of the set $\{i : X_i = Y_i, 1 \leq i \leq n\}$. If m is the observed value of M, then which one of the following statements is true?

    a) The likelihood ratio test rejects H_0 if $m > c$ for some c
    b) The likelihood ratio test rejects H_0 if $m < c$ for some c
    c) The likelihood ratio test rejects H_0 if $c_1 < m < c_2$ for some c_1 and c_2
    d) The likelihood ratio test rejects H_0 if $m < c_1$ or $m > c_2$ for some c_1 and c_2

43) Let $g(x) = f(x) + f(2 - x)$ for all $x \in (0, 2)$, where $f\colon\ (0, 2) \to \mathbb{R}$ is continuous on $(0, 2)$ and twice differentiable on $(0, 2)$. If g' denotes the derivative of g and f'' denotes the second derivative of f, then which one of the following statements is NOT true?

    a) There exists $c \in (0, 2)$ such that $g'(c) = 0$
    b) If $f'' > 0$ on $(0, 2)$, then g is strictly decreasing on $(0, 1)$
    c) If $f'' < 0$ on $(0, 2)$, then g is strictly increasing on $(1, 2)$
    d) If $f'' = 0$ on $(0, 2)$, then g is a constant function

44) For any subset U of $\mathbb{R}^n$, let $L(U)$ denote the span of U. For any two subsets T and S of $\mathbb{R}^n$, which one of the following statements is NOT true?

    a) If T is a proper subset of S, then $L(T)$ is a proper subset of $L(S)$
    b) $L(L(S)) = L(S)$

c) $L(T \cup S) = \{u + v \colon u \in L(T), v \in L(S)\}$
d) If α, β, and γ are three vectors in $\mathbb{R}^n$ such that $\alpha + 2\beta + 3\gamma = 0$, then $L(\{\alpha, \beta\}) = L(\{\beta, \gamma\})$

45) Let f be a continuous function from $[0,1]$ to the set of all real numbers. Then which one of the following statements is NOT true?
a) For any sequence $\{x_n\}_{n\geq 1}$ in $[0,1]$, $\sum_{n=1}^{\infty} \frac{f(x_n)}{n^2}$ is absolutely convergent.
b) If $|f(x)| = 1$ for all $x \in [0,1]$, then $\left|\int_0^1 f(x)dx\right| = 1$.
c) If $\{x_n\}_{n\geq 1}$ is a sequence in $[0,1]$ such that $\{f(x_n)\}_{n\geq 1}$ is convergent, then $\{x_n\}_{n\geq 1}$ is convergent.
d) If f is also monotonically increasing, then the image of f is given by $[f(0), f(1)]$.

46) Let X be a random variable with cumulative distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < -1 \\ \frac{1}{4}(x+1) & \text{if } -1 \leq x < 0 \\ \frac{1}{4}(x+3) & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x \geq 1. \end{cases} \tag{4}$$

Which one of the following statements is true?
a) $\lim_{n\to\infty} P\left(-\frac{1}{2} + \frac{1}{n} < X < -\frac{1}{n}\right) = \frac{5}{8}$
b) $\lim_{n\to\infty} P\left(-\frac{1}{2} - \frac{1}{n} < X < \frac{1}{n}\right) = \frac{5}{8}$
c) $\lim_{n\to\infty} P\left(X = \frac{1}{n}\right) = \frac{1}{2}$
d) $P(X = 0) = \frac{1}{3}$

47) Let (X, Y) have joint probability mass function

$$p(x,y) = \begin{cases} \frac{c}{2^{x+y+2}} & \text{if } x = 0,1,2,...;\ y = 0,1,2,...;\ x \neq y \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

Then which one of the following statements is true?
a) $c = \frac{1}{2}$
b) $c = \frac{1}{4}$
c) $c > 1$
d) X and Y are independent

48) Let $X_1, X_2, ..., X_{10}$ be a random sample of size 10 from a $N_3(\mu, \Sigma)$ distribution, where μ and non-singular Σ are unknown parameters. If

$$\overline{X}_1 = \frac{1}{5}\sum_{i=1}^{5} X_i, \quad \overline{X}_2 = \frac{1}{5}\sum_{i=6}^{10} X_i, \tag{6}$$

$$S_1 = \frac{1}{4}\sum_{i=1}^{5}\left(X_i - \overline{X}_1\right)\left(X_i - \overline{X}_1\right)', \quad S_2 = \frac{1}{4}\sum_{i=6}^{10}\left(X_i - \overline{X}_2\right)\left(X_i - \overline{X}_2\right)', \tag{7}$$

then which one of the following statements is NOT true?
a) $\frac{5}{6}\left(\overline{X}_1 - \mu\right)' S_1^{-1}\left(\overline{X}_1 - \mu\right)$ follows an F-distribution with 3 and 2 degrees of freedom.
b) $\frac{6}{\left(\overline{X}_1-\mu\right)' S_1^{-1}\left(\overline{X}_1-\mu\right)}$ follows an F-distribution with 2 and 3 degrees of freedom.
c) $4(S_1 + S_2)$ follows a Wishart distribution of order 3 with 8 degrees of freedom.
d) $5(S_1 + S_2)$ follows a Wishart distribution of order 3 with 10 degrees of freedom.

49) Which of the following sets is/are countable?

a) The set of all functions from $\{1, 2, 3, ..., 10\}$ to the set of all rational numbers
b) The set of all functions from the set of all natural numbers to $\{0, 1\}$
c) The set of all integer valued sequences with only finitely many non-zero terms
d) The set of all integer valued sequences converging to 1

50) For a given real number a, let $a^+ = \max\{a, 0\}$ and $a^- = \max\{-a, 0\}$. If $\{x_n\}_{n\geq 1}$ is a sequence of real numbers, then which of the following statements is/are true?

a) If $\{x_n\}_{n\geq 1}$ converges, then both $\{x_n^+\}_{n\geq 1}$ and $\{x_n^-\}_{n\geq 1}$ converge
b) If $\{x_n\}_{n\geq 1}$ converges to 0, then both $\{x_n^+\}_{n\geq 1}$ and $\{x_n^-\}_{n\geq 1}$ converge to 0
c) If both $\{x_n^+\}_{n\geq 1}$ and $\{x_n^-\}_{n\geq 1}$ converge, then $\{x_n\}_{n\geq 1}$ converges
d) If $\left\{x_n^2\right\}_{n\geq 1}$ converges, then both $\{x_n^+\}_{n\geq 1}$ and $\{x_n^-\}_{n\geq 1}$ converge

51) Let A be a 3×3 real matrix such that $A\begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\4\end{pmatrix}$, $A\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}4\\0\\0\end{pmatrix}$, $A\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}0\\4\\0\end{pmatrix}$ Then which of the following statements is/are true?

a) $A\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\2\\-2\end{pmatrix}$
b) $A\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}2\\-2\\2\end{pmatrix}$
c) $A\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}2\\0\\2\end{pmatrix}$
d) $A\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}8\\4\\0\end{pmatrix}$

52) Let X be a positive valued continuous random variable with finite mean. If $Y = [X]$, the largest integer less than or equal to X, then which of the following statements is/are true?

a) $P(Y \leq u) \leq P(X \leq u)$ for all $u \geq 0$
b) $P(Y \geq u) \leq P(X \geq u)$ for all $u \geq 0$
c) $E(X) < E(Y)$
d) $E(X) > E(Y)$