

9.5.7

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Question:

Find the solution of the pair of equations

$$\frac{3}{x} + \frac{8}{y} = -1, \frac{1}{x} - \frac{2}{y} = 2, x, y \neq 0 \quad (0.1)$$

Solution:

Variable	Description
x	Original variable representing the first unknown in the equations.
y	Original variable representing the second unknown in the equations.
a	Substituted variable representing $\frac{1}{x}$. Used to transform the equations.
b	Substituted variable representing $\frac{1}{y}$. Used to transform the equations.

TABLE 0: Variables Used

To solve using matrices, let's rewrite the equations in a simpler form by letting:

$$a = \frac{1}{x}, b = \frac{1}{y}$$

The equations become:

$$3a + 8b = -1 \quad (0.2)$$

$$a - 2b = 2 \quad (0.3)$$

We can represent the system in matrix form as:

$$\begin{pmatrix} 3 & 8 \\ 1 & -2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad (0.4)$$

The system can be represented in augmented matrix form as:

$$\begin{pmatrix} 3 & 8 & -1 \\ 1 & -2 & 2 \end{pmatrix} \xrightarrow{R_2 \rightarrow 3R_2 - R_1} \begin{pmatrix} 3 & 8 & -1 \\ 0 & -14 & 7 \end{pmatrix} \quad (0.5)$$

$$\begin{pmatrix} 3 & 8 & -1 \\ 0 & -14 & 7 \end{pmatrix} \xrightarrow{R_2 \rightarrow \frac{R_2}{-7}} \begin{pmatrix} 3 & 8 & -1 \\ 0 & 2 & -1 \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 - 4R_2} \begin{pmatrix} 3 & 0 & 3 \\ 0 & 2 & -1 \end{pmatrix} \quad (0.6)$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -\frac{1}{2} \end{pmatrix} \quad (0.7)$$

Therefore ;

$$a = 1, b = -\frac{1}{2} \quad (0.8)$$

As $a = \frac{1}{x}$, $b = \frac{1}{y}$ we get :

$$x = 1, y = -2 \quad (0.9)$$

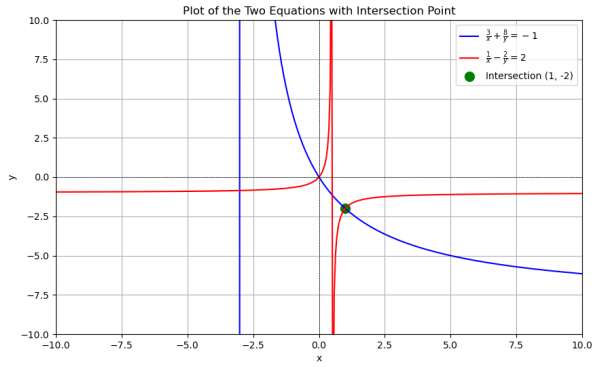


Fig. 0.1: System of equations