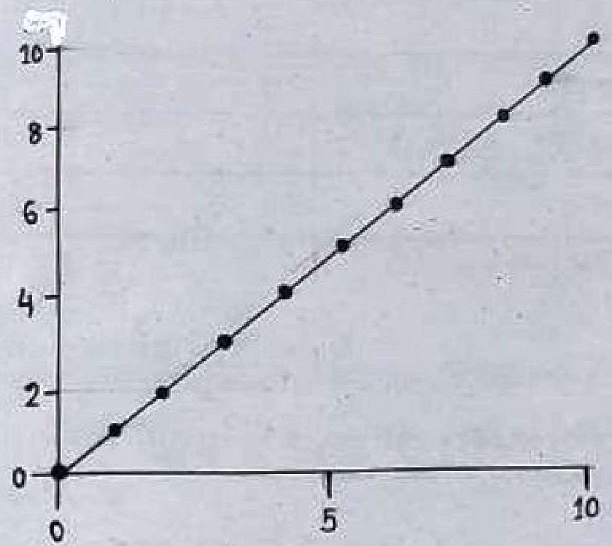
Linear Functions

- Linear means straight.
- A linear function is a straight line.
- A linear graph represents a linear function.

Linear Functions

A Function is special relationship where each input has an output.
A function is often written as $f(x)$ where x is input :

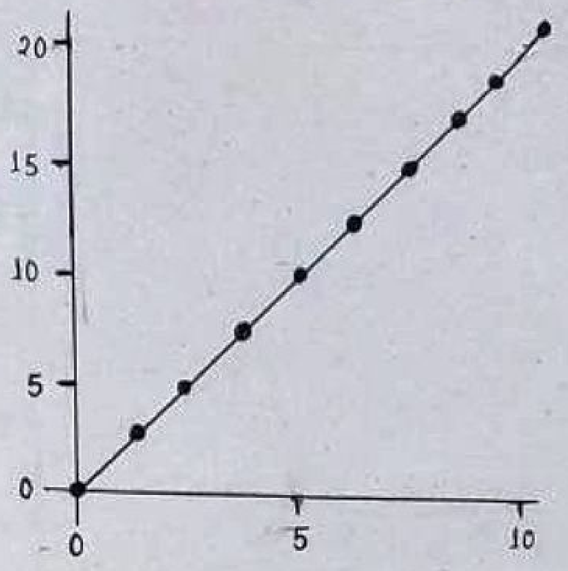
$$f(x) = x$$



Results from $f(x) = x$

x	y	$y = x$
1	1	$y = x = 1$
2	2	$y = x = 2$
3	3	$y = x = 3$
4	4	$y = x = 4$
5	5	$y = x = 5$

$f(x) = 2x$



Results from $f(x) = 2x$

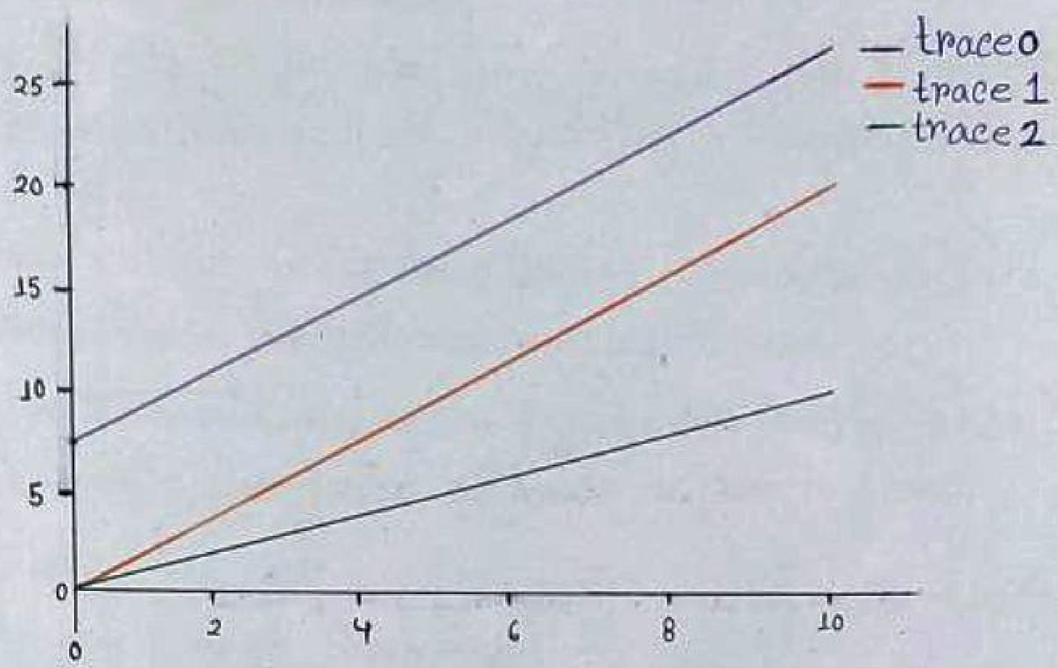
x	y	$y = 2x$
1	2	$y = 2x = 2$
2	4	$y = 2x = 4$
3	6	$y = 2x = 6$
4	8	$y = 2x = 8$
5	10	$y = 2x = 10$

Linear Equations

A Linear Equation is an equation for a straight line:

- $y = x$
- $y = x*2$
- $y = x*2 + 7$
- $y = ax + b$
- $5x = 3y$
- $y/2 = 6$

[y = x*2+7] [y = x*2] [y = x]



Non-Linear Equations

A Linear Equation can NOT contain exponents or Square roots:

- y = x**2
- y = Math.sqrt(x)
- y = Math.sin(x)

Linear Regression

A Linear regression tries to model the relationship between two variables by fitting a linear graph to data.
one variable (x) is considered to be data, and the Other (y) is Considered to be dependent.
For example, a Linear Regression can be a model to relate the price of house to their size.

Linear Least Squares

Linear algebra is used to solve Linear Equations.
Linear Least Squares (LLS) is a set of formulations for solving statistical problems involved in Linear Regression.

Linear Algebra

Linear Algebra is the branch of mathematics that concerns linear equations (and linear maps) and their representations in vector space and through matrices.

Linear Algebra is one of the most important part for machine learning.

Scalars

In linear algebra, a scalar is a single number.

$x = 1$

Vectors

Vectors are 1-dimentional Arrays.

Vectors have a Magnitude and a Direction.

Vectors typically describes Motion or Force.

Vector Notation

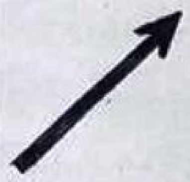
Vectors can be written in many ways. The most common are:

$$V = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

or:

$$V = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Vectors in Geometry



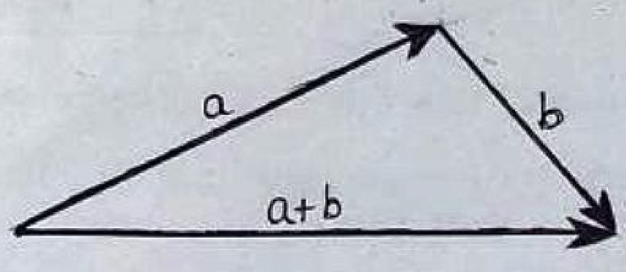
The image to the left is a Vector.
The Length shows the Magnitude.
The Arrow shows the Direction.

Motion

Vectors are the building blocks of Motion
In geometry, a vector can describe a movement from one point to another.
The vector [3.2] Says go 3 right and 2 up.

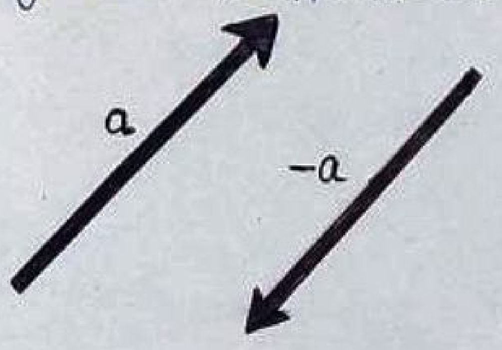
Vector Addition

The sum of two vectors (a+b) is found by moving the vector b until the tail meets the head of vector a. (This does not change vector b).
Then, the line from the tail of a to the head of b is the vector a+b:



Vector Subtraction

Vector -a is the opposite of +a.
This means that vector a and vector -a has the Same magnitude in opposite directions:



Scalar Operations

Vector can be modified by adding, subtracting, or multiplying a Scalar (number) from all the Vector values:

a = [1 1 1]

a + 1 = [2 2 2]

[1 2 3] + 1 = [2 3 4]

Vector multiplications has much of the Same properties as normal multiplication:

[2 2 2] * 3 = [6 6 6]

[6 6 6] / 3 = [2 2 2]

Force

Force is a vector.
Force is a vector with a Magnitude and a Direction.

Velocity

Velocity is a vector.
Velocity is a vector with a Magnitude anda Direction.

Matrices

A matrix is Set of Numbers.
A matrix is an Rectangular Array.
A matrix is arranged in Rows and Columns.

Matrix Dimensions

This Matrix has 1 row and 3 Columns:

$$C = \begin{bmatrix} 2 & 5 & 3 \end{bmatrix}$$

The Dimension of the matrix is (1x3).

This matrix has 2 rows and 3 columns.

$$C = \begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix}$$

The dimension of the matrix is (2x3).

Square Matrices

A square Matrix is a matrix with the Same number of rows and columns.

An n-by-n matrix is known as a Square matrix of order n.

A 2-by-2 matrix (square matrix of order 2):

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

A 4-by-4 matrix (square matrix of order 4):

$$C = \begin{bmatrix} 1 & -2 & 3 & 4 \\ 5 & 6 & -7 & 8 \\ 4 & 3 & 2 & -1 \\ 8 & 7 & 6 & -5 \end{bmatrix}$$

Diagonal Matrices

A Diagonal Matrix has values on the diagonal entries, and zero on the rest:

$$C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Scalar Matrices

A Scalar Matrix has equal diagonal entries and zero on the rest.

$$C = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

The Identity Matrix

The Identity Matrix has 1 on the diagonal and 0 on the rest.
This is the matrix equivalent of 1. The Symbol is I.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

If you multiply any matrix with the identity matrix, the result equals the original.

The Zero Matrix

The zero Matrix (Null Matrix) has only zeros.

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Equal Matrices

Matrices are Equal if each element correspond:

$$\begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix}$$

Negative Matrices

The Negative of a matrix is easy to understand:

$$-\begin{bmatrix} -2 & 5 & 3 \\ -4 & 7 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -5 & -3 \\ 4 & -7 & -1 \end{bmatrix}$$

Adding Matrices

If two matrices have the same dimension, we can add them:

$$\begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 7 & 1 \\ 2 & 5 & 3 \end{bmatrix} = \begin{bmatrix} 6 & 12 & 4 \\ 6 & 12 & 4 \end{bmatrix}$$

Subtracting Matrices

If two matrices have the same dimension, we can subtract them:

$$\begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 7 & 1 \\ 2 & 5 & 3 \end{bmatrix} = \begin{bmatrix} -2 & -2 & 2 \\ 2 & 2 & -2 \end{bmatrix}$$

Scalar Multiplication

while numbers in rows and columns are called Matrices, single numbers are called scalars.
It is easy to multiply a matrix with a scalar. Just multiply each number in the matrix with the scalar:

$$\begin{bmatrix} 2 & 5 & 3 \\ 4 & 7 & 1 \end{bmatrix} \times 2 \quad \begin{bmatrix} 4 & 10 & 6 \\ 8 & 14 & 2 \end{bmatrix}$$

Transpose a Matrix

To transpose a matrix, means to replace rows with columns.
when you swap rows and columns, you rotate the matrix around it's diagonal.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Multiplying Matrices

Multiplying matrices is more difficult.
We can only multiply two matrices if the numbers of rows in matrix A is the Same as the number of Columns in matrix B.
Then, we need to compile a "dot product":
we need to multiply the numbers in each row of A with the numbers in each column of B,

and then add the product:

$$\underset{A}{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}} \times \underset{B}{\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}} = \underset{C}{\begin{bmatrix} 1\times1 + 2\times1 + 3\times1 \\ 1\times2 + 2\times2 + 3\times2 \\ 1\times3 + 2\times3 + 3\times3 \end{bmatrix}} = \underset{C}{\begin{bmatrix} 6 \\ 12 \\ 18 \end{bmatrix}}$$

Tensors

A Tensor is a N-dimensional Matrix:

- A Scalar is a 0-dimensional tensor
- A vector is a 1-dimensional tensor
- A Matrix is a 2-dimensional tensor

A Tensor is a generalization of vectors and Matrices to higher dimensions.

Scalar

1

Vector(s)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

Tensor

$$\left[\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} \right]$$

Tensor Ranks

The number of directions a tensor can have in a N-dimensional space, is called the Rank of the tensor.

The Rank is denoted R.

A Scalar is a single number.

- It has 0 Axes
- It has a Rank of 0
- It is a 0-dimensional Tensor

A vector is an array of numbers.

- It has 1 Axis
- It has a Rank of 1
- It is a 1-dimensional Tensor

A Matrix is a 2-dimensional array.

- It has 2 Axis
- It has a Rank of 2
- It is a 2-dimensional Tensor

Real Tensors

Technically, all of the above are tensors, but when we speak of tensors, we generally speak of matrices with a dimension larger than 2 (R>2).