Assignment 1

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Download all C codes from

https://github.com/pranayEE11009/C_and_DataStructures/tree/main/Assignment_1/codes

and latex-tikz codes from

https://github.com/pranayEE11009/C_and_DataStructures/tree/main/Assignment_1

1 Problem

Consider the following ANSI C function:

```
int SimpleFunction(int Y[], int n, int x)
{
int total = Y[0], loopIndex;
for (loopIndex=1; loopIndex<=n−1; loopIndex++)
    {
    total=x∗total +Y[loopIndex];
    }
return total;
}
```

Let Z be an array of 10 elements with Z[i]=1, for all i such that 0≤i≤9. The value returned by SimpleFunction(Z,10,2) is ?

2 Solution

Solution: 1023

Code to generate the solution:

```
#include <stdio.h>
int SimpleFunction(int Y[], int n, int x){
    int total = Y[0], loopIndex;
    for( loopIndex = 1; loopIndex<=n−1;
        loopIndex++){
        total = x∗total + Y[loopIndex];
    }return total;
}
int main()
{
    int Z[10] = {1,1,1,1,1,1,1,1,1,1};
    printf("%d", SimpleFunction(Z, 10, 2));
    return 0;
}
```

The function **S**impleFunction of the C code in the question takes an integer type array (Y[]), and two integer variables (n and x) as the inputs and returns an integer as the output.

The inputs of the SimpleFunction are:

1) integer type array, Z[i] = 1 for all 0≤i≤9
   i,e., Z = [1,1,1,1,1,1,1,1,1,1]
2) integer n = 10
3) integer x = 2

In the function SimpleFunction(Z,n,x) a "for loop" is run for n-1 iterations and in each iteration the integer variable "**total**", which is initiated with 1, is recursively multiplied with 2 and added to 1.

$$total = x * total + Z[loopIndex] \tag{2.0.1}$$

Since, Z[i] is always 1 and $x = 2$.

$$total = 2 * total + 1 \tag{2.0.2}$$

The values of total for n-1 iterations are,
initially total = 1
for loopIndex = 1, total = 2*(1) + 1 = 3
for loopIndex = 2, total = 2*(3) + 1 = 7
for loopIndex = 3, total = 2*(7) + 1 = 15
for loopIndex = 4, total = 2*(15) + 1 = 31
for loopIndex = 5, total = 2*(31) + 1 = 63
for loopIndex = 6, total = 2*(63) + 1 = 127
for loopIndex = 7, total = 2*(127) + 1 = 255
for loopIndex = 8, total = 2*(255) + 1 = 511
for loopIndex = 9, total = 2*(511) + 1 = 1023

The for loop terminates at loopIndex = 9, and the SimpleFunction returns the final value of total, which is equal to **1023**.

Now, as we observe the values of "total" (3,7,15...,1023), we can observe that each value of "total" is one less than some integer exponential of 2. For example;

for loopIndex = 1, total = $3 = 2^2 - 1$
for loopIndex = 2, total = $7 = 2^3 - 1$ and so on.

So, in the SimpleFunction for a given array Y[i] = y for 0≤i≤n-1, int n and int x, the "total value" (T) for a given loopIndex (m) can be written as ,

$$T(m) = x * T(m-1) + y \tag{2.0.3}$$

Note: *Here, we are considering array Y[] to have the same value(int y) for the whole array. If array Y[] consists of random integer values then there won't be any pattern in the recursive values of total and it is not possible to find a general equation for total(T).*

Now, let us consider all the recursive values of **T** till the base case(initial value of T, $T_o = 1$)

$$T(m) = xT(m-1) + y \tag{2.0.4}$$
$$T(m-1) = xT(m-2) + y \tag{2.0.5}$$
$$T(m-2) = xT(m-3) + y \tag{2.0.6}$$
$$\vdots \tag{2.0.7}$$
$$T(1) = xT_o + y \tag{2.0.8}$$

$\because loopIndex \in [1, n-1]$
$\implies m \geq 1$

Now, to get a equation for T, we multiply the above equations with suitable coefficients and add the equations,

$$T(m) = xT(m-1) + y \tag{2.0.9}$$
$$xT(m-1) = x * (xT(m-2) + y) \tag{2.0.10}$$
$$x^2 * T(m-2) = x^2 * (xT(m-3) + y) \tag{2.0.11}$$
$$\vdots \tag{2.0.12}$$
$$x^{m-1} * T(1) = x^{m-1} * (x + y) \tag{2.0.13}$$
$$[\because T_o = 1] \tag{2.0.14}$$

Now, on adding all the above equations we get rid of all the T(m-i) form values except T(m), T_o and we are left with all the integer y's from each equation,

$$T(m) = y + yx + yx^2 + ... + yx^{m-1} + x^m \tag{2.0.15}$$
$$T(m) = y\frac{(x^m - 1)}{x - 1} + x^m \tag{2.0.16}$$

So, We can find "total" value corresponding to a particular iterative loopIndex using the equation below, where m is the loopIndex.

$$T(m) = y\frac{(x^m - 1)}{x - 1} + x^m \tag{2.0.17}$$

Since the SimpleFunction returns the "total" value corresponding for the loopIndex = n-1.
So, for m = n-1, we have

$$T(n-1) = y\frac{(x^{n-1} - 1)}{x - 1} + x^{n-1} \tag{2.0.18}$$

Finally, we can find the final output of the SimpleFunction directly using the equation 2.0.18 for a given integer array Y[], integer n and integer x.

In the given question, the array Y[] = Z[i] = 1 for 0≤i≤9, n=10 and x = 2;

$$total = 1 * \frac{(2^{10-1} - 1)}{2 - 1} + 2^{10-1} \tag{2.0.19}$$
$$= 2^9 - 1 + 2^9 \tag{2.0.20}$$
$$= 2^{10} - 1 \tag{2.0.21}$$
$$= 1024 - 1 \tag{2.0.22}$$
$$= \mathbf{1023} \tag{2.0.23}$$