EE3025 - Assignment 1

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Download all python codes from

```
https://github.com/pranayEE11009/EE3025_IDP/
    tree/main/Assignment_1/codes
```

and latex-tikz codes from

```
https://github.com/pranayEE11009/EE3025_IDP/
    tree/main/Assignment_1
```

1 Problem

1.1. Let

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \tag{1.1.1}$$

and

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2)$$
$$y(n) = 0, n < 0 \tag{1.1.2}$$

Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{1.1.3}$$

and H(k) using h(n).

2 Solution

2.1. Since, We know that the Impulse response of an LTI system is the output of the system when an Unit Impulse signal is given as the input to the system.

Now, from equation (1.1.2) the impulse response of the system can be defined as,

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2) \tag{2.1.1}$$

$$\implies h(n) = \delta(n) + \delta(n-2) - \frac{1}{2}h(n-1) \tag{2.1.2}$$

2.2. **Computing the DFT manually:**
The DFT of the Input Signal x(n) is given by :

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.2.1}$$

Similarly, the DFT of the Impulse Response h(n) is given by :

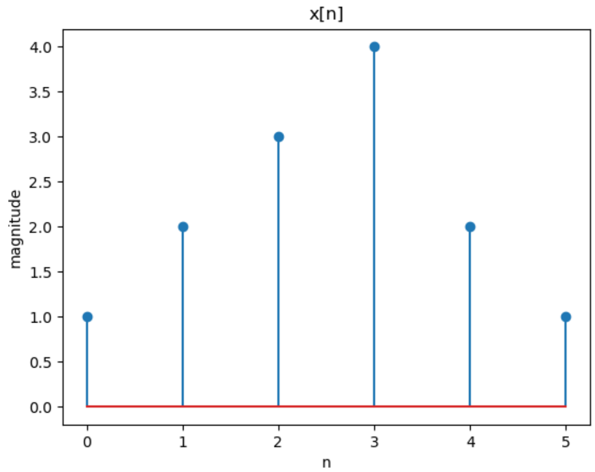
$$H(k) \triangleq \sum_{n=0}^{N-1} h(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.2.2}$$

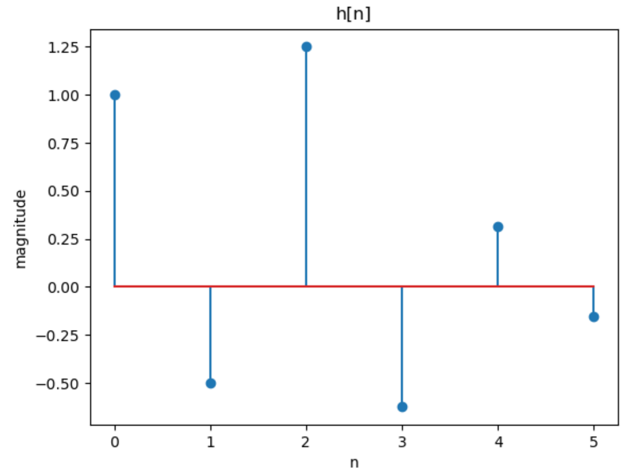
The following python code computes the DFT of x(n) and h(n).

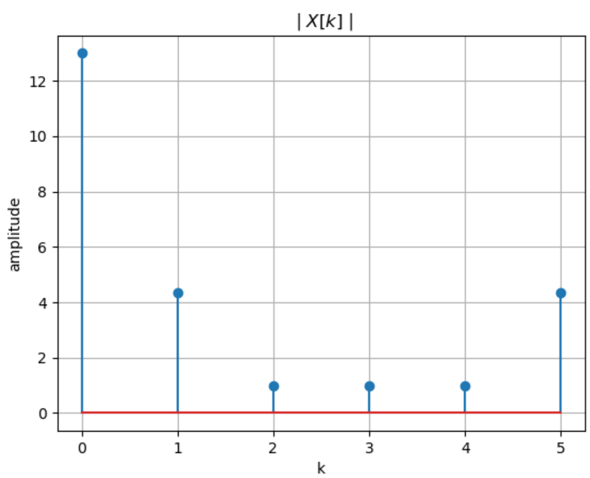
```
https://github.com/pranayEE11009/
    EE3025_IDP/tree/main/Assignment_1/
    codes
```

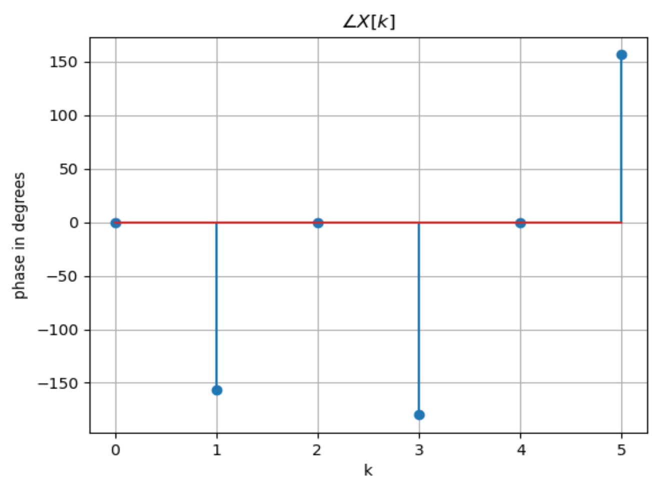
The plots are in

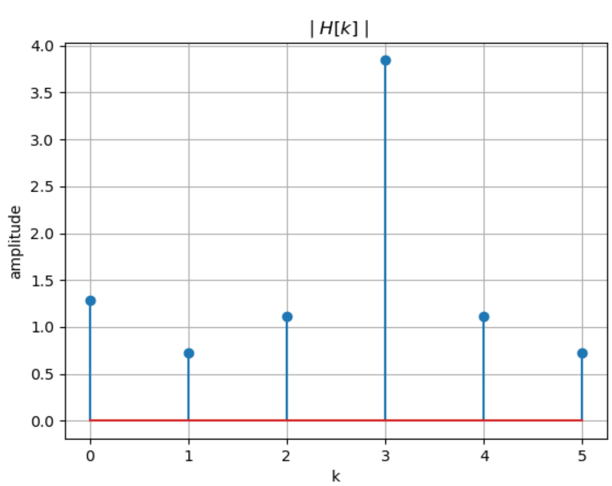
```
https://github.com/pranayEE11009/
    EE3025_IDP/tree/main/Assignment_1/
    figs
```

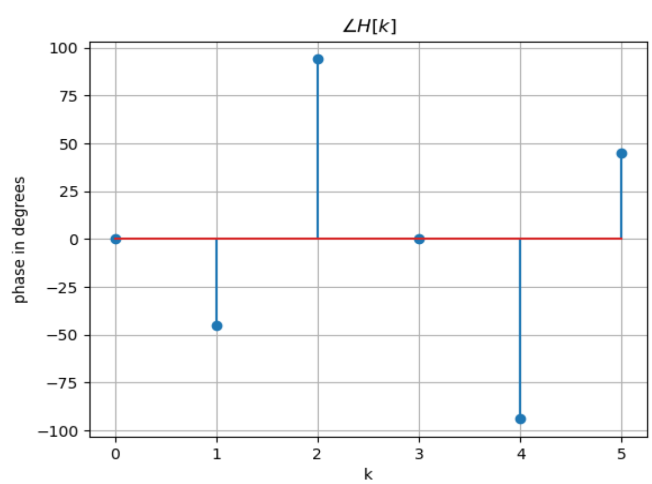

x[n]
magnitude
n
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
0 1 2 3 4 5


h[n]
magnitude
n
−0.50 −0.25 0.00 0.25 0.50 0.75 1.00 1.25
0 1 2 3 4 5


$|X[k]|$
amplitude
k
0 2 4 6 8 10 12
0 1 2 3 4 5


$\angle X[k]$
phase in degrees
k
−150 −100 −50 0 50 100 150
0 1 2 3 4 5


$|H[k]|$
amplitude
k
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0
0 1 2 3 4 5


$\angle H[k]$
phase in degrees
k
−100 −75 −50 −25 0 25 50 75 100
0 1 2 3 4 5

2.3. Computing the DFT manually using matrix multiplication:

We know that DFT of x(n) is,

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.3.1}$$

Let us define a matrix "W" such that,

$$DFT(x) = X = W.x \tag{2.3.2}$$

Where (.) represents the matrix multiplication of W and x.
Each point of the matrix W is defined as,

$$W_N^{nk} = e^{-j2\pi kn/N} \tag{2.3.3}$$
$$\tag{2.3.4}$$

where n and k are column and row indices of the matrix respectively.

Now from equation: (2.3.2),

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix} \tag{2.3.5}$$

Given, x(n) = $\left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\}$ and N=6
we have,

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1+2+3+4+2+1 \\ 1+(2)e^{-j\pi/3} + ... + (1)e^{-j5\pi/3} \\ 1+(2)e^{-2j\pi/3} + ... + (1)(e^{-2j5\pi/3} \\ 1+(2)e^{-3j\pi/3} + ... + (1)e^{-3j5\pi/3} \\ 1+(2)e^{-4j\pi/3} + ... + (1)e^{-4j5\pi/3} \\ 1+(2)e^{-5j\pi/3} + ... + (1)e^{-5j5\pi/3} \end{bmatrix} \tag{2.3.6}$$

On Solving we get,

$$X(0) = 13 + 0j \tag{2.3.7}$$
$$X(1) = -4 - 1.732j \tag{2.3.8}$$
$$X(2) = 1 + 0j \tag{2.3.9}$$
$$X(3) = -1 + 0j \tag{2.3.10}$$
$$X(4) = 1 + 0j \tag{2.3.11}$$
$$X(5) = -4 + 1.732j \tag{2.3.12}$$

We can observe that these values are same as the values that we obtained from the plots.

Similarly, the DFT of h(n) can be defined as,

$$DFT(h) = H = W.h \tag{2.3.13}$$

So, we have,

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} h(0) \\ h(1) \\ h(2) \\ h(3) \\ h(4) \\ h(5) \end{bmatrix} \tag{2.3.14}$$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} h(0)+h(1)+h(2)+h(3)+h(4)+h(5) \\ h(0)+h(1)e^{-j\pi/3} + ... + h(5)e^{-j5\pi/3} \\ h(0)+h(1)e^{-2j\pi/3} + ... + h(5)e^{-2j5\pi/3} \\ h(0)+h(1)e^{-3j\pi/3} + ... + h(5)e^{-3j5\pi/3} \\ h(0)+h(1)e^{-4j\pi/3} + ... + h(5)e^{-4j5\pi/3} \\ h(0)+h(1)e^{-5j\pi/3} + ... + h(5)e^{-5j5\pi/3} \end{bmatrix} \tag{2.3.15}$$

Now, from equation: (2.1.2) we know that,

$$h(n) = \delta(n) + \delta(n-2) - \frac{1}{2}h(n-1) \tag{2.3.16}$$

for N=6,

$$h(n) = \left\{\underset{\uparrow}{1}, -0.5, 1.25, -0.625, 0.3125, -0.15625\right\} \tag{2.3.17}$$

Therefore, on solving we get,

$$H(0) = 1.28125 + 0j, \tag{2.3.18}$$
$$H(1) = 0.515625 - 0.5142026j, \tag{2.3.19}$$
$$H(2) = -0.078125 + 1.109595j, \tag{2.3.20}$$
$$H(3) = 3.84375 + 0j, \tag{2.3.21}$$
$$H(4) = -0.078125 - 1.109595j. \tag{2.3.22}$$
$$H(5) = 0.515625 + 0.51420256j \tag{2.3.23}$$

We can observe that these values are same as the values that we obtained from the plots.