# Control Systems

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Contents



***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

## 1 Stability

## 2 Bode Plot

### *2.1 Example*

2.1.1. The asymptotic Bode magnitude plot of minimum phase transfer function G(s) is show below. Express $20\log|G(j\omega)|$ as a function of $\omega$ using Fig 2.1.1.

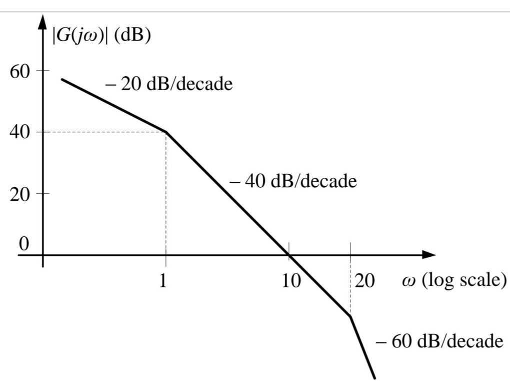


Fig. 2.1.1

**Solution:**

$$20\log|G(j\omega)| = \begin{cases} 60 - 20(\log(\omega) - \log(0.1)) \\ 0.1 < \omega < 1 \\ 80 - 40(\log(\omega) - \log(0.1)) \\ 1 < \omega < 20 \\ 126.02 - 60(\log(\omega) - \log(0.1)) \\ 20 < \omega \end{cases} \tag{2.1.1.1}$$

2.1.2. Express the slope of $20\log|G(j\omega)|$ as a function of $\omega$.
**Solution:** The desired slope is

$$\nabla 20\log|G(j\omega)| = \frac{d(20\log|G(j\omega)|)}{d(\log(\omega))} \tag{2.1.2.1}$$

$$\nabla 20\log|G(j\omega)| = \begin{cases} -20 & \omega < 1 \\ -40 & 1 < \omega < 20 \\ -60 & 20 < \omega \end{cases} \tag{2.1.2.2}$$

2.1.3. Express the change of slope of $20\log|G(j\omega)|$ as a function of $\omega$.
**Solution:**
$\Delta(\nabla 20\log|G(j\omega)|)$ = change of slope of $20\log|G(j\omega)|$ at $\omega$

$$\Delta(\nabla 20\log|G(j\omega)|) = \begin{cases} -20 & \omega = 0 \\ -20 & \omega = 1 \\ -20 & \omega = 20 \end{cases} \tag{2.1.3.1}$$

2.1.4. Find the poles and zeros of G(s).
**Solution:**
Since, each pole corresponds to -20 dB/decade and each zero corresponds to +20 dB/decade. there is a decrease in 20 dB/decade for each value of $\omega$ at 0,1,20.
$\therefore$ there exists **3 poles** and **no zeros**.

2.1.5. Find G(s)
**Solution:**

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From the solution of the problem 2.1.4 we can obtain G(s)

$$G(s) = \frac{k}{s(1+s)(20+s)} \tag{2.1.5.1}$$

2.1.6. Obtain the Bode plot of G(s) through a python code and compare with the line plot of the expression that you obtained in Problem 2.1.1
**Solution:** Fig 2.1.6 shows the Bode plot of the transfer function obtained.
The **Line plot** is the approximation of the **caluculated bode plot**.

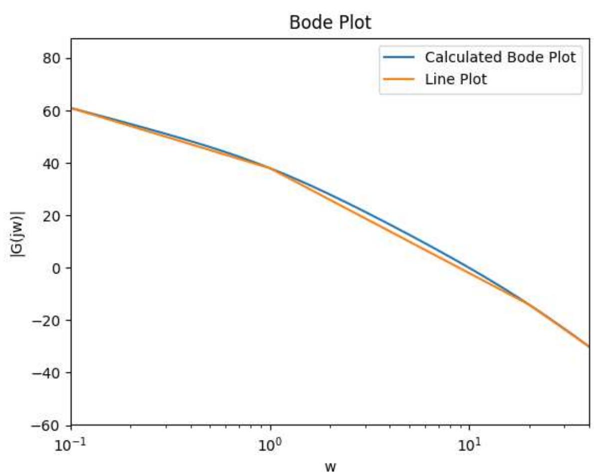


Fig. 2.1.6

2.1.7. Verify if at very high frequency ($\omega \to \infty$), the phase angle $\angle G(j\omega) = -3\pi/2$
**Solution:**
**Calculating phase:**

Since we know that,
phase $\phi$ is the sum of all the phases corresponding to each pole and zero.
phase corresponding to pole is =

$$-tan^{-1}(\frac{imaginary}{real}) \tag{2.1.7.1}$$

phase corresponding to zero is =

$$tan^{-1}(\frac{imaginary}{real}) \tag{2.1.7.2}$$

Now take,

$$s = j\omega \tag{2.1.7.3}$$

$$\Rightarrow G(j\omega) = \frac{k}{j\omega(1+j\omega)(20+j\omega)} \tag{2.1.7.4}$$

Therefore,

$$\phi = -tan^{-1}(\frac{\omega}{0}) - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20}) \tag{2.1.7.5}$$

$$\phi = -90^\circ - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20}) \tag{2.1.7.6}$$

$$\because \omega \to \infty \tag{2.1.7.7}$$

$$\phi = -90^\circ - 90^\circ - 90^\circ \tag{2.1.7.8}$$

$$\phi = -270^\circ \tag{2.1.7.9}$$

$$\phi = -3\pi/2 \tag{2.1.7.10}$$

$\therefore$ at very high frequency ($\omega \to \infty$), the phase angle $\angle G(j\omega) = -3\pi/2$.

2.1.8.

## 3 Routh Hurwitz Criterion

## 4 Compensators

## 5 Nyquist Plot