Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

1 Stability

2 Bode Plot

2.1 Introduction

2.1. For an LTI system, the Bode plot for its gain defined as

$$G(s) = 20 \log |H(s)| \tag{2.1.1}$$

is as illustrated in the Fig. 2.1. Express G(f) in terms of f.
Solution:

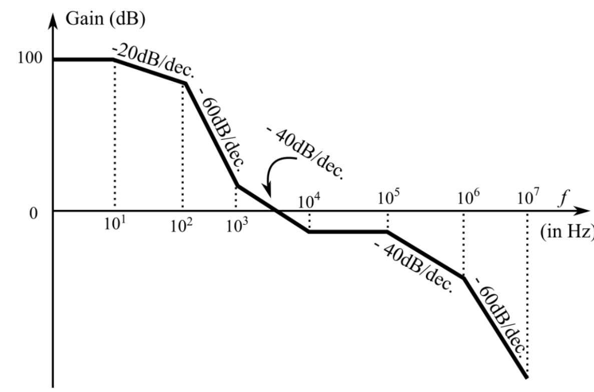

Fig. 2.1

2.2. Express the slope of G(f) in terms of f.
Solution:

$$Slope = \frac{d(20 \log H(f))}{df} \tag{2.2.1}$$

$$Slope = \begin{cases} 0 & 0 < f < 10^1 \\ -20 & 10 < f < 10^2 \\ -60 & 10^2 < f < 10^3 \\ -40 & 10^3 < f < 10^4 \\ 0 & 10^4 < f < 10^5 \\ -40 & 10^5 < f < 10^6 \\ -60 & 10^6 < f < 10^7 \end{cases} \tag{2.2.2}$$

2.3. Express the change of slope of G(f) in terms of f.
Solution: Δ Slope = Change in slope at f

$$\Delta Slope = \begin{cases} -20 & f = 10^1 \\ -40 & f = 10^2 \\ +20 & f = 10^3 \\ +40 & f = 10^4 \\ -40 & f = 10^5 \\ -20 & f = 10^6 \end{cases} \tag{2.3.1}$$

2.4. Find the number of poles and zeros of H(s).
Solution:

2.5. Find the location of the poles and zeros of H(s). The number of system poles N_p and number of system zeros N_z in the frequency range 1 Hz $\leq$ f $\leq 10^7$ Hz is

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2.6. Obtain the transfer function of H(s).

2.7. Obtain the Bode plot and the slope plot for H(s) and verify with Fig. 2.1

Solution: Let us consider a generalized transfer gain

$$H(s) = k\frac{(s-z_1)(s-z_2)...(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)....(s-p_{n-1})(s-p_n)} \tag{2.7.1}$$

$$Gain = 20\log|H(s)| = 20\log|k| + 20\log|s-z_1| + 20\log|s-z_2| + \cdots + 20\log|s-z_m| - 20\log|s-p_1| - 20\log|s-p_2| - \cdots - 20\log|s-z_n| \tag{2.7.2}$$

Let us consider a $20\log|s-z_1|$
Let s = jω

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| \tag{2.7.3}$$

Based on log scale plot approximations,to the left of $z_1\omega << z_1$ and towards right $\omega >> z_1$
For $\omega < z_1$

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|z_1| \quad = constant \tag{2.7.4}$$

$$i.e.Slope = 0 \tag{2.7.5}$$

For $\omega > z_1$

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|\omega| \tag{2.7.6}$$

$$i.eSlope = 20 \tag{2.7.7}$$

When a zero is encountered the slope always increases by 20 dB/decade
Doing similar analysis for $-20\log|s-p_1|$ We conclude
When a pole is encountered the slope always decreases by 20 dB/decade

Final Transfer function is

$$H(f) = \frac{K(f+10^3)(f+10^4)^2}{(f+10^1)(f+10^2)^2(f+10^5)^2(f+10^6)1} \tag{2.7.8}$$

$$N_p = 6 \tag{2.7.9}$$

$$N_z = 3 \tag{2.7.10}$$

Python plot of the obtained transfer function is shown in 2.2.3

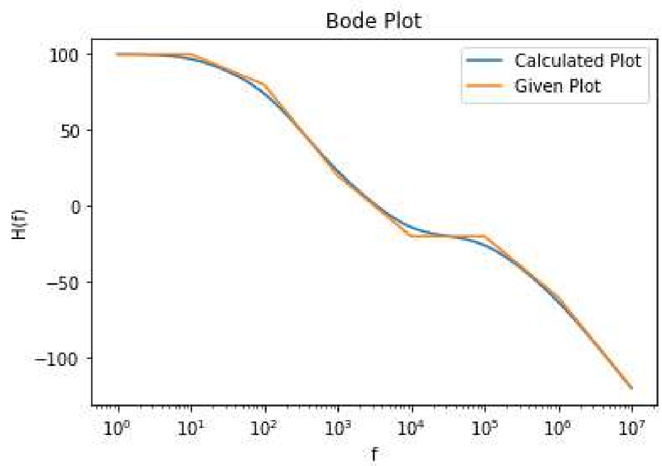

Fig. 2.7

2.2 Example

2.2.1. The asymptotic Bode magnitude plot of minimum phase transfer function G(s) is show below.

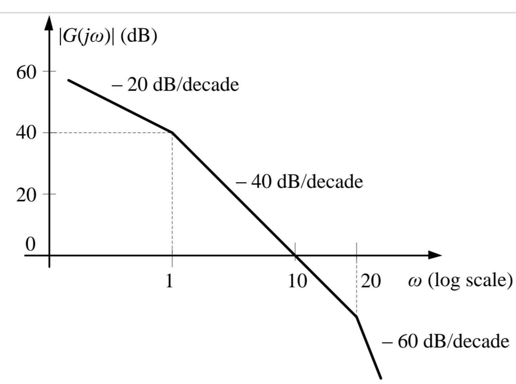

Fig. 2.2.1

2.2.2. Verify if the transfer function G(s) has 3 poles and one zero.

2.2.3. Verify if at very high frequency ($\omega \to \infty$), the phase angle $\angle$ G(jω)=-3π/2

Solution:

Since, each pole corresponds to -20 dB/decade and each zero corresponds to +20 dB/decade. Therefore, from the given Bode plot we can get the Transfer equation,

$$G(s) = \frac{k}{s(1+s)(20+s)} \tag{2.2.3.1}$$

Python Plot of the transfer function is as shown in the figure

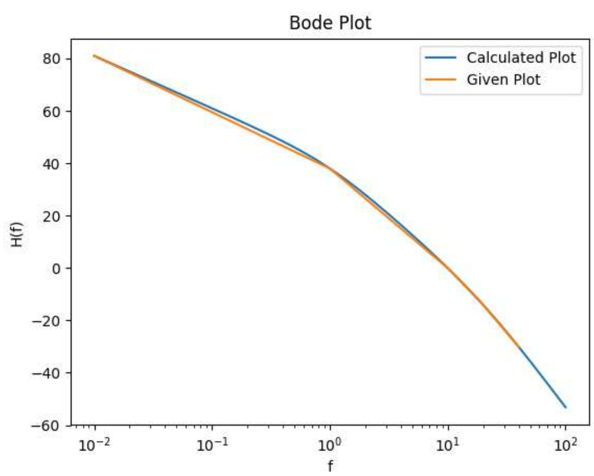

Fig. 2.2.3

Now, from the Transfer equation we can conclude that, there are three poles (0, -1 and -20) and no zeros.

$\therefore$ Statement 1 is false ....(1)

Calculating phase:

Since we know that,
phase ϕ is the sum of all the phases corresponding to each pole and zero.
phase corresponding to pole is =

$$-tan^{-1}(\frac{imaginary}{real}) \tag{2.2.3.2}$$

phase corresponding to zero is =

$$tan^{-1}(\frac{imaginary}{real}) \tag{2.2.3.3}$$

Now take,

$$s = j\omega \tag{2.2.3.4}$$

$$\Rightarrow G(j\omega) = \frac{k}{j\omega(1+j\omega)(20+j\omega)} \tag{2.2.3.5}$$

Therefore,

$$\phi = -tan^{-1}(\frac{\omega}{0}) - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20}) \tag{2.2.3.6}$$

$$\phi = -90° - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20}) \tag{2.2.3.7}$$

$$\because \omega \to \infty \tag{2.2.3.8}$$

$$\phi = -90° - 90° - 90° \tag{2.2.3.9}$$

$$\phi = -270° \tag{2.2.3.10}$$

$$\phi = -3\pi/2 \tag{2.2.3.11}$$

$\therefore$ Statement 2 is true ....(2)
thus, from (1) and (2) option (B) is correct.

3 Routh Hurwitz Criterion

4 Compensators

5 Nyquist Plot