Control Systems

G V V Sharma*

Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

1 Stability

1.1 Bode Plot

1.1. The asymptotic Bode magnitude plot of minimum phase transfer function G(s) is show below.
Consider the following two statements.

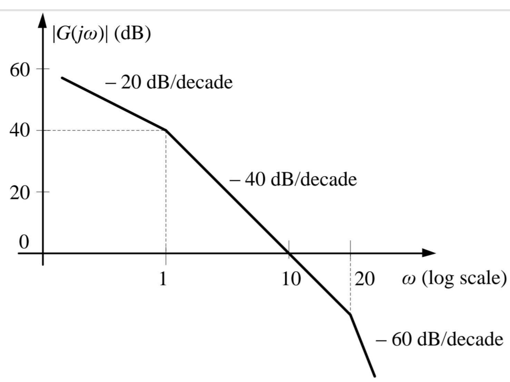

Fig. 1.1

Statement 1: Transfer function G(s) has 3 poles and one zero
Statement 2: At very high frequency ($\omega \to \infty$), the phase angle $\angle G(j\omega) = -3\pi/2$

Which of the following is correct ?
(A) Statement 1 is true and Statement 2 is false.
(B) Statement 1 is false and Statement 2 is true.
(C) Both the statements are true.
(D) Both the statements are false.

Solution:
Since, each pole corresponds to -20 dB/decade and each zero corresponds to +20 dB/decade. Therefore, from the given Bode plot we can get the Transfer equation,

$$G(s) = \frac{k}{s(1+s)(20+s)} \tag{1.1.1}$$

Now, from the Transfer equation we can conclude that, there are three poles (0, -1 and -20) and no zeros.

$\therefore$ Statement 1 is false ....(1)

Calculating phase:

Since we know that,
phase ϕ is the sum of all the phases corresponding to each pole and zero.
phase corresponding to pole is =

$$-tan^{-1}(\frac{imaginary}{real}) \tag{1.1.2}$$

phase corresponding to zero is =

$$tan^{-1}(\frac{imaginary}{real}) \tag{1.1.3}$$

Now take,

$$s = j\omega \tag{1.1.4}$$

*The author is with the Department of Electrical Engineering, Indian Institute of Technology, Hyderabad 502285 India e-mail: gadepall@iith.ac.in.

$$\Rightarrow G(j\omega) = \frac{k}{j\omega(1+j\omega)(20+j\omega)} \tag{1.1.5}$$

Therefore,

$$\phi = -tan^{-1}(\frac{\omega}{0}) - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20}) \tag{1.1.6}$$

$$\phi = -90^\circ - tan^{-1}(\omega) - tan^{-1}(\frac{\omega}{20}) \tag{1.1.7}$$

$$\because \omega \to \infty \tag{1.1.8}$$

$$\phi = -90^\circ - 90^\circ - 90^\circ \tag{1.1.9}$$

$$\phi = -270^\circ \tag{1.1.10}$$

$$\phi = -3\pi/2 \tag{1.1.11}$$

$\therefore$ Statement 2 is true ....(2)
thus, from (1) and (2) option (B) is correct.

1.2. For an LTI system, the Bode plot for its gain is as illustrated in the Fig.2.1 The number of system poles N_p and number of system zeros N_z in the frequency range 1 Hz $\leq$ f $\leq$ 10^7 Hz is

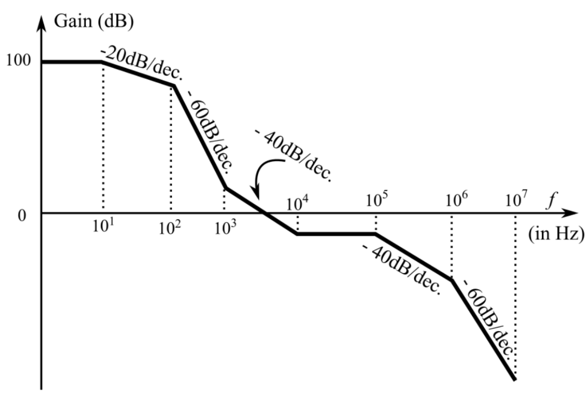

Fig. 1.2

Solution: Let us consider a generalized transfer gain

$$H(s) = k\frac{(s-z_1)(s-z_2)...(s-z_{m-1})(s-z_m)}{(s-p_1)(s-p_2)....(s-p_{n-1})(s-p_n)} \tag{1.2.1}$$

$$Gain = 20\log|H(s)| = 20\log|k| + 20\log|s-z_1| + 20\log|s-z_2| + \cdots + 20\log|s-z_m| - 20\log|s-p_1| - 20\log|s-p_2| - \cdots - 20\log|s-z_n| \tag{1.2.2}$$

Let us consider a $20\log|s-z_1|$
Let $s = j\omega$

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| \tag{1.2.3}$$

Based on log scale plot approximations,to the left of z_1 $\omega << z_1$ and towards right $\omega >> z_1$

For $\omega < z_1$

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|z_1| \quad = constar \tag{1.2.4}$$

i.e. $Slope = 0$
For $\omega > z_1$

$$20\log|s-z_1| = 20\log\left|\sqrt{\omega^2+z_1^2}\right| = 20\log|\omega| \tag{1.2.5}$$

i.e $Slope = 20$
When a zero is encountered the slope always increases by 20 dB/decade

Doing similar analysis for $-20\log|s-p_1|$
We conclude
When a pole is encountered the slope always decreases by 20 dB/decade

$$Slope = \frac{d(20\log H(f))}{df} \tag{1.2.6}$$

$$Slope = \begin{cases} 0 & 0 < f < 10^1 \\ -20 & 10 < f < 10^2 \\ -60 & 10^2 < f < 10^3 \\ -40 & 10^3 < f < 10^4 \\ 0 & 10^4 < f < 10^5 \\ -40 & 10^5 < f < 10^6 \\ -60 & 10^6 < f < 10^7 \end{cases} \tag{1.2.7}$$

Δ Slope = Change in slope at f

$$\Delta Slope = \begin{cases} -20 & f = 10^1 \\ -40 & f = 10^2 \\ +20 & f = 10^3 \\ +40 & f = 10^4 \\ -40 & f = 10^5 \\ -20 & f = 10^6 \end{cases} \tag{1.2.8}$$

Final Transfer function is

$$H(f) = \frac{K(f+10^3)(f+10^4)^2}{(f+10^1)(f+10^2)^2(f+10^5)^2(f+10^6)1} \tag{1.2.9}$$

$$N_p = 6 \tag{1.2.10}$$

$$N_z = 3 \tag{1.2.11}$$

Python plot of the obtained transfer function is shown in fig 2.2

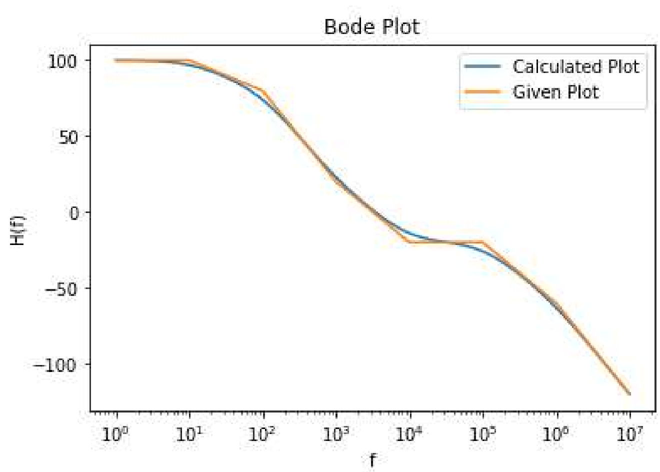

Fig. 1.2

2 Routh Hurwitz Criterion

3 Compensators

4 Nyquist Plot