EE5609: Matrix Theory Assignment-18

Y.Pranaya
AI20MTECH14014

***Abstract*—This document contains a solution for standard inner product.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment18

1 Problem

Let $(|)$ be the standard inner product on $\mathbb{R}^2$.

(a) Let

$$\alpha = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \beta = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{1.0.1}$$

If γ is a vector such that $(\alpha^T\gamma) = -1$ and $(\beta^T\gamma) = 3$. Find γ

(b) Show that for any α in $\mathbb{R}^2$ we have

$$\alpha = (\alpha^T\epsilon_1)\epsilon_1 + (\alpha^T\epsilon_2)\epsilon_2 \tag{1.0.2}$$

2 Solution

(a) From $\alpha = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $(\alpha^T\gamma) = -1$ we get,

$$(\alpha^T\gamma) = -1 \tag{2.0.1}$$
$$\implies \alpha_1\gamma_1 + \alpha_2\gamma_2 = -1 \tag{2.0.2}$$
$$\implies \gamma_1 + 2\gamma_2 = -1 \tag{2.0.3}$$

from $\beta = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $(\beta^T\gamma) = 3$ we get,

$$(\beta^T\gamma) = 3 \tag{2.0.4}$$
$$\implies \beta_1\gamma_1 + \beta_2\gamma_2 = 3 \tag{2.0.5}$$
$$\implies -\gamma_1 + \gamma_2 = 3 \tag{2.0.6}$$

using (2.0.3) and (2.0.6),

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{2.0.7}$$

row reductions,

$$\begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 3 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 + R_1} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 2 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow{R_2 \to \frac{1}{3}R_2} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & \frac{2}{3} \end{pmatrix} \xleftrightarrow{R_1 \to R_1 - 2R_2} \begin{pmatrix} 1 & 0 & \frac{-7}{3} \\ 0 & 1 & \frac{2}{3} \end{pmatrix} \tag{2.0.9}$$

Hence $\gamma = \begin{pmatrix} \frac{-7}{3} \\ \frac{2}{3} \end{pmatrix}$

(b) let $\epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are standard basis vectors,

$$(\alpha^T\epsilon_1) = \alpha_1 + 0.\alpha_2 = \alpha_1 \tag{2.0.10}$$
$$\implies (\alpha^T\epsilon_1)\epsilon_1 = \begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix} \tag{2.0.11}$$
$$(\alpha^T\epsilon_2) = 0.\alpha_1 + \alpha_2 = \alpha_2 \tag{2.0.12}$$
$$\implies (\alpha^T\epsilon_2)\epsilon_2 = \begin{pmatrix} 0 \\ \alpha_2 \end{pmatrix} \tag{2.0.13}$$

using (2.0.11) and (2.0.13) we get,

$$\implies (\alpha^T\epsilon_1)\epsilon_1 + (\alpha^T\epsilon_2)\epsilon_2 = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \alpha \tag{2.0.14}$$

Hence proved