EE5609: Matrix Theory Assignment-13

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***Abstract*—This document contains a solution for a given function over a $n \times n$ diagonal matrix**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment13

1 PROBLEM

Let $\mathbf{A}$ be an $n \times n$ diagonal matrix over the field $\mathbf{F}$, i.e., a matrix satisfying $\mathbf{A}_{ij} = 0$ for $i \neq j$. Let f be the polynomial over $\mathbf{F}$ defined by $f = (x - A_{11})...(x - A_{nn})$. What is the matrix $f(\mathbf{A})$?

2 SOLUTION

Given $\mathbf{A}$ is a diagonal matrix. let, diagonal elements be,

$$A_{ij} = a_{ij} \quad i = j \quad i, j = 1, 2, ..., n \tag{2.0.1}$$

Characteristic equation of $\mathbf{A}$,

$$\left|A - \lambda\mathbf{I}\right| = 0 \tag{2.0.2}$$

$$\implies (a_{11} - \lambda)(a_{22} - \lambda)...(a_{nn} - \lambda) = 0 \tag{2.0.3}$$

$$\implies (A_{11} - \lambda)(A_{22} - \lambda)...(A_{nn} - \lambda) = 0 \tag{2.0.4}$$

from (2.0.4) we see $f(x) = 0$ is the characteristic equation of $\mathbf{A}$. Hence using Cayley-Hamilton theorem we get $f(\mathbf{A}) = 0$.