EE5609: Matrix Theory Assignment-14

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***Abstract*—This document contains a solution for calculating $E_i = P_i(\mathbf{A})$ for lagrange polynomials.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment14

1 Problem

Let $\mathbf{F}$ be the field of real numbers,

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{1.0.1}$$

$$p = (x-2)(x-3)(x-1) \tag{1.0.2}$$

Let P_1, P_2, P_3 be the Lagrange polynomials for $t_1 = 2,\ t_2 = 3,\ t_3 = 1$. Compute $E_i = P_i(\mathbf{A}) \quad i = 1, 2, 3$.

2 Solution

Lagrange polynomials are given by,

$$P_i = \prod_{i \neq j} \frac{x - t_j}{t_i - t_j} \tag{2.0.1}$$

$$P_1 = \frac{(x-3)(x-1)}{(2-3)(2-1)} = -(x-3)(x-1) \tag{2.0.2}$$

$$P_2 = \frac{(x-2)(x-1)}{(3-2)(3-1)} = \frac{1}{2}(x-2)(x-1) \tag{2.0.3}$$

$$P_3 = \frac{(x-2)(x-3)}{(1-2)(1-3)} = \frac{1}{2}(x-2)(x-3) \tag{2.0.4}$$

Now calculating E_i,

$$E_1 = P_1(\mathbf{A}) = -(\mathbf{A} - 3\mathbf{I})(\mathbf{A} - \mathbf{I}) \tag{2.0.5}$$

$$= -\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.6}$$

$$\implies E_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.7}$$

$$E_2 = P_2(\mathbf{A}) = \frac{1}{2}(\mathbf{A} - 2\mathbf{I})(\mathbf{A} - \mathbf{I}) \tag{2.0.8}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.9}$$

$$\implies E_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{2.0.10}$$

$$E_3 = P_3(\mathbf{A}) = \frac{1}{2}(\mathbf{A} - 2\mathbf{I})(\mathbf{A} - 3\mathbf{I}) \tag{2.0.11}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix} \tag{2.0.12}$$

$$\implies E_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.13}$$