EE5609: Matrix Theory Assignment-18

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***Abstract*—This document contains a proof for linear operators.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment18

1 Problem

Let $(|)$ be the standard inner product on $\mathbb{R}^2$.

(a) Let $\alpha = \begin{pmatrix}1\\2\end{pmatrix}$, $\beta = \begin{pmatrix}-1\\1\end{pmatrix}$. If γ is a vector such that $(\alpha|\gamma) = -1$ and $(\beta|\gamma) = 3$. Find γ

(b) Show that for any α in $\mathbb{R}^2$ we have $\alpha = (\alpha|\epsilon_1)\epsilon_1 + (\alpha|\epsilon_2)\epsilon_2$.

2 Solution

(a) From $\alpha = \begin{pmatrix}1\\2\end{pmatrix}$ and $(\alpha|\gamma) = -1$ we get,

$$(\alpha|\gamma) = -1 \tag{2.0.1}$$
$$\implies \alpha_1\gamma_1 + \alpha_2\gamma_2 = -1 \tag{2.0.2}$$
$$\implies \gamma_1 + 2\gamma_2 = -1 \tag{2.0.3}$$

from $\beta = \begin{pmatrix}-1\\1\end{pmatrix}$ and $(\beta|\gamma) = 3$ we get,

$$(\beta|\gamma) = 3 \tag{2.0.4}$$
$$\implies \beta_1\gamma_1 + \beta_2\gamma_2 = 3 \tag{2.0.5}$$
$$\implies -\gamma_1 + \gamma_2 = 3 \tag{2.0.6}$$

using (2.0.3) and (2.0.6),

$$\begin{pmatrix}1 & 2\\-1 & 1\end{pmatrix}\begin{pmatrix}\gamma_1\\\gamma_2\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix} \tag{2.0.7}$$

row reductions,

$$\begin{pmatrix}1 & 2 & -1\\-1 & 1 & 3\end{pmatrix} \xleftrightarrow{R_2 \to R_2 + R_1} \begin{pmatrix}1 & 2 & -1\\0 & 3 & 2\end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow{R_2 \to \frac{1}{3}R_2} \begin{pmatrix}1 & 2 & -1\\0 & 1 & \frac{2}{3}\end{pmatrix} \xleftrightarrow{R_1 \to R_1 - 2R_2} \begin{pmatrix}1 & 0 & \frac{-7}{3}\\0 & 1 & \frac{2}{3}\end{pmatrix} \tag{2.0.9}$$

Hence $\gamma = \begin{pmatrix}\frac{-7}{3}\\\frac{2}{3}\end{pmatrix}$

(b) let $\epsilon_1 = \begin{pmatrix}1\\0\end{pmatrix}$ and $\epsilon_2 = \begin{pmatrix}0\\1\end{pmatrix}$ are standard basis vectors,

$$(\alpha|\epsilon_1) = \alpha_1 + 0.\alpha_2 = \alpha_1 \tag{2.0.10}$$
$$\implies (\alpha|\epsilon_1)\epsilon_1 = \begin{pmatrix}\alpha_1\\0\end{pmatrix} \tag{2.0.11}$$
$$(\alpha|\epsilon_2) = 0.\alpha_1 + \alpha_2 = \alpha_2 \tag{2.0.12}$$
$$\implies (\alpha|\epsilon_2)\epsilon_2 = \begin{pmatrix}0\\\alpha_2\end{pmatrix} \tag{2.0.13}$$

using (2.0.11) and (2.0.13) we get,

$$\implies (\alpha|\epsilon_1)\epsilon_1 + (\alpha|\epsilon_2)\epsilon_2 = \begin{pmatrix}\alpha_1\\\alpha_2\end{pmatrix} = \alpha \tag{2.0.14}$$

Hence proved