# EE5609: Matrix Theory Assignment-10

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***Abstract*—This document checks the isomorphism of V of complex numbers on $\mathbb{R}^2$.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment10

## 1 Problem

Let **V** be the set of complex numbers and let **F** be the field of real numbers. With the usual operations, **V** is a vector space over **F**. Describe explicitly an isomorphism of this space onto $\mathbb{R}^2$.

## 2 Solution

Let,

$$\mathbf{T} : \mathbf{V} \to \mathbb{R}^2 \tag{2.0.1}$$

$$\mathbf{T}(x + iy) = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.0.2}$$

$$x, y \in \mathbb{R} \quad i \in \mathbb{C} \tag{2.0.3}$$

Consider two vectors,

$$\mathbf{u}, \mathbf{v} \in \mathbf{V} \tag{2.0.4}$$

Using (2.0.2) if **T** is Linear Transformation.

$$\mathbf{T}(\mathbf{u} + c\mathbf{v}) = \mathbf{T}(\mathbf{u}) + \mathbf{T}(c\mathbf{v}) = \mathbf{T}(\mathbf{u}) + c\mathbf{T}(\mathbf{v}) \tag{2.0.5}$$

Hence this is a Linear transformation. Now, checking if **T** is one-one. let,

$$\mathbf{u} = 0 = 0 + j0 \tag{2.0.6}$$

$$\tag{2.0.7}$$

from (2.0.2),

$$\mathbf{T}(\mathbf{u}) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.8}$$

From (2.0.8) we see **T** is one-one. Now, checking if **T** is onto. let,

$$\mathbf{T}(\mathbf{u}) = \begin{pmatrix} a \\ b \end{pmatrix} \tag{2.0.9}$$

where, $a$, $b$ are scalars. Using (2.0.2), we see there exists a solution for (2.0.8) for all $a$, $b$. Hence **T** is onto. Therefore **T** is isomorphic over **V** and $\mathbb{R}^2$