EE5609: Matrix Theory
Assignment-11

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***Abstract*—This document has an example of finding the Transpose or adjoint $\mathbf{T}^t$ of a linear operator T for a given function.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment12

1 PROBLEM

Let, $\mathbf{V}$ be the space of all $n \times n$ matrices over a field $\mathbf{F}$ and let B be a fixed $n \times n$ matrix. If T is the linear operator on $\mathbf{V}$ defined by $\mathbf{T}(\mathbf{A}) = \mathbf{AB} - \mathbf{BA}$, and if f is the trace function, what is $\mathbf{T}^t f$?

2 SOLUTION

$$\mathbf{T}^t f(\mathbf{A}) = f(\mathbf{T}(\mathbf{A})) \tag{2.0.1}$$
$$= f(\mathbf{AB} - \mathbf{BA}) \tag{2.0.2}$$
$$= trace(\mathbf{AB} - \mathbf{BA}) \tag{2.0.3}$$

using $trace(\mathbf{AB}) = trace(\mathbf{BA})$ in (2.0.3),

$$\mathbf{T}^t f(\mathbf{A}) = trace(\mathbf{AB}) - trace(\mathbf{BA}) = 0 \tag{2.0.4}$$

Hence $\mathbf{T}^t f = 0$