EE5609: Matrix Theory Challenge Question

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Abstract—**This document contains a proof for a theorem related to triangle.**

Download the python codes from

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https://github.com/pranaya14014/EE5609/tree/master/Assignment3/code
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and latex-tikz codes from

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https://github.com/pranaya14014/EE5609/tree/master/Assignment3
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1 PROBLEM

Sides opposite to equal angles of a triangle are equal.

2 SOLUTION

Given: Consider a $\triangle ABC$ with sides AB, BC, AC. Let direction vectors of AB, BC and CA be $\mathbf{a} = (\mathbf{A} - \mathbf{B})$, $\mathbf{b} = (\mathbf{B} - \mathbf{C})$, and $\mathbf{c} = (\mathbf{A} - \mathbf{C})$. Let $\angle ABC = \angle ACB = \theta$
Need to prove: $\|\mathbf{a}\| = \|\mathbf{c}\|$
Proof: Using cosine formula for $\angle ABC$,

$$\cos\theta = \frac{\mathbf{a}^{\mathbf{T}}\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|} \tag{2.0.1}$$

Using cosine formula for $\angle ACB$,

$$\cos\theta = \frac{\mathbf{c}^{\mathbf{T}}\mathbf{b}}{\|\mathbf{b}\|\,\|\mathbf{c}\|} \tag{2.0.2}$$

From (2.0.1) and (2.0.2),

$$\frac{\mathbf{a}^{\mathbf{T}}\mathbf{b}}{\|\mathbf{a}\|\,\|\mathbf{b}\|} = \frac{\mathbf{c}^{\mathbf{T}}\mathbf{b}}{\|\mathbf{b}\|\,\|\mathbf{c}\|} \tag{2.0.3}$$

$$\frac{\mathbf{a}^{\mathbf{T}}\mathbf{b}}{\|\mathbf{a}\|} = \frac{\mathbf{c}^{\mathbf{T}}\mathbf{b}}{\|\mathbf{c}\|} \tag{2.0.4}$$

$$\|\mathbf{c}\|\,\mathbf{a}^{\mathbf{T}}\mathbf{b} = \|\mathbf{a}\|\,\mathbf{c}^{\mathbf{T}}\mathbf{b} \tag{2.0.5}$$

$$\mathbf{b}\left(\|\mathbf{c}\|\,\mathbf{a}^{\mathbf{T}} - \|\mathbf{a}\|\,\mathbf{c}^{\mathbf{T}}\right) = 0 \tag{2.0.6}$$

From the above either $\mathbf{b} = 0$ or $\left(\|\mathbf{c}\|\,\mathbf{a}^{\mathbf{T}} - \|\mathbf{a}\|\,\mathbf{c}^{\mathbf{T}}\right) = 0$ or both are 0. From given as $\mathbf{b} = (\mathbf{B} - \mathbf{C}) = 0$ but this can't be true as $\mathbf{B}$ and $\mathbf{C}$ are points on a triangle. So,

$$\left(\|\mathbf{c}\|\,\mathbf{a}^{\mathbf{T}} - \|\mathbf{a}\|\,\mathbf{c}^{\mathbf{T}}\right) = 0 \tag{2.0.7}$$

From given we can write (2.0.7) as,

$$\|\mathbf{c}\|\,(\mathbf{A} - \mathbf{B})^T - \|\mathbf{a}\|\,(\mathbf{A} - \mathbf{C})^T = 0 \tag{2.0.8}$$

$$\|\mathbf{c}\|\left(\mathbf{A}^{\mathbf{T}} - \mathbf{B}^{\mathbf{T}}\right) - \|\mathbf{a}\|\left(\mathbf{A}^{\mathbf{T}} - \mathbf{C}^{\mathbf{T}}\right) = 0 \tag{2.0.9}$$

$$(\|\mathbf{c}\| - \|\mathbf{a}\|)\,\mathbf{A}^{\mathbf{T}} - \|\mathbf{c}\|\,\mathbf{B}^{\mathbf{T}} + \|\mathbf{a}\|\,\mathbf{C}^{\mathbf{T}} = 0 \tag{2.0.10}$$

As the $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are points on the triangle and using the Linear independence in (2.0.10) we get,

$$\|\mathbf{c}\| = \|\mathbf{a}\| \tag{2.0.11}$$

Hence Proved

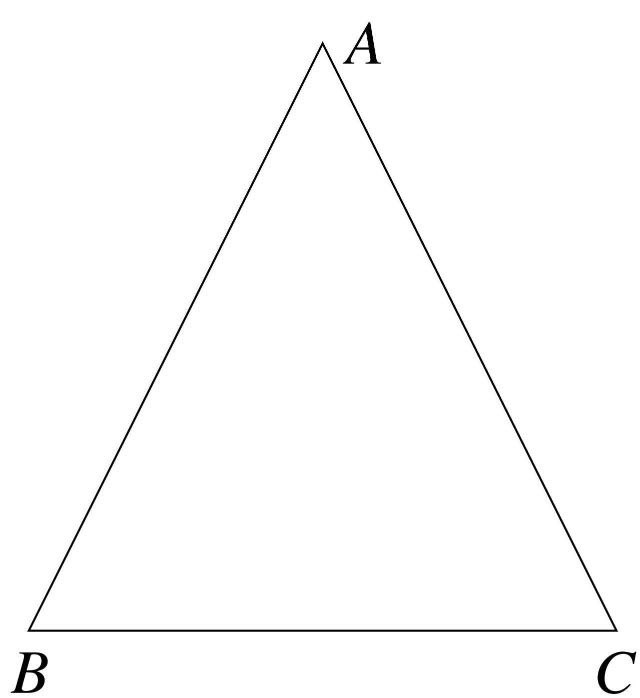

Fig. 0: Triangle