EE5609: Matrix Theory Assignment-3

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***Abstract*—This document contains a proof for a theorem related to triangle.**

Download the python codes from

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https://github.com/pranaya14014/EE5609/tree/master/Assignment3/code
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and latex-tikz codes from

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https://github.com/pranaya14014/EE5609/tree/master/Assignment3
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1 Problem

A line through the mid-point of a side of a triangle parallel to another side bisects the third side.

2 Solution

Given: Consider a $\triangle ABC$ with sides AB, BC, AC. Let $\mathbf{D}$ be the mid-point of AB, $\mathbf{E}$ be a point on AC and $DE \parallel BC$
Need to prove: $\mathbf{E} = \frac{A+C}{2}$
Proof:

$$\mathbf{D} = \frac{A+B}{2} \tag{2.0.1}$$

$$DE \parallel BC \tag{2.0.2}$$

Let direction vectors be $\mathbf{m}_{DE}$ and $\mathbf{m}_{BC}$ for line segment DE and BC respectively.

$$\mathbf{m}_{DE} = \mathbf{D} - \mathbf{E} = \frac{\mathbf{A}+\mathbf{B}}{2} - \mathbf{E} \tag{2.0.3}$$

$$\mathbf{m}_{BC} = \mathbf{B} - \mathbf{C} \tag{2.0.4}$$

From (2.0.2) we can write the following with $\mathbf{k}$ being a real value,

$$\mathbf{m}_{DE} = k\mathbf{m}_{BC} \tag{2.0.5}$$

Using (2.0.3) and (2.0.4) in the above equation,

$$\frac{\mathbf{A}+\mathbf{B}}{2} - \mathbf{E} = k(\mathbf{B} - \mathbf{C}) \tag{2.0.6}$$

Let $\mathbf{E} = \frac{m\mathbf{A}+C}{m+1}$ and Substitute $\mathbf{E}$ in (2.0.6)

$$\frac{\mathbf{A}+\mathbf{B}}{2} - \frac{m\mathbf{A}+C}{m+1} = k(\mathbf{B} - \mathbf{C}) \tag{2.0.7}$$

$$\left(\frac{1}{2} - \frac{m}{m+1}\right)\mathbf{A} + \left(\frac{1}{2} - k\right)\mathbf{B} + \left(k - \frac{1}{m+1}\right)\mathbf{C} = 0 \tag{2.0.8}$$

Since $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are points on a triangle and hence they are Linearly dependent which implies :

$\left(\frac{1}{2} - \frac{m}{m+1}\right)$=0 and $\left(\frac{1}{2} - k\right) = 0$ and $\left(k - \frac{1}{m+1}\right)$ =0

Therefore we get k = $\frac{1}{2}$ and m = 1
Substituting m value in $\mathbf{E}$ we get

$$\mathbf{E} = \frac{\mathbf{A}+\mathbf{C}}{2} \tag{2.0.9}$$

Hence Proved

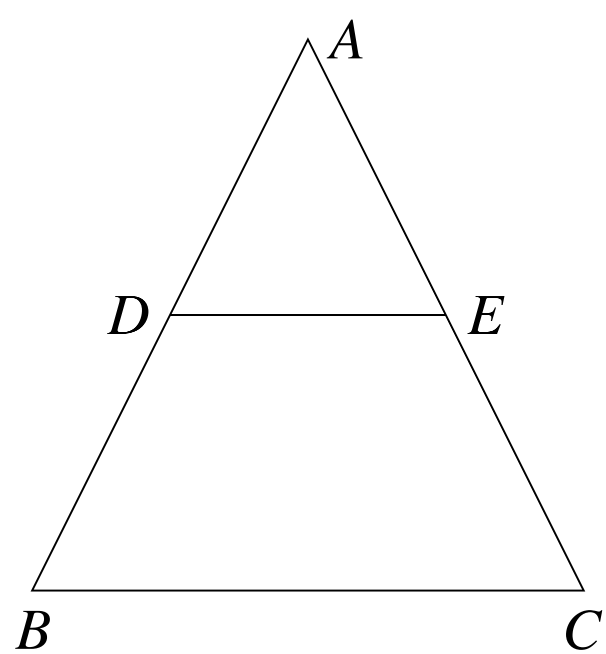

Fig. 0: Triangle