

EE5609: Matrix Theory

Assignment-3

Y.Pranaya
AI20MTECH14014

Abstract—This document contains a proof for a theorem related to triangle.

Download the python codes from

<https://github.com/pranaya14014/EE5609/tree/master/Assignment3/code>

and latex-tikz codes from

<https://github.com/pranaya14014/EE5609/tree/master/Assignment3>

1 PROBLEM

A line through the mid-point of a side of a triangle parallel to another side bisects the third side.

2 SOLUTION

Given: Consider a $\triangle ABC$ with sides AB, BC, AC. Let **D** be the mid-point of AB, **E** be a point on AC and $DE \parallel BC$

Need to prove: $\mathbf{E} = \frac{\mathbf{A} + \mathbf{C}}{2}$

Proof:

$$\mathbf{D} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (2.0.1)$$

$$DE \parallel BC \quad (2.0.2)$$

Let direction vectors be \mathbf{m}_{DE} and \mathbf{m}_{BC} for line segment DE and BC respectively.

$$\mathbf{m}_{DE} = \mathbf{D} - \mathbf{E} = \frac{\mathbf{A} + \mathbf{B}}{2} - \mathbf{E} \quad (2.0.3)$$

$$\mathbf{m}_{BC} = \mathbf{B} - \mathbf{C} \quad (2.0.4)$$

From (2.0.2) we can write the following with **k** being a real value,

$$\mathbf{m}_{DE} = k\mathbf{m}_{BC} \quad (2.0.5)$$

Using (2.0.3) and (2.0.4) in the above equation,

$$\frac{\mathbf{A} + \mathbf{B}}{2} - \mathbf{E} = k(\mathbf{B} - \mathbf{C}) \quad (2.0.6)$$

Let $\mathbf{E} = \frac{m\mathbf{A} + \mathbf{C}}{m+1}$ and Substitute **E** in (2.0.6)

$$\frac{\mathbf{A} + \mathbf{B}}{2} - \frac{m\mathbf{A} + \mathbf{C}}{m+1} = k(\mathbf{B} - \mathbf{C}) \quad (2.0.7)$$

$$\left(\frac{1}{2} - \frac{m}{m+1}\right)\mathbf{A} + \left(\frac{1}{2} - k\right)\mathbf{B} + \left(k - \frac{1}{m+1}\right)\mathbf{C} = 0 \quad (2.0.8)$$

Since **A**, **B** and **C** are points on a triangle and hence they are Linearly dependent which implies :

$$\left(\frac{1}{2} - \frac{m}{m+1}\right) = 0 \text{ and } \left(\frac{1}{2} - k\right) = 0 \text{ and } \left(k - \frac{1}{m+1}\right) = 0$$

Therefore we get $k = \frac{1}{2}$ and $m = 1$. Substituting **m** value in **E** we get,

$$\mathbf{E} = \frac{\mathbf{A} + \mathbf{C}}{2} \quad (2.0.9)$$

Hence Proved

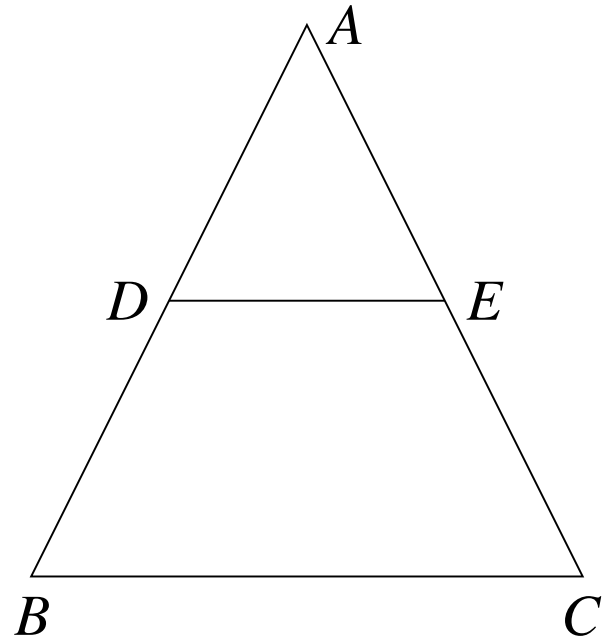


Fig. 0: Triangle