EE5609: Matrix Theory Assignment-10

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***Abstract*—This document checks the isomorphism of V of complex numbers on $\mathbb{R}^2$.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment10

1 PROBLEM

Let $\mathbf{V}$ be the set of complex numbers and let $\mathbf{F}$ be the field of real numbers. With the usual operations, $\mathbf{V}$ is a vector space over $\mathbf{F}$. Describe explicitly an isomorphism of this space onto $\mathbb{R}^2$.

2 SOLUTION

Let,

$$\mathbf{T} : \mathbf{V} \to \mathbb{R}^2 \tag{2.0.1}$$

$$\mathbf{T}(x + iy) = \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.0.2}$$

$$x, y \in \mathbb{R} \quad i \in \mathbb{C} \tag{2.0.3}$$

Consider two vectors,

$$\mathbf{u}, \mathbf{v} \in \mathbf{V} \tag{2.0.4}$$

Using (2.0.2) we see above vectors preserves multiplication and scalar multiplication as,

$$\mathbf{T}(\mathbf{u} + c\mathbf{v}) = \mathbf{T}(\mathbf{u}) + \mathbf{T}(c\mathbf{v}) = \mathbf{T}(\mathbf{u}) + c\mathbf{T}(\mathbf{v}) \tag{2.0.5}$$

Hence this is a Linear transformation and the inverse is $\mathbf{T}^{-1}(x, y) = (x + iy)$. Hence the two spaces are isomorphic.