EE5609: Matrix Theory Assignment-18

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***Abstract*—This document contains a solution for standard inner product.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment18

1 Problem

Let $(|)$ be the standard inner product on $\mathbb{R}^2$.

(a) Let

$$\alpha = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \beta = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{1.0.1}$$

If γ is a vector such that $(\alpha^T\gamma) = -1$ and $(\beta^T\gamma) = 3$. Find γ

(b) Show that for any α in $\mathbb{R}^2$ we have

$$\alpha = (\alpha^T\epsilon_1)\epsilon_1 + (\alpha^T\epsilon_2)\epsilon_2 \tag{1.0.2}$$

2 Solution

(a) From $\alpha = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $(\alpha^T\gamma) = -1$ we get,

$$(\alpha^T\gamma) = -1 \tag{2.0.1}$$

$$\implies \begin{pmatrix} 1 \\ 2 \end{pmatrix}^T \gamma = -1 \tag{2.0.2}$$

from $\beta = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $(\beta^T\gamma) = 3$ we get,

$$(\beta^T\gamma) = 3 \tag{2.0.3}$$

$$\implies \begin{pmatrix} -1 \\ 1 \end{pmatrix}^T \gamma = 3 \tag{2.0.4}$$

using (2.0.2) and (2.0.4),

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \gamma \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \tag{2.0.5}$$

row reductions,

$$\begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 3 \end{pmatrix} \xleftrightarrow{R_2 \to R_2 + R_1} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 2 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_2 \to \frac{1}{3}R_2} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & \frac{2}{3} \end{pmatrix} \xleftrightarrow{R_1 \to R_1 - 2R_2} \begin{pmatrix} 1 & 0 & \frac{-7}{3} \\ 0 & 1 & \frac{2}{3} \end{pmatrix} \tag{2.0.7}$$

Hence $\gamma = \begin{pmatrix} \frac{-7}{3} \\ \frac{2}{3} \end{pmatrix}$

(b) Here ϵ_1, ϵ_2 are standard basis vector in $\mathbb{R}^2$. As $\alpha \in \mathbb{R}^2$ we can write it as,

$$\alpha = \alpha_1\epsilon_1 + \alpha_2\epsilon_2 = \alpha^T \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \tag{2.0.8}$$

using this we can write,

$$(\alpha^T\epsilon_1) = \left(\alpha^T \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}\right)^T \epsilon_1 = \alpha^T \begin{pmatrix} \epsilon_1^T \\ \epsilon_2^T \end{pmatrix} \epsilon_1 \tag{2.0.9}$$

$$= \alpha^T \begin{pmatrix} \epsilon_1^T\epsilon_1 \\ \epsilon_2^T\epsilon_1 \end{pmatrix} = \alpha^T \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.10}$$

$$\implies (\alpha^T\epsilon_1)\epsilon_1 = \alpha^T \begin{pmatrix} \epsilon_1 \\ 0 \end{pmatrix} \tag{2.0.11}$$

$$(\alpha^T\epsilon_2) = \left(\alpha^T \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix}\right)^T \epsilon_2 = \alpha^T \begin{pmatrix} \epsilon_1^T \\ \epsilon_2^T \end{pmatrix} \epsilon_2 \tag{2.0.12}$$

$$= \alpha^T \begin{pmatrix} \epsilon_1^T\epsilon_2 \\ \epsilon_2^T\epsilon_2 \end{pmatrix} = \alpha^T \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.13}$$

$$\implies (\alpha^T\epsilon_2)\epsilon_2 = \alpha^T \begin{pmatrix} 0 \\ \epsilon_2 \end{pmatrix} \tag{2.0.14}$$

using (2.0.11) and (2.0.14) we can write,

$$(\alpha^T\epsilon_1)\epsilon_1 + (\alpha^T\epsilon_2)\epsilon_2 = \alpha^T \begin{pmatrix} \epsilon_1 \\ 0 \end{pmatrix} + \alpha^T \begin{pmatrix} 0 \\ \epsilon_2 \end{pmatrix} \tag{2.0.15}$$

$$\implies (\alpha^T\epsilon_1)\epsilon_1 + (\alpha^T\epsilon_2)\epsilon_2 = \alpha^T \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \tag{2.0.16}$$

hence using (2.0.8)and (2.0.16) we get,

$$\alpha = (\alpha^T \epsilon_1)\epsilon_1 + (\alpha^T \epsilon_2)\epsilon_2 \tag{2.0.17}$$

Hence proved