EE5609: Matrix Theory Assignment-15

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***Abstract*—This document contains a solution for finding the monic generator of the ideal.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment15

1 PROBLEM

Let $\mathbf{F}$ be a subfield of complex numbers, and let

$$\mathbf{A} = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix} \tag{1.0.1}$$

Find the monic generator of the ideal of all polynomials f in $F[z]$ such that $f(\mathbf{A}) = 0$.

2 SOLUTION

let, ideal $I = f(x) \in F[x]$ such that $f(\mathbf{A}) = 0$. where,

$$f(x) = \sum_{i=0}^{n} a_i x^i \quad a_n = 1 \tag{2.0.1}$$

Computing $f(\mathbf{A})$ for $deg(f) \leq 1$ we get,

$$\mathbf{A}^0 = \mathbf{I} \tag{2.0.2}$$

$$\mathbf{A}^1 = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix} \tag{2.0.3}$$

$$f(\mathbf{A}) = a_0 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + a_1 \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix} \tag{2.0.4}$$

As $\mathbf{I}$ and $\mathbf{A}$ are linearly independent so from (2.0.4) we see $f(\mathbf{A}) = 0$ only if $a_1 = 0$ and $a_0 = 0$ but this can't happen from (2.0.1). Hence for $deg(f) \leq 1$, $f(\mathbf{A}) \neq 0$. Hence for any $f(x) \in I$ such that $deg(f) = 2$ then f is Monic generator or minimal polymial. We can write,

$$f(x) = x^2 + a_1 x^1 + a_0 \tag{2.0.5}$$

Minimal polynomial or monic generator can be found using Characteristic equation,

$$\left| \mathbf{A} - \mathbf{I}\lambda \right| = 0 \tag{2.0.6}$$

$$\begin{vmatrix} 1-\lambda & -2 \\ 0 & 3-\lambda \end{vmatrix} = 0 \tag{2.0.7}$$

$$(1-\lambda)(3-\lambda) = 0 \tag{2.0.8}$$

$$\implies \lambda^2 - 4\lambda + 3 = 0 \tag{2.0.9}$$

comparing (2.0.5) and (2.0.9) we get,

$$f(x) = x^2 - 4x + 3 = 0 \tag{2.0.10}$$

using Cayley-Hamilton equation,

$$f(\mathbf{A}) = \mathbf{A}^2 - 4\mathbf{A} + 3 = 0 \tag{2.0.11}$$

Hence, $f = x^2 - 4x + 3$ is the monic generator.