EE5609: Matrix Theory Assignment-11

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***Abstract*—This document has a proof for trace(AB) = trace(BA) and for similar matrices have the same trace.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment11

1 Problem

If $\mathbf{A}$ and $\mathbf{B}$ are $n \times n$ matrices over the field $\mathbf{F}$, show that trace($\mathbf{AB}$) = trace($\mathbf{BA}$). Now show that similar matrices have the same trace.

2 Solution

$$trace(\mathbf{AB}) = \sum_{i=1}^{n} (\mathbf{AB})_{ii} \tag{2.0.1}$$
$$= \sum_{i=1}^{n}\sum_{k=1}^{n} \mathbf{A}_{ik}\mathbf{B}_{ki} \tag{2.0.2}$$
$$= \sum_{i=1}^{n}\sum_{k=1}^{n} \mathbf{B}_{ki}\mathbf{A}_{ik} \tag{2.0.3}$$
$$= \sum_{i=1}^{n} (\mathbf{BA})_{kk} \tag{2.0.4}$$
$$= trace(\mathbf{BA}) \tag{2.0.5}$$

Hence proved trace($\mathbf{AB}$) = trace($\mathbf{BA}$). Let $\mathbf{A}$ and $\mathbf{B}$ be similar matrices then $\exists$ $\mathbf{S}$ such that,

$$\mathbf{A} = \mathbf{S}^{-1}\mathbf{BS} \tag{2.0.6}$$

Taking trace on both sides

$$trace(\mathbf{A}) = trace(\mathbf{S}^{-1}\mathbf{BS}) = trace(\mathbf{S}^{-1}(\mathbf{BS})) \tag{2.0.7}$$

Using (2.0.2) in (2.0.4)

$$trace(\mathbf{A}) = trace(\mathbf{S}^{-1}\mathbf{SB}) \tag{2.0.8}$$
$$= trace((\mathbf{S}^{-1}\mathbf{S})\mathbf{B}) \tag{2.0.9}$$
$$= trace(\mathbf{IB}) \tag{2.0.10}$$
$$= trace(\mathbf{B}) \tag{2.0.11}$$

Hence Proved