EE5609: Matrix Theory Assignment-9

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***Abstract*—This document proves the given function is Linear Transformation**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment9

1 Problem

$$\mathbf{T}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_1 \end{pmatrix} \tag{1.0.1}$$

Does function $\mathbf{T}$ from $\mathbb{R}^2$ into $\mathbb{R}^2$ is Linear Transformation.

2 Solution

Let,

$$\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \tag{2.0.2}$$

Applying transformation on $\mathbf{T}$,

$$\mathbf{T} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{T}(\mathbf{x}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_1 \end{pmatrix} \tag{2.0.4}$$

$$\mathbf{T}(\mathbf{y}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} y_2 \\ y_1 \end{pmatrix} \tag{2.0.5}$$

With c being a scalar,

$$\mathbf{T}(c\mathbf{x} + \mathbf{y}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} cx_1 + y_1 \\ cx_2 + y_2 \end{pmatrix} \tag{2.0.6}$$

$$= \begin{pmatrix} cx_2 + y_2 \\ cx_1 + y_1 \end{pmatrix} \tag{2.0.7}$$

$$= \begin{pmatrix} cx_2 \\ cx_1 \end{pmatrix} + \begin{pmatrix} y_2 \\ y_1 \end{pmatrix} \tag{2.0.8}$$

$$= c\begin{pmatrix} x_2 \\ x_1 \end{pmatrix} + \begin{pmatrix} y_2 \\ y_1 \end{pmatrix} \tag{2.0.9}$$

From (2.0.4), (2.0.5) and (2.0.9) we get,

$$\mathbf{T}(c\mathbf{x} + \mathbf{y}) = c\mathbf{T}(\mathbf{x}) + \mathbf{T}(\mathbf{y}) \tag{2.0.10}$$

Hence from (2.0.10) we can say $\mathbf{T}$ is a Linear Transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$