

EE5609: Matrix Theory

Assignment-18

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Abstract—This document contains a proof for linear operators.

Download the latex-tikz codes from

<https://github.com/pranaya14014/EE5609/tree/master/Assignment18>

1 PROBLEM

Let $(\cdot|\cdot)$ be the standard inner product on \mathbb{R}^2 .

(a) Let

$$\alpha = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \beta = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (1.0.1)$$

If γ is a vector such that $(\alpha|\gamma) = -1$ and $(\beta|\gamma) = 3$. Find γ

(b) Show that for any α in \mathbb{R}^2 we have

$$\alpha = (\alpha|\epsilon_1)\epsilon_1 + (\alpha|\epsilon_2)\epsilon_2 \quad (1.0.2)$$

2 SOLUTION

(a) From $\alpha = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and $(\alpha|\gamma) = -1$ we get,

$$(\alpha|\gamma) = -1 \quad (2.0.1)$$

$$\Rightarrow \alpha_1\gamma_1 + \alpha_2\gamma_2 = -1 \quad (2.0.2)$$

$$\Rightarrow \gamma_1 + 2\gamma_2 = -1 \quad (2.0.3)$$

from $\beta = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ and $(\beta|\gamma) = 3$ we get,

$$(\beta|\gamma) = 3 \quad (2.0.4)$$

$$\Rightarrow \beta_1\gamma_1 + \beta_2\gamma_2 = 3 \quad (2.0.5)$$

$$\Rightarrow -\gamma_1 + \gamma_2 = 3 \quad (2.0.6)$$

using (2.0.3) and (2.0.6),

$$\begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix} \quad (2.0.7)$$

row reductions,

$$\begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 3 \end{pmatrix} \xrightarrow{R_2 \rightarrow R_2 + R_1} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & 2 \end{pmatrix} \quad (2.0.8)$$

$$\xrightarrow{R_2 \rightarrow \frac{1}{3}R_2} \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & \frac{2}{3} \end{pmatrix} \xrightarrow{R_1 \rightarrow R_1 - 2R_2} \begin{pmatrix} 1 & 0 & -\frac{7}{3} \\ 0 & 1 & \frac{2}{3} \end{pmatrix} \quad (2.0.9)$$

$$\text{Hence } \gamma = \begin{pmatrix} -\frac{7}{3} \\ \frac{2}{3} \end{pmatrix}$$

(b) let $\epsilon_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\epsilon_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are standard basis vectors,

$$(\alpha|\epsilon_1) = \alpha_1 + 0.\alpha_2 = \alpha_1 \quad (2.0.10)$$

$$\Rightarrow (\alpha|\epsilon_1)\epsilon_1 = \begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix} \quad (2.0.11)$$

$$(\alpha|\epsilon_2) = 0.\alpha_1 + \alpha_2 = \alpha_2 \quad (2.0.12)$$

$$\Rightarrow (\alpha|\epsilon_2)\epsilon_2 = \begin{pmatrix} 0 \\ \alpha_2 \end{pmatrix} \quad (2.0.13)$$

using (2.0.11) and (2.0.13) we get,

$$\Rightarrow (\alpha|\epsilon_1)\epsilon_1 + (\alpha|\epsilon_2)\epsilon_2 = \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} = \alpha \quad (2.0.14)$$

Hence proved