EE5609: Matrix Theory Assignment-2

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***Abstract*—This document contains a solution for solving the determinants using determinant properties.**

Download the python codes from

```
https://github.com/pranaya14014/EE5609/tree/
    master/Assignment2/code
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and latex-tikz codes from

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https://github.com/pranaya14014/EE5609/tree/
    master/Assignment2
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1 PROBLEM

Without expanding the determinant, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & b^3 \end{vmatrix} \tag{1.0.1}$$

2 PROPERTIES OF DETERMINANTS

Properties required for solving this problem:
Property.1: If all the elements of row/column are multiplied by a scalar then the value of new determinant is equal to multiplying the old determinant by that scalar.
Property.2: If any two rows/columns are exchanged then the sign of determinant changes.

3 SOLUTION

Proceeding to the solution by taking the left hand side matrix of (1.0.1) and applying properties of determinants.

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} \tag{3.0.1}$$

Using **Property.1** performing the following operations:

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} \xleftrightarrow{R_1 \to aR_1} \frac{1}{a} \begin{vmatrix} a^2 & a^3 & abc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} \tag{3.0.2}$$

$$\begin{vmatrix} a^2 & a^3 & abc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} \xleftrightarrow{R_2 \to bR_2} \frac{1}{ab} \begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c & c^2 & ab \end{vmatrix} \tag{3.0.3}$$

$$\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c & c^2 & ab \end{vmatrix} \xleftrightarrow{R_3 \to cR_3} \frac{1}{abc} \begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} \tag{3.0.4}$$

As the last column has similar elements,

$$\frac{abc}{abc} \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} \leftrightarrow \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} \tag{3.0.5}$$

Applying **Property.2** on (3.0.5):

$$\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} \xleftrightarrow{C_2 \to C_3} - \begin{vmatrix} a^2 & 1 & a^3 \\ b^2 & 1 & b^3 \\ c^2 & 1 & c^3 \end{vmatrix} \xleftrightarrow{C_1 \to C_2} \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \tag{3.0.6}$$

The above determinant is equal to right hand side determinant of equation (1.0.1).
Hence proved:

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & b^3 \end{vmatrix}$$