EE5609: Matrix Theory Assignment-2

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Abstract—**This document contains a solution for solving the determinants using determinant properties.**

Download the python codes from

```
https://github.com/pranaya14014/EE5609/tree/
    master/Assignment2/code
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and latex-tikz codes from

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https://github.com/pranaya14014/EE5609/tree/
    master/Assignment2
```

1 Problem

Without expanding the determinant, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & b^3 \end{vmatrix} \tag{1.0.1}$$

2 Solution

Proceeding to the solution by taking the left hand side matrix of (1.0.1) and applying properties of determinants.

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} \tag{2.0.1}$$

Using the property of determinants that multiplication of a constant to elements of any row/column is equal to multiplying the determinant by that constant. $\left|\mathbf{A_1}\right| = \mathbf{c}\left|\mathbf{A}\right|$ where $\mathbf{A_1}$ is the determinant of a matrix with a one row/column of matrix $\mathbf{A}$ multiplied by a constant $\mathbf{c}$.
Based on the above property performing the following operations:
Multiplying and dividing by $\mathbf{a}$

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \frac{1}{a}\begin{vmatrix} a^2 & a^3 & abc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} \tag{2.0.2}$$

Multiplying and dividing by $\mathbf{b}$

$$\begin{vmatrix} a^2 & a^3 & abc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \frac{1}{ab}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c & c^2 & ab \end{vmatrix} \tag{2.0.3}$$

Multiplying and dividing by $\mathbf{c}$

$$\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c & c^2 & ab \end{vmatrix} = \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & abc \\ c^2 & c^3 & abc \end{vmatrix} \tag{2.0.4}$$

As the last column has similar elements using the same property of determinant performing the following operation.

$$\frac{abc}{abc}\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} = \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} \tag{2.0.5}$$

Based on the property of determinants that if any two rows/columns are exchanged then the signs of determinant changes performing the following operations on (2.0.5):

$$C_2 \leftrightarrow C_3; C_1 \leftrightarrow C_2$$

$$\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix} = -\begin{vmatrix} a^2 & 1 & a^3 \\ b^2 & 1 & b^3 \\ c^2 & 1 & c^3 \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \tag{2.0.6}$$

The above determinant is equal to right hand side determinant of equation (1.0.1).
Hence proved:

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & b^3 \end{vmatrix}$$