

EE5609: Matrix Theory

Assignment-6

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Abstract—This document contains QR-decomposition of a 2x2 matrix. The above values are given by,

Download the latex-tikz codes from

<https://github.com/pranaya14014/EE5609/tree/master/Assignment6/>

1 PROBLEM

Find the QR decomposition of

$$\mathbf{A} = \begin{pmatrix} 8 & 5 \\ 3 & 2 \end{pmatrix} \quad (1.0.1)$$

2 SOLUTION

The matrix \mathbf{A} can be written as,

$$\mathbf{A} = (\mathbf{a} \quad \mathbf{b}) \quad (2.0.1)$$

where \mathbf{a} and \mathbf{b} are column vectors. From (1.0.1)

$$\mathbf{a} = \begin{pmatrix} 8 \\ 3 \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{b} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} \quad (2.0.3)$$

The QR decomposition of the given matrix is given by

$$\mathbf{A} = \mathbf{QR} \quad (2.0.4)$$

here \mathbf{R} is a upper triangular matrix and

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \quad (2.0.5)$$

where

$$\mathbf{Q} = (\mathbf{q}_1 \quad \mathbf{q}_2) \quad \mathbf{R} = \begin{pmatrix} r_1 & r_2 \\ 0 & r_3 \end{pmatrix} \quad (2.0.6)$$

$$r_1 = \|\mathbf{a}\| \quad (2.0.7)$$

$$\mathbf{q}_1 = \frac{\mathbf{a}}{r_1} \quad (2.0.8)$$

$$r_2 = \frac{\mathbf{q}_1^T \mathbf{b}}{\|\mathbf{q}_1\|^2} \quad (2.0.9)$$

$$\mathbf{q}_2 = \frac{\mathbf{b} - r_2 \mathbf{q}_1}{\|\mathbf{b} - r_2 \mathbf{q}_1\|} \quad (2.0.10)$$

$$r_3 = \mathbf{q}_2^T \mathbf{b} \quad (2.0.11)$$

Substituting (2.0.2) and (2.0.3) we get

$$r_1 = \sqrt{8^2 + 3^2} = \sqrt{73} \quad (2.0.12)$$

$$\mathbf{q}_1 = \frac{1}{\sqrt{73}} \begin{pmatrix} 8 \\ 3 \end{pmatrix} = \begin{pmatrix} \frac{8}{\sqrt{73}} \\ \frac{3}{\sqrt{73}} \end{pmatrix} \quad (2.0.13)$$

$$r_2 = \frac{1}{\left(\sqrt{\frac{64}{73} + \frac{9}{73}}\right)^2} \begin{pmatrix} \frac{8}{\sqrt{73}} & \frac{3}{\sqrt{73}} \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \frac{46}{\sqrt{73}} \quad (2.0.14)$$

$$\mathbf{q}_2 = \frac{1}{\sqrt{73}} \left(\begin{pmatrix} 5 \\ 2 \end{pmatrix} - \frac{46}{\sqrt{73}} \begin{pmatrix} \frac{8}{\sqrt{73}} \\ \frac{3}{\sqrt{73}} \end{pmatrix} \right) = \begin{pmatrix} \frac{-3}{73\sqrt{73}} \\ \frac{8}{73\sqrt{73}} \end{pmatrix} \quad (2.0.15)$$

$$r_3 = \begin{pmatrix} \frac{-3}{73\sqrt{73}} & \frac{8}{73\sqrt{73}} \end{pmatrix} \begin{pmatrix} 5 \\ 2 \end{pmatrix} = \frac{1}{73\sqrt{73}} \quad (2.0.16)$$

Hence substituting these values in (2.0.6) and then back in (2.0.4) we get,

$$\mathbf{A} = \begin{pmatrix} \frac{8}{\sqrt{73}} & \frac{-3}{73\sqrt{73}} \\ \frac{3}{\sqrt{73}} & \frac{8}{73\sqrt{73}} \end{pmatrix} \begin{pmatrix} \sqrt{73} & \frac{46}{\sqrt{73}} \\ 0 & \frac{1}{73\sqrt{73}} \end{pmatrix} \quad (2.0.17)$$

Hence QR decomposition is,

$$\begin{pmatrix} 8 & 5 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} \frac{8}{\sqrt{73}} & \frac{-3}{73\sqrt{73}} \\ \frac{3}{\sqrt{73}} & \frac{8}{73\sqrt{73}} \end{pmatrix} \begin{pmatrix} \sqrt{73} & \frac{46}{\sqrt{73}} \\ 0 & \frac{1}{73\sqrt{73}} \end{pmatrix} \quad (2.0.18)$$