

EE5609: Matrix Theory

Assignment-11

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Abstract—This document has a proof for $\text{trace}(\mathbf{AB}) = \text{trace}(\mathbf{BA})$ and for similar matrices have the same trace. Using (2.0.5) in (2.0.7)

Download the latex-tikz codes from

<https://github.com/pranaya14014/EE5609/tree/master/Assignment11>

$$\text{trace}(\mathbf{A}) = \text{trace}(\mathbf{S}^{-1}\mathbf{SB}) \quad (2.0.8)$$

$$= \text{trace}((\mathbf{S}^{-1}\mathbf{S})\mathbf{B}) \quad (2.0.9)$$

$$= \text{trace}(\mathbf{IB}) \quad (2.0.10)$$

$$= \text{trace}(\mathbf{B}) \quad (2.0.11)$$

Hence Proved

1 PROBLEM

If \mathbf{A} and \mathbf{B} are $n \times n$ matrices over the field \mathbf{F} , show that $\text{trace}(\mathbf{AB}) = \text{trace}(\mathbf{BA})$. Now show that similar matrices have the same trace.

2 SOLUTION

$$\text{trace}(\mathbf{AB}) = \sum_{i=1}^n (\mathbf{AB})_{ii} \quad (2.0.1)$$

$$= \sum_{i=1}^n \sum_{k=1}^n \mathbf{A}_{ik} \mathbf{B}_{ki} \quad (2.0.2)$$

$$= \sum_{i=1}^n \sum_{k=1}^n \mathbf{B}_{ki} \mathbf{A}_{ik} \quad (2.0.3)$$

$$= \sum_{i=1}^n (\mathbf{BA})_{ii} \quad (2.0.4)$$

$$= \text{trace}(\mathbf{BA}) \quad (2.0.5)$$

Hence proved $\text{trace}(\mathbf{AB}) = \text{trace}(\mathbf{BA})$. Let \mathbf{A} and \mathbf{B} be similar matrices then $\exists \mathbf{S}$ such that,

$$\mathbf{A} = \mathbf{S}^{-1}\mathbf{BS} \quad (2.0.6)$$

Taking trace of \mathbf{A} we get,

$$\text{trace}(\mathbf{A}) = \text{trace}(\mathbf{S}^{-1}\mathbf{BS}) = \text{trace}(\mathbf{S}^{-1}(\mathbf{BS})) \quad (2.0.7)$$