EE5609: Matrix Theory Assignment-16

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Abstract—**This document contains a solution for finding the monic generator of the ideal.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment16

1 PROBLEM

Let

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix} \tag{1.0.1}$$

and $\mathbf{I}$ be the 3×3 identity matrix. If

$$\mathbf{A}^{-1} = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I} \tag{1.0.2}$$

for $a, b, c \in \mathbb{R}$ then (a,b,c) equals

2 SOLUTION

First finding $\mathbf{A}^{-1}$

$$\left|\mathbf{A}\right| = \begin{vmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{vmatrix} = 6 \tag{2.0.1}$$

$$6\mathbf{A}^{-1} = \frac{6}{6}\begin{pmatrix} 6 & 0 & 4 \\ 3 & -3 & 2 \\ 0 & 0 & -2 \end{pmatrix} = \begin{pmatrix} 6 & 0 & 4 \\ 3 & -3 & 2 \\ 0 & 0 & -2 \end{pmatrix} \tag{2.0.2}$$

now finding $\mathbf{A}^2$,

$$\mathbf{A}^2 = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -4 \\ -1 & 4 & 2 \\ 0 & 0 & 9 \end{pmatrix} \tag{2.0.3}$$

$$a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I} = \begin{pmatrix} a & 0 & -4a \\ -a & 4a & 2a \\ 0 & 0 & 9a \end{pmatrix} + \begin{pmatrix} b & 0 & 2b \\ b & -2b & 0 \\ 0 & 0 & -3b \end{pmatrix} + \begin{pmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{pmatrix} \tag{2.0.4}$$

$$= \begin{pmatrix} a+b+c & 0 & -4a+2b \\ -a+b & 4a-2b+c & 2a \\ 0 & 0 & 9a-3b+c \end{pmatrix} \tag{2.0.5}$$

equating (2.0.2) and (2.0.5)

$$\begin{pmatrix} 6 & 0 & 4 \\ 3 & -3 & 2 \\ 0 & 0 & -2 \end{pmatrix} = \begin{pmatrix} a+b+c & 0 & -4a+2b \\ -a+b & 4a-2b+c & 2a \\ 0 & 0 & 9a-3b+c \end{pmatrix} \tag{2.0.6}$$

Equating both sides,

$$2a = 2 \quad -a+b = 3 \quad a+b+c = 6 \tag{2.0.7}$$

$$\implies a = 1 \quad b = 4 \quad c = 1 \tag{2.0.8}$$

Hence $(a, b, c) = (1, 4, 1)$