EE5609: Matrix Theory Assignment-16

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***Abstract*—This document contains a solution for finding the monic generator of the ideal.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment16

1 PROBLEM

Let

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{pmatrix} \tag{1.0.1}$$

and $\mathbf{I}$ be the 3×3 identity matrix. If

$$6\mathbf{A}^{-1} = a\mathbf{A}^2 + b\mathbf{A} + c\mathbf{I} \tag{1.0.2}$$

for $a, b, c \in \mathbb{R}$ then (a,b,c) equals

$$1. \quad (1, 2, 1) \tag{1.0.3}$$
$$2. \quad (1, -1, 2) \tag{1.0.4}$$
$$3. \quad (4, 1, 1) \tag{1.0.5}$$
$$4. \quad (1, 4, 1) \tag{1.0.6}$$

2 SOLUTION

Finding the characteristic equation,

$$\left|\mathbf{A} - \lambda\mathbf{I}\right| = \begin{vmatrix} 1-\lambda & 0 & 2 \\ 1 & -2-\lambda & 0 \\ 0 & 0 & -3-\lambda \end{vmatrix} \tag{2.0.1}$$

$$\implies (1-\lambda)(-2-\lambda)(-3-\lambda) = 0 \tag{2.0.2}$$
$$\implies (\lambda^2 + \lambda - 2)(-3-\lambda) = 0 \tag{2.0.3}$$
$$\implies \lambda^3 + 4\lambda^2 + \lambda - 6 = 0 \tag{2.0.4}$$

Using Cayley-Hamilton Theorem we get,

$$\mathbf{A}^3 + 4\mathbf{A}^2 + \mathbf{A} - 6\mathbf{I} = 0 \tag{2.0.5}$$
$$\implies \mathbf{A}^3 + 4\mathbf{A}^2 + \mathbf{A} = 6\mathbf{I} \tag{2.0.6}$$
$$\implies \mathbf{A}(\mathbf{A}^2 + 4\mathbf{A} + \mathbf{I}) = 6\mathbf{I} \tag{2.0.7}$$

$\left|\mathbf{A}\right| = 6 \neq 0$ hence inverse exists. Hence (2.0.7) we get,

$$6\mathbf{A}^{-1} = \mathbf{A}^2 + 4\mathbf{A} + \mathbf{I} \tag{2.0.8}$$

Comparing (1.0.2) and (2.0.8) we get,

$$a = 1 \quad b = 4 \quad c = 1 \tag{2.0.9}$$

Hence $(a, b, c) = (1, 4, 1)$