EE5609: Matrix Theory Assignment-3

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 $\begin{subarray}{c} Abstract — This document contains a proof for a theorem related to triangle. \end{subarray}$

Download the python codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment3/code

and latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment3

1 PROBLEM

A line through the mid-point of a side of a triangle parallel to another side bisects the third side.

2 SOLUTION

Given: Consider a $\triangle ABC$ with sides AB, BC, AC. Let **D** be the mid-point of AB, **E** be a point on AC and $DE \parallel BC$

Need to prove: $E = \frac{A+C}{2}$ Proof:

$$\mathbf{D} = \frac{A+B}{2} \tag{2.0.1}$$

$$DE \parallel BC \tag{2.0.2}$$

Let direction vectors be \mathbf{m}_{DE} and \mathbf{m}_{BC} for line segment DE and BC respectively.

$$\mathbf{m}_{DE} = \mathbf{D} - \mathbf{E} = \frac{\mathbf{A} + \mathbf{B}}{2} - \mathbf{E}$$
 (2.0.3)

$$\mathbf{m}_{BC} = \mathbf{B} - \mathbf{C} \tag{2.0.4}$$

From (2.0.2) we can write the following with \mathbf{k} being a real value,

$$\mathbf{m}_{DE} = k\mathbf{m}_{BC} \tag{2.0.5}$$

Using (2.0.3) and (2.0.4) in the above equation,

$$\frac{\mathbf{A} + \mathbf{B}}{2} - \mathbf{E} = k(\mathbf{B} - \mathbf{C}) \tag{2.0.6}$$

From the above we get $k = \frac{1}{2}$ Substituting k value in (2.0.6)

$$\frac{\mathbf{A}}{2} - \mathbf{E} = -\frac{1}{2}\mathbf{C} \tag{2.0.7}$$

$$\mathbf{E} = \frac{\mathbf{A} + \mathbf{C}}{2} \tag{2.0.8}$$

Hence Proved

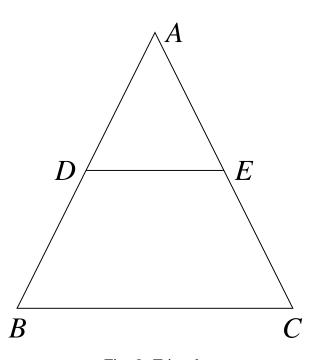


Fig. 0: Triangle