

EE5609: Matrix Theory

Assignment-13

Y.Pranaya
AI20MTECH14014

Abstract—This document contains a solution for a given function over a $n \times n$ diagonal matrix

Download the latex-tikz codes from

<https://github.com/pranaya14014/EE5609/tree/master/Assignment13>

similarly, each diagonal entry of $(\mathbf{A} - \mathbf{A}_{11}) \dots (\mathbf{A} - \mathbf{A}_{nn})$ is a product of numbers one of which is zero. Hence, $f(\mathbf{A})$ is a zero matrix.

1 PROBLEM

Let \mathbf{A} be an $n \times n$ diagonal matrix over the field \mathbf{F} , i.e., a matrix satisfying $\mathbf{A}_{ij} = 0$ for $i \neq j$. Let f be the polynomial over \mathbf{F} defined by $f = (x - \mathbf{A}_{11}) \dots (x - \mathbf{A}_{nn})$. What is the matrix $f(\mathbf{A})$?

2 SOLUTION

Given \mathbf{A} is a diagonal matrix. let, diagonal elements be,

$$\mathbf{A}_{ij} = a_{ij} \quad i = j \quad i, j = 1, 2, \dots, n \quad (2.0.1)$$

$$\mathbf{A} - \mathbf{A}_{11} = \begin{pmatrix} 0 & \dots & \dots \\ 0 & (a_{22} - a_{11}) & \dots \\ \vdots & \vdots & \vdots \\ 0 & 0 & (a_{nn} - a_{11}) \end{pmatrix} \quad (2.0.2)$$

$$\mathbf{A} - \mathbf{A}_{22} = \begin{pmatrix} (a_{11} - a_{22}) & \dots & \dots \\ 0 & 0 & \dots \\ \vdots & \vdots & \vdots \\ 0 & 0 & (a_{nn} - a_{22}) \end{pmatrix} \quad (2.0.3)$$

similarly, $\mathbf{A} - \mathbf{A}_{ii}$ is a diagonal matrix with i, i element zero.

$$(\mathbf{A} - \mathbf{A}_{11})(\mathbf{A} - \mathbf{A}_{22}) =$$

$$\begin{pmatrix} 0 & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots \\ \vdots & (a_{ii} - a_{11})(a_{ii} - a_{22}) & \vdots & \vdots \\ 0 & 0 & \dots & (a_{nn} - a_{11})(a_{nn} - a_{22}) \end{pmatrix} \quad (2.0.4)$$