

EE5609: Matrix Theory

Assignment-17

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Abstract—This document contains a proof for linear operators.

Download the latex-tikz codes from

<https://github.com/pranaya14014/EE5609/tree/master/Assignment17>

1 PROBLEM

Let E_1, \dots, E_k be linear operators on the space \mathbf{V} such that $E_1 + \dots + E_k = \mathbf{I}$.

a) Prove that if $E_i E_j = 0$ for $i \neq j$, then $E_i^2 = E_i$ for each i .

b) In the case $k = 2$, prove the converse of (a). That is, if $E_1 + E_2 = \mathbf{I}$ and $E_1^2 = E_1$, $E_2^2 = E_2$, then $E_1 E_2 = 0$

2 SOLUTION

a) From the given,

$$E_1 + \dots + E_k = \mathbf{I} \quad (2.0.1)$$

$$\implies E_i = \mathbf{I} - E_1 - \dots - E_{i-1} - E_{i+1} - \dots - E_k \quad (2.0.2)$$

$$\implies E_i^2 = E_i (\mathbf{I} - E_1 - \dots - E_{i-1} - E_{i+1} - \dots - E_k) \quad (2.0.3)$$

$$\begin{aligned} \implies E_i^2 &= E_i - E_i E_1 - \dots - E_i E_{i-1} - E_i E_{i+1} - \dots \\ &\quad - E_i E_k \end{aligned} \quad (2.0.4)$$

$$\implies E_i^2 = E_i - \sum_{i \neq j} E_i E_j \quad (2.0.5)$$

substituting $E_i E_j = 0$ for $i \neq j$ in the above equation we get,

$$E_i^2 = E_i - 0 \quad (2.0.6)$$

$$\implies E_i^2 = E_i \quad (2.0.7)$$

Hence proved if $E_i E_j = 0$ for $i \neq j$, then $E_i^2 = E_i$
b) Using,

$$E_1 + E_2 = \mathbf{I} \quad (2.0.8)$$

Multiplying both sides by E_1 ,

$$E_1(E_1 + E_2) = E_1 \quad (2.0.9)$$

$$\implies E_1^2 + E_1 E_2 = E_1 \quad (2.0.10)$$

Substituting $E_1^2 = E_1$ in (2.0.8) we get,

$$\implies E_1 + E_1 E_2 = E_1 \quad (2.0.11)$$

$$\implies E_1 E_2 = 0 \quad (2.0.12)$$

Similarly multiplying on both sides of (2.0.8) and substituting $E_2^2 = E_2$ we get,

$$\implies E_1 E_2 + E_2^2 = E_2 \quad (2.0.13)$$

$$\implies E_1 E_2 + E_2 = E_2 \quad (2.0.14)$$

$$\implies E_1 E_2 = 0 \quad (2.0.15)$$

Hence proved from (2.0.12) and (2.0.15) that if $E_1 + E_2 = \mathbf{I}$ and $E_1^2 = E_1$, $E_2^2 = E_2$, then $E_1 E_2 = 0$