# EE5609: Matrix Theory
# Challenge Question

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***Abstract*—This document contains a solution to a inverse of rectangular matrix.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Challenge1

## 1 PROBLEM

$$\begin{pmatrix} 4 & -2+\sqrt{3} \\ 3 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -5 \\ -2 \\ 2+\sqrt{3} \end{pmatrix} \tag{1.0.1}$$

Find $\mathbf{c}$

## 2 SOLUTION

let,

$$\mathbf{A} = \begin{pmatrix} 4 & -2+\sqrt{3} \\ 3 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{b} = \begin{pmatrix} -5 \\ -2 \\ 2+\sqrt{3} \end{pmatrix} \tag{2.0.2}$$

Then,

$$\mathbf{c} = \mathbf{A^{-1}b} \tag{2.0.3}$$

Using left inverse,

$$\mathbf{A^{-1}} = (\mathbf{A^T A})^{-1}\mathbf{A^T} \tag{2.0.4}$$

Using (2.0.1) in (2.0.4)

$$\mathbf{A^T} = \begin{pmatrix} 4 & 3 & -\sqrt{3} \\ -2+\sqrt{3} & -\sqrt{3} & 1 \end{pmatrix} \tag{2.0.5}$$

$$\mathbf{A^T A} = \begin{pmatrix} 28 & -8 \\ -8 & 17-4\sqrt{3} \end{pmatrix} \tag{2.0.6}$$

$$(\mathbf{A^T A})^{-1} = \frac{1}{444-112\sqrt{3}} \begin{pmatrix} 17-4\sqrt{3} & 8 \\ 4 & 28 \end{pmatrix} \tag{2.0.7}$$

$$\mathbf{A^{-1}} = (\mathbf{A^T A})^{-1}\mathbf{A^T} = \frac{1}{444-112\sqrt{3}} \begin{pmatrix} 52-8\sqrt{3} & 51-20\sqrt{3} & -17\sqrt{3}+20 \\ -40+28\sqrt{3} & 12-28\sqrt{3} & -4\sqrt{3}+28 \end{pmatrix} \tag{2.0.8}$$

Subtituting (2.0.2) and (2.0.8) in (2.0.3)

$$\mathbf{c} = \frac{1}{444-112\sqrt{3}} \begin{pmatrix} -373+66\sqrt{3} \\ 220+216\sqrt{3} \end{pmatrix} \tag{2.0.9}$$