EE5609: Matrix Theory
Assignment-8

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***Abstract*—This document contains a proof to show that upper-triangular matrix is invertible if and only if diagonal elements are not 0.**

Download the latex-tikz codes from

https://github.com/pranaya14014/EE5609/tree/master/Assignment8

1 PROBLEM

An $n \times n$ matrix $\mathbf{A}$ is called upper-triangular if $\mathbf{A_{ij}} = 0$ for $i > j$, that is, if every entry below the main diagonal is 0. Prove that an upper-triangular (square) matrix is invertible if and only if every entry on its main diagonal is different from 0.

2 SOLUTION

Considering $\mathbf{A}$, an upper triangular matrix. Using the property that determinant of upper triangular matrix is the product of diagonal elements,

$$\left|\mathbf{A}\right| = a_{1,1}a_{2,2}...a_{(n-1),(n-1)}a_{n,n} \tag{2.0.1}$$

If $\mathbf{A}$ be invertible then $\left|\mathbf{A}\right| \neq 0$. Hence from (2.0.1) we get,

$$a_{1,1}a_{2,2}...a_{(n-1),(n-1)}a_{n,n} \neq 0 \tag{2.0.2}$$

if any diagonal element is 0 then (2.0.2) won't be right hence no diagonal elements should be 0.
Hence Proved.