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exercise 1

$N = 30 \qquad p = 3 \quad q = 5 \qquad m = 2$

a) Model V2:

$H_0: \gamma_{ij} = 0 \quad \forall i,j \qquad H_1: \gamma_{ij} \neq 0$ for at least one (i,j)

Test statistic $R = \frac{2}{8} \cdot \frac{\hat{D}_R - \hat{D}_{1,2}}{\hat{D}_{1,2}} \sim F_{2,14,15}$

we accept H_0 if $R \leq f$ as f is the $(1-\alpha)$ quantile of the $F_{8,15}$ distribution.

b) filter data for one seed and make 3 one factor anova tests of the form: (with β_i are the factors due to fertilizers

$H_0: \beta_1 = \dots = \beta_5 = 0 \qquad H_1: \beta_i \neq 0$ for at least one $i \in \{1, \dots, 5\}$

$R = \frac{5}{4} \frac{\hat{D}_B}{\hat{D}_I}$

accept H_0 if $R \leq f$ where f is the $(1-\alpha)$-quantile of the $F_{4,5}$ distribution

c) make the 2 factor anova test where α_i are the factors due to seeds

$H_0: \alpha_1 = \alpha_2 = \alpha_3 = 0 \qquad H_1: \alpha_i \neq 0$ for at least one $i \in \{1,2,3\}$

$R = \frac{23}{2} \cdot \frac{\hat{D}_1}{\hat{D}_R}$

accept H_0 if $R \leq f$ where f is the $(1-\alpha)$ Quantil of the $F_{2,23}$ distrib.

exercise 4

$Y_1, \dots, Y_N$ ind. $\mathcal{L}(Y_i) = \text{Pois}(\lambda_i)$ $\quad g = \log$

$$\ell(b \mid Y_1, \dots, Y_N) \underset{\substack{\uparrow \\ \text{def of} \\ \text{Pois distrib.}}}{=} \sum_{i=1}^{N} \log\left(e^{-\lambda_i} \frac{\lambda_i^{Y_i}}{Y_i!}\right) = \sum_{i=1}^{N} -\lambda_i + Y_i \log(\lambda_i) - \underbrace{\log(Y_i!)}_{\substack{\text{no dependence} \\ \text{on } \lambda_i \rightsquigarrow \text{ can be} \\ \text{omitted for maximizing} \\ \ell}}$$

for the saturated model compute b_{max}:

$$\hat{b}_{max} = \underset{b}{\arg\max} \sum_{i=1}^{N} \underbrace{(-b_i + Y_i \cdot \log(b_i))}_{=: G(b_i)}$$

$$G' = -1 + Y_i \cdot \frac{1}{b_i} \overset{!}{=} 0 \iff b_i = Y_i \Rightarrow \hat{b}_{max} = \begin{pmatrix} Y_1 \\ \vdots \\ Y_N \end{pmatrix}$$

$$\Rightarrow D = 2 \cdot \left(\ell(\hat{b}_{max} \mid Y_1, \dots, Y_N) - \ell(\hat{b} \mid Y_1, \dots, Y_N)\right)$$

$$= 2 \sum_{i=1}^{N} -Y_i + Y_i \log(Y_i) + \hat{\lambda}_i - Y_i \log(\hat{\lambda}_i)$$

$$= 2 \sum_{i=1}^{N} \hat{\lambda}_i - Y_i + Y_i \log\left(\frac{Y_i}{\hat{\lambda}_i}\right) = 2 \sum_{i=1}^{N} \hat{\lambda}_i + Y_i \cdot \left(\log\left(\frac{Y_i}{\hat{\lambda}_i}\right) - 1\right)$$