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Ex7.1

a) $\hat{D}_B = \sum_{j=1}^{p=3} n_j (\bar{Y}_{j\cdot} - \bar{Y}_{\cdot\cdot})^2 = n_1(\bar{Y}_{1\cdot} - \bar{Y}_{\cdot\cdot})^2 + n_2(\bar{Y}_{2\cdot} - \bar{Y}_{\cdot\cdot})^2 + n_3(\bar{Y}_{3\cdot} - \bar{Y}_{\cdot\cdot})^2$

$\bar{Y}_{1\cdot} = \frac{1.52 + 1.38 + 1.29 + 1.48 + 1.63}{5} = \frac{73}{50} = 1.46$

$\bar{Y}_{2\cdot} = \frac{1.63 + 1.82 + 1.35 + 1.03 + 2.30 + 1.45}{6} = \frac{479}{300} = 1.5967$

$\bar{Y}_{3\cdot} = \frac{2.56 + 3.32 + 2.76 + 2.63 + 2.12 + 2.78}{6} = \frac{539}{200} = 2.695$

$\bar{Y}_{\cdot\cdot} = \frac{1}{N} \sum_{j=1}^{p=3} \sum_{k}^{n_j} Y_{jk} = \left(\frac{1}{17}\right)\left(33.05\right) = \frac{330.5}{170} = \frac{661}{340}$

$= 1.9441$

$\Rightarrow \hat{D}_B = 5\left(\frac{73}{50} - \frac{661}{340}\right)^2 + 6\left(\frac{479}{300} - \frac{661}{340}\right)^2 + 6\left(\frac{539}{200} - \frac{661}{340}\right)^2$

$= 5.27913$

$\hat{D}_E = \sum_{j=1}^{p=3} \sum_{k=1}^{n_j} (Y_{jk} - \bar{Y}_{j\cdot})^2 = \sum_{k=1}^{5} (Y_{1k} - \bar{Y}_{1\cdot})^2 + \sum_{k=1}^{6} (Y_{2k} - \bar{Y}_{2\cdot})^2 + \sum_{k=1}^{6} (Y_{3k} - \bar{Y}_{3\cdot})^2$

$= (1.52 - 1.46)^2 + (1.38 - 1.46)^2 + \dots + (1.63 - 1.5967)^2 + \dots + (2.56 - 2.695)^2$

$= 1.77248$

$R = \frac{N-p}{p-1} \cdot \frac{\hat{D}_B}{\hat{D}_E} = \frac{17-3}{3-1} \cdot \frac{5.27913}{1.77248} = 20.0487$

(for H_0: T1, T2, T3 have no contribution to the mean)

for $\alpha = 0.01$, $F_{1-\alpha, p-1, N-p} = F_{0.99, 2, 14} = 6.51488$

$\Rightarrow R > F_{0.99,2,14} \Rightarrow H_0$ is rejected, so the amount of rainfall truly does have an effect on soil moisture deficit

b) For pairwise tests $p=2$

i) H_0: there is no difference b/w the two treatments T1 & T2

$$\hat{D}_B = \sum_{j=1}^{p=2} n_j (\bar{Y}_{j\cdot} - \bar{Y}_{\cdot\cdot})^2 = n_1(\bar{Y}_{1\cdot} - \bar{Y}_{\cdot\cdot})^2 + n_2(\bar{Y}_{2\cdot} - \bar{Y}_{\cdot\cdot})^2$$

$$\bar{Y}_{\cdot\cdot} = \frac{1}{N} \sum_{j=1}^{p=2} \sum_{k=1}^{n_j} Y_{jk} = \left(\frac{1}{11}\right)(16.88) = \frac{422}{275} = 1.5345$$

$$\hat{D}_B = 5\left(\frac{73}{50} - \frac{422}{275}\right)^2 + 6\left(\frac{479}{300} - \frac{422}{275}\right)^2 = 0.0509379$$

$$\hat{D}_I = \sum_{j=1}^{p=2} \sum_{k=1}^{n_j} (Y_{jk} - \bar{Y}_{j\cdot})^2 = \left(1.52 - \frac{73}{50}\right)^2 + \left(1.58 - \frac{73}{50}\right)^2$$

$$+ \left(1.29 - \frac{73}{50}\right)^2 + \left(1.48 - \frac{73}{50}\right)^2 + \left(1.63 - \frac{73}{50}\right)^2$$

$$+ \left(1.63 - \frac{479}{300}\right)^2 + \left(1.82 - \frac{479}{300}\right)^2 + \left(1.35 - \frac{479}{300}\right)^2$$

$$+ \left(1.03 - \frac{479}{300}\right)^2 + \left(2.30 - \frac{479}{300}\right)^2 + \left(1.45 - \frac{479}{300}\right)^2$$

$$= 0.0682 + 0.9491333$$

$$= 1.017333$$

$$R = \frac{N-p}{p-1} \cdot \frac{\hat{D}_B}{\hat{D}_I} = \frac{(11-2)}{(2-1)} \cdot \frac{(0.0509379)}{(1.017333)}$$

$$= 0.45$$

$$F_{1-\alpha, p-1, N-p} = F_{0.99, 2-1, 11-2} = 10.56$$

$$\Rightarrow R < F_{0.99, 2-1, 11-2} \Rightarrow \boxed{H_0 \text{ is accepted}}$$

ii) H_0: there is no difference b/w the two treatments T2 & T3

$$\hat{D}_B = \sum_{j=2}^{3} n_j (\bar{Y}_{j\cdot} - \bar{Y}_{\cdot\cdot}) = n_2 (\bar{Y}_{2\cdot} - \bar{Y}_{\cdot\cdot}) + n_3 (\bar{Y}_{3\cdot} - \bar{Y}_{\cdot\cdot})^2$$

$$\bar{Y}_{\cdot\cdot} = \frac{1}{N} \sum_{j=2}^{3} \sum_{k=1}^{n_j} Y_{jk} = \left(\frac{1}{12}\right)(25.75) = \frac{103}{48} = 2.1458$$

$$\hat{D}_B = 6\left(\frac{479}{300} - \frac{103}{48}\right)^2 + 6\left(\frac{539}{200} - \frac{103}{48}\right)^2$$

$$= 3.61900$$

$$\hat{D}_I = \sum_{j=2}^{3} \sum_{k=1}^{n_j} (Y_{jk} - \bar{Y}_{j\cdot})^2 = \left(1.63 - \frac{479}{300}\right)^2 + \left(1.82 - \frac{479}{300}\right)^2$$

$$+ \left(1.38 - \frac{479}{300}\right)^2 + \left(1.03 - \frac{479}{300}\right)^2 + \left(2.30 - \frac{479}{300}\right)^2$$

$$+ \left(1.45 - \frac{479}{300}\right)^2$$

$$+ \left(2.56 - \frac{539}{200}\right)^2 + \left(3.32 - \frac{539}{200}\right)^2 + \left(2.76 - \frac{539}{200}\right)^2$$

$$+ \left(2.63 - \frac{539}{200}\right)^2 + \left(2.12 - \frac{539}{200}\right)^2 + \left(2.78 - \frac{539}{200}\right)^2$$

$$= 0.9491333 + 0.755115 = 1.707283$$

$$R = \frac{N-p}{p-1} \cdot \frac{\hat{D}_B}{\hat{D}_I} = \frac{(12-2)}{(2-1)} \cdot \frac{(3.61900)}{(1.707283)} = 19.111262$$

$$F_{1-\alpha, p-1, N-p} = F_{0.99, 2-1, 12-2} = 10.044429$$

$$\Rightarrow R > F_{0.99, 2-1, 12-2} \Rightarrow \boxed{H_0 \text{ is rejected}}$$

iii) H_0: there is no difference b/w treatments T1 and T3

$$\hat{D}_B = \sum_{j \in \{1,3\}} n_j (\bar{Y}_{j\cdot} - \bar{Y}_{\cdot\cdot}) = n_1 (\bar{Y}_{1\cdot} - \bar{Y}_{\cdot\cdot}) + n_3 (\bar{Y}_{3\cdot} - \bar{Y}_{\cdot\cdot})$$

$$\bar{Y}_{\cdot\cdot} = \frac{1}{N} \sum_{j \in \{1,3\}} \sum_{k=1}^{n_j} Y_k = \left(\frac{1}{11}\right)\left(23.47\right) = \frac{2347}{1100}$$

$$= 2.13363$$

$$\hat{D}_B = 5\left(\frac{73}{50} - \frac{2347}{1100}\right)^2 + 6\left(\frac{539}{200} - \frac{2347}{1100}\right)^2$$

$$= 4.16970$$

$$\hat{D}_I = \sum_{j \in \{1,3\}} \sum_{k=1}^{n_j} (Y_{jk} - \bar{Y}_{j\cdot})^2$$

$$= \left(1.52 - \frac{73}{50}\right)^2 + \left(1.30 - \frac{73}{50}\right)^2 + \left(1.29 - \frac{73}{50}\right)^2 + \left(1.48 - \frac{73}{50}\right)^2$$

$$+ \left(1.63 - \frac{73}{50}\right)^2 + \left(2.56 - \frac{539}{200}\right)^2 + \left(3.32 - \frac{539}{200}\right)^2$$

$$+ \left(2.76 - \frac{539}{200}\right)^2 + \left(2.63 - \frac{539}{200}\right)^2 + \left(2.12 - \frac{539}{200}\right)^2$$

$$+ \left(2.78 - \frac{539}{200}\right)^2 = 0.0682 + 0.75515$$

$$= 0.82335$$

$$R = \frac{N-p}{p-1} \cdot \frac{\hat{D}_B}{\hat{D}_I} = \frac{(11-2)}{(2-1)} \cdot \frac{(4.15970)}{(0.82335)} = 45.46948$$

$$F_{1-\alpha, p-1, N-p} = F_{0.99, 2-1, 11-2} = 10.56$$

$$\Rightarrow R > F_{0.99, 2-1, 11-2} \Rightarrow \boxed{H_0 \text{ is rejected}}$$