Assignment 3 in LaTeX

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Assignment 3

Problem 12.13.5.10 : A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

1) At least once
2) Exactly once
3) At least twice?

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Answer 12.13.5.10:
Let X represent the number of prizes winning in 50 lotteries and the trials are Bernoulli trials
Here clearly, we have X is a binomial distribution where

$$n = 50, p = \frac{1}{100} \tag{1}$$

Now, we know

$$q = 1 - p \tag{2}$$
$$= 1 - \frac{1}{100} \tag{3}$$
$$= \frac{99}{100} \tag{4}$$

The Binomial distribution of X is given by,

$$P(X = r) = \binom{n}{r} p^r q^{n-r} \tag{5}$$
$$= \binom{50}{r} \left(\frac{1}{100}\right)^r \left(\frac{99}{100}\right)^{n-r} \tag{6}$$

1) At least once
   Probability of winning lottery at least once is $P(X \geq 1)$

$$Pr(X \geq 1) = \sum_{x=1}^{\infty} Pr(X = x) \tag{7}$$
$$= 1 - Pr(X < 1) \tag{8}$$
$$= 1 - Pr(X = 0) \tag{9}$$
$$= 1 - \binom{50}{0} \left(\frac{99}{100}\right)^{50} \tag{10}$$
$$= 1 - \left(\frac{99}{100}\right)^{50} \tag{11}$$

2) Exactly once

Probability of winning in lottery exactly once is $Pr(X = 1)$

$$Pr(X = 1) = \sum_{x=1}^{1} Pr(X = x) \tag{12}$$

$$= \binom{50}{1}\left(\frac{1}{100}\right)\left(\frac{99}{100}\right)^{49} \tag{13}$$

$$= \left(\frac{1}{2}\right)\left(\frac{99}{100}\right)^{49} \tag{14}$$

3) At least twice
Probability of winning lottery at least twice is $Pr(X \geq 2)$

$$Pr(X \geq 2) = \sum_{x=2}^{\infty} Pr(X = x) \tag{15}$$

$$= 1 - Pr(X < 2) \tag{16}$$

$$= 1 - Pr(X = 0) - Pr(X = 1) \tag{17}$$

$$= 1 - \left(\frac{99}{100}\right)^{50} - \left(\frac{1}{2}\right)\left(\frac{99}{100}\right)^{49} \tag{18}$$

$$= 1 - \left(\frac{149}{100}\right)\left(\frac{99}{100}\right)^{49} \tag{19}$$