

Assignment 3 in L^AT_EX

Pranay Jain

Assignment 3

Problem 12.13.5.10 : A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

- 1) At least once
- 2) Exactly once
- 3) At least twice?

Answer 12.13.5.10:

Let X represent the number of prizes winning in 50 lotteries and the trials are Bernoulli trials. Here clearly, we have X is a binomial distribution where

$$n = 50, p = \frac{1}{100} \quad (1)$$

Now, we know

$$q = 1 - p \quad (2)$$

$$= 1 - \frac{1}{100} \quad (3)$$

$$= \frac{99}{100} \quad (4)$$

The Binomial distribution of X is given by,

$$P(X = r) = \binom{n}{r} p^r q^{n-r} \quad (5)$$

$$= \binom{50}{r} \left(\frac{1}{100}\right)^r \left(\frac{99}{100}\right)^{n-r} \quad (6)$$

- 1) At least once

Probability of winning lottery at least once is $P(X \geq 1)$

$$Pr(X \geq 1) = 1 - Pr(X < 1) \quad (7)$$

$$= 1 - Pr(X = 0) \quad (8)$$

$$= 1 - \binom{50}{0} \left(\frac{1}{100}\right)^0 \left(\frac{99}{100}\right)^{50} \quad (9)$$

$$= 1 - \left(\frac{99}{100}\right)^{50} \quad (10)$$

- 2) Exactly once

Probability of winning in lottery exactly once is $Pr(X = 1)$

$$Pr(X = 1) = \binom{50}{1} \left(\frac{1}{100} \right) \left(\frac{99}{100} \right)^{49} \quad (11)$$

$$= \left(\frac{1}{2} \right) \left(\frac{99}{100} \right)^{49} \quad (12)$$

3) At least twice

Probability of winning lottery at least twice is $Pr(X \geq 2)$

$$Pr(X \geq 2) = 1 - Pr(X < 2) \quad (13)$$

$$= 1 - Pr(X = 0) - Pr(X = 1) \quad (14)$$

$$= 1 - \left(\frac{99}{100} \right)^{50} - \left(\frac{1}{2} \right) \left(\frac{99}{100} \right)^{49} \quad (15)$$

$$= 1 - \left(\frac{149}{100} \right) \left(\frac{99}{100} \right)^{49} \quad (16)$$