Module III

AC Fundamentals

Alternating Quantity

Any quantity whose magnitude has alternation value with respect to time.

Wave form

Shape of the curve obtained when we plot the values of alternative quantity with time.

Instantaneous value.

Value/magnitude of a quantity at any instant of time.

Cycle.

Each repetition of a set of values.

Half cycle.

One set of positive value or one set of negative value of a symmetric waveform.

Time interval (T)

- Fixed interval at which values are repeated.
- Time taken to complete one cycle.

Frequency

- No. of cycles per second.
- $f = 1/T$

Angular velocity (w)

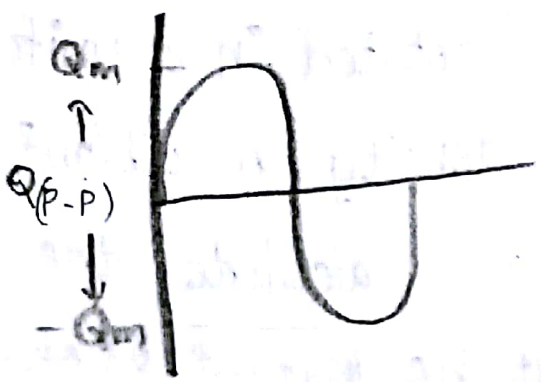
Speed at which rotates

$$w = \frac{2\pi}{T} = 2\pi f$$

Peak value

Maximum instantaneous value (+ve or -ve) of a symmetric waveform is called peak value.

Peak to peak.



Distance b/w +ve maximum and -ve max value.

$Q_{(P-P)} = 2Q_P$

Equations of Alternating Current & Voltage

$$\phi = \phi_m \cos\theta$$

$$e = -\frac{d}{dt}(N\phi)$$

$$= -\frac{d}{dt}(N\phi_m \cos\theta) \qquad \theta = \omega t$$

$$= -N\frac{d}{dt}\phi_m \cos\omega t$$

$$= \omega N\phi_m \sin\omega t$$

When $\sin\omega t = 1$

A rectangular coil of size 5cm x 10cm has 50 turns. The coil is rotated in a uniform magnetic field of flux density $0.1\ wb/m^2$ at a speed of 1000 rpm. Calculate the maximum emf and the emf at the instant when the plane of coil makes an angle of 45° to the field direction.

$A = 5 \times 10$ cm

$= 50$ cm

$N = 50$

$B = 0.1$ wb/m2

Speed = 1000 rpm.

$\theta = 45°$.

$E_m = (2\pi f) N (BA)$ $(\because \omega = 2\pi f$ & $\phi = BA)$

$f = \frac{1000}{60}$ cycles/s or Hz

$E_m = 2 \times 3.14 \times \frac{100}{6} \times 50 \times 0.1 \times .5 \times 10^{-2}$

$= \underline{\underline{2.62}}$

$e = E_m \sin\theta$

$= 2.62 \times \sin 45$

$= \underline{\underline{1.85V}}$.

An AC of frequency 60Hz has maximum value of 120A. Find equ^n of its instantaneous value. Also calculate instantaneous value after 1/360 s and time taken to reach 96 A after first time after 0 crossing.

$f = 60Hz$ $\quad I = 120A$ $\quad \omega = 2\pi \times 60$
$= 120 \times 3.14$

i) $i = I_m \sin\omega t$

$= 120 \times \sin(120\pi t)$

ii) $i = I_m \sin\omega t$

$= 120 \sin\left(120\pi \frac{1}{360}\right)$

$= 120 \sin \pi/3$

$= 120 \times \sqrt{3}/2 = 60\sqrt{3} = \underline{103.92 A}$

$96 = 120 \times \sin(120\pi t)$

$\frac{96}{120} = \sin(120\pi t)$

$\sin^{-1}(96/120) = 120\pi t$

$t = \frac{53.13}{120 \times 3.14} \times \frac{\pi}{180} = \underline{\cancel{0.14160}}$ $2.45\,ms$

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RMS value

square root of mean of squares of value.

That value of dc current which produces same heating effect as a.c in the circuit is same resistance and time.

$$I_{dc}^2 Rt = \frac{i_1^2 Rt + i_2^2 Rt + \dots + i_n^2 Rt}{n}.$$

$$I_{dc}^2 = \frac{i_1^2 + i_2^2 + \dots i_n^2}{n}$$

$$I_{dc} = \sqrt{\frac{i_1^2 + i_2^2 + i_3^2 + \dots + i_n^2}{n}}$$

$$V_{rms} = \sqrt{\frac{V_1^2 + V_2^2 + \dots V_n^2}{n}}$$

RMS of sinusoidal AC.

$i = I_m \sin\theta$.

$$RMS = \sqrt{\int_0^{2\pi} \frac{i^2}{2\pi - 0}\, d\theta}$$

$$\sqrt{\int_0^{2\pi} \frac{Im^2 \sin^2\theta}{2\pi} \, d\theta}$$

$$\sqrt{\frac{Im^2}{2\pi} \int_0^{2\pi} \sin^2\theta \, d\theta} \, .$$

$$\sqrt{\frac{Im^2}{2\pi} \int_0^{2\pi} \cancel{(1-\cos^2\theta) d\theta}} \, .$$

$$\sqrt{\frac{Im^2}{4\pi} \int_0^{\pi} 1 - \cos 2\theta}$$

$$\sqrt{\frac{Im^2}{4\pi} [\theta]_0^{2\pi} - \left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}} \, .$$

$$= \frac{Im}{2} \sqrt{\frac{1}{\cancel{\pi}} [2\cancel{\pi} - 0]} \, .$$

$$= \frac{Im}{\sqrt{2}}$$

$$P = I_{rms}^2 R \, .$$

$$= \left(\frac{Im}{\sqrt{2}}\right)^2 R = \frac{Im^2 R}{2}$$

Average value of sinusoidal ac

$\Rightarrow$ dc which produces the same 'charge as that of ac.

$$I_{av} \rightarrow \frac{i_1 + i_2 + \dots + i_n}{n}$$

$$I_{av} = \int_0^{\pi} \frac{i}{\pi} d\theta$$

$$= \int_0^{\pi} \frac{i_m \sin\theta}{\pi} d\theta$$

$$= \frac{I_m}{\pi} \int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi} [-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi} [-\cos\pi + \cos 0]$$

$$= \frac{I_m}{\pi} [1 + 1]$$

$$I_{av} = \frac{2I_m}{\pi}$$

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$$\text{Form factor } (K_f) = \frac{\text{RMS value}}{\text{Avg value}}$$

$$I = \frac{I_m/\sqrt{2}}{2 I_m/\pi}$$

$$= \underline{\underline{1.11}}$$

$$I_{RMS} = 1.11 \times \text{Avg value}$$

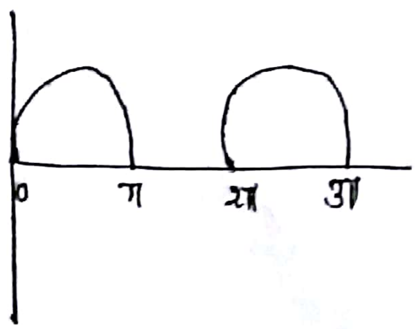
$$\text{Peak factor } (k_p) = \frac{\text{Max. value}}{\text{RMS value}}$$

$$= \frac{I_m}{I_m/\sqrt{2}} = \sqrt{2} = \underline{\underline{1.414}}$$

$$\text{Power Factor} = \cos\phi$$

ϕ = Phase dif b/w voltage & current

Half Wave Rectifier



$$I_{rms} = \sqrt{\int_0^{\pi} \frac{i^2}{2\pi}\, d\theta}$$

$$= \sqrt{\int_0^{\pi} \frac{I_m^2 \sin^2\theta}{2\pi}\, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{\pi} \sin^2\theta\, d\theta}$$

$$= \sqrt{\frac{I_m^2}{2\pi} \int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \left[\pi - \frac{\sin 2\pi}{2}\right]}$$

$$= \sqrt{\frac{I_m^2}{4\cancel{\pi}} [\cancel{\pi}]}$$

$$= \frac{I_m}{2}$$

$$I_{avg} = \int_0^{\pi} \frac{i}{2\pi} d\theta$$

$$= \int_0^{\pi} \frac{I_m \sin\theta \, d\theta}{2\pi}$$

$$= \frac{I_m}{2\pi} [-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{2\pi} [-\cos\pi - \cos 0]$$

$$= \frac{I_m}{2\pi} 2 = \frac{2 I_m}{2\pi} = \frac{I_m}{\pi}$$

Find the form factor of half-wave rectified ac.

$$\text{Form factor} = \frac{RMS}{Avg}$$

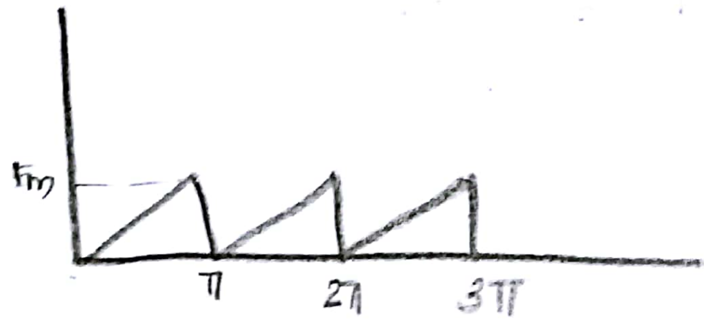
$$= \frac{I_m/2}{I_m/\pi}$$

$$= \pi/2 = 1.57$$

Find the average value, effective value and form factor of full wave rectified sine wave.

aw- Tooth Waveform .

Calculate average, RMS value and form factor.



$$e = m\theta = \frac{E_m}{\pi}\theta$$

$$y = mx$$

$$m = y/x = \frac{E_m}{\pi}$$

$$E_{rms} = \sqrt{\int_0^{2\pi} \frac{e^2}{\pi} d\theta}$$

$$= \sqrt{\int_0^{\pi} \frac{E_m^2 \theta^2 d\theta}{\pi^3}}$$

$$= \sqrt{\frac{E_m^2}{\pi^3} \int_0^{\pi} \theta^2 d\theta}$$

$$= \sqrt{\frac{E_m^2}{\pi^3} \left[\left[\frac{\theta^3}{3}\right]_0^{\pi} \right]}$$

$$= \sqrt{\frac{E_m^2}{3\pi^3} [\pi^3 - 0]} = \frac{E_m}{\sqrt{3}}$$

$$E_{avg} = \frac{\text{Area under the wave over the first cycle.}}{\text{base}}$$

$$= \frac{\frac{1}{2} bh}{\text{base}} = \frac{\frac{1}{2} \pi E_m}{\pi}$$

$$= \frac{E_m}{2}$$

$$\text{Form factor} = \frac{RMS}{Avg}$$

$$= \frac{E_m/\sqrt{3}}{E_m/2}$$

$$= \frac{2}{\sqrt{3}} = 1.154$$

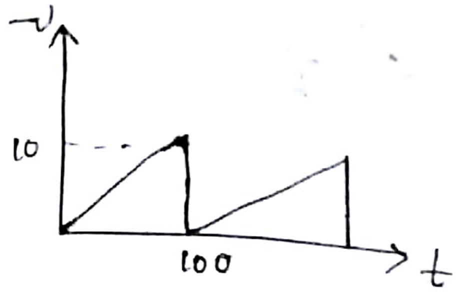
$$\text{Peak factor} = \frac{E_m}{E_{rms}}$$

$$= \frac{E_m}{E_m/\sqrt{3}}$$

$$= \sqrt{3} = 1.732$$

Find freq, wave equation, V_{rms}, V_{avg} & K_f

~~a) freq~~ =



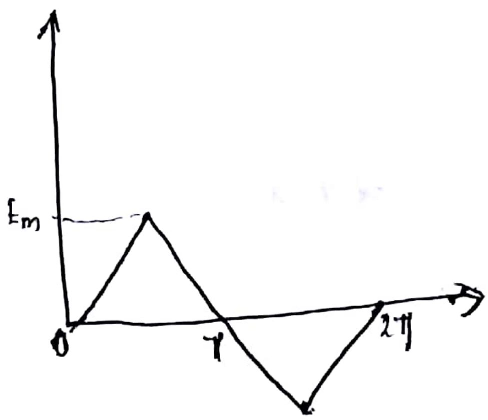
$$Freq = \frac{1}{T} = \frac{1}{100} = 0.01\,Hz$$

$$V = m\theta$$

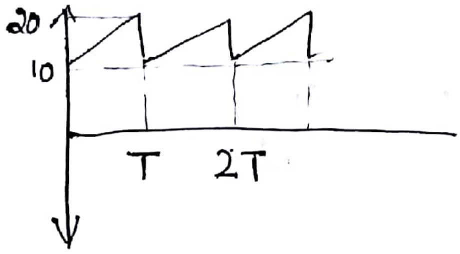
$$= \frac{V_m}{100} = \frac{10}{100} = \underline{\underline{.1}}$$

$$V_{rms} = \frac{E_m}{\sqrt{3}} = 10/\sqrt{3} = \underline{\underline{5.773\,V}}$$

$$V_{avg} = \frac{E_m}{2} = \frac{10}{2} = \underline{\underline{5V}}$$



Find form factor.



$$y = mt + c$$

$$= \left(\frac{y_2 - y_1}{x_2 - x}\right) t + c$$

$$= \left(\frac{20 - 10}{(T - 0)}\right) t + 10$$

$$= \frac{10}{T} t + 10$$

$$y_{avg} = \int_0^T \frac{y}{T} \, dt = 15$$

$$y_{rms} = \sqrt{\int_0^T \frac{y^2}{T} \, dt} = 15.275$$

$$k_f = 1.018$$

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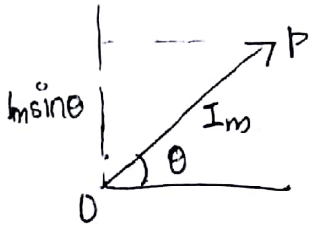
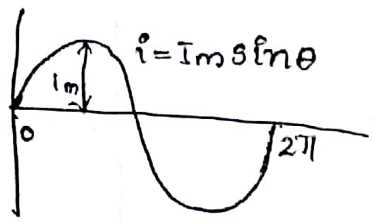
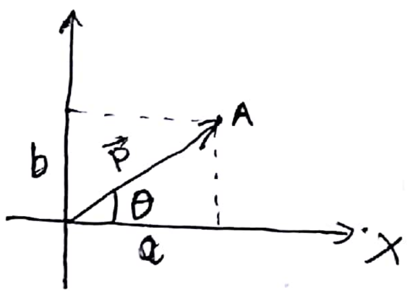
Phasor.





Phasor Algebra.

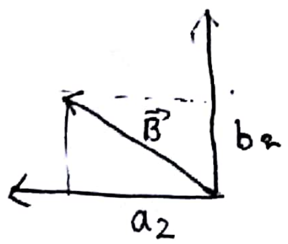
1. Rectangular (cartesian) form.
2. Trigonometric form.
3. Exponential form.
4. Polar form.



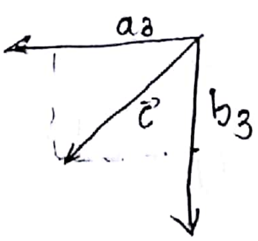
$\vec{P} = a + jb$

$a \rightarrow$ inphase component

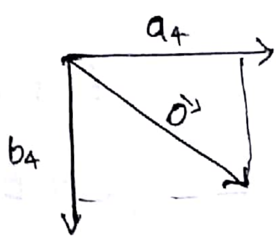
$b \rightarrow$ quadrature component.



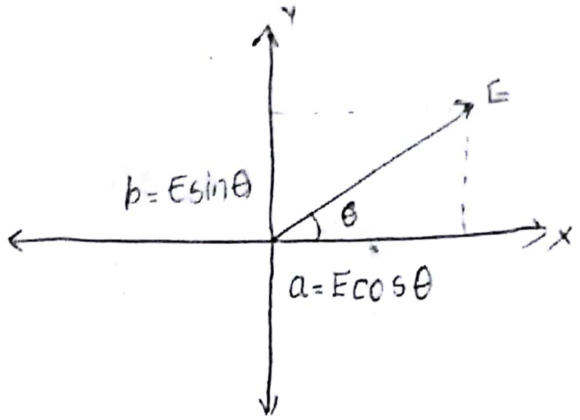
$\vec{B} = -a_2 + jb_2$.



$\vec{C} = -a_3 - jb_3$



$\vec{D} = a_4 - jb_4$



$$\vec{E} = E\cos\theta + jE\sin\theta$$

$$(\vec{E} = a + jb)$$

$$\vec{E} = 3 + j4$$

$$|E| = \sqrt{4^2 + 3^2} = 5$$

$$\theta = \tan^{-1}(4/3) = 53$$

$$\vec{E} = E\cos\theta + jE\sin\theta$$

$$= 5\cos 53 + j5\sin 53$$

$$\cos\theta + j\sin\theta = e^{j\theta}$$

$$\cos\theta - j\sin\theta = e^{-j\theta}$$

$$E = E\cos\theta + jE\sin\theta$$

$$= E(\cos\theta + j\sin\theta)$$

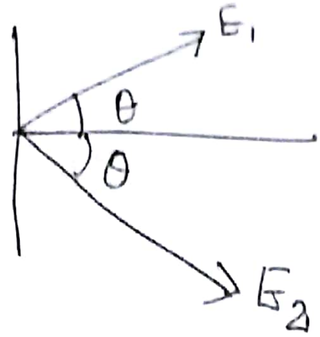
$$= Ee^{j\theta}$$

Polar form

$\vec{E_1} = |\vec{E}| \angle +\theta$

E angle θ.

$E_2 = E \angle -\theta$



$$E = \sqrt{x^2+y^2}$$

$$\theta = \tan^{-1}\frac{y}{x}$$

$$\vec{E_1} + \vec{E_2} = (a_1+a_2) + j(b_1+b_2)$$

$$\vec{E_1} - \vec{E_2} = (a_1+a_2) - j(b_2+b_1)$$

$$E_1 \times E_2 = a_1a_2 + ja_1b_2 + ja_2b_2 + j^2 b_1b_2$$
$$= E_1E_2 \angle (\theta_1+\theta_2)$$

$$\frac{E_1}{E_2} = \frac{(a_1+jb_1)}{(a_2+jb_2)} \times \frac{(a_2-jb_2)}{(a_2-jb_2)} = \frac{E_1}{E_2} \angle \theta_1-\theta_2$$

Q. $A = 16+j12$ $B = -6+j10$.

i) Find $A+B$, $A-B$, AB & $\frac{A}{B}$ in rectangular form & polar form.

ii) Find polar forms of A and B.

$A+B = 10 + j22$

$A-B = 22+2j$

$AB =$

$A = \sqrt{16^2+12^2} = 20$

$B = \sqrt{6^2+10^2} = 11.6$

$\theta_1 = \tan^{-1}\left(\frac{12}{16}\right) = 37°$

$\theta_2 = \tan^{-1}\left(\frac{10}{6}\right) = 59°$

Polar form

$A = 20 < 37°$

$B = 11.6 < 59°$

$AB = 233 < 96$

$\frac{A}{B} = 1.715 < -22°$

$A+B = \sqrt{10^2+22^2} < \tan^{-1}\left(\frac{22}{10}\right)$

$A-B = \sqrt{22^2+2^2} < \tan^{-1}\left(2/22\right) =$

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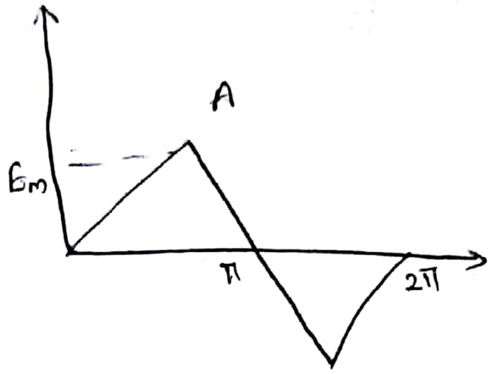


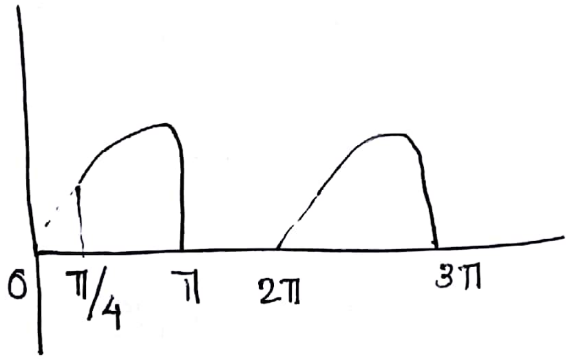
$$E_{avg} = \frac{E_m \times 2\theta}{\pi}$$

$$E_{rms} = \frac{2}{\sqrt{3}} e_m$$

$$E_{avg} = e_m$$

Peak factor $= \sqrt{3}/2$

Form factor $= 2/\sqrt{3}$



$i = I_m \sin\theta$

$$I_{rms} = \sqrt{\int_{\pi/4}^{\pi} \frac{I_m^2 \sin^2\theta \, d\theta}{2\pi}}$$

$$= \sqrt{\frac{I_m^2}{4\pi} \int_{\pi/4}^{\pi} (1 - \cos 2\theta)\, d\theta}$$

$$\frac{I_m}{2}\sqrt{\frac{1}{\pi}\left(\frac{3\pi}{4}+\frac{1}{2}\right)}$$

$$= \frac{I_m}{2}\sqrt{\frac{3}{4}+\frac{1}{2\pi}}$$

$$= \left(-\cos\theta\right)_{\pi/4}^{\pi} \times \frac{I_m}{2\pi}$$

$$= \frac{I_m}{2\pi}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$FF = \frac{\pi\sqrt{\frac{3}{4}+\frac{1}{2\pi}}}{1+\frac{1}{\sqrt{2}}}$$

$$PK = \frac{2}{\sqrt{3/4+1/2\pi}}$$

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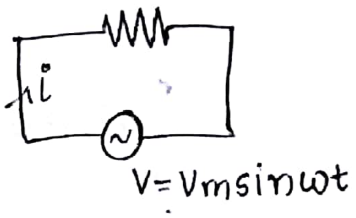
Power in AC circuits.

AC through resistance:

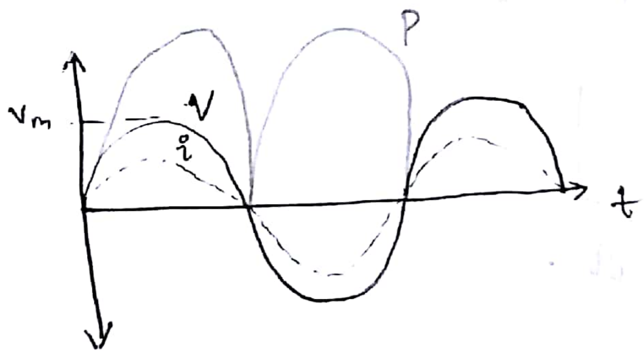


$$V = V_m \sin\omega t = iR$$

$$i = \frac{V_m}{R}\sin\omega t$$

$$= I_m \sin\omega t$$

where $I_m = \frac{V_m}{R}$.



Time diagram.

Phasor diagram. $\xrightarrow[I]{} V$

Power, $P = vi$

$$= V_m \sin\omega t \, I_m \sin\omega t$$

$$= V_m I_m \sin^2\omega t$$

$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2}(\cos 2\omega t)$$

$$= \left(\frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}\right)$$

$$\left[\sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$$

$$P_{avg} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$$

$$P = VI$$

$$= \underline{I R^2}$$

AC through pure Inductor.

$V = V_m \sin\omega t$.

$$V = \frac{L\,di}{dt} = V_m \sin\omega t$$

$$di = \frac{V_m}{L} \sin\omega t \, dt.$$

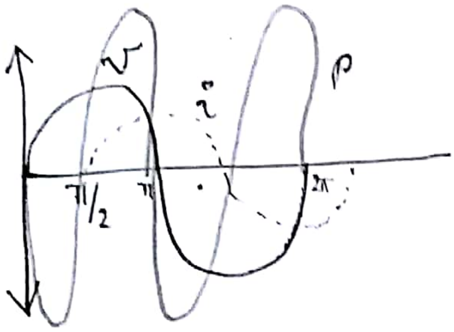
$$\int di = \int \frac{V_m}{L} \sin\omega t \, dt$$

$$I = \frac{V_m}{L}\left(\frac{-\cos\omega t}{\omega}\right)$$

$$= \frac{V_m}{L\omega} (\sin(\omega t - \pi/2))$$

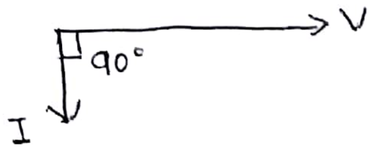
where $L\omega = X_L$ and it is called inductive reactance.

$$i = I_m \sin(\omega t - \pi/2).$$



Current lags behind Voltage.

Phasor diagram.



$$P = V_m \sin\omega t \cdot I_m \sin(\omega t - \pi/2).$$

$$= V_m I_m \sin\omega t \cdot -\cos\omega t$$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$.

$$= -V_m I_m \sin\omega t \cdot \cos\omega t$$

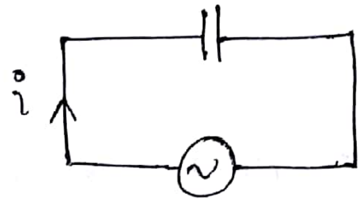
$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

$Power_{avg}$ in induction circuit is zero. Why?

$$P_{avg} = \int_0^T -\frac{V_m I_m}{2} \sin 2\omega t$$

$$= \underline{\underline{0}}$$

AC through capacitance



$v = V_m \sin\omega t$

$$v = V_m \sin\omega t$$

$$q = Cv$$

$$i = \frac{dq}{dt}$$

$$= \frac{d\,Cv}{dt}$$

$$= C\frac{dv}{dt}$$

$$= C\frac{d}{dt}(V_m \sin\omega t)$$

$$= C V_m \cos\omega t \cdot \omega$$

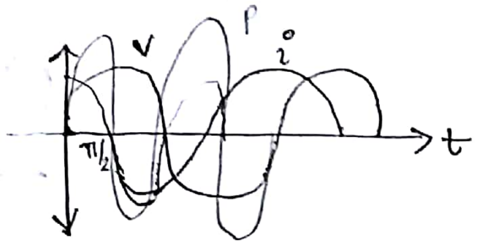
$$= C\omega V_m \sin(\omega t + \pi/2)$$

$$= \frac{V_m}{1/C\omega} \sin(\omega t + \pi/2)$$

$$= i_m \sin(\omega t + \pi/2).$$

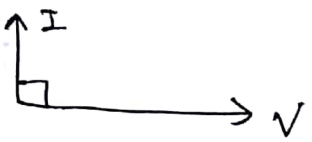
~~Since~~ $1/C\omega = X_C$ and it is called capacitative reactance.

Time diagram:



Current is ahead of voltage.

Phasor diagram:



$$P = vi$$

$$= V_m \sin\omega t \; i_m \sin(\omega t + \pi/2)$$

$$= V_m I_m \sin\omega t \sin(\omega t + \pi/2)$$

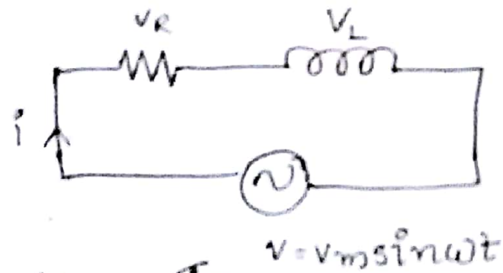
$$= V_m I_m \sin\omega t \cdot \cos\omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

$$P_{avg} = \int_0^T P = \underline{\underline{0}}$$

AC series current

R-L circuit

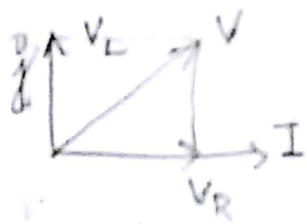


$$V_R = IR$$

$$V_L = jI X_L$$

$$\vec{V} = \vec{V}_R + \vec{V}_L$$

~~= iR + in~~



$$\tan\phi = \frac{V_L}{V_R}$$

$$\phi = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$V = \sqrt{V_R^2 + V_L^2}$$

$$= \sqrt{(IR)^2 + (I X_L)^2}$$

$$= I\sqrt{R^2 + X_L^2}$$

$$V = IR + jIX_L = I(R + jX_L)$$

$$I = \frac{V}{R + jX_L} = \frac{V}{Z}$$

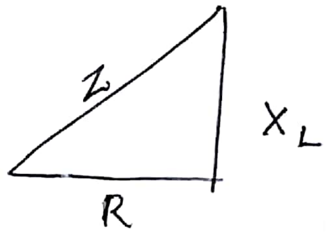
$Z \rightarrow$ Impedance (Ω)

$$|z| = \sqrt{R^2 + X_L^2}$$

An inductive circuit draws 10A and 1kW from 200V, 50Hz AC circuit. Determine the impedance of the circuit

$$Z = \frac{V_{rms}}{I} = \frac{200}{10} = \underline{\underline{20\,\Omega}}$$

Impedance Triangle.



$P = VI$

$\phi \rightarrow$ Power factor angle.

$\cos\phi \Rightarrow$ Power factor.

Apparent power $S = V \times I$.

Actual (Real/Active/True) Power $= VI\cos\phi$

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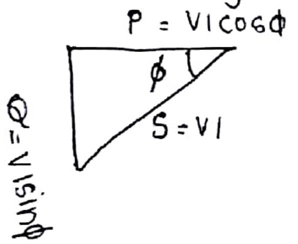
Reactive Power

Power in the inductive component or reactant

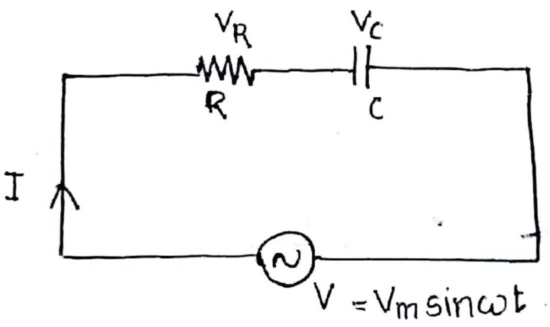
$Q = VI\sin\phi$ VAR (or KVAR)

$$\boxed{\begin{aligned} P &= VI\cos\phi \\ Q &= VI\sin\phi \\ S &= VI \end{aligned}}$$

Power triangle



R-C Circuit



$V_R = IR$

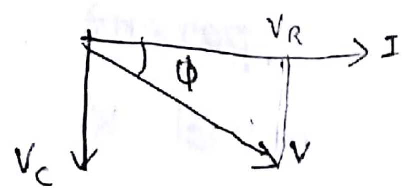
$V_c = IX_c$

$X_c = \frac{1}{C\omega} = \frac{1}{2\pi f C}$

$\vec{V} = \vec{V}_R + \vec{V}_c$

$\vec{V} = IR - jIX_c$

$= I(R - jX_c)$



Current leads voltage by 90°.

$$I = \frac{V}{R - jX_C} = \frac{V}{Z}.$$

$$Z = R - jX_C$$

$$= R + \frac{1}{j\omega C}$$

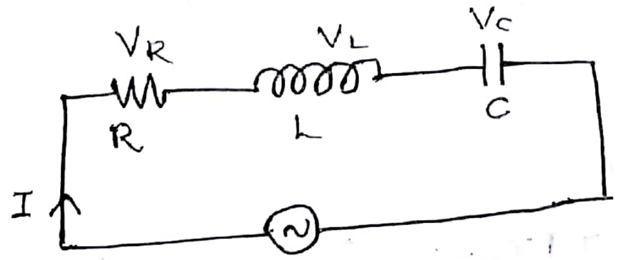
$$p.f = \cos\phi = R/Z.$$

$$P = VI\cos\phi$$

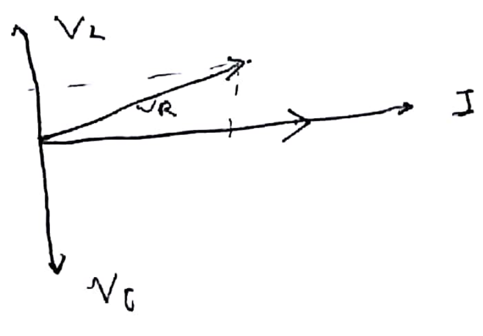
$$= IZ \times \frac{IR}{Z}$$

$$= \underline{\underline{I^2 R}}$$

<u>R - L - C circuit.</u>



$V = V_m \sin \omega t$



$$V_R = IR$$

$$V_L = IX_L$$

$$V_C = IX_C$$

$$\vec{V} = \vec{V_R} + \vec{V_L} + \vec{V_C}$$

$$= \vec{I}R + \vec{I}X_L - jIX_C$$

$$\vec{V} = \vec{I}\,(R + j\,(X_L - X_C))$$

$$\vec{V} = \vec{I}\,Z$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Current will lead when capacitance is high.

Current will lag when inductance is high.

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

When $X_L = X_C$

$$\underline{Z = R}$$

$$I = V/R$$

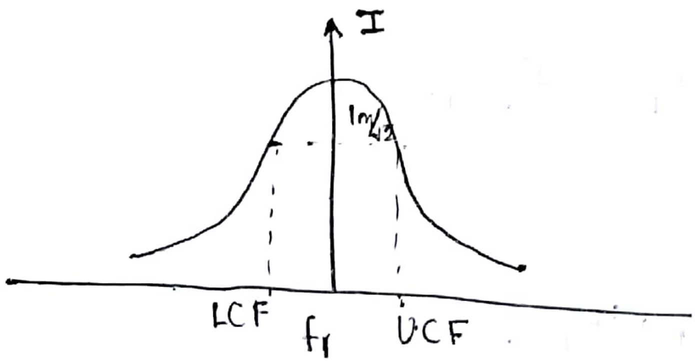
$$X_L = WL = 2\pi f L$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$X_L = X_C .$$

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$ Resonant frequency.



It is called half power frequency.

$$P_{max} = I_m^2 R .$$

$$P_{h.p} = \left(\frac{I_m}{\sqrt{2}}\right)^2 R$$

$$= \frac{I_m^2 R}{2}$$

$$= \frac{P_{max}}{2}$$

$$\text{Quality factor} = \frac{\text{Reactive Power}}{\text{Avg Power}}.$$

$$Q = \frac{I^2 X_L}{I^2 R} \text{ or } \frac{I^2 X_C}{I^2 R}$$

$$Q = \frac{X_L}{R} \text{ or } \frac{X_C}{R}$$

$$= \frac{\omega L}{R} \text{ or } \frac{1}{C\omega R}$$

$$= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{f_r}{B_W}.$$

Y → Admittance
G → Conductance
B → Susceptance

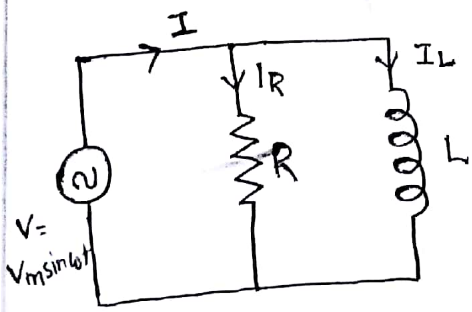
$$\boxed{Y = G + jB}$$

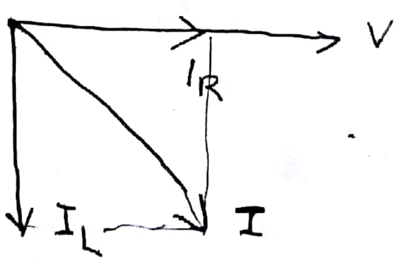
$1/Z$ → Admittance $= Y$

$$B_W = f_2 - f_1 = f_{UCF} - f_{LCF}.$$

Parallel AC circuits.

R-L





$$\vec{I} = \vec{I_R} + I_L$$

$$= \frac{\vec{V}}{R} + \frac{\vec{V}}{X_L}$$

$$= \frac{\vec{V}}{R} - j\frac{V}{X_L}$$

$$= V\left(\frac{1}{R} - j\frac{1}{X_L}\right)$$

$$= V\left(\frac{1}{R} + \frac{1}{j\omega L}\right)$$

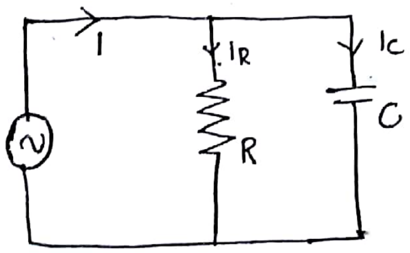
$$Z = \frac{1}{\left(\frac{1}{R} - j\frac{1}{X_L}\right)}$$

$$\frac{1}{Z} = \frac{1}{R} - j\frac{1}{X_L} = Y$$

$$\phi = \tan^{-1}\frac{I_L}{I_R} = \tan^{-1}\frac{V/X_L}{V/R}$$

$$= \tan^{-1} R/X_L = \tan^{-1}(R/L\omega)$$

R-C Circuit



$$\vec{I} = \vec{I}_R + \vec{I}_C = \frac{V}{R} + j\frac{V}{X_C} = V\left(\frac{1}{R} + j\frac{1}{X_C}\right) = V/Z = VY$$

$$\left(\frac{1}{R} + j\frac{1}{X_C}\right) \qquad \left(\frac{1}{R} + j\omega C\right)$$

$$|Z| = \sqrt{\frac{1}{R^2} + \frac{1}{X_C^2}}$$

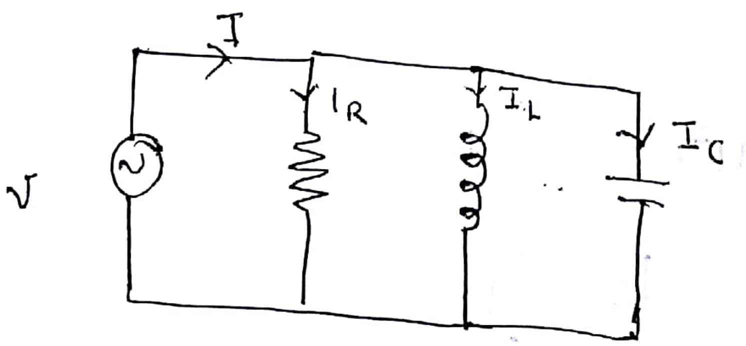
$$\tan\phi = \frac{I_C}{I_R}$$

$$\phi = \tan^{-1}\left(\frac{I_C}{I_R}\right)$$

$$= \tan^{-1}\left(\frac{V/X_C}{V/R}\right)$$

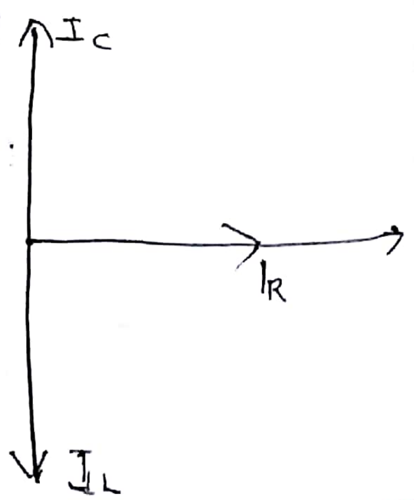
$$= \tan^{-1}(\omega CR)$$

R-L-C Circuit.



$$I = \vec{I_R} + \vec{I_L} + I_C$$

$$I = \frac{V}{R} - j\frac{V}{X_L} + j\frac{V}{X_C}$$



$$= V\left(\frac{1}{R} - j\frac{1}{X_L} + j\frac{1}{X_C}\right)$$

$$= \frac{V}{Z} = VY$$

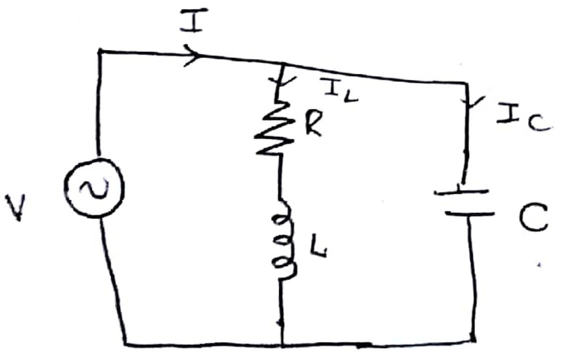
$$\tan\phi = \frac{(I_C \sim I_L)}{I_R}.$$

$$\therefore \phi = \tan^{-1}\left(\frac{I_C \sim I_L}{I_R}\right)$$

$$= \tan^{-1}\frac{V(1/X_L - 1/X_C)}{V/R}.$$

$$= \tan^{-1}\left(\frac{R}{(1/X_L - 1/X_C)}\right)$$

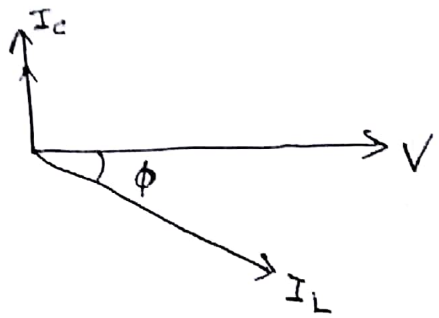
Resonance in Parallel Circuit.



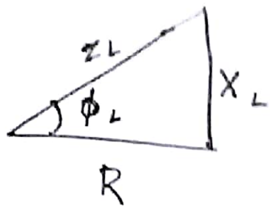
$$Z_L = \sqrt{R^2 + X_L^2}$$

$$I_L = \frac{V}{Z_L}$$

$$I_C = \frac{V}{X_C}$$



$$I_C = I_L \sin\phi_L$$



$$\frac{V}{X_C} = \frac{V}{Z_L} \cdot \frac{X_L}{Z_L}$$

$$X_L X_C = Z_L^2$$

$$\cancel{\omega} L \frac{1}{\cancel{\omega} C} = Z_L^2$$

$$Z_L^2 = \frac{L}{C}$$

$$R^2 + X_L^2 = \frac{L}{C}$$

$$R^2 + (2\pi f_r L)^2 = \frac{L}{C}$$

$$R^2 + 4\pi^2 f_r^2 L^2 = \frac{L}{C}$$

$$f_r^2 = \frac{\frac{L}{C} - R^2}{4\pi^2 L^2}$$

$$\frac{\cancel{CR^2}}{C} \times \frac{1}{4\pi^2 L^2}$$

$$\boxed{f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}}$$

Impedance

Resonant current, $I_r = I_L \cos\phi_L$

$$\frac{\cancel{V}}{Z_r} = \frac{\cancel{V}}{Z_L}\frac{R}{Z_L}$$

$$\frac{1}{Z_r} = \frac{R}{Z_L^2}$$

$$\frac{1}{Z_r} = \frac{R}{L/C}$$

$$\boxed{Z_r = \frac{L}{CR}}$$

→ Impedance in parallel RLC circuit..

A voltmeter and ammeter are connected in AC circuit and shows 220V and 12A respectively. Calculate the maximum value and average value of current.

$V_{rms} = 220V$

$I_{rms} = 12A$

$V_m = \sqrt{2}\, V_{rms}$

$= \underline{\underline{311.126V}}$

$V_{avg} = \dfrac{2V_m}{\pi}$

$= \underline{\underline{198.17V}}$

$I_m = \sqrt{2} \times I_{rms}$

$= \sqrt{2} \times 12$

$= 16.97A$

$I_{avg} = \dfrac{2I_m}{\pi}$

$= \cancel{8.14.8A}\ \underline{\underline{10.80A}}$

A 60Hz voltage of 115V(rms) is impressed on a 100Ω resistance. Write the time eqns for voltage and current. Draw the phosor diagram.

$f = 60Hz$ $\quad V_{rms} = 115V$ $\quad R = 100\Omega$ $\quad \omega = 376.8$ $(2\pi f)$

~~Vrms2~~ $V_m = \sqrt{2} \times V_{rms} = 162.63V$

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$$I_m = \cancel{\sqrt{2}}\,\frac{V_m}{R}$$

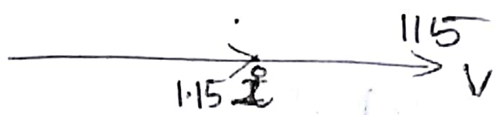
$$= \frac{162.63}{100}$$

$$= \underline{\underline{1.62A}}$$

$$V = 162.63 \sin(376.8t) = 163 \cdot \sin(377t)$$

$$i = 1.62 \sin(376.8t) = 1.63 \sin(377t)$$

Phasor



A 60Hz voltage of 230V effective is impressed on an inductance of 0.265 H i) Write the time equ^n v and I

ii) show v and i in phasor diagram.

iii) Find max energy stored in the inductor.

$$\omega = 2\pi f = 377$$

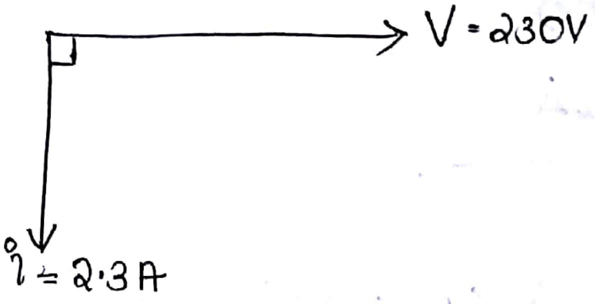
$$V_m = \sqrt{2} \times 230 = \underline{\underline{325.26V}}$$

$$I_m = \frac{V_m}{L\omega} = \frac{325.26}{0.265 \times 376.8} = \underline{\underline{3.25A}}$$

i) $V = 325 \sin(377t)$

$i = 3.25 \sin(377t - \pi/2)$

ii)



iii) $E = \frac{1}{2} L I^2$

$= \frac{1}{2} \times 0.265 \times 3.25^2$

$= \underline{\underline{1.4\ J}}$

A 50Hz of 230V effective value is impressed on a capacitance of 26.5 μF.

i) Write time equⁿˢ for v and i

ii) Show v and i in phasor diagram

iii) Show v and i in time diagram

iv) max energy stored in capacitor.

$\omega = 314$

$X_C = \frac{1}{C\omega} = \frac{1}{26.5 \times 10^{-6} \times 314} = \underline{\underline{120.17\ \Omega}}$

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$V_m = \sqrt{2} \times 230$

$= 325.27\ V$

$I_m = \dfrac{V_m}{X_C} = \dfrac{325.27}{120.17}$

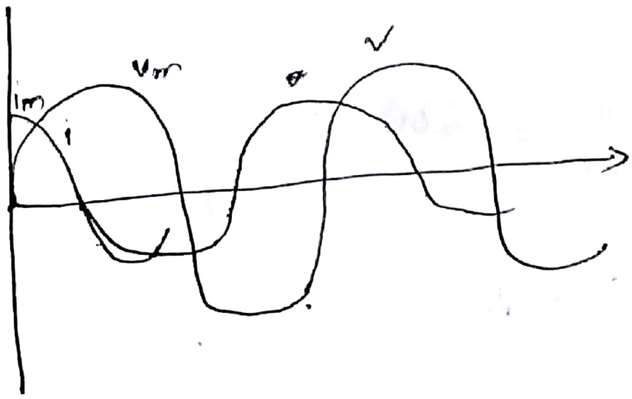
$= 2.70\ A$

i) $V = 325 \sin(314t)$

$i = 2.7 \sin(314t + \pi/2)$

ii)

iii)



iv) $E = \frac{1}{2} C V^2$

$= \frac{1}{2}\, 26.5 \times 10^{-6} \times 325.27^2$

$= \underline{1.4\ J}$

A 230V, 50Hz is applied to coil of resistance $R = 10\Omega$ and inductance 0.2H. Calculate

i) Reactance and impedance

ii) Current and its phase angle relative to its applied voltage.

$V = 230V \quad f = 50Hz \quad R = 10\Omega \quad . \quad L = 0.2H$

i) $X_L = L\omega = 314 \times 0.2$

$= \underline{\underline{62.8\,\Omega}}$.

$Z = \sqrt{R^2 + X_L^2}$

$= \sqrt{100 + 62.8^2}$

$= \underline{\underline{63.59\,\Omega}}$.

ii) $I_{rms} = \frac{V_{rms}}{Z} = \frac{230}{63.59}$

$= \underline{\underline{3.61A}}$.

$\tan\phi = \frac{X_L}{R}$

$\phi = \tan^{-1}\left(\frac{62.8}{10}\right) = \underline{\underline{81^\circ}}$

or

A 100Ω resistance in series with 120μF capacitor is connected to 230V, 50Hz supply.

Find i) circuit impedance

ii) current.

iii) Power factor.

iv) Phase angle.

v) Voltage across R.

vi) Voltage across C.

i) $X_C = \frac{1}{C\omega} = \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}}$

$= 26.54\Omega$

$Z = \sqrt{R^2 + X_C^2}$

$= \sqrt{100^2 + 26.54^2}$

$= \underline{\underline{103.46\Omega}}$

ii) $I = \frac{V}{Z} = \frac{230}{103.46} = \underline{\underline{2.223A}}$.

iii) $pf = \cos\phi = \underline{\underline{.96}}$

iv) $\phi = \tan^{-1}\left(\frac{26.54}{100}\right) = 14^\circ 50'$

v) V across R

$V = 2.223 \times 100$

$= \underline{\underline{222.3\,V}}$

vi) $V = 2.223 \times 26.54$

$= \underline{\underline{58.83\,V}}$

A resistor of $10\,\Omega$ and inductance of $\cdot 3H$ and a capacitance of $100\,\mu F$ are connected in series across 230V, 50Hz mains. Calculate

i) Impedance

ii) Current

iii) Voltage across R, V_L, V_C

iv) Power in watts

v) pf.

i) $X_L = \omega L = 314 \times \cdot 03$

$= \underline{\underline{94.2\,\Omega}}$

$X_C = \frac{1}{\omega C} = \frac{1}{314 \times 100 \times 10^{-6}} = 31.84\,\Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{100 + (94.2 - 31.84)^2}$$

$$= 63.15\ \Omega$$

ii) $I = \dfrac{230}{63.16} = \underline{\underline{3.64\ A}}$.

iii) $V_R = IR = 3.64 \times 10 = \underline{\underline{36.4\ V}}$

$V_L = 3.64 \times 94.2 = \underline{\underline{343.05\ V}}$

$V_C = 3.64 \times 31.84 = \underline{\underline{115.89\ V}}$

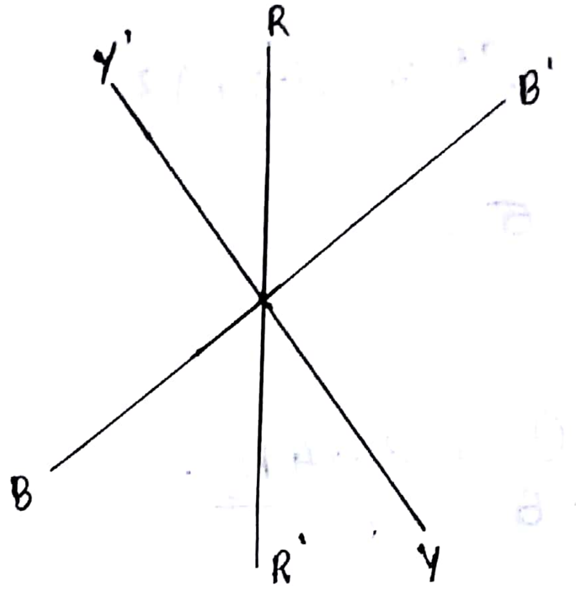
iv) $P = V_{rms} I_{rms} \cos\phi$

$$= 230 \times 3.64 \times \frac{10}{63.16}$$

$$= 132.56\ W$$

v) $\cos\phi = R/Z = \dfrac{10}{63.15} = 0.158$

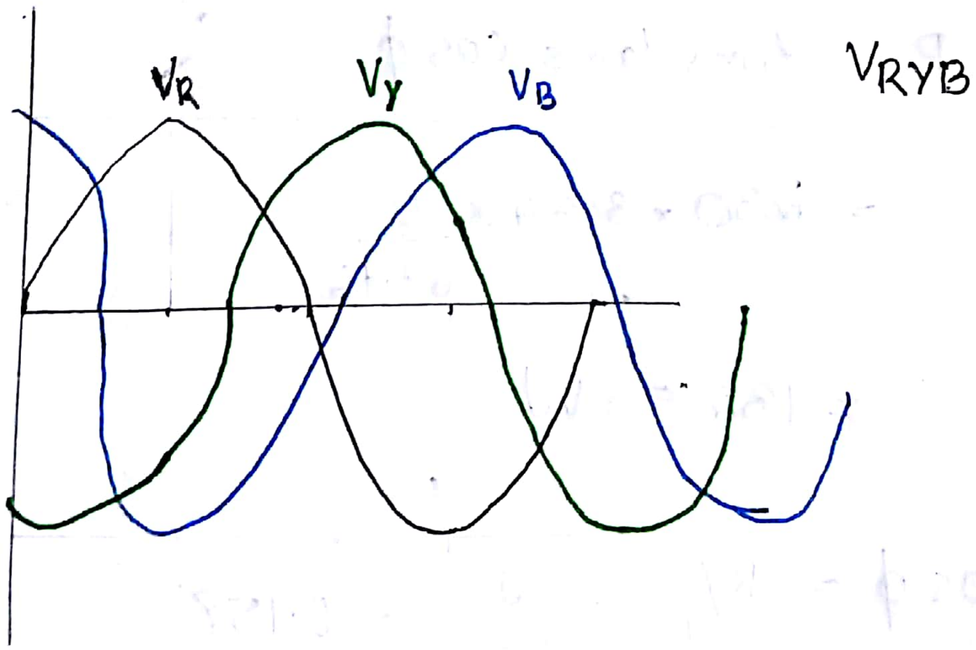
Three Phase Systems



$V_R = V_{Rm} \sin \omega t = V_{Rm} \sin \theta$

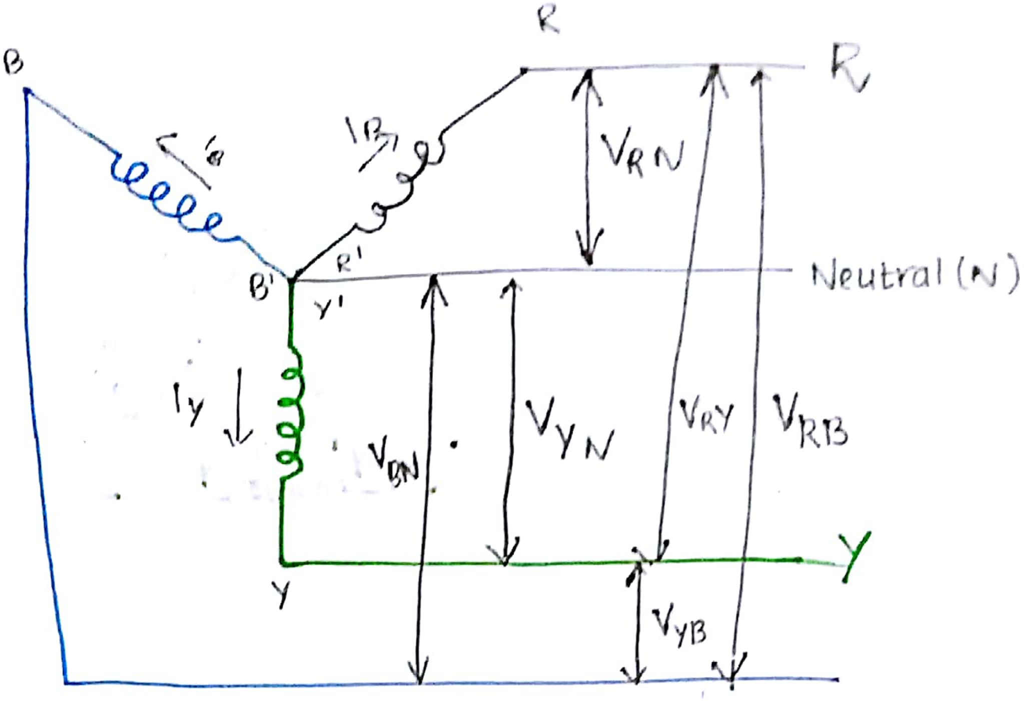
$V_y = V_{ym} \sin(\omega t - 120) = V_{ym} \sin(\theta - 120°)$

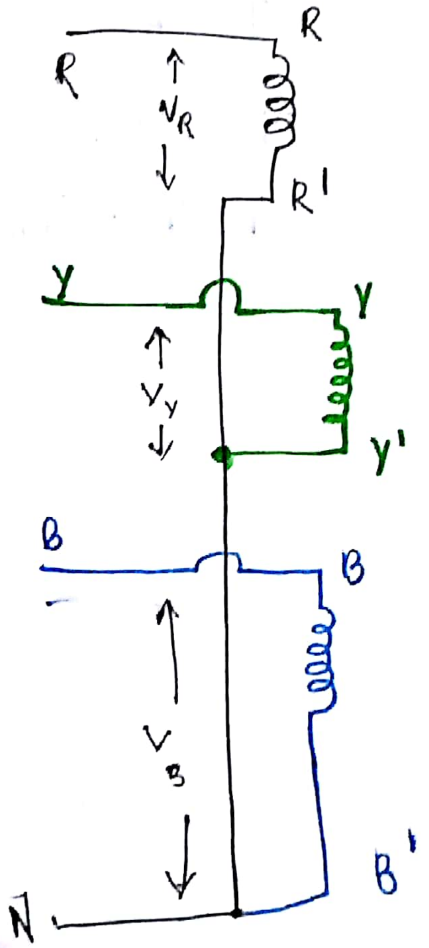
$V_B = V_{Bm} \sin(\omega t - 240) = V_{Bm} \sin(\theta - 240)$.



STAR - Y - 'wye' - ⅄

DELTA - △





$V_{RN} = V_{BN} = V_{YN} =$ Phase voltages.

R - phase

Y - phase

B - phase.

$V_{RY} - V_{YB} = V_{BR} =$ Line voltages.

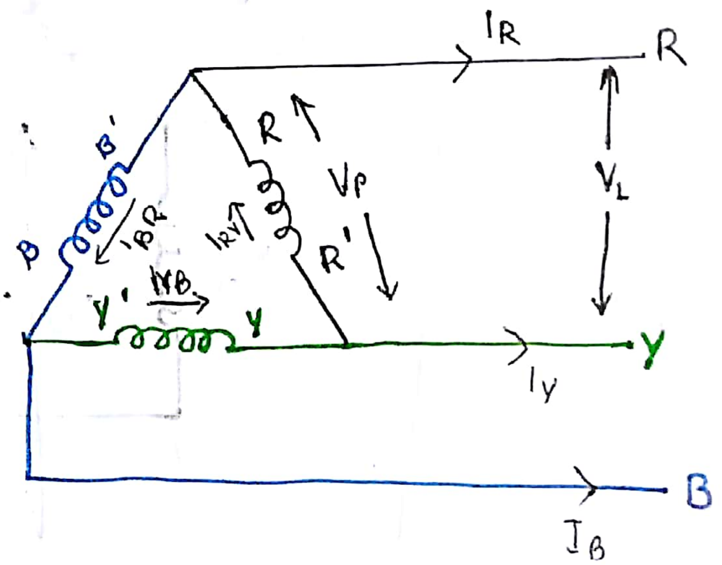
$V_{RY} = V_R - V_y$

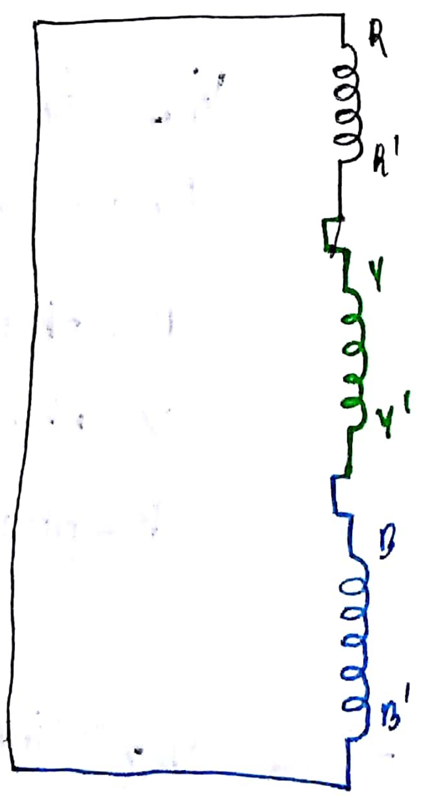
$V_{YB} = V_y - V_B$

$V_{BR} = V_B - V_R$.

$V_L = \sqrt{3}\, V_P$.

DELTA





Line currents

$I_L = I_R = I_Y = I_B$

Phase currents

$I_P = I_{RY} = I_{BR} = I_{YB}$

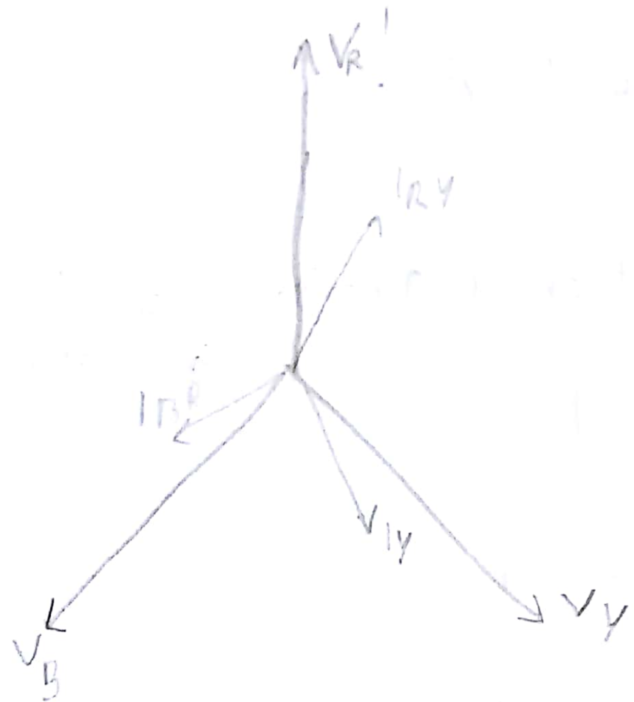
$$V_{RY} = V_R - V_y$$

$$= 2V_R \cos 30$$

$$= 2V_p \cos 30$$

$$= 2V_p \frac{\sqrt{3}}{2}$$

$$= \underline{\underline{\sqrt{3}\, V_p}} .$$



$$I_L = I_{RY} - I_{BR}$$

$$= I_R + (-I_{BR})$$

$$= 2 I_{RY} \cos 30$$

$$= 2 I_p \cdot \frac{\sqrt{3}}{2}$$

$$I_L = \sqrt{3}\, I_p$$

Power in 3 ϕ circuits

$$P_{1-\phi} = V_\phi I \cos\phi .$$

$$P_{3\phi} = 3 P_{1-\phi}$$

Assuming balanced system

$$P_{3\phi} = 3 V_p I_p \cos\phi$$

Star Connection ($V_L = \sqrt{3} V_p$, $I_L = I_p$) (4 wire system)

$$P_{3\phi} = 3 V_p I_p \cos\phi$$

$$= 3 \frac{V_L}{\sqrt{3}} I_L \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi$$

Delta Connection ($V_p = V_L$, $I_L = \sqrt{3} I_p$)
(3 wire system)

$$P_{3\phi} = 3 V_p I_p \cos\phi$$

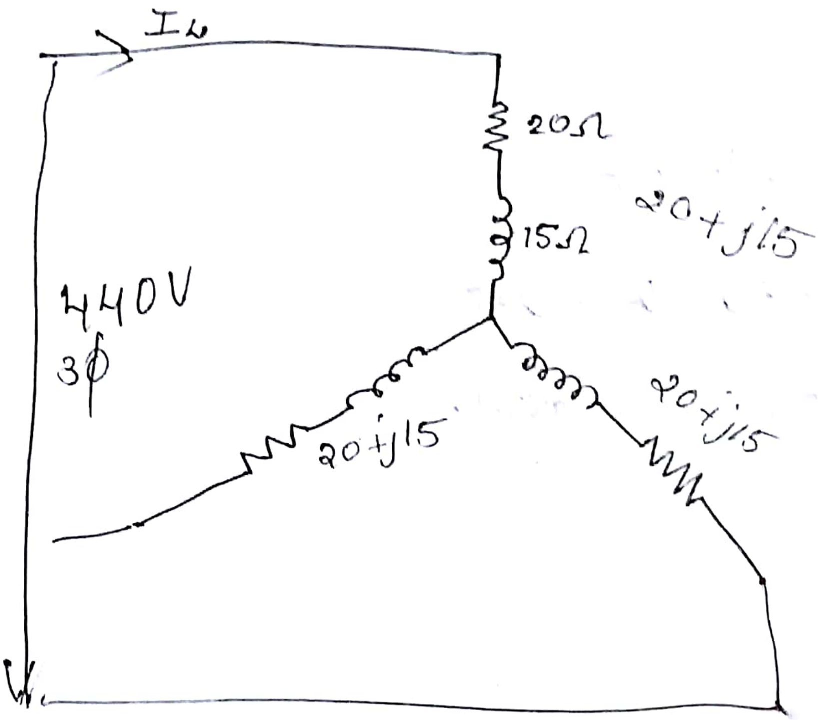
$$= 3 V_L \frac{I_L}{\sqrt{3}} \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi$$

Three impedances each having resistance 20Ω and inductive reactance 15Ω are connected in star across a 440V 3ϕ AC supply! Calculate a) line current

b) Power factor.

c) Total power.



$V_L = 440$

$V_p = 440/\sqrt{3}$

$= 254\ V$

$Z_p = \sqrt{R^2 + X_L^2}$

$= \sqrt{20^2 + 15^2} = \underline{\underline{25\Omega}}$

$I = \frac{V}{Z} = \frac{254}{25} = \underline{\underline{10.16\ A}}$

$$pf = \cos\phi = \frac{R}{Z}$$

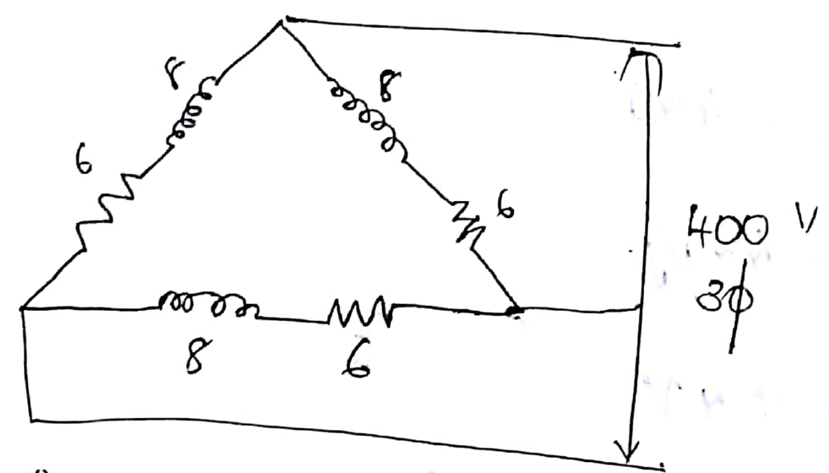
$$= \frac{20}{25}$$

$$= 0.8$$

$$P = \sqrt{3}\, V_L I_L \cos\phi$$

$$= \sqrt{3} \times 440 \times 10.1 \times 0.8$$

$$= 6157.78 \text{ Watts.}$$



Find line & phase current, p.f, active power, reactive power.

Star

$V_L = \sqrt{3}\, V_P$

$I_L = I_P$

Delta

$V_L = V_P$

$I_L = \sqrt{3}\, I_P$

$$I_L = \sqrt{3}\, I_p$$

$$= \sqrt{3} \times 40 = 69.28\,A$$

$$Z_p = 6 + j8$$

$$|Z_p| = \sqrt{6^2 + 8^2} = 10\,\Omega$$

$$\cos\phi = R/Z = 6/10 = 0.6$$

$$P_{active} = \sqrt{3} \times 400 \times 69.28 \times 0.6$$

$$= 28799\,W$$

$$P_{reactive} = \sqrt{3}\, V_L\, I_L \sin\phi$$

$$= 38398\,W$$

Three elements of a resistance of 100Ω, an inductance 0.1 H and capacitance 150 μF are connected in parallel to a 230V, 50Hz supply. Calculate current in each element ~~to~~ ii) supply current.

iii) phase angle b/w ~~b-a~~ V and I

$V = 230\Omega$ $\quad R = 100\Omega$ $\quad L = 0.1H$ $\quad C = 150\mu F$

i) $I_R = \frac{V}{R} = \frac{230}{100} = \underline{2.3A}$

$I_L = \frac{V}{X_L} = \frac{230}{0.1 \times 314} = \underline{7.32A}$

$I_C = \frac{V}{X_C} = \frac{230}{1/150 \times 10^{-6} \times 314} = \underline{10.83A}$

ii) $Z = \sqrt{R^2 + (X_L \sim X_C)^2}$

$= \sqrt{100^2 + (31.4 - 21.23)^2}$

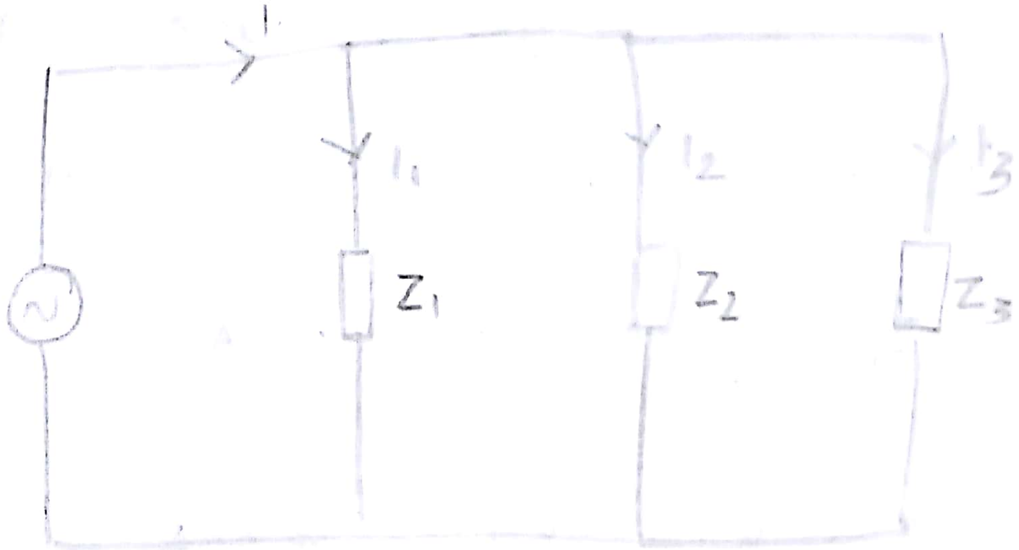
$= 100.51$

$I = \frac{V}{Z} = \frac{230}{100.51} = 2.28$

$I = \sqrt{I_R^2 + (I_C - I_L)^2} = \underline{4.196A}$.

iii) $\phi = \tan^{-1}\left(\frac{X_L \sim X_C}{R}\right)$

$= 56.76°$



Admittance, $Y = \frac{1}{Z}$

$Y_{eq} = Y_1 + Y_2 + Y_3$

$\vec{I} = \vec{I_1} + \vec{I_2} + \vec{I_3}$

$= \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3}$

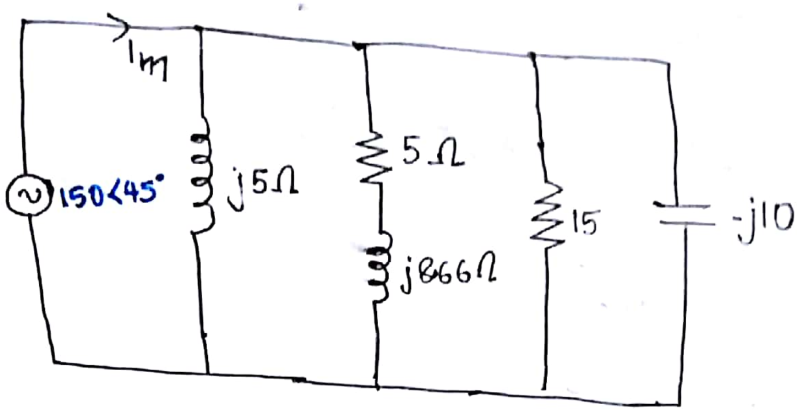
$= VY_1 + VY_2 + VY_3$

$= V(Y_1 + Y_2 + Y_3)$

$= VY_{eq}$.

$= V\frac{1}{Z_{eq}}$.

Scanned by CamScanner



Find the total current & equivalent impedance of the circuit.

$Z_1 = 0 + j5$

$Y_1 = \frac{1}{Z_1} = \frac{1}{J5} = -j\,0.2$

$Z_2 = 5 + j8.66$

$Y_2 = \frac{1}{5+j8.66}$

Convert to polar form.

$\sqrt{5^2 + 8.66^2} \angle \tan^{-1} \frac{8.66}{5}$

$= 10 \angle 60^\circ$

$\frac{1}{Z_2} = \frac{1}{10\angle 60^\circ} = \frac{1\angle 0}{10\angle 60}$

$= .1\angle -60^\circ \Rightarrow 0.1\cos -60 + j\,0.1\sin -60$

$= 0.05 - j0.0866$

$Z_3 = 15\Omega$

$Y_3 = \frac{1}{15} = \underline{\underline{0.0675 + j0}}$.

$Z_4 = 0 - j10$

$Y_4 = \frac{1}{Z_4} = \frac{1}{-j10} = \underline{\underline{j\,0.1}}$

$Y_{eq} = Y_1 + Y_2 + Y_3 + Y_4$

$= (0 - j0.2) + (0.05 - j0.0866) + (0.0675 + j0)$
$+ (0 + j0.1)$

$= 0.1175 - j\,0.1866$. $Z = \frac{1}{Y_{eq}} = \underline{\underline{4.55<}}$

Polar form.

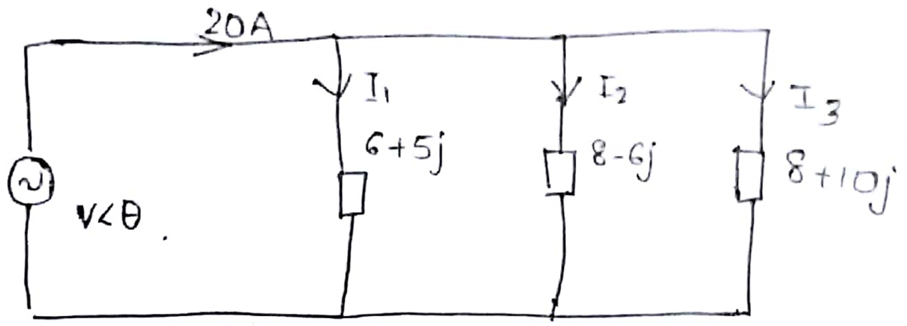
$\sqrt{(117)^2 + (0.1866)^2} \quad < \tan^{-1}\left(\frac{-0.1866}{0.1175}\right)$

$= 0.2202 < -57.911$

$I = V \cdot Y_{eq}$

$= 150 < 45^\circ \times 0.22 < -58^\circ$

$= 33 < -13^\circ$.



Calculate current in each branch.

$I = V\,Y_{eq}$

Division, multi → polar
add, minus → rect

$Z_1 = 6 + 5j$

$\sqrt{36+25} \angle \tan^{-1}(5/6)$

$7.81 \angle 39^\circ 48'$

$$Y_1 = \frac{1 \angle 0^\circ}{7.81 \angle 39^\circ 48'}$$

$= 0.128 \angle -39^\circ 48''$

$= 0.128 \cos(-40) + j\,0.128 \sin(-40^\circ).$

$= \underline{\underline{\cancel{0.0154}}}\ 0.098 - j\,0.082$

$Z_2 = 8 - 6j$

$\sqrt{64+36} = \sqrt{100} = 10$

$\tan^{-1}(-6/8) = -37$ $\qquad 10 \angle -37$