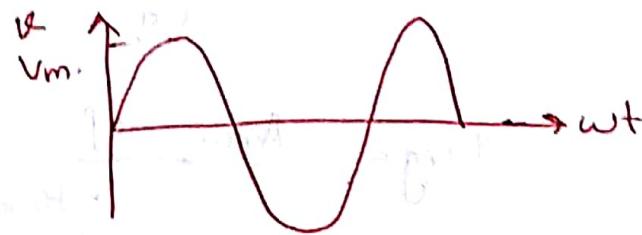


## Sinusoidal alternating waveforms

1. Cycle:



2. Frequency : The

no. of cycles in second.

3. Amplitude

value at particular

4. Instantaneous value : - value at instant.

### Average value

It is defined as steady direct current which transfers across any circuit, the same amount of charge as is transferred by the alternating current during same time.

### Determination of Avg. value

1. Mid ordinate method

→ by taking arithmetic mean of ordinates at equal intervals

$$i_{av} = \frac{i_1 + i_2 + \dots + i_n}{n}$$



### Analytical Method

Eq.

$$I_{avg} = \frac{\text{Area of first Repeating Cycle}}{\text{Base length}}$$

- \* In case of symmetrical alternating current (sinusoidal), average value over complete cycle is zero. In such case avg. value is found by considering one half cycle only.



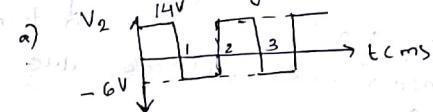
$$V_{avg} = \frac{\text{Area of first half cycle of sine wave}}{\text{Base length}}$$

$$\begin{aligned} &= \frac{1}{\pi} \int_0^{\pi} V_m \sin wt dt \\ &= \frac{V_m}{\pi} - [\cos wt]_0^{\pi} = \frac{2V_m}{\pi} \end{aligned}$$

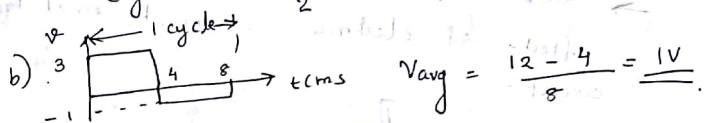
- \* Average value of symmetrical wave = Area of one alteration

$$\begin{aligned} &= \frac{\text{Base length} \times \text{Area over one cycle}}{\text{Base length}} \\ &= \text{Area over one cycle} \end{aligned}$$

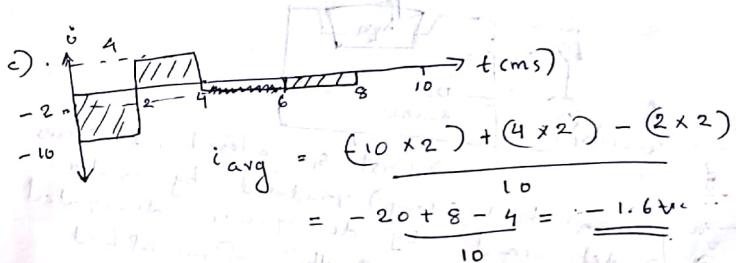
2.) Find the Average value of the waveform.



$$V_{avg} = \frac{(14 \times 1) - (6 \times 1)}{2} = \frac{4}{2} = 4V$$



$$V_{avg} = \frac{12 - 4}{8} = \frac{8}{8} = 1V$$



$$V_{avg} = \frac{(10 \times 2) + (4 \times 2) - (2 \times 2)}{10} = \frac{20 + 8 - 4}{10} = 1.6V$$

\* Why sine waveform?

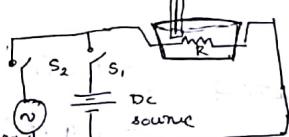
- 1) In ac machines sinusoidal voltages and currents respectively produces least iron and copper losses for given output. The efficiency is better.
- 2) Sinusoidal voltage & currents produce less interference (noise) on telephone lines.
- 3) The sine wave form produces the least disturbance in electrical circuit and smoothest & efficient waveforms.

### RMS Value

An alternating voltage or current can be expressed in terms of (i) instantaneous value. (ii) Peak value.

#### (iii) RMS value.

→ RMS value is based on its heating effect of electric current in electric circuit.



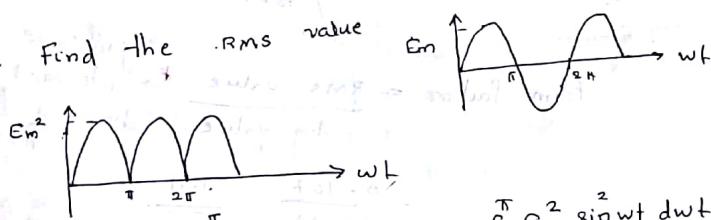
First  $S_1$  closed,  $I$  flows and heat (temp) produced by water determined by de power dissipated in resistor b. in the form of heat. Switch  $S_2$  closed,  $S_1$  open, AC it varied until temp. reaches same as that of dc it.

∴ RMS value is defined as that equivalent steady direct current which when flowing through a given resistance, for a given time produces the same amount of heat energy as produced by alternating current when flowing through the resistance for the same time

$$I_{rms} = \sqrt{\frac{i_1^2 + i_2^2 + \dots + i_n^2}{n}} R$$

$$I_{rms}^2 = \frac{\text{Area under squared waveform or curve}}{\text{Base.}}$$

1. Find the RMS value



$$\begin{aligned} E_{rms}^2 &= \frac{\int_0^{\pi} E_m^2 d(wt)}{\pi} = \frac{1}{\pi} \int_0^{\pi} E_m^2 \sin^2 wt dwt \\ &= \frac{E_m^2}{\pi} \int_0^{\pi} \frac{[1 + \cos 2wt]}{2} dwt = \frac{E_m^2}{2\pi} [(\pi - 0) - (\sin 2w\pi)] \\ &= \frac{E_m^2}{2} \end{aligned}$$

$$I_{rms} = \frac{E_m}{\sqrt{2}}$$

2. Find  $E_{rms}$ ,  $\arg$ .

$$E_{arg} = \frac{1}{2\pi} \int_0^{\pi} E_m \sin wt dwt$$

$$= \frac{E_m}{2\pi} - [\cos wt]_0^\pi = \frac{E_m}{2\pi} \times 2 = \frac{E_m}{\pi}$$

$$E_{rms}^2 = \frac{1}{2\pi} \int_0^{\pi} E_m^2 \sin^2 wt dwt = \frac{E_m^2}{2\pi} \int_0^{\pi} \frac{[1 - \cos 2wt]}{2} dwt$$

$$= \frac{E_m^2}{2\pi} \left[ \frac{1 - \cos 2w\pi}{2} \right]_0^\pi = \frac{E_m^2}{4\pi} [\pi - 0] = \frac{E_m^2}{4}$$

$$E_{rms} = \underline{\underline{\frac{E_m}{2}}}$$

$$\text{Peak Factor} = \frac{\text{Max. Value}}{\text{RMS value}}$$

$$\text{Sine wave} = \frac{V_m}{0.707 V_m} = 1.412.$$

Significant form factor =  $\frac{\text{RMS value}}{\text{Avg. value}}$  determines the sharpness of wave form

$$= \frac{0.707}{0.637} = \underline{\underline{1.11}}$$

for square wave, rectangular wave,  $\text{Form Factor} = 1$

$\rightarrow$  Frequency spectrum

E LF - 30Hz - 3 kHz  $\rightarrow$  Extremely low freq.

VLF - 3 kHz - 30 kHz  $\rightarrow$  Very low freq.

LF - 30 kHz - 300 kHz  $\rightarrow$  Low freq.

MF - 300 kHz - 3 MHz  $\rightarrow$  Medium freq.

HF - 3 MHz - 30 MHz  $\rightarrow$  High freq.

VHF - 30 MHz - 300 MHz  $\rightarrow$  Very high freq.

RF - 300 MHz - 3 GHz  $\rightarrow$  Ultra high freq.

SHF - 3 GHz - 30 GHz  $\rightarrow$  Super high freq.

EHF - 30 GHz - 300 GHz  $\rightarrow$  Extremely high freq.

Audio freq.

15 Hz - 20 kHz

Radio freq.  $\rightarrow$  3 kHz - 3000 MHz

Micro wave  $\rightarrow$  300 MHz - 300 GHz  
- even - 3000 - 30000 GHz

WIFI  $\rightarrow$  2.4 GHz - 2.56 GHz

A rotating vector is called phasor.



V or I can be represented by a line of definite length rotating in anticlockwise at constant angular velocity ( $\omega$ ).

Length of phasor = Max. value of alternating quantity.

Phase and phase angle. Phase of an alternating quantity

is the fraction of time period that has elapsed since it last passed from origin.

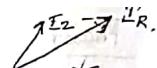
Phase angle ( $\phi$ )  $\rightarrow$  equivalent to phase in radians or degree.

Phase difference.

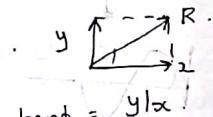
If 1 & 2 are alternating quantities of same frequency, have different zero points; they are said to have a phase difference.

### Addition of phasors

1. parallelogram method



2. Method of Components.



$$R = \sqrt{x^2 + y^2}$$

$$\tan \phi = y/x$$

1. Two currents represented by

$$i_1 = 15 \sin(\omega t + 15^\circ)$$

$$i_2 = 25 \sin(\omega t + 45^\circ)$$

Find total current.

to common conductor. Find total current.  
Then find the energy loss in 10 hours, if the  
circuit is provided with  $2\Omega$  resistance



$$I_{\text{result}} = \sqrt{I_1^2 + I_2^2 + 2I_1 I_2 \cos 45^\circ}$$

$$\phi = \tan^{-1} \frac{80}{25.17} = 50.7^\circ$$

Resultant phasor

$$I_{\text{resultant}} = 39.9 (\sin(\omega t + 50.7))$$

$$\text{maximum current} = \frac{I_{\text{max}}}{T_2} = I_{\text{rms}}$$

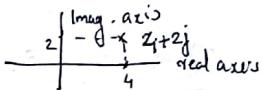
$$H = \frac{I^2}{2\pi f} R t$$

$$= \left( \frac{39.9}{\sqrt{2}} \right)^2 \times 2 \times 10 \times 60 \times 60$$

$$= 57.31 \times 10^6 \text{ Joules}$$

operators j  $j = \sqrt{-1}$

operator  $j$  when applied to a phasor  $E$  gives a new phasor  $jE$ , which is displaced by  $90^\circ$  in anticlockwise direction from  $E$ .



Complex no.

1. Rect. form  $x + jy$ .

$$\text{Trigonometric form: } r[\cos \theta + j \sin \theta]$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} y/x$$

3. Exponential form:  $r e^{j\theta}$

4. Polar form:

$$r \angle \pm \theta$$

Operation.

Division,

$$\text{where } r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} y/x$$

1. Convert  $3+4j$  to polar  $\rightarrow 5 \angle 53.13$

2.  $10 \angle 45^\circ$  to rectangular  $\rightarrow 10 \cos 45 + j 10 \sin 45$

$$\rightarrow 7.07 + j 7.07$$

Complex notation of AC circuit

$$X_C = \frac{1}{j\omega C} = -j\omega C$$

$$X_L = j\omega L$$

Response of  $R$ ,  $L$ ,  $C$  elements to sinusoidal voltage

1. AC circuit contain  $R$  only

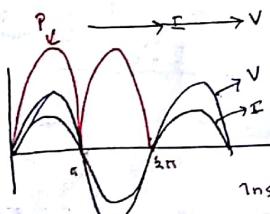


$$V = IR$$

$$V_m \sin wt = IR$$

$$I = \frac{V_m \sin wt}{R} = I_m \sin wt$$

phasor



$$\text{or } \overset{I}{\longrightarrow} \overset{V}{\longrightarrow}$$

$$\text{Instantaneous Power} = V_m \sin wt \cdot I_m \sin wt$$

$$\text{Average Power over a complete cycle} = \frac{\text{Area under Curve}}{\text{Base}} = \frac{1}{\pi} \int_0^{\pi} V_m \sin wt \cdot I_m \sin wt dt$$

$$= \frac{1}{\pi} V_m I_m \int_0^{\pi} \sin^2 wt dt = \frac{V_m I_m}{2\pi} [\pi] = \frac{V_m I_m}{2}$$

$$= \frac{V_m I_m}{2} = \frac{V_m I_m}{2}$$

\* Power is +ve, never zero. This means power is always absorbed in a resistive circuit and the same is dissipated as heat.

2. AC through purely inductive circuit

$$V = V_m \sin wt$$

$$= L \frac{di}{dt}$$

$$\frac{di}{dt} = \frac{V_m}{L} \sin wt$$

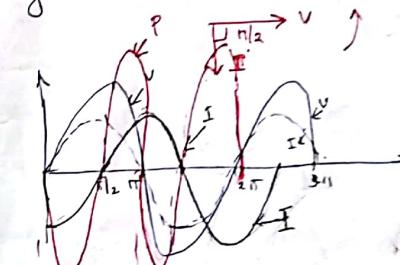
$$\therefore i = \int \frac{V_m}{L} \sin wt dt = \frac{V_m}{L\omega} [\cos wt]$$

$$i = \frac{V_m}{L\omega} [\sin(\omega t - \pi/2)]$$

$$\rightarrow \sin(\omega t - \pi/2) = -\sin(\omega t - \pi) = \cos \omega t$$

$$i_{\max} = \frac{V_m}{L\omega}$$

where  $L\omega$  is called inductive reactance unit ohm.  
 $\propto L = L\omega \rightarrow$  it is the opposition offered by inductance to alternating current flow.



$$\text{Instantaneous Power} = P = VI = V_m \sin wt \cdot I_m \sin(wt - \pi/2)$$

$$= V_m \sin wt \cdot I_m \cos wt = V_m I_m \cos^2 wt$$

Average Power for a Complete Cycle  $C = 1/2$

$$= \frac{1}{\pi} \int_0^{\pi} V_m I_m \cos^2 wt dt = \frac{V_m I_m}{4\pi} [0] = 0$$

Avg. power demand by pure inductive circuit is zero.

$$\therefore \text{Max. value of int. power} = \frac{1}{2} V_m I_m$$

$$= \frac{1}{\pi} \int_0^{\pi} V_m I_m \cos^2 wt dt = \frac{V_m I_m}{2\pi} [0] = 0$$

3. Ac through purely capacitive

$$V_c = V = V_m \sin \omega t$$

$$V_c = \omega/C = \int \frac{dq}{C}$$

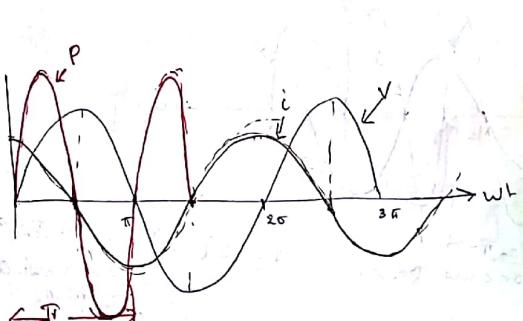
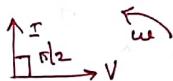
$$\rightarrow C = \frac{dq}{dt} \quad q = CV$$

$$= \frac{dq}{dt} \left[ V_m C \sin \omega t \right]$$

$$i = V_m C \omega \cos \omega t$$

$$V_c = \frac{V_m}{C} \sin(\omega t + \pi/2)$$

$$I = I_m \sin(\omega t + \pi/2)$$



Instantaneous Power =  $V_c = V_m \sin \omega t \cdot I_m \cos \omega t = \frac{V_m I_m}{2} \sin 2\omega t$

Avg. power over whole cycle =  $\frac{V_m I_m}{2} \int_0^{\pi} \sin 2\omega t d\omega t = 0$

Avg. power demand in purely capacitive circuit is zero.

Max. value of inst. Power  $\frac{V_m^2}{2}$

$$P_{avg} = \frac{1}{2} \int_0^{\pi} \sin 2\omega t = \frac{\sin 2\pi - \sin 0}{2} = 0$$

### Power triangle

Active power  $\rightarrow$  Power actually consumed in an AC circuit.  $\text{kw} \rightarrow VI \cos \phi$



$$\text{Power Factor } \cos \phi = \frac{VI \cos \phi}{VI}$$

$$= \frac{\text{Active Power}}{\text{Apparent Power}}$$

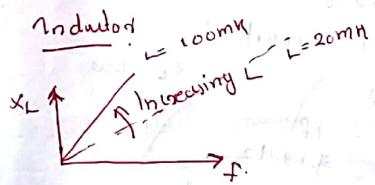
Cosine of angle between voltage & current in the circuit.

### Frequency Response of Basic elements

Resistor R



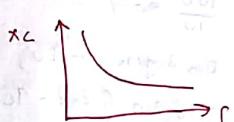
Inductor



$$f_0 = \frac{1}{2\pi L} \quad f = \text{very high freq.}$$

$$X_L = L\omega = L 2\pi f \quad I = V/X_L$$

Capacitor



$$X_C = \frac{1}{C \omega}$$

### Impedance

a) Ratio of phasor voltage to phasor current as "impedance". Complex quantity having dimension of ohms. Impedance is not a phasor.

Current through  $100\text{ }\mu\text{F}$  capacitor is given  
 Find expression for voltage across  $C_{\text{load}}$   
 $c = 40 \sin(500t + 60^\circ)$ .  
 $= 20\Omega$ .

$$X_C = \frac{1}{\omega C} = \frac{1}{500 \times 100 \times 10^{-6}}$$

$$V = I_m X_C = 40 \times 20 = 800 \text{ V}$$

$$V(t) = 800 \sin(500t - 30^\circ)$$

2. The voltage across  $0.5\text{ H}$  coil is  
 $v(t) = 100 \sin 20t$ . What is the sinusoidal expression for current?

$$L = 0.5\text{ H}$$

$$\omega = 20, \quad f = 2\pi f \\ \therefore f = 3.18\text{ Hz}$$

$$X_L = L\omega = 20 \times 0.5 = 10\Omega$$

$$I_m = \frac{V_m}{X_L} = \frac{100}{10} = 10\text{ A}$$

since  $L \rightarrow$

$$I = I_m \sin(\omega t - 90^\circ) \\ = 10 \sin(20t - 90^\circ)$$

3. Current through  $5\Omega$  resistor is  
 Given  $i(t) = 20 \sin(377t + 30^\circ)$ . Find  
 expression for voltage across resistor.

$$V_m = \frac{40 \times 5}{\theta} = 200\text{ V}$$

$$V(t) = 200 \sin(377t + 30^\circ)$$

Average power.

$$P = \frac{V_m I_m}{2} \cos \phi$$

$$P_{\text{avg}} = V_{\text{rms}} I_{\text{rms}} \cos \phi$$

$\phi \rightarrow$  angle b/w  $V$  and  $i$

Find the average power dissipated in a network whose ip  $I$  and  $V$ .

$$i = 5 \sin(\omega t + 40^\circ)$$

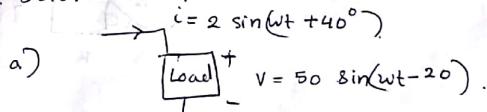
$$v = 10 \sin(\omega t + 40^\circ)$$

Circuit is purely  $R$ .

$$P = \frac{V_m I_m}{2} = \frac{10 \times 5}{2} = 25\text{ W}$$

$$= I_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R}$$

2. Determine power factor.



$$PF = \cos \phi = \cos 60^\circ = 0.5 \text{ leading current}$$

b).

$$V = 120 \sin(\omega t + 80^\circ)$$

$$i = 5.0 \sin(\omega t + 30^\circ)$$

$$\cos \phi = \cos 50^\circ = 0.64 \text{ lagging.}$$

Q.  $P = 100\text{ W}$

$$V_{\text{eff}} = 20\text{ V}$$

$$I_{\text{eff}} = 5\text{ A}$$

cos

Ac through  $RL$

Applied voltage  $V_m \sin \omega t$

$I$

By KVL  $v(t) = V_R + V_L$

$V = iR + jI X_L$

$V_R = IR$

$V_L = jI X_L$

$\bar{V} = \bar{V}_R + j\bar{V}_L$

$\bar{V} = \sqrt{V_R^2 + V_L^2}$

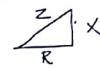
$I = \frac{V}{Z}$

$Z = \sqrt{R^2 + X_L^2}$   $Z = R + jX_L$  is the impedance of series  $RL$  circuit.

$$\phi = \tan^{-1} \frac{V_L}{V_R}$$

$$= \tan^{-1} \frac{Z \cdot X_L}{R}$$

$$\omega \sin \phi = R/Z$$



Series RLC circuit

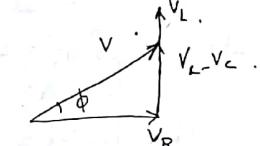
$V = V_R + V_L + V_C$

$= iR + jI X_L - jI X_C$

$V_L > V_C$

$|V| = I \sqrt{R^2 + (X_L - X_C)^2}$

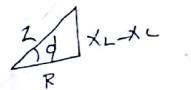
$\bar{V} = \bar{V}_R + j(\bar{V}_L - \bar{V}_C)$



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= I \sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$



$$\phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right)$$

$$V = V_m \sin \omega t$$

$$i = I_m \sin (\omega t \pm \phi)$$

$X_C > X_L \rightarrow +ve$  sign

$X_L > X_C \rightarrow -ve$  sign

$$P = VI \cos \phi$$

$$\cos \phi = \frac{R}{Z}$$

Ac through RC

By KVL  $v(t) = V_R + V_C$

$V = iR - jI X_C$

$V_R = IR$

$V_C = -jI X_C$

$\bar{V} = \bar{V}_R - j\bar{V}_C$

$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Applied voltage  $V = \bar{V}_R + \bar{V}_C = jR - jI X_C \therefore I = CR - jX_C$

$$I = \frac{V}{R - jX_C} = \frac{V}{Z}$$

where  $Z = R - jX_C$  is the impedance of circuit.

$$|Z| = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{(2\pi f C)^2}}$$

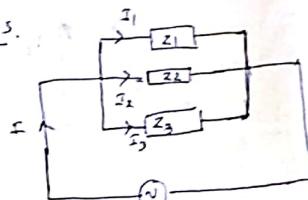
### Admittance ( $\gamma$ )

If  $\gamma$  is the reciprocal of impedance.  
Its unit is siemen or mho.

$$\begin{aligned} \gamma &= \frac{1}{Z} \\ &= \frac{1}{R+jX} \\ &= \frac{R-jX}{R^2+X^2} = \frac{R}{R^2+X^2} - j\frac{X}{R^2+X^2} \\ &\quad \text{Conductance} \quad \text{Reactance} \end{aligned}$$

In general  $\gamma = G \pm jB$ .

### Parallel Circuits.



$$I = I_1 + I_2 + I_3$$

$$I_1 = \frac{V}{Z_1}$$

$$I_2 = \frac{V}{Z_2}$$

$$I_3 = \frac{V}{Z_3}$$

Admittance  $\gamma = \frac{1}{Z}$ , mho.

$$\therefore \gamma_1 = \frac{1}{Z_1} = \frac{I_1}{V}$$

$$\therefore I_1 = \gamma_1 V$$

$$\text{Similarly } I_2 = \gamma_2 V$$

$$I_3 = \gamma_3 V$$

$$\therefore I = I_1 + I_2 + I_3 = V [\gamma_1 + \gamma_2 + \gamma_3]$$

$$\boxed{I = V \gamma_{\text{equ}}}$$

$$\gamma_{\text{equ}} = \gamma_1 + \gamma_2 + \gamma_3$$

### Effect of freq. variation in RLC Series Circuit

$$X_L = L\omega$$

$$X_C = \frac{1}{\omega C}$$

$$\text{Resultant } X = X_L - X_C = 2\pi f L - \frac{1}{2\pi f C}$$

If  $X_L > X_C$ ,  $X$  is +ve

If  $X_L < X_C$ ,  $X$  is -ve.

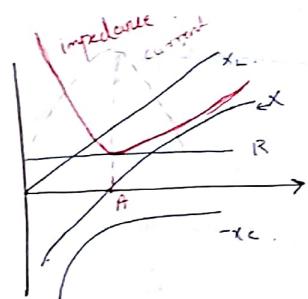
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (2\pi f L - \frac{1}{2\pi f C})^2}$$

$X_L \rightarrow L\omega$  so  $X_L$  vs  $f$ .

$X_C$

straight line.  
 $X_C$  by rectangle hyperbola.



$$Z = R + jX_e$$

$$X = X_L - X_C$$

At A  $\Rightarrow X_L = X_C$ . Impedance  $X = 0$ .  
 $I = V/X$  is max.

Effect of freq. variation in Parallel circuit.

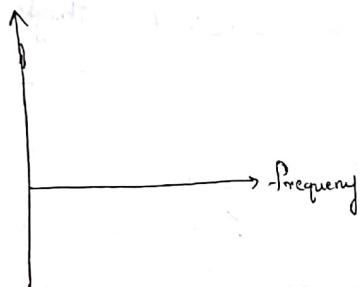
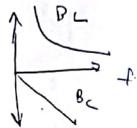
$$\text{Inductive Susceptance} = \frac{1}{X_L} = \frac{1}{2\pi f L}$$

$$X_L \propto f$$

$$B_L \propto \frac{1}{f}$$

$$\text{Capacitive Susceptance} = \frac{1}{X_C} = 2\pi f C$$

$$B_C \propto f$$



When AC voltage of 250V is applied across the circuit, the current in the circuit is found to be 25A. If the current is 0.8 Pf lagging, find impedance in the circuit.

$$V = 250V$$

$$\cos \phi = 0.8$$

$$\phi = 36.86^\circ$$

$$\sqrt{36.86^\circ}$$

$$|I| = 25A$$

$$\angle \phi = 36.86^\circ$$

$$I = 25 \angle 36.86^\circ$$

$$\boxed{x+iy = r \angle \phi \\ \text{where } r = \sqrt{x^2+y^2} \\ \phi = \tan^{-1}(y/x)}$$

$$\begin{aligned} r \angle \phi &= r \angle y/x \\ x^2 + y^2 &= r^2 \\ \tan \phi &= y/x \end{aligned}$$

$$= 25 \cos \phi + j 25 \sin \phi$$

$$I = 20 - j 14.99$$

$$Z = \frac{V}{I} = \frac{250}{20-j15} = \underline{\underline{8+j6}} \Omega$$

$$|Z| = \underline{\underline{10}} \Omega$$

2) A  $100\ \Omega$  resistor is in series with  $120\text{mF}$  capacitor, is connected to  $230V, 50\text{Hz}$  supply. Find (i) circuit impedance, (ii) current (iii)  $\text{PF}$ . (iv) voltage across

$$R = 100\ \Omega \quad f = 50\text{Hz}$$

$$C = 120 \times 10^{-6}\text{F}$$

$$X_C = \frac{1}{C\omega} = \frac{1}{120 \times 10^{-6} \times 2\pi \times 50} = 26.5\ \Omega$$

Impedance  $Z = \sqrt{R^2 + X_C^2} = \underline{103.45\ \Omega}$

$$\text{Current } I = \frac{V}{Z} = \frac{230}{103.45} = \underline{2.223\text{A}}$$

$$\phi = \tan^{-1} \frac{X_C}{R} = 14.84^\circ$$

$$I = 2.23 \times \underline{14.84} \text{A}$$

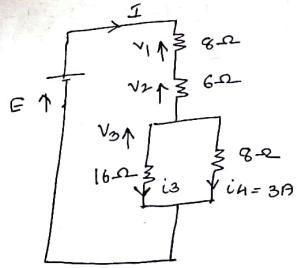
$$\cos \phi = 0.966$$

$$V_R = IR = 222.3\text{V}$$

$$V_C = I \times X_C = \underline{58.90\text{V}}$$

KCL and KVL

✓ D Find supply current & source emf



$$\begin{aligned}V_3 &= i_4 R_4 \\&= 8 \times 3 = 24 \text{ V} \\24 &= i_3 \times 16 \\i_3 &= 1.5 \text{ A}\end{aligned}$$

$$I = i_3 + i_4$$

$$= \underline{\underline{4.5 \text{ A}}}$$

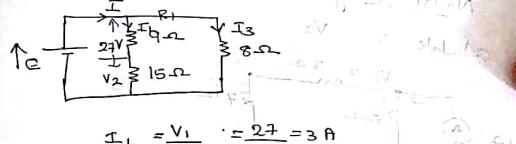
$$V_1 = IR_1 = 4.5 \times 8 = \underline{\underline{36 \text{ V}}}$$

$$V_2 = IR_2 = 4.5 \times 6 = 27 \text{ V}$$

by KVL

$$E = V_1 + V_2 + V_3 = \underline{\underline{87 \text{ V}}}$$

2) Determine  $I_1, E, I_3$  and  $I$



$$I_1 = \frac{V_1}{R_1} = \frac{27}{9} = 3 \text{ A}$$

$$V_2 = I_1 \cdot 16 = 48 \text{ V}$$

$$45 + 27 \text{ V} = E = \underline{\underline{72 \text{ V}}}$$

$$I_3 = \frac{72}{8} = 9 \text{ A}$$

$$I = I_1 + I_3 = 3 + 9 = \underline{\underline{12 \text{ A}}}$$

Mesh

$$15A$$

$$\frac{\Delta V - IV}{E} + \frac{IV}{\Delta} = n$$

$$\frac{\Delta V}{E} + \left( \frac{1}{\Delta} - \frac{1}{n} \right) IV = 1$$

$$\frac{\Delta V}{E} + \frac{IV}{\Delta} - \frac{IV - \Delta V}{E} = 1$$

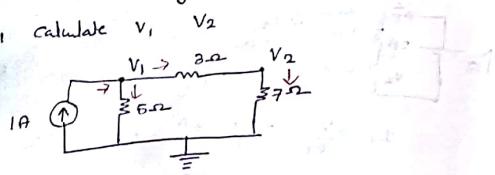
$$\frac{IV}{E} = \left[ \frac{1}{\Delta} - \frac{1}{n} \right] \Delta V$$

$$\frac{IV}{E} = \frac{\Delta V}{\Delta^2 + 1}$$

$$IV = \frac{E}{\Delta^2 + 1} \Delta V$$

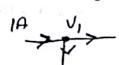
### Nodal Analysis

1. calculate  $V_1, V_2$



Step 1:  $V_1, V_2$  : 2 nodes.

Step 2: Assign current



Step 3: Applying KCL at node 1

$$I = \frac{V_1}{5} + \frac{V_1 - V_2}{3}$$

$$I = V_1 \left( \frac{1}{5} + \frac{1}{3} \right) - \frac{V_2}{3} \quad \text{--- (1)}$$

$$I = V_1 \frac{8}{15} - \frac{V_2}{3} \quad \rightarrow V_2$$

node 2

$$\frac{V_2 - V_1}{3} = \frac{V_2}{7}$$

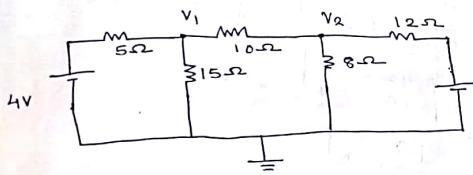
$$V_2 \left[ \frac{1}{3} - \frac{1}{7} \right] = \frac{V_1}{3} \quad \text{--- (2)}$$

$$(1 + \frac{1}{3}) V_2 = V_1 \frac{8}{15} \quad \text{--- (3)}$$

$$V_1 \frac{2}{5} \left[ \frac{1}{21} \right] = V_1$$

$$V_2 = \frac{V_1 \cdot 8 \cdot 5}{15 \cdot 3}$$

2. Using node voltage calculate  $V_1, V_2$  and calc. Current in  $8\Omega$



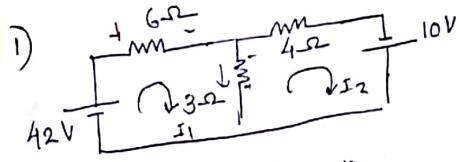
$$V_1 = 2.88 V$$

$$V_2 = 2.55 V$$

$$I \text{ in } 8\Omega = \frac{V_2}{8} = 0.32 A$$

### Nodal Analysis

### Mesh - solution Using matrix method



$$42 - 6I_1 - 3(I_1 - I_2) = 0 \quad \text{--- (1)}$$

$$42 + 9I_1 - 3I_2 = 42 \quad \text{--- (2)}$$

$$10 - 4I_2 - 3(I_2 - I_1) = 0 \quad \text{--- (3)}$$

$$+3I_1 - 7I_2 = -10 \quad \text{--- (4)}$$

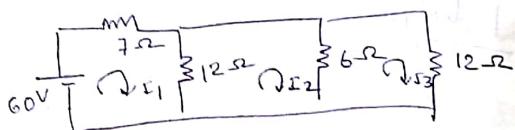
$$-3I_1 + 7I_2 = 10 \quad \text{--- (5)}$$

$$\begin{bmatrix} 9 & -3 \\ -3 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 42 \\ 10 \end{bmatrix}$$

$$I_1 = \frac{1}{9-3} \begin{vmatrix} 42 & -3 \\ 10 & -9 \end{vmatrix} = 6A$$

$$I_2 = \frac{1}{9-3} \begin{vmatrix} 9 & 42 \\ -3 & 10 \end{vmatrix} = 4A$$

2) Now calculate Current  $I_1$ , shown using mesh



$$19I_1 - 12I_2 = 60$$

$$12I_1 - 18I_2 + 6I_3 = 0$$

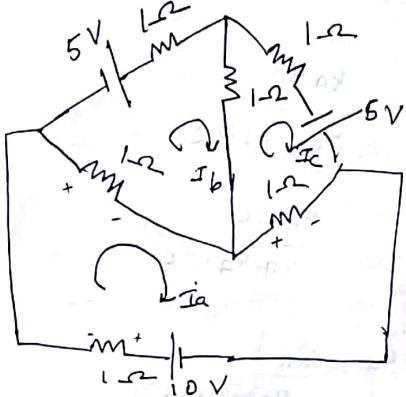
$$6I_2 - 18I_3 = 0 \quad \text{---}$$

$$\begin{bmatrix} 19 & -12 & 0 \\ 12 & 18 & -6 \\ 0 & 6 & 18 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 60 \\ 0 \\ 0 \end{bmatrix}$$

$$I_1 = \frac{1}{19-12} \begin{bmatrix} 60 & -12 & 0 \\ 0 & 18 & -6 \\ 0 & -6 & 18 \end{bmatrix} \div \begin{bmatrix} 19 & -12 & 0 \\ 12 & 18 & -6 \\ 0 & 6 & 18 \end{bmatrix}$$

$$I_1 = \underline{\underline{6A}}$$

3.



$$I_a = 7.5A$$

$$I_b = 6.25A$$

$$I_c = 6.25A$$

$$10 - I_a - (I_a - I_b) - (I_a - I_c) = 0$$

$$10 - 3I_a + I_b + I_c = 0 \quad \text{--- (1)}$$

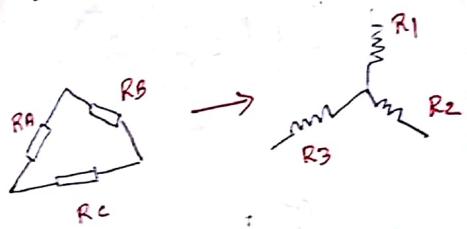
$$5 - I_b - (I_b - I_c) - (I_b - I_a) = 0$$

$$5 - 3I_b + I_c + I_a = 0 \quad \text{--- (2)}$$

$$5 - I_c - (I_c - I_a) - (I_c - I_b) = 0$$

$$5 - 3I_c + I_a + I_b = 0 \quad \text{--- (3)}$$

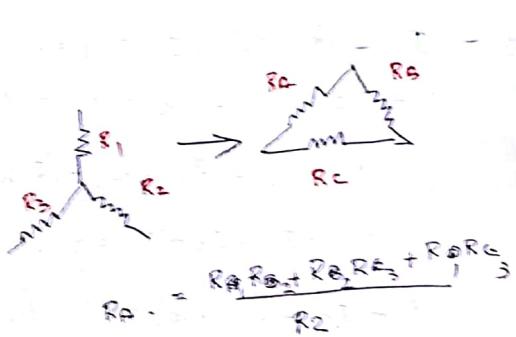
Star - Delta  $\rightarrow$



$$R_s = \frac{R_A R_B}{R_A + R_B + R_C}$$

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$



$$R_s = \frac{R_A R_B + R_B R_C + R_A R_C}{3}$$

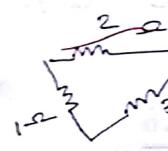
$$R_1 = \frac{R_A R_B + R_B R_C + R_A R_C}{R_A}$$

$$R_2 = \frac{R_A R_B + R_B R_C + R_A R_C}{R_B}$$

1. Convert



2)

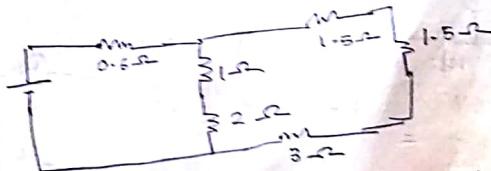
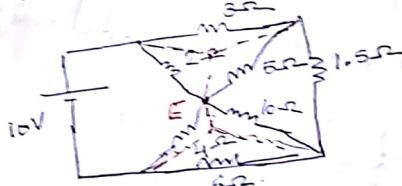


$$R_1 = \frac{2}{6} = \frac{1}{3} \Omega$$

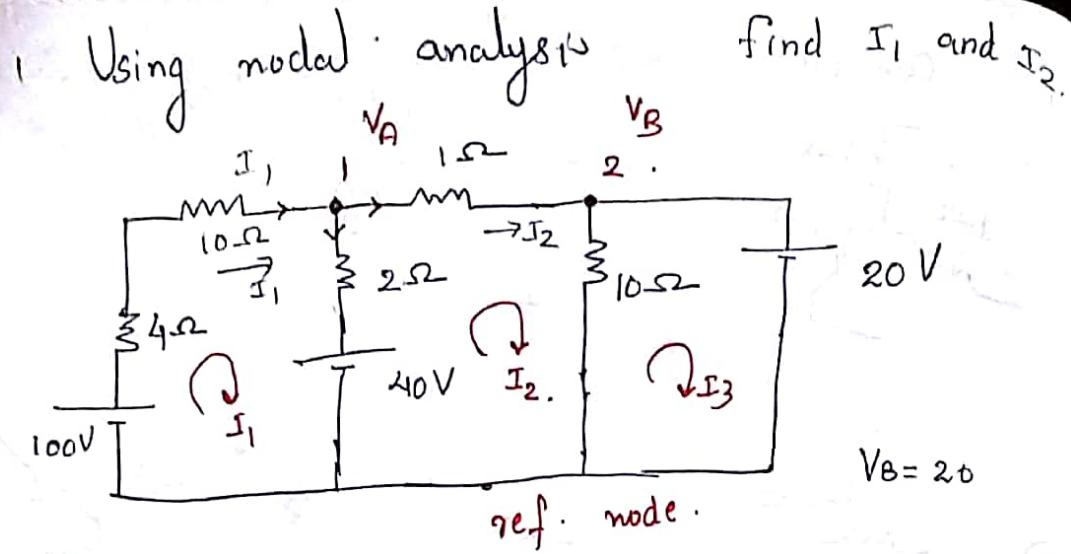
$$R_2 = \frac{(2 \times 3)}{6} = 1 \Omega$$

$$R_3 = \frac{3 \times 1}{6} = \frac{1}{2} \Omega$$

1. Using delta - star transformation



$$I = 10 / S.S = 2.63 A$$



$I_1$

$$\frac{100 - V_A}{14} = \frac{V_A - 40}{2} + \frac{V_A - 20}{1}$$

$$100 - V_A = 9V_A - 280 + 14V_A - 280$$

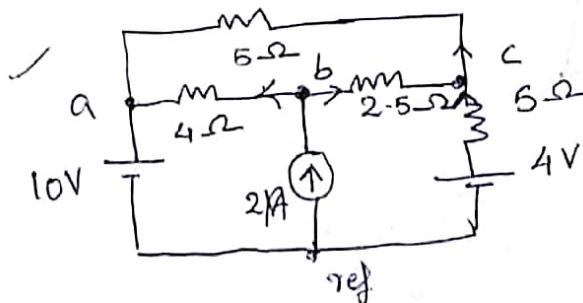
$$660 = 22V_A$$

$$V_A = \frac{660}{22} = \underline{\underline{30V}}$$

$$I_1 = \frac{100 - V_A}{14} = \frac{60}{14} = \underline{\underline{5A}}$$

$$I_2 = \frac{V_A - 20}{1} = \underline{\underline{30A}}$$

2. Determine  $V_b$  and  $V_c$  using nodal analysis



$$V_a = 10V \quad \text{--- (1)}$$

$$\text{At } b, \quad 2 = \frac{V_b - 10}{4}$$

$$\frac{2 - V_c}{5} + \frac{V_b - V_c}{2.5} = \frac{V_c - 10}{5} \quad \text{--- (2)}$$

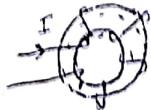
At c:

$$\frac{2 - V_c}{5} + \frac{V_b - V_c}{2.5} = \frac{V_c - 10}{5} \quad \text{--- (3)}$$

$$\frac{mmf}{AT}$$

$$N \cdot I^2$$

Magnetic circuit: The complete closed loop path followed by any group of magnetic flux lines.



One of the simplest form of magnetic circuit is the ring as shown.

where the steel ring provides the space in which magnetic flux is created. Most rings are made like anchor rings in that their cross section is circular such rings called toroid.

$$mmf = NI \quad \text{Amperes turns}$$

$$\text{Magnetic field strength} \quad H = \frac{NI}{l}$$

$$B = \mu_0 H A$$

$$\mu_0 = \frac{B}{H} \quad \text{unit Henry/m A/m}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\text{Reluctance: } s = \frac{mmf}{\text{flux}} = \frac{NI}{B l} = \frac{NI}{\mu_0 H A}$$

$$\boxed{s = \frac{l}{\mu_0 H A}}$$

$$AT/Wb$$

$$\text{Permeability} = \frac{1}{\text{reluctance}} = \frac{\phi}{mmf}$$

- 1) A mild steel ring having cross-sectional area  $10 \text{ cm}^2$  and mean circumferential length  $20 \text{ mm}$  has a coil of 200 turns wound uniformly around it. Calculate  
 a) reluctance  
 b) current required to produce flux 800 mwb.

$$s = \frac{l}{area \cdot \mu_0 H} = \frac{0.4}{500 \times 10^{-6} + 4\pi \times 10^{-7} \times 380} = 1.68 \times 10^6 \text{ AT/Wb}$$

$$b) \approx 6.7 \text{ A}$$

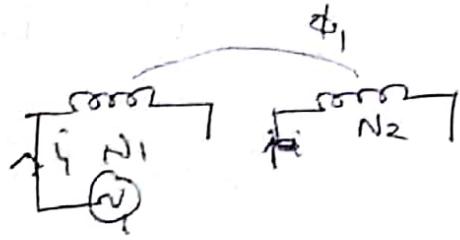
Permeability - the ability of a material to conduct magnetic flux through it. ( $\mu$ ) unit  $\text{H/m}$  (Henry/m)

- 2) A flux of 0.04 wb is produced in a solenoid of axial length 25cm with 500 turns carrying a current of 4A. Find reluctance of magnetic circuit?

$$s = \frac{mmf}{\phi} = \frac{NI}{\phi} = \frac{500 \times 4}{0.04} = 50,000 \text{ AT/Wb}$$

$$L = \frac{N_1 \phi_1}{I_1}$$

$$M = \frac{N_2 \phi}{I_1}$$



- Coupling
- Coupling Coefficient

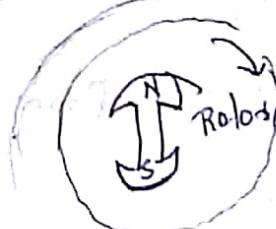
$$k = \frac{M}{\sqrt{L_1 L_2}}$$

Generation of alternating voltages

- An alternating voltage may be generated
- (i) by rotating a coil at constant angular velocity in a uniform magnetic field



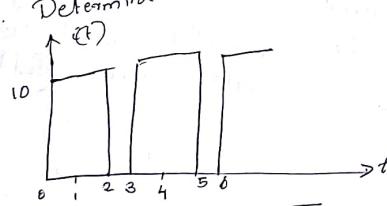
- (ii) by rotating mag. field at cons. angular velocity with in stationary coil



1. Find RMS value  
 $i = 10 + 10 \sin \omega t$

$$\begin{aligned} I_{rms} &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} i^2 dt} \\ &= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (10 + 10 \sin \omega t)^2 dt} \\ &= \sqrt{\frac{100}{\pi} \int_0^{2\pi} (1 + \sin \omega t)^2 dt} \\ &= \sqrt{\frac{50}{\pi} \left[ \omega - 2 \cos \omega t + \frac{1}{2} \sin 2\omega t \right]_0^{2\pi}} \\ &= \sqrt{\frac{50}{\pi} (2\pi + \pi)} \\ &= \sqrt{150} = 12.247 A \end{aligned}$$

2. Determine rms value, avg current



$$I_{rms} = \sqrt{\left(\frac{10^2 \times 2}{3}\right)} = 8.16 A$$

$$I_{av} = \frac{10^2 \times 2}{3} = 6.67 A$$

$$f_{rms} = \frac{I_{rms}}{I_{av}} = \frac{8.16}{6.67} = 1.2$$

In an AC circuit consisting of 2 elements in series, the equations of voltage and current are given by  $e = 150 \sin(200\pi t - 25^\circ)$  and  $i = 8 \sin(200\pi t + 5^\circ)$ . Calculate the frequency, Pf. and values of circuit constants.

$$\omega = 2\pi f = 200 \text{ rad/s}$$

$$f = \frac{200 \text{ rad/s}}{2\pi} = 31.843 \text{ Hz}$$

Angle b/w  $e$  and  $i$  is  $-25 + 5 = 30^\circ$

$$Pf = \cos 30 = \frac{\sqrt{3}}{2} = 0.866, \text{ leading}$$

$$E_{rms} = 50 V, I_{rms} = \frac{8}{\sqrt{2}} = 5.65 A$$

$$C_{rms} = \frac{50}{\sqrt{2}} = 35.35 V$$

$$Impedance Z = \frac{E_{rms}}{I_{rms}} = 6.25 \Omega$$

$$Z = \frac{E_{rms}}{I_{rms}} = \frac{50}{8.16} = 6.125 \Omega$$

$$Z = \frac{8.16 \angle -25^\circ}{8.16 \angle 5^\circ} = 8.16 \angle -30^\circ$$

$$Z = 5.41 - j3.125 \Omega$$

$$R = 5.41 \Omega$$

$$X_C = 3.125 = \frac{1}{C \omega}$$

$$C = 1.6 \times 10^{-4} F$$

$$= 0.16 mF$$

### Advantage of 3<sup>rd</sup> system over (1<sup>st</sup> 1m)

1. o/p of 3<sup>rd</sup> machine generating electricity is more than the o/p of a single phase machine of same size.
2. The 3<sup>rd</sup> induction motors are self starting. For 1<sup>st</sup> motors, a separate starting winding is required.
3. 3<sup>rd</sup> power transmission is more economical.
4. PF of 3<sup>rd</sup> system is better than 1<sup>st</sup> systems.
5. 1<sup>st</sup> power supply can also be obtained from a 3<sup>rd</sup> supply.
6. The inst power in a 1<sup>st</sup> system is fluctuating with time giving rise to noisy performance of 1<sup>st</sup> motors. The power o/p is steady.
7. For rectification of AC into DC, DC voltage becomes less fluctuating.

Power measurement using 2 wattmeter

$$W_1 + W_2 \rightarrow 3\phi \text{ Power}$$

$$W_1 - W_2 = \sqrt{3} V_{ph} I_{ph} \cos \phi$$

$$\tan \phi = \frac{W_1 - W_2}{\sqrt{3} (W_1 + W_2)}$$

$$W_1 = V_{ph} I_{ph} \cos(\phi - d)$$

$$W_2 = V_{ph} I_{ph} \cos(30 + d)$$

case 1) At  $\phi = 0$

$$W_1 = W_2, \text{ equal \& +ve}$$

$$\cos \phi = 0.5 \Rightarrow \phi = 60^\circ$$

$$W_2 = 0, \quad W_1 = V_{ph} I_{ph} \cos 30^\circ$$

3) When  $d > 60^\circ$ ,  $W_1$  will be -ve reading,  $W_2$  will give +ve reading.

4) When  $d = 90^\circ$ ,  $W_1$  is purely inductive and  $W_2$  is capacitive.

$$W_1 = V_{ph} I_{ph} \cos 60^\circ$$

$$W_2 = V_{ph} I_{ph} \cos (120^\circ)$$

Both wattmeter reading equal but opposite  
Total power = 0

$$\underline{W_1 = -W_2}$$

1. Draw the connection diag. for 2 W/m<sub>s</sub> method. The load connected was 30kW at 0.7 Pf lagging. Find reading of each wattmeter.

$$W_1 + W_2 = 30 \text{ kW}$$

$$\phi = 45.57^\circ \text{ lagging}$$

$$\sqrt{3} V_L I_L \cos\phi = 30 \text{ kW}$$

$$V_L I_L = 24743.88 \text{ VA}$$

$$= 24743 \text{ kVA}$$

$$W_1 = V_L I_L \cos(30 - \phi)$$

$$W_2 = V_L I_L \cos(30 + \phi)$$

$$W_1 = 24743 \times 10^3 \cos(30 - 45.57^\circ)$$

$$= 23.835 \text{ kW}$$

$$W_2 = 6.165 \text{ kW}$$

$$W_1 + W_2 = 30 \text{ kW}$$

2. A 3φ balanced load connected across a 30/1400V ac supply draws a line current of 10A. Two W/m<sub>s</sub> are used to measure 1P power. Ratio of W/m reading to 2 W/m<sub>s</sub> is 2:1. Find readings of 2 W/m<sub>s</sub>.

Ratio of W/m reading to 2 W/m<sub>s</sub> is 2:1. Find

readings of 2 W/m<sub>s</sub>.

A balanced 3φ star-connected supply of  $8+j6\Omega$  per phase is supplied by 400V<sub>LS</sub>. Calculate  $I_L$ ,  $P_f$ ,  $P$ ,  $Q$ .

$$V_{ph} = \frac{230.94}{\sqrt{3}} = 133.94 \text{ V}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{133.94}{8+j6} = 16.4 \angle -36^\circ$$

$$PF = 0.8006 \quad \text{laying.}$$

$$\text{Active power} = \sqrt{3} V_L I_L \cos \phi$$

$$= 12.776 \text{ kW}$$

$$\text{Reactive power} = \sqrt{3} V_L I_L \sin \phi$$

$$= 9.627 \text{ kVAR}$$

2. The input power to a load meter was measured by 2 WLM method. The readings were 5.2 kW and -1.7 kW and the line voltage was 400V. Calculate  $P$ ,  $PF$ ,  $I_L$ .

$$W_1 = \sqrt{3} V_L I_L \cos \phi (30^\circ - \phi)$$

$$W_2 = V_L I_L \cos (30^\circ + \phi)$$

$$\begin{matrix} 5.2 \\ -1.7 \\ \hline 6.9 \\ 3.5 \end{matrix}$$

$$\tan \phi = \sqrt{3} \left[ \frac{5.2 + 1.7}{5.2 - 1.7} \right]$$

$$\phi = 93.67^\circ$$

$$PF = \cos \phi = \frac{6.9}{7.6} = 0.88 \quad \text{or} \quad 0.28$$

$$I_L = \frac{W_1}{V_L \cos (30^\circ - \phi)} = \frac{5200}{400 \times \cos 26.57^\circ} = 17.99$$

\* principle system of power system Substation.

V.K Mehta At some places in the line of power system, it may be desirable to convert large quantities of ac power to dc power e.g. for traction, electroplating. This job is again performed by a suitable apparatus called substation. It is clear that equipment needed in a substation will depend upon service requirement.

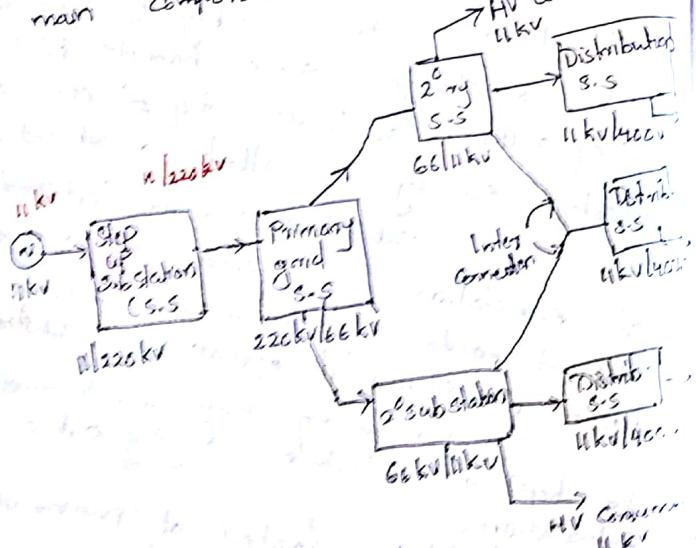
Substation: "The assembly of apparatus used to change some characteristics (e.g. voltage, ac to dc, freq. etc.) of electric supply is called a substation.

Substation is an important part of substation. The continuity of supply depends upto a considerable extent upon the successful operation of substation. The following imp. points which must be kept in view while laying out a substation;

- 1) It should be located at proper site.
- 2) " provide safe & reliable arrangement.

It should be easily operable  
It should involve min. capital cost

- For beyond 66 kV, equipment is installed out doors; so outdoor substation is used.
- pole mounted substation → cheapest substation for voltages not exceeding 11 kV (6932)
- The majority of substation are transformer substation (→ they are concerned with changing of voltage level of electric supply). Reserve power at some voltage & delivers power at some other voltage e.g. Transformers will be the main component in such substation



### Equipments in a transformer substation

#### 1. Bus bars

Cu or Al bars (generally rectangular cross section) and operate at constant voltage. The incoming and outgoing lines in a substation are connected to the bus bars.

#### 2. Insulators

They serve 2 purposes

- a) Support the conductors (on bus bars)
- b) Confine the current to the conductors.
- Porcelain (used for manuf. of insulators)
- Several types of insulators are used
  - a) pin type
  - b) suspension type
  - c) post insulators etc.

#### 3. Isolating switches

In substation, it is often desired to disconnect a part of system for general maintenance and repairs. This is accomplished by an isolating switch or isolator; it is designed to open a circuit under no load i.e. isolator switches are operated only when the lines in which they are connected carry no current.

4. Circuit Breaker: It is an equipment which can open or close a circuit under normal as well as fault condition. CB is designed to operate a) manually (or by remote control) and b) automatically under fault condition. For this relay is used with CB.

5. Power Transformers: are used in a substation to step up or step down the voltage.

6. Instrument Transformers

a) CT → step up transformer  
→ one or more turns thick wire connected in series with low voltage coil.  
→ large no. of fine wires.

Rating:  $100/5$  A for low resistance.

b) PT or voltage transformer  
→ steps down transformer  
→ primary balance  
eg:  $66 \text{ kV} / 110 \text{ V}$

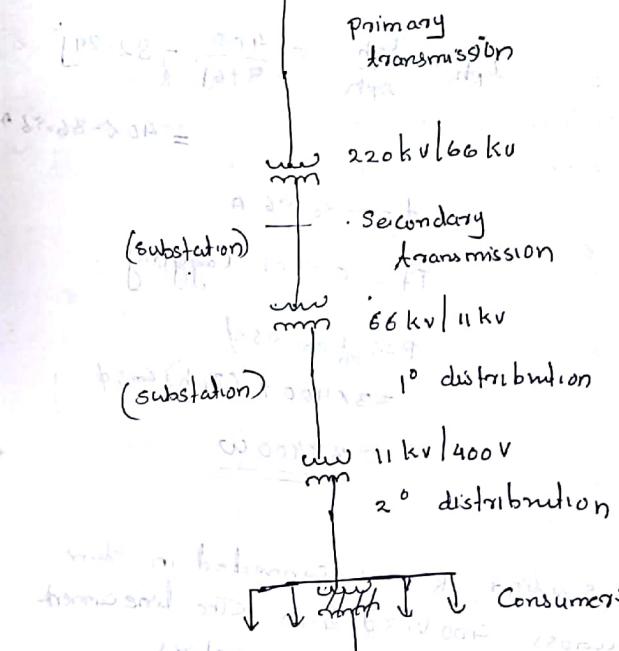
7. Metering & indicating instruments:

eg: vlm, Alm, energy meters

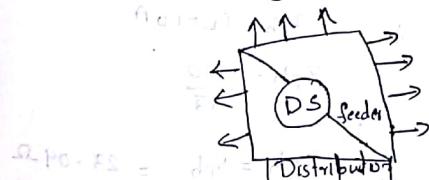
8. Miscellaneous equipment:

→ Fuse, carrier-current equipment, substation auxiliary supplies

Layout of typical power supply scheme by single line diagram



Secondary distribution — It consists of feeders, distributor and service main.



A balanced  $\Delta$  connection is supplied from a  $400V$  supply. Find the line current,  $P_f$ , and total  $P$ .

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{400}{8+6j} = 32-24j$$

$$= 40 \angle -36.86^\circ$$

$$\phi = -36.86^\circ$$

$$P_f = 0.8001 \text{ lagging}$$

$$P = 3V_{ph} I_{ph} \cos \phi$$

$$= 3 \times 400 \times (I_{ph}) \cos \phi$$

$$= 38400 \text{ W}$$

2. 3 similar R. are connected in star across  $400V$  3 $\delta$  lines. The line current is  $10A$ . If the same resistors are connected in  $\Delta$  across same supply calculate current drawn from the supply.

$$\text{In } \Delta, I_{ph} = I_L = 10A$$

$$V_{ph} = \frac{400}{\sqrt{3}}$$

$$Z_{ph} = \frac{V_{ph}}{I_{ph}} = 23.09 \Omega$$

$$\text{In } \Delta, V_{ph} = 400V = \sqrt{3} V_L$$

$$I_{ph} = \frac{V_{ph}}{R} = \frac{400}{23.09} = 10\sqrt{3}$$

$$I_L = \sqrt{3} I_{ph} = 30A$$

3. 3 similar inductive coils are connected in  $\Delta$  to a  $3\phi$ , 4 wire,  $415V$ ,  $50Hz$  supply. The line current is  $4A$  at  $P_f = 0.6$  lagging. Calculate the resistance and L of coil?

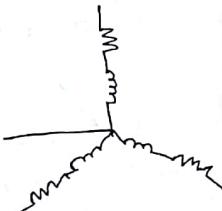
$$V_L = 415V$$

$$V_{ph} = \frac{V_L}{\sqrt{3}}$$

$$I_L = 4A = I_{ph}$$

$$Z_{ph} = \frac{289.6}{4\angle 53.13} = \frac{V_{ph}}{I_{ph}}$$

$$= 59 \angle 53.13^\circ$$



$$\cos \phi = R/Z$$

$$I = \frac{I_L}{\sqrt{3} \angle 53.13^\circ}$$

$$\phi = \cos^{-1}(0.6)$$

$$= 4 \angle -53.13^\circ$$

$$= 53.13^\circ$$

$$Z = R + jX_L = 35.4 + j47.19$$

$$R = 35.4 \Omega$$

$$\omega = 2\pi f = 100\pi$$

$$X_L = 47.19 = L\omega$$

$$L = 0.15 H$$

4. A  $3\phi$ ,  $\Delta$  connected load has a pf 0.8 lag and power consumed by it is 4.35 kW. Calculate current drawn from supply.

$$\text{V}_{ph} = \underline{\underline{V_L}} = 415 \text{ V}$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

$$I_L = \frac{4350}{415 \times 0.8 \times \sqrt{3}}$$

$$= \underline{\underline{7.56 \text{ A}}}$$

DC transmission

1) chandrapur to ridge  
Mumbai  
- 1500 MW at  $\pm 500 \text{ kV}$

2) mumbai to delhi

3) talchak to kolar

### Transformer

#### Losses -

a) Core losses  $\rightarrow$  Eddy current loss or iron losses.  $\rightarrow$  hysteresis losses  $\text{kWh/Bm}^2 \text{ W/m}^3$

#### b) Copper losses.

$$I_1^2 R_1 + I_2^2 R_2$$

### Construction of transformer

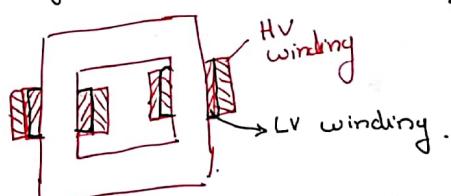
1. Core is made of si. steel which has low hysteresis loss and high permeability. Core is laminated in order to reduce eddy current loss.

2. Instead of placing  $1^\circ$  on one limb and  $2^\circ$  on the other, it is a usual practice to wind one-half of each winding on one limb. This ensure tight coupling b/w the 2 windings. Consequently leakage flux is considerably reduced.

### Types of transformers

- a. Core type
- b. Shell type.

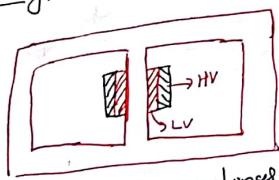
### Core type



- windings surrounds the core.

Windings are glued around each limb.

### Shell type transformer



text  
electrical  
machined  
by N.K. Metha.

Core surrounds  
windings

Both the windings are placed  
around the central limb

→ Core type - for H.V (voltage  $> 11 \text{ kV}$ )  
shell type suitable for L.V and upto  
small oil

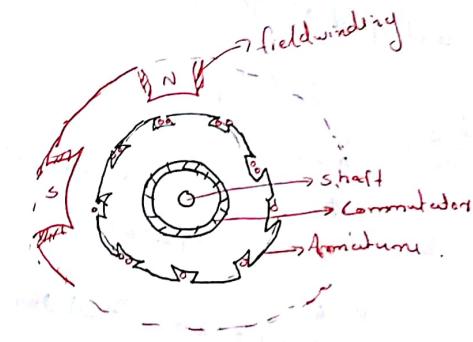
3d transformer 3d transformer  
In 3d sm is used to generate  
transmit electric power. 3 phase rotors  
are caused or lowered by means  
of 3d transformers.

### DC Machines

#### Construction

Stator: stationary part of a m/c.

Rotor: This part is free to rotate  
and normally inner part of  
machine.



Find the useful flux per pole on no load of a 250V, 6 pole shunt motor having wave connected arm winding with 110 turns. The armature resistance including brush is 0.2. The arm current is 13.3 A at no load speed of 908 rpm.

$$E_g = \frac{4ZNP}{60A}$$

$$A = 2 \text{ (wave)}$$

$$\therefore E_g = V - i_a R_a - 250 - (13.3 \times 0.2) \\ = 247.36 \text{ V}$$

$$P = 6$$

$$Z = 110 \times 2 = 220 \text{ conductors}$$

$$\therefore \phi = \frac{E_g + i_a R_a}{ZNP} \\ = \frac{247.36 \times 2}{220 \times 908 \times 6} = \underline{\underline{24.8 \text{ mW}}}$$

## Machines

### 3φ Induction motor

#### Universal motor

Can operate on DC supply or AC supply at approximately the same speed and output. It is smaller version of DC series motor, suitably modified for operation on AC supply.

$$P \rightarrow 5 \text{ W} - 200 \text{ W} \\ N \rightarrow 6000 - 8000 \text{ rpm}$$

Construction: is similar to that of a DC series motor.

### Necessity of earthing

By earthing we mean a metallic connection of the body to the earth (considered to be at zero potential) for the safety of human body from shocks.

All metallic covers of machines starters or sheathing of wiring, etc. are generally dead but can become alive due to failure of insulation or bad workmanship. When a person touches such part of machine or installation, gets a serious shock. To avoid from such severe shocks, all the metallic covers and frames of machines are earthed.

A good earthing should have very low resistance and easily allow the leakage current through.

### Systems of earthing

#### 1. Pipe earthing

In this method a galvanised iron (GI) pipe of  $(38\text{ mm dia}, 6\frac{1}{2}\text{'' length})$  having  $\frac{1}{2}\text{''}$  mm diameter holes on its surface and is buried vertically in the wet earth to work as an earth electrode. The lower end of pipe is made tapered to make driving easy.

If soil is dry then the length of pipe varies from ~~2 m~~ to  $2.75\text{ m}$ .

The depth to which GI pipe should be embedded depends upon the moisture in the soil. It should be normally be  $4.75\text{ m}$ , but this is not a hard and fast rule.

Another pipe of  $19\text{ mm diameter}$  and of sufficient length is connected to the earth electrode through a reducing socket. A funnel having a wire mesh is provided at the top of  $19\text{ mm diameter}$  pipe. During summer season water is poured in the funnel for maintaining earth connection in good condition. The funnel is enclosed in a concrete box with a cover fitted with a hinge.

The earth pipe is surrounded by  $15\text{ cm (6'')}$  thick alternate layers of salt and charcoal powder. The reason for adding the salt is that it attracts the moisture from the soil. Charcoal retains the moisture near the earth electrode which decreases the earth resistance (the GI earth wire (earth continuity conductor) of suitable size to carry the fault current safely is copper connected).

to 19mm dia pipe. Further, continuity conductor is brought to main switch, distribution box and individual machines for earthing.

### plate earthing

In this system earthing pit is dug in the ground until sufficient moisture is available in the soil. (3m approx.) Then Cr or Cu plate is connected to the earth continuity conductor.

Cr plate  $60\text{cm} \times 60\text{cm} \times 6.35\text{ mm}$

and for Cu plate  $60\text{cm} \times 60\text{cm} \times 3.16\text{ mm}$

This plate is plated at

the bottom of pit and is covered with 15 cm thick alternative layers of salt and charcoal.

→ MCB provides over load protection  
→ fuse offers short circuit protection only

\* For low protective fault current pipe earthing used.  
For large protective fault current plate electrodes are used.

### Conduit wiring

3 types of conduit wiring

a) Concealed Conduit

b) Surface Conduit

c) flexible conduit wiring

### Metal halide lamp

It is an electric lamp producing light by an electric arc through a gaseous mixture of mercury and metal halides (Compounds of metals with Bromine or Iodine).

or

Tariff It is the rate at which electrical energy is supplied to consumers. (Tariff includes cost of power generation, transmission and profit to the power company).

Consumers →  
 a) Domestic  
 b) Commercial (shops)  
 c) Industrial (manufact. unit).

Lowest ratings of Tariffs are given to residential and highest rate for industrial consumers.

Tariff divided into 2 parts  
 Fixed charge for supply of energy to your premises, the variable charge for the amount of energy used.

### Types of Tariff

1. Uniform Tariff / Simple Tariff  
 — Here there is fixed rate per unit of energy consumed.

In this tariff, the price charged per unit of energy consumption is constant and the price of tariff does not vary with no. of units of energy consumed.

### Differential Tariff

Price/unit of energy varies with consumption.

During peak time tariff high during low energy consumption " low. This helps to balance the rate at which power is used and created.

### Objectives of Tariff

1. It should compensate the cost of generation of power.

2. It is " " transmission & distribution operation cost and maintenance cost

3. To calculate revenue

Traffic	RS
0 - 50	2.8
51 - 100	3.2
101 - 150	4.2
151 - 200	5.8
201 - 250	7

(1) Differential and simple Tariff  
 returns for 1.7 VPP or (DPP)  
 maximum 61 min rate has  
 minimum 61 min rate has  
 higher rate against unit price  
 maximum also same term as  
 tariff (units, month etc.)

Power Consumers: Consumers of electricity are called power consumers.

HT Consumers: Large consumers, electric traction systems, large housing colony, airport etc., taking electric connection at 11 kV upto 33 kV and called high tension consumers.

eg: Domestic HT: Bulk supply for residential colonies.

Commercial HT for bigger offices, film studios, etc

Industrial HT: for heavy industries.

Agriculture HT: large farms or estate.

#### LT Consumers

In India Low Tension (LT) voltage (supply) is 400V for 3φ connection and 230V for 1φ connection. Consumers of electricity using low tension voltage are called LT consumers.

eg: Most small scale consumers like houses, shops, offices.

Domestic LT - most individual residential connection.

Commercial LT - Small shops and offices

Industrial LT - small manufacturing units.

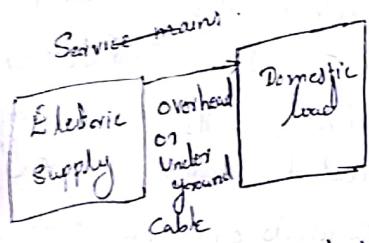
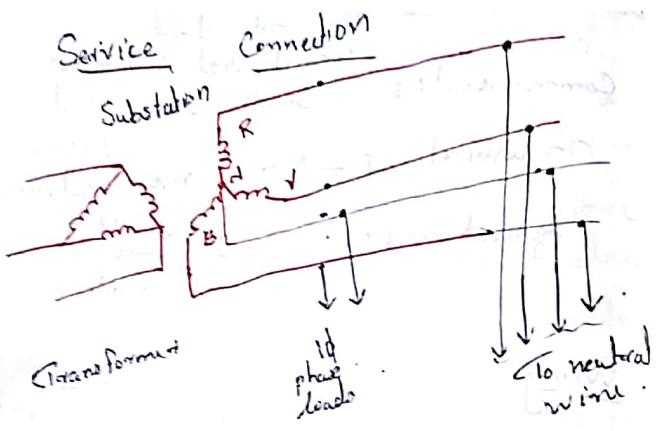
Agriculture LT: - Small farms and estates.

#### Wiring

Conduit wiring consists of PVC wires taken through either steel conduit pipes or through PVC conduit. Conduits can run over surface of walls and ceiling or are concealed under masonry work. When conduits are run over the surface of walls, the wiring is called surface conduit wiring.

When the conduits are run inside the walls → concealed conduit wiring

Surface Conduit wiring is used in factories for installation of heavy motors & other electrical equipment.



Typical effects of electric shock currents on human

50 Hz current	effect
0-1 mA	No sensation
0-3 mA	Mild sensation
3-5 mA	Pain or annoyance
5-10 mA	Painful shock.
10-15 mA	Local Muscle contraction
30-50 mA	Breathing difficult
50-100 mA	Ventricular fibrillation
100-200 mA	of heart certain

Fuse  
It is a small piece of wire connected in b/w 2 terminals mounted insulated base and connected in series with the circuit. It is the simplest form of protection and used for protecting low voltage equipment against over loads or short circuit.

Fuse carry the normal working current safely without overheating, and during fault (Overload / s.c) it gets heated up to melting point rapidly. Materials normally used: tin, lead, silver, Zn, Al, Cu etc. For small value of current ( $\leq 15\text{ A}$ ) alloy of tin and lead ( $63\%$ ,  $37\%$ ) is used.

for  $\phi > 15\text{ A}$  greater than  $15\text{ A}$ , silver is found to be quite satisfactory as a fuse material because it is not subjected to oxidation and its oxides are unstable. Draw back: Relatively high cost. So tin-lead used for low range current

Fuse: Used to protect electrical equipment from over loads & short circuit.

Fuse rating: It is the value of current which when flows through the element, does not melt it.

$$\text{Fusing factor} = \frac{\text{min. fusing current}}{\text{Fuse rating}}$$

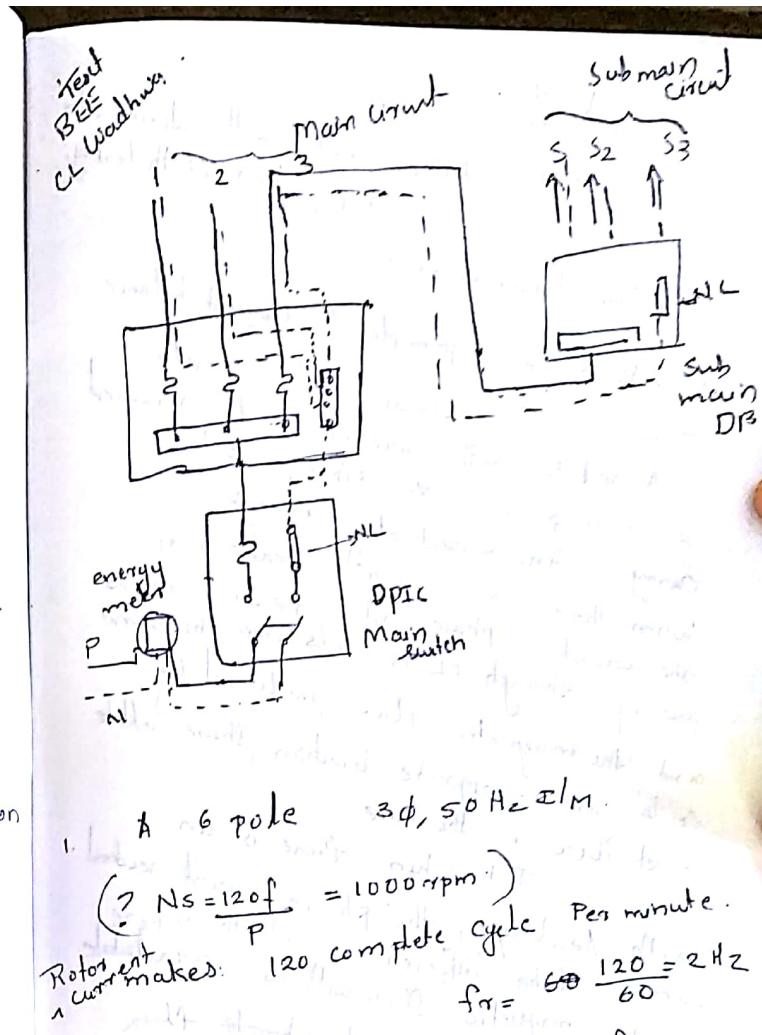
Fusing factor is greater than unity.

### Distribution board

The cable from electric utility is connected to energy meter first before taking any other connection. From the local terminals of energy meter it is taken to distribution board.

(Double pole main switch through DPIC)

Then it goes to consumer end through consumer fuse and consumer switch.



A 6 pole  $3\phi, 50\text{Hz I/M}$ .

$$(N_s = \frac{120f}{P} = 1000 \text{ rpm})$$

Rotor makes: 120 complete cycles per minute.  
 $f_r = \frac{60}{60} = 1 \text{ Hz}$

$$f_r = s f_n \Rightarrow s = \frac{f_r}{f_n} = \frac{2}{50} = 4\%$$

### EICB

protective device, which will automatically trip when earth leakage.

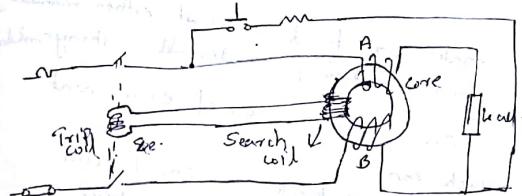
Also known as RCCB.

Works on principle of current balance.

Two coils with same no. of turns wound on a magnetic core. These coils

carry phase & neutral current. When there is no earth leakage in the circuit, phase and neutral currents passing through EICB coils are the same and the magnetic flux produced by coils are in opposite direction, there will be no net flux in the core.

But when there is an earth leakage, the phase and neutral currents will be different and therefore a net magnetic flux will be set up in the core. This coil activates flux in the core. This coil induces a voltage in search coil wound on the core and activates the trip circuit.



## Circuit breakers (CB)

A circuit breaker can make or break a circuit either manually or automatically under all working conditions. CB is a piece of equipment.

which can  
(i) make or break a circuit either manually or by remote control

under normal conditions.

Power system (ii) break a circuit automatically under fault conditions.

(iii) make a circuit either manually or by remote control under fault conditions.

operating principle

Consist of a fixed and

moving contacts.

Under normal working condition these contacts remain closed.

and will not open automatically until and unless it becomes faulty

When a fault occurs the trip coil of CB get energized and moving contacts are pulled apart by some mechanism.

### Circuit

Non-Linear Circuit : The circuit whose parameters change with application of voltage or current.

Linear : Parameters are constant

Bilateral circuit : It's properties are same in either direction.

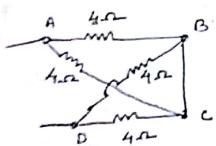
Eg: Transmission line

Unilateral circuit : properties changes with direction of its operation

Eg: Diode

Node : It is a junction in a circuit where 2 or more circuit elements are connected together.

Calculate R b/w P and Q.

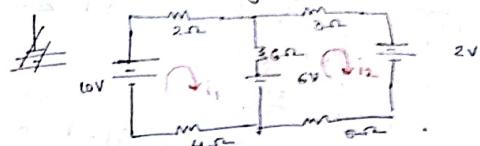


$$2+2 = 4\Omega$$

D. 9

### Mesh analysis

Find I<sub>1</sub> current through Gnd?



$$10 - 2I_1 - 6(I_1 - I_2) - 4 = 0$$

$$4 + 6I_2 - 12I_1 = 0$$

$$2 + 3I_2 - 6I_1 = 0$$

$$6I_1 - 3I_2 = 2 \quad \text{--- (1)}$$

$$6(I_1 - I_2) + 6 - 3I_2 - 2 - 5I_2 = 0$$

$$6I_1 - 14I_2 + 4 = 0$$

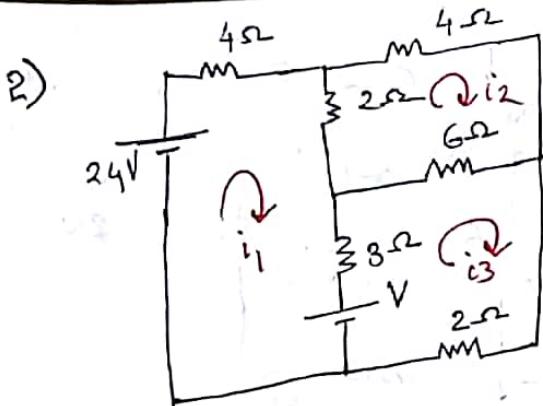
$$2I_1 - 7I_2 = -2 \quad \text{--- (2)}$$

$$\text{check } I_1 = \frac{20}{23} \text{ A}$$

$$I_1 - I_2 = \frac{2}{23}$$

$$0.87 - 0.17 = 0.70 \text{ A}$$

$$0.87 - 0.17 = 0.70 \$$



Determine  
such that  $I_1$   
will be zero

$$I_1 = 0$$

$$24 - 4I_1 - 2(I_1 - I_2) - 3(I_1 - I_3) - V = 0 \quad \text{--- (1)}$$

$$24 - V + 2I_2 + 3I_3 = 0$$

In mesh ②

$$-4I_2 - 6(I_2 - I_3) - 2(I_2 - I_1) = 0$$

$$-4I_2 - 12I_2 + 6I_3 = 0$$

$$2I_2 = I_3 \quad \text{--- (2)}$$

In mesh ③

$$-3(I_3 - I_1) - 2I_3 - 6(I_3 - I_2) + V = 0$$

$$-11I_3 + 6I_2 + V = 0$$

$$V = 11I_3 - 6I_2 \quad \text{--- (3)}$$

from eqn ②

sub in eqn ③

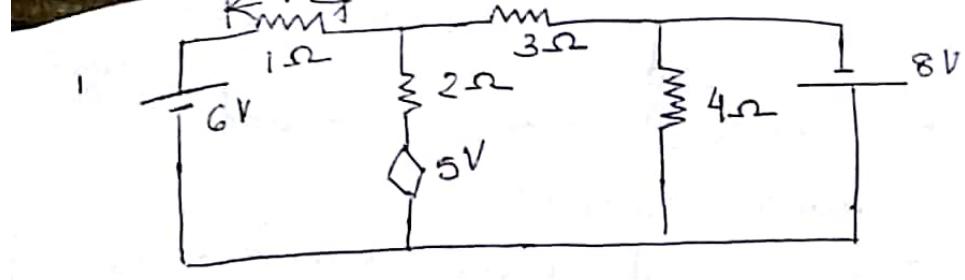
$$V = 11I_3 - 3I_3 = 8I_3 \quad \text{--- (a)}$$

put eqn ② in eqn ①

$$24 - V + 2I_3 + 3I_3 = 0$$

$$24 - V + 4I_3 = 0$$

$$V = 24 + 4I_3 \quad \text{--- (b)}$$



Find the total current in the circuit.

Given:  $I_1 = 6V / 1\Omega = 6A$

$I_2 = 5V / 2\Omega = 2.5A$

$I_3 = 8V / 4\Omega = 2A$

Total current =  $I_1 + I_2 + I_3 = 6A + 2.5A + 2A = 10.5A$

Ans: Total current =  $10.5A$

Ques: Find the total current in the circuit.

Given:  $I_1 = 6V / 1\Omega = 6A$

$I_2 = 5V / 2\Omega = 2.5A$

$I_3 = 8V / 4\Omega = 2A$

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Total current =  $I_1 + I_2 + I_3 = 6A + 2.5A + 2A = 10.5A$

Ans: Total current =  $10.5A$

Ques: Find the total current in the circuit.

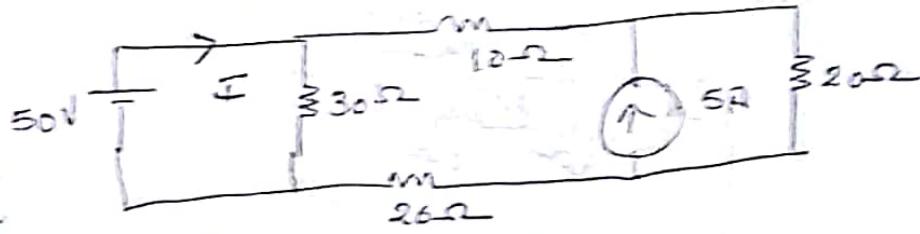
Given:  $I_1 = 6V / 1\Omega = 6A$

$I_2 = 5V / 2\Omega = 2.5A$

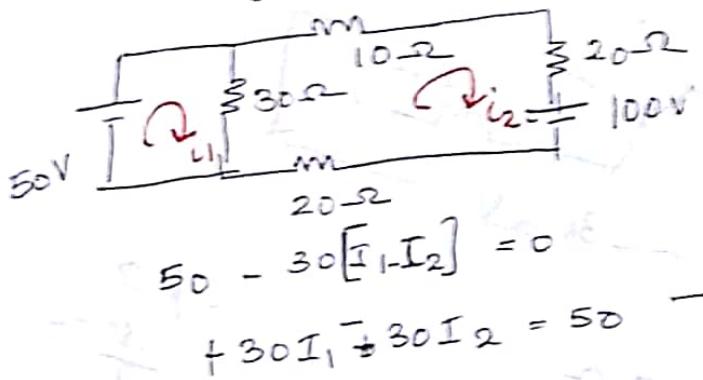
$I_3 = 8V / 4\Omega = 2A$

Total current =  $I_1 + I_2 + I_3 = 6A + 2.5A + 2A = 10.5A$

Find current in Resistor



; using source transformation;

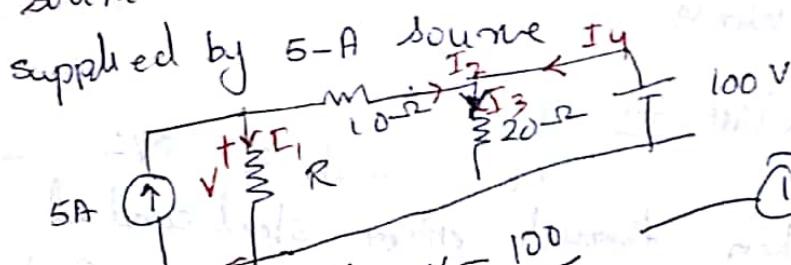


$$\text{In mesh 2: } -50I_2 + 30(I_2 - I_1) - 100 = 0$$

$$-50I_2 + 30(I_2 - I_1) - 100 = 0 \quad (1)$$
$$30I_1 - 80I_2 = 100 \quad (2)$$

$$(2) - (1)$$
$$100 - 50I_2 = 50$$
$$I_2 = \frac{50}{50} = 1 \text{ A}$$
$$I_2 = 1 \text{ A}$$

2. Find the value of  $R$  in the circuit such that power supplied by 100V source to network is same as power supplied by 5-A source



$$R = 20\Omega$$

$$\text{At node 1: } 5 = \frac{V}{R} + V = \frac{100}{10} + V \quad (1)$$

$$\text{At node 2: } \frac{100 - V}{10} + I_4 = \frac{100}{20} = 5 \quad (2)$$

$$100I_4 = 5V$$
$$R = 20\Omega$$

Find the current in 5Ω resistor

$$R_{eq} = 3\Omega$$

$$I_{total} = \frac{36}{3} = 12A$$

$$I_{5\Omega} = 12 \times \frac{3}{9} = 4A$$

### System of Wiring Various types

1. cleat wiring
2. Wood casing wiring
3. batten wiring
4. Conduit wiring

### Conduit wiring

Conduit wiring consists of PVC wires taken through either steel conduit pipes or through PVC conduit pipes. Conductors are run over the surface of walls, the wiring is called surface conduit.

wiring : When the conductors are run inside the walls, the wiring is called surface concealed wiring.

Surface conduit wiring is used in factories for installation of heavy motors and other electrical equipment. The system is water proof and replacement of defective wires is easy.

In concealed conduit wiring, a chase or groove is cut on the wall to place conduit pipes. In case of building under construction the chase should be provided on the wall and ceilings for laying the conduit pipes before plastering of walls and ceiling done. Suitable inspection boxes are provided to permit the inspection and replacement of wires, if necessary.

Concealed conduit wiring almost all modern residential, commercial and public buildings from the appearance of buildings from

KVL

$$-10I_1 - 25(I_1 + I_2)$$

$$+90 = 0$$

$$-35I_1 - 25I_2 = -90 \quad \text{--- (1)}$$

$$35I_1 + 25I_2 = 90$$

$$5I_2 - 125 + 25(I_1 + I_2) = 0 \quad \text{--- (2)}$$

$$+25I_1 + 30I_2 = 125$$

$$\begin{bmatrix} 35 & 25 \\ 25 & 30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 90 \\ 125 \end{bmatrix}$$

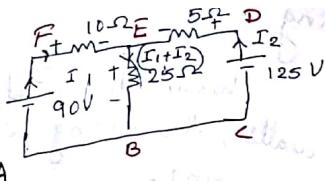
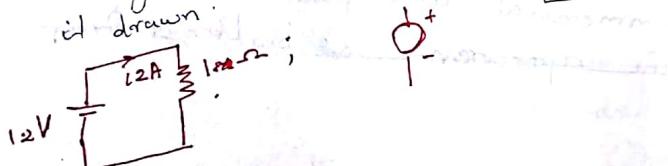
$$I_1 = -1A$$

$$I_2 = 5A$$

Voltage source

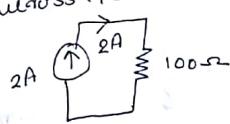
The term voltage source is used to describe a source of energy which establishes a potential difference across its terminals.  
eg: batteries, dc generators, alternators.

Ideal voltage source (const. Volt. Source) is one that maintains a constant terminal voltage, no matter how much current is drawn.

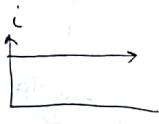


ideal current source

an I<sub>H</sub> will supply the same current to any resistance (load) connected across its terminals.



eg: solar cells

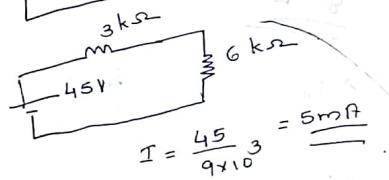
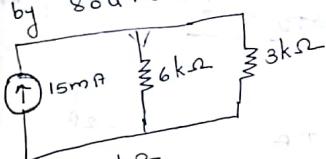


Real Current source

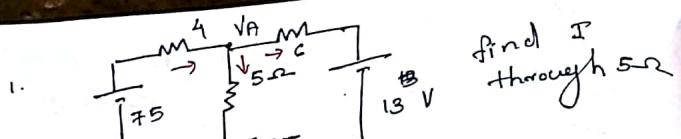


$$I_L = I - \frac{V}{R_{int}}$$

- Find the current in  $6\text{k}\Omega$  resistor by source conversion.



$$I = \frac{45}{9 \times 10^3} = 5\text{mA}$$

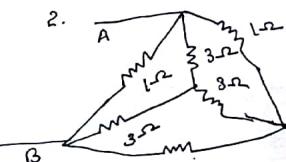


$$\begin{aligned} \frac{75 - V_A}{4} &= \frac{V_A - 65}{6} + \frac{V_A - 13}{6} \\ &= \frac{6V_A - 390}{30} + \frac{5V_A - 65}{30} \quad \text{or} \quad \frac{11V_A - 455}{30} \\ -30V_A + 2250 &= 4(11V_A - 455) \\ +2250 + 1820 &= 44V_A \quad \text{or} \quad 44V_A = 4070 \\ \therefore V_A &= \frac{4070}{44} = 92.05V \\ i_5 &= \frac{+55}{5} = \frac{-10}{5} = -2A \end{aligned}$$

$$i_5 = -2A$$

so  $\uparrow i_5$

$$\frac{75 - V_A}{4} = V_A$$

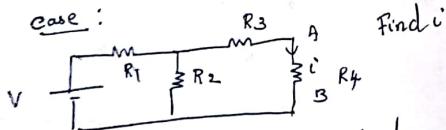


### Thevenin's Theorem

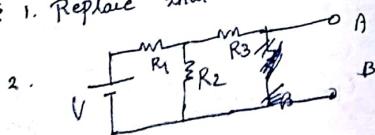
"Any 2 terminal linear bilateral network having a number of voltage, current sources and resistance can be replaced by a simple equivalent circuit having a single voltage source consisting of a single voltage source in series with a resistance."

- The value of voltage source = open circuit voltage across two terminals of network
- Resistance = equivalent resistance measured b/w terminals with all the energy sources are replaced with their internal resistance.

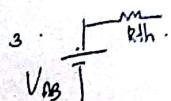
case:



steps 1. Replace that branch element



Find  $V_{AB}$ ,  $R_{Th}$ .



3.

4. Again join removed branch & find  $i$

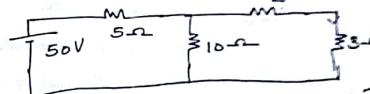
$$\text{Circuit diagram: } V_{Th} \text{ is in series with } R_{Th} \text{ and } R_4. \text{ Current } i = \frac{V_{th}}{R_{th} + R_4}.$$

To find  $R_{th}$

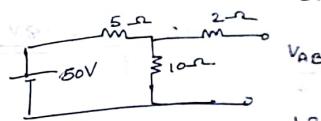
→ short circuit, voltage source.

→ open circuit, current source.

- Find current in  $3\Omega$ ? Use Thevenin's theorem.



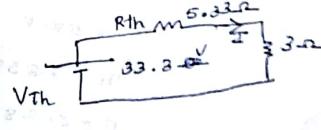
step 1: Remove  $3\Omega$ .



$$V_{th} = V_{AB} = \frac{50 \times 10}{15} = 33.3V$$

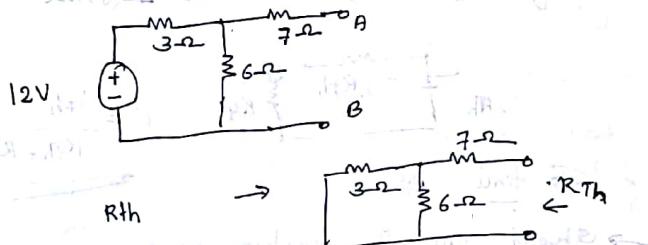
$$R_{AB} = 2 + \frac{5 \times 10}{15} = \underline{\underline{5.33\Omega}}$$

Redo Again connect  $3\Omega$  to Thevenin circuit & find  $I$ .



$$I = \frac{33.3}{8.33} = \underline{\underline{4A}}$$

2i. Find Thevenin equivalent



(3//6) reduces to  $7\Omega$

$$R_{Th} = \frac{9\Omega}{2}$$

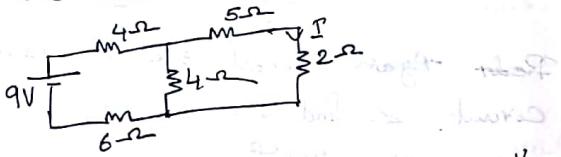
$$V_{AB} = \text{Voltage across } 6\Omega$$

(Since no current through  $7\Omega$ )

$$= 12 \times \frac{6}{9} = \frac{8V}{1}$$



3. Use thevenin theorem find I in  $2\Omega$ .



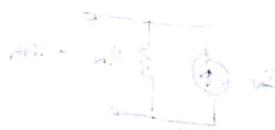
$$V_{Th} = 2.57V$$

$$R_{Th} = 7.85\Omega$$

$$I = 260.8mA$$

3. A 12V battery has an internal resistance of  $2\Omega$ . It is connected to a load of  $7\Omega$ . Calculate the power delivered by the battery.

Answer: 10.2W



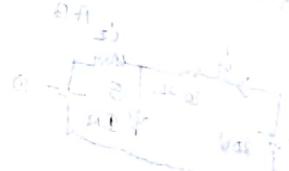
total power delivered =  $\frac{V^2}{R_{Total}}$



current through load



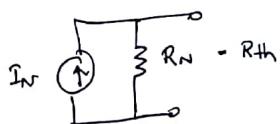
power delivered =  $\frac{V^2}{R_{Total}}$



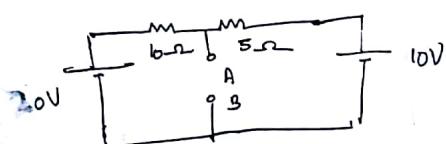
$$\frac{12^2}{2+7} = 10.2W$$

### Norton's Theorem

Any 2 terminal linear network with current source, voltage & resistance can be replaced by an equivalent circuit consisting of a current source in parallel with resistance.



1. Determine Norton's equivalent



$$R_N = R_{Th}$$

short voltage source

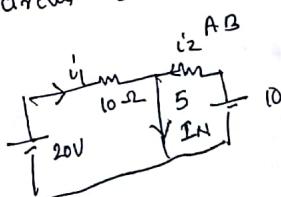


$$R_N = \frac{50}{15} = \frac{10}{3} = 3.33\Omega$$

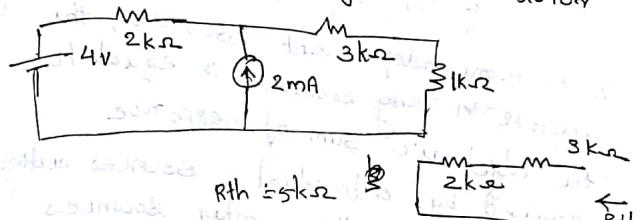
$i_N$  = short circuit current through

$$i_N = i_1 + i_2$$

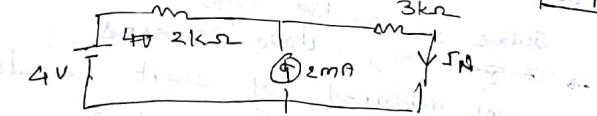
$$= \frac{20}{210} + \frac{10}{5} = 4A$$



1. Find Norton equivalent circuit for the network faced by  $1\text{k}\Omega$  resistor



To find  $i_N$ , use superposition



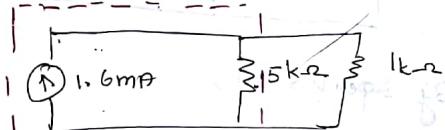
when 4V alone acting;

$$i_1 = \frac{4}{5 \times 10} = \frac{4}{50} = 0.08A$$

when 2mA alone acting;

$$i_2 = \frac{2 \times 2}{5} = \frac{4}{5} = 0.8A$$

$$\therefore i_N = i_1 + i_2 = \frac{8}{5} = 1.6mA$$



Norton equivalent

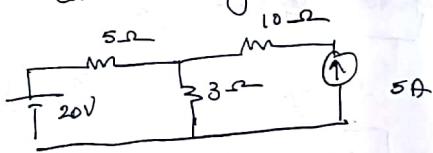
$$i_N = \frac{20}{210} + \frac{10}{5} = 4A$$

### Superposition theorem

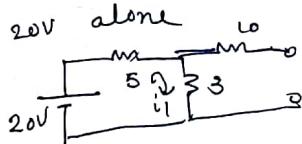
"In any linear nw containing 2 or more independent sources, the response in any element is equal to the algebraic sum of response caused by individual sources acting alone, while the other sources are non operative."

Source non operative means  
 $\rightarrow$  problem voltage sources are replaced by short circuits, and independent current sources replaced by open circuit.

1. Find current through  $3\Omega$

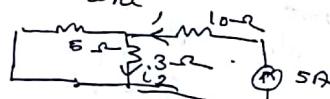


By superposition,



$$i_1 = \frac{20}{8} = 2.5A$$

when 5A alone:



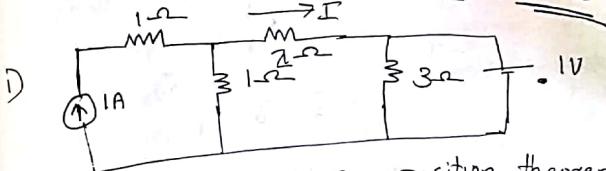
$$i_2 = 5 \times \frac{5}{8}$$

$$= 3.125A$$

By superposition:

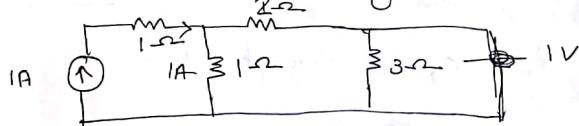
$$\text{Total current } i = i_1 + i_2$$

$$= 5.625$$

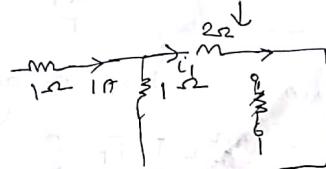


find I using superposition theorem?

When 1A, source acting:

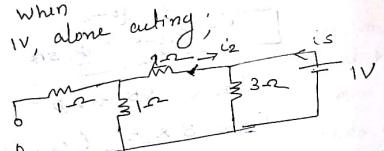


so ineffective  
 $3\Omega$  will cut as  
short circuit



current through  $2\Omega$

$$i_1 = 1 \cdot \frac{1}{3} = \frac{1}{3}A$$



When 1V alone acting;  
Total resistance seen from 1V,

$$3\Omega \parallel 3\Omega = 1.5\Omega$$

$$I_s = \frac{1}{1.5} = 0.66A$$

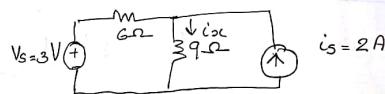
$$-i_2 = I_s \times \frac{3}{1.5} = \frac{3}{2}$$

$$= \frac{2}{3} \times \frac{3}{6} = \frac{1}{3}$$

$$i_2 = -\frac{1}{3}$$

$$\text{So total current} = \frac{1}{3} - \frac{1}{3} = 0$$

3. Find Current through 9Ω.

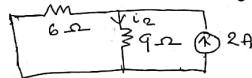


When 3V alone acting;



$$i_1 = \frac{3}{15} = \frac{1}{5}$$

When 2A alone acting;



$$i_2 = \frac{2 \times 6}{15} = \frac{4}{5}$$

$$\therefore i_s = i_1 + i_2 = \underline{\underline{1A}}$$

Coulomb's law

This law states that like charged repel and opposite charges attract, with a force proportional to product of charges and inversely proportional to square of distance b/w

$$F = k_e \frac{q_1 q_2}{r^2} = \frac{q_1 q_2}{4\pi \epsilon_0 r^2}$$

$$k_e = 8.99 \times 10^9 \text{ N m}^2/\text{C}^{-2}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}}{\text{N} \cdot \text{m}^2}$$

Electric field strength  $\Rightarrow E = F/q$   
 $\Rightarrow V/m$

Electric flux density  $D = \epsilon E$   
charge density built up on a test capacitor and given in units of Coulombs/ square meter

$$E = \frac{q_1}{r^2} = \epsilon E \quad D = \frac{q_1 q_2}{4\pi \epsilon_0 r^2}$$

$$E = \frac{q_1}{4\pi \epsilon_0 r^2} r^2$$

$$D = \epsilon E = \frac{q_1}{4\pi \epsilon_0 r^2}$$

$$\text{Capacitance} = \frac{Q}{V}$$

$$E = \frac{1}{2} CV^2 \quad Q = C V$$

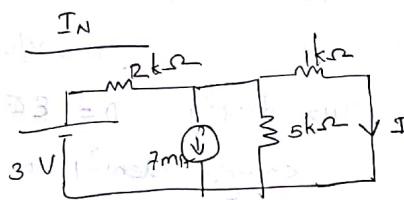
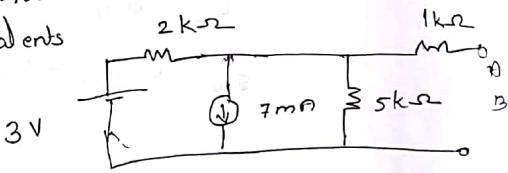


$$Q = C V$$

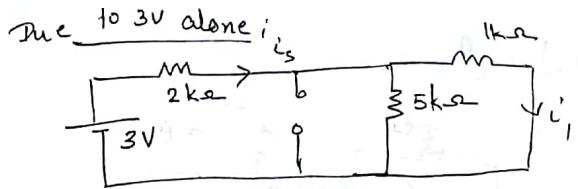
$$I = C \frac{dV}{dt}$$

$$C = \frac{A \epsilon_0}{d}$$

Determine Thvenin & Norton,  
equivalents  $2 \text{ k}\Omega$

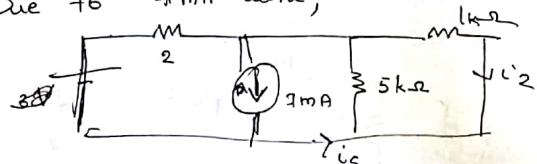
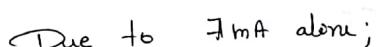


To find  $\Gamma_N$  apply Superposition theorem.



$$i_s = \frac{s}{2 + (1/15)} = \frac{15}{17} A$$

$$i_1 = i_s \times \frac{5}{6} = \frac{15}{17} \times \frac{5}{6} = \frac{25}{17} A$$



$$Is = \frac{7 \times 2}{1 + 1} \times 6 \times 10^3 \quad (\text{Current division rule})$$

$$= \frac{14 \times 6}{17} \times 10^{-3}$$

$$i_2 = - \left( \frac{14 \times 6}{17} \right) \cdot 50 \times 10^{-3}$$

Current division rule.

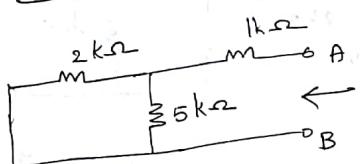
$$= \frac{14 \times 5}{17} = \frac{-70}{17} \times 10^{-3}$$

$$i_1 + i_2 = \frac{15}{17} - \frac{70}{17}$$

$$= \frac{-55}{17} \times 10^{-3} = -3.285 \text{ mA}$$

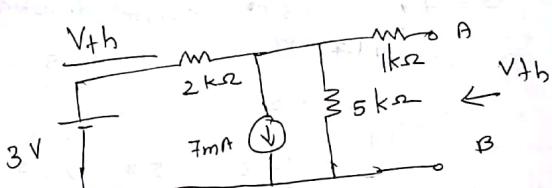
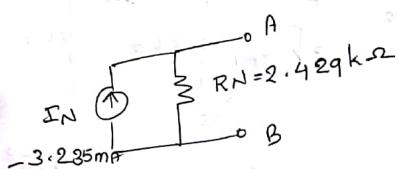
To find

$R_N$



$$(2 \parallel 5) + 1$$

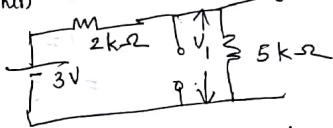
$$\frac{10}{7} + 1 = \frac{17}{7} \text{ k}\Omega$$
$$R_N = \underline{\underline{2.429 \text{ k}\Omega}}$$



$$V_{th} = V \text{ across } 5 \text{ k}\Omega$$

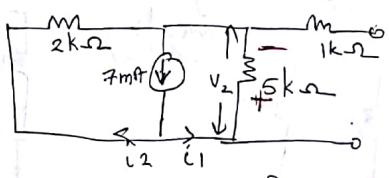
Apply superposition;

When 3V alone acting



$$V_1 = \frac{3 \times 5}{7} = \frac{15}{7} \text{ V}$$

When 9mA alone acting,



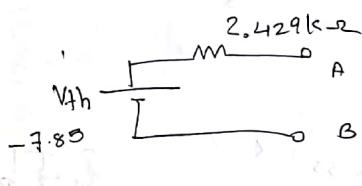
$$i_1 = \frac{(9 \times 10)^3 \cdot 2}{7} \rightarrow \text{Current division rule}$$
$$= 2 \text{ mA}$$

$$\therefore V_2 = i_1 \times 5 \text{ k}\Omega$$
$$= \underline{\underline{-10 \text{ V}}}$$

$V_{th}$

$$= V_1 + V_2$$

$$= \frac{15}{7} - 10 = \underline{\underline{-7.85 \text{ V}}}$$

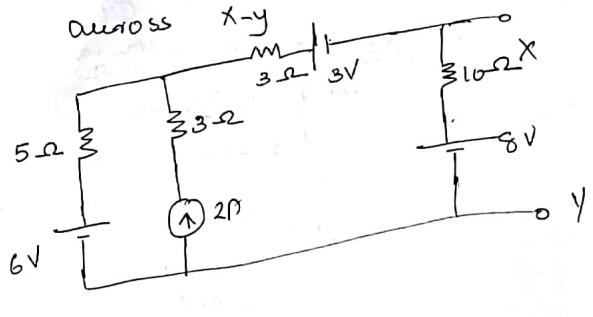


∴  $V_{th} = 2.429 \text{ k}\Omega \cdot 7.85 \text{ V}$

$= 18.85 \text{ V}$

$\therefore V_{th} = 18.85 \text{ V}$

Q9 Obtain Thvenin's equivalent circuit across X-Y



R<sub>th</sub>

The diagram shows the circuit with the 6V source removed. The 5Ω and 3Ω resistors are in series. The 2Ω resistor is in parallel with the 10Ω resistor. The 8V source is in series with the 10Ω resistor. The calculation for R<sub>th</sub> is:

$$\frac{8 \times 10}{18} = \frac{80}{18} = \frac{40}{9} = 4.44\Omega$$

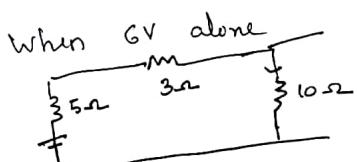
V<sub>th</sub>

Voltage across X-Y

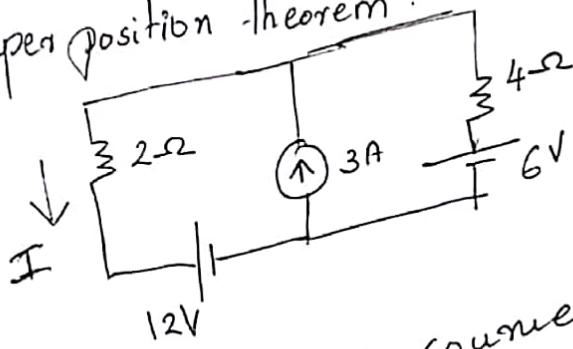
The diagram shows the circuit with the 6V source removed. The 5Ω and 3Ω resistors are in series. The 2Ω resistor is in parallel with the 10Ω resistor. The 8V source is in series with the 10Ω resistor. The calculation for V<sub>th</sub> is:

$$= 8V + V_{\text{across } 10\Omega}$$

Current through 10Ω theorem. Use superposition

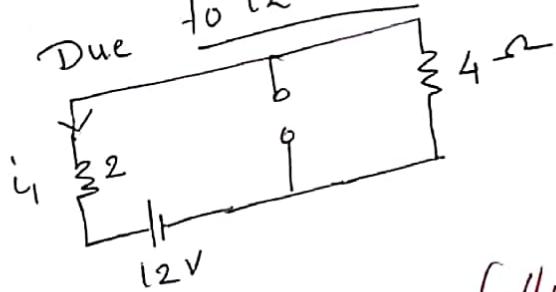


2. Find current through  $2\Omega$ , using superposition theorem.



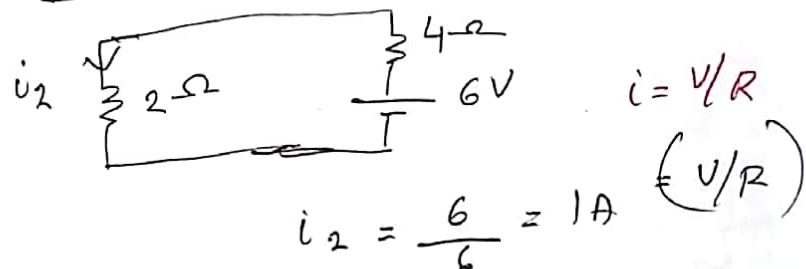
$I$

source,



$$i_1 = \frac{12}{6} = \underline{\underline{2A}} \quad (\text{V}/R)$$

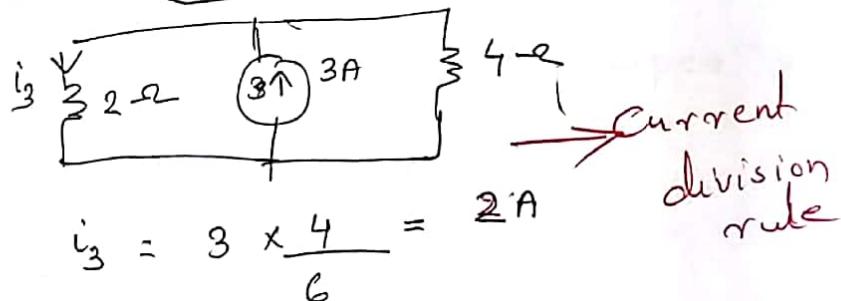
Due to  $-6V$  source,



$$i = V/R$$

$$i_2 = \frac{6}{6} = \underline{\underline{1A}} \quad (\text{V}/R)$$

Due to  $3A$  source;



$$i_3 = 3 \times \frac{4}{6} = \underline{\underline{2A}}$$

current  
division  
rule

$$I = i_1 + i_2 + i_3$$

$$= \underline{\underline{5A}}$$

$$V_R = \frac{(9.44 \times 80.5)}{19.44 \times 80.5}^{\circ} \quad (1<0)$$

$$V_C = 15.72 \\ = (9.44 \angle -80^\circ) C_{6.2 \angle -90^\circ} \\ = 11.8.44 \angle -9.46^\circ$$

$$\therefore I_c = \frac{V_c}{Z_c} = \frac{V_c}{2L - 90} = 59.2 A$$

Complex no. Conversion blw  
forms.

Rectangular to polar

$$r = \sqrt{x^2 + y^2}$$

$$\Theta = \tan^{-1} y / x$$

Rectangular

Oct 14

polar form

polar to rectangular

$$x = a \cos \vartheta$$

$$y = r \sin \theta$$

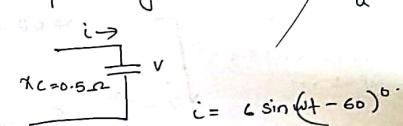
1. convert 10430

$$x = 10 \cos 30^\circ, y = 10 \sin 30^\circ$$

$$x+iy = -6.42 - j 7.6$$

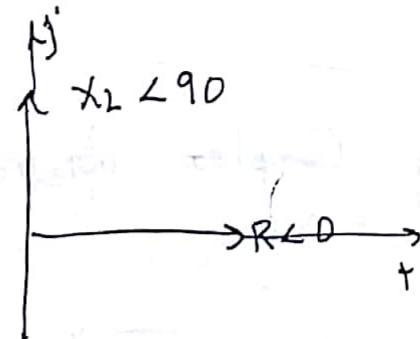
## Series & Net in Circuits

1. Using Complex algebra, find the voltage  $u$



$$V = I \cdot Z_C = I \cdot L_Q \quad (x < L_Q).$$

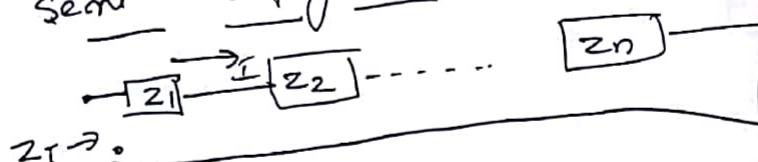
## Impedance diagram.



For any phasor

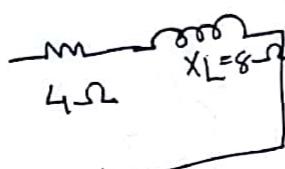
the resistance will always appear on the +ve real axis,  $XL$  on +ve imaginary axis, &  $XC$  on -ve imaginary axis. The result is an "impedance diagram" that can reflect the individual & total impedance levels of an ac phasor.

### series configuration

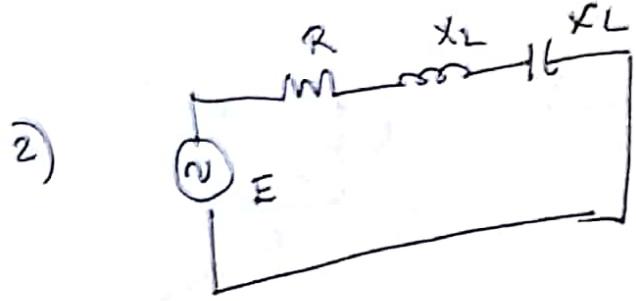


$$Z_T = z_1 + z_2 + \dots + z_n$$

Over all properties of series ac circuits are the same as those for dc circuits.

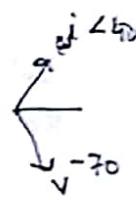


$$\begin{aligned} Z_T &= R < 0 + XL < 90 \\ &= R + jXL \\ &= 4 + j8 \Omega \\ &= 8.9 L 63.43 \end{aligned}$$

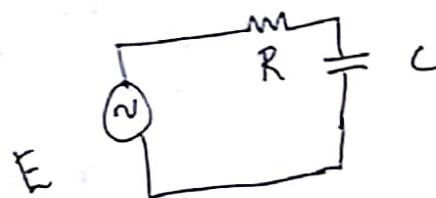


$$V_R = \frac{E \cdot Z_R}{Z_R + Z_L + Z_C}$$

$$V_L = \frac{Z_L \cdot E}{Z_R + Z_L + Z_C}$$



Freq. response of  $RC$  n/w



$f = 0 \text{ to } 20 \text{ kHz}$

At low f  $X_C$  will be high  
so impedance will be capacitive

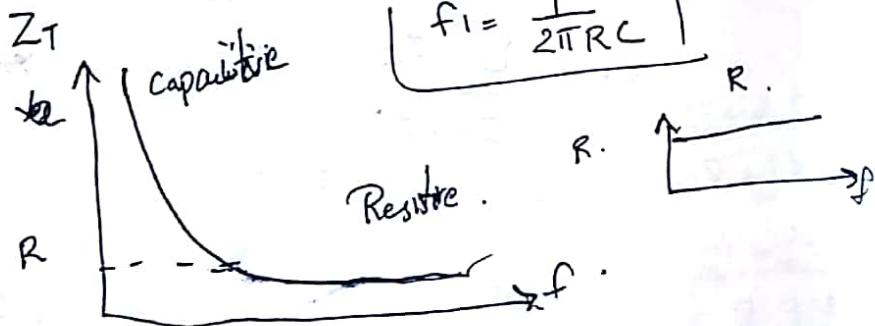
in nature. At high freq.  $X_C$  will drop  
below  $R$  and the n/w will shift towards  
one of a purely resistive nature.

The freq. at which  $X_C = R$  can be determined

by

$$X_C = \frac{1}{2\pi f_1 C} = R$$

$$f_1 = \frac{1}{2\pi R C}$$

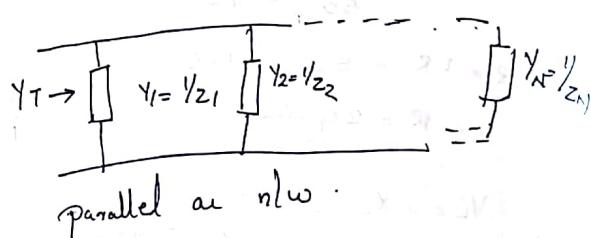


### Admittance

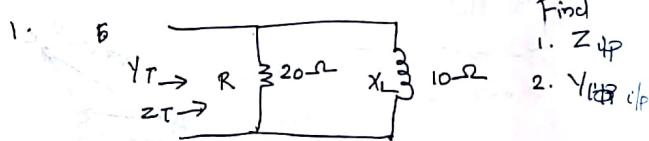
$$Y = \frac{1}{Z}$$

unit Siemens

Admittance is a measure of how well an ac circuit will admit or allow current to flow in the circuit.



→ Reciprocal of reactance ( $1/X$ ) is called Susceptance.



$$Y_R = \frac{1}{R} = \frac{1}{20\Omega} = 0.05$$

$$Y_L = \frac{1}{jX_L} = \frac{1}{j10} \angle -90^\circ$$

$$= \frac{1}{10} \angle -90^\circ$$

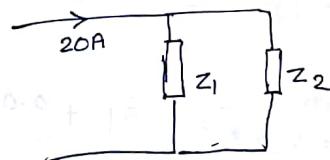
$$= 0.1 \angle -90^\circ = -j0.1$$

$$Y_{IP} = Y_R + Y_L$$

$$= 0.05 - j0.1$$

$$Z_{IP} = \frac{1}{Y_{IP}} = 8.93\Omega \angle 63.43^\circ$$

- u.a  
1. Two circuits, the impedances of which are given by  $Z_1 = (5+j10)\Omega$  and  $Z_2 = (8-j6)\Omega$  connected in parallel. If the total current supplied is 20A, what is the power taken by each branch.



$$P_1 = I_1^2 R_1$$

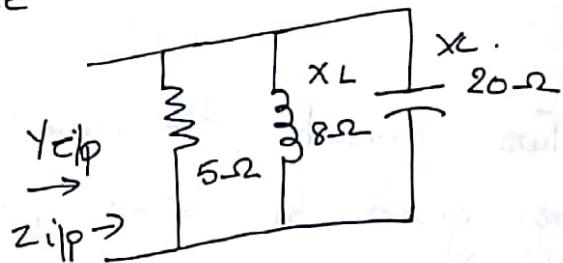
$$P_2 = I_2^2 R_2$$

$$I_1 = \frac{I}{Z_1 + Z_2} = \frac{20}{23 + 4j}$$

$$I_2 = \frac{I}{Z_1 + Z_2} = \frac{20(95 + j10)}{23 + 4j}$$

20.

2. Find i/p impedance and admittance



$$\begin{aligned}
 Y_{ip} &= \frac{1}{5} + \frac{1}{8j} - \frac{1}{20j} \\
 &= \frac{1}{5} + 0.2 + 0.125 \angle 90^\circ \\
 &\quad + 0.05 \angle 90^\circ \\
 &= 0.2 - 0.125j + 0.05j \\
 &= 0.2 - 0.075j
 \end{aligned}$$

$$\begin{aligned}
 Z_{ip} &= \frac{1}{Y_{ip}} = \frac{1}{0.2 - 0.075j} = 4.383 + 1.64j \\
 &= \underline{\underline{4.68 \angle 20^\circ}}
 \end{aligned}$$

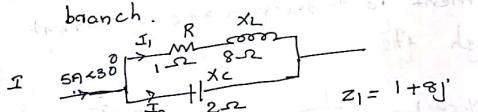
Find admittance

$$Y = \frac{1}{Z} = \frac{1}{R + jX_L} = \frac{1}{6 + j8} = 0.06 - j0.08$$

$$Z = R - jX_L = 6 - j8$$

$$Y = \frac{1}{Z} = 0.06 + j0.08$$

2. Find current through each parallel branch.



$$Z_1 = 1 + j8$$

$$Z_2 = -j2$$

$$I_1 = \frac{I \cdot Z_{02}}{Z_1 + Z_2} = \frac{(5 \angle 30^\circ)(-j2)}{1 + j8} = 1.64 A \angle -140^\circ$$

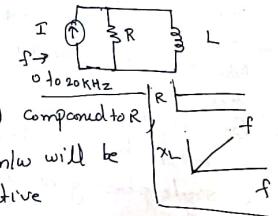
$$I_2 = \frac{I \cdot Z_1}{Z_1 + Z_2} = \frac{(5 \angle 30^\circ)(1 + j8)}{1 + j8}$$

$$= 6.62 A \angle 32^\circ$$

$$\text{Total current } I = I_1 + I_2 = 8.26 A \angle -140^\circ$$

Freq. response of parallel elements

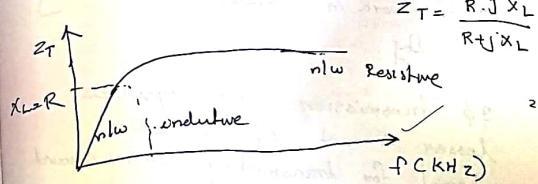
parallel RL nlw



At low freq.

$X_L$  is very small compared to R

so low freq. nlw will be primarily inductive



$$Z_T = \frac{R \cdot jX_L}{R + jX_L}$$

$\frac{\phi_{ZNP}}{GOA}$   
dp.

R.F.  
T.D.

$3\phi$  power is common form of electrical power & popular method of electric power transmission and distribution. This is due to Adv. of  $3\phi$  power over  $1\phi$  power.

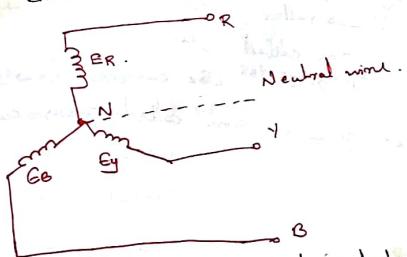
- For a given size of frame,  $3\phi$  generator or motor has greater eff than that of a single phase generator.
- $3\phi$  gen work in Parallel without any difficulty.
- $3\phi$  transmission line requires lesser amount of conductor material for transmitting same amount of Power over a  $1\phi$  line.
- $3\phi$  m/s possess uniform torque; whereas  $1\phi$  m/s possess a pulsating torque.
- Polyphase induction m/s are self starting whereas  $1\phi$  ac m/s are not self starting.

### Methods of connection of $3\phi$ s.m

2 methods of interconnecting the 3 phases.

#### 1. Star Connection

Similar ends of  $3\phi$  phases are joined together with in the alternator and 3 lines are run from free ends as shown.

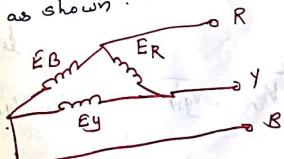


$$\frac{1}{dt} \frac{\phi_{ZNP}}{\phi_{GA}} = \frac{dp}{dp}$$

If a neutral conductor is present, it is called  $3\phi$ , 4 wire system.  
→  $3\phi$ , 4 wire system used in industrial & large consumers.

#### 2. Delta Connection

Dissimilar ends of phases are joined to form a closed mesh and 3 lines are run from junction points as shown.



Analysis - star connection

Similar ends of

3 phases are joined together to form  
a common junction called star

or neutral point

$V_{ph}$   $\rightarrow$  voltage b/w any line & neutral

point ie voltage across each winding

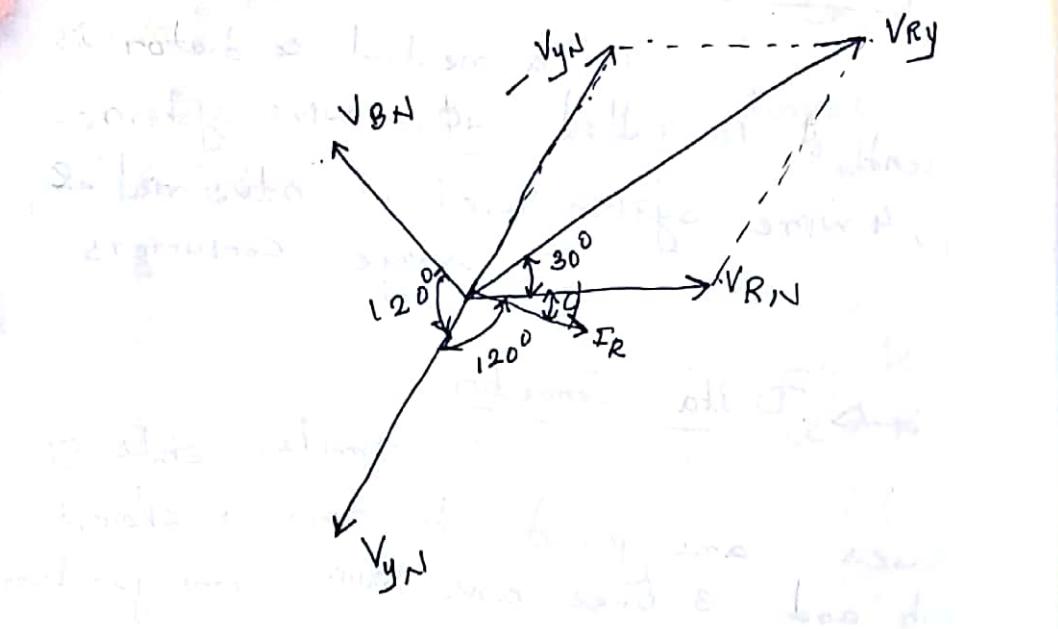
e.g.:  $V_{RN}$ ,  $V_{YN}$  b/w any 2 lines.

$V_L \rightarrow$  voltage line voltage.

are called

e.g.:  $V_{RY}$ ,  $V_{YB}$ ,  $V_{BR}$  The currents flowing

in phases are called phase current



$V_{RY}$  line voltage

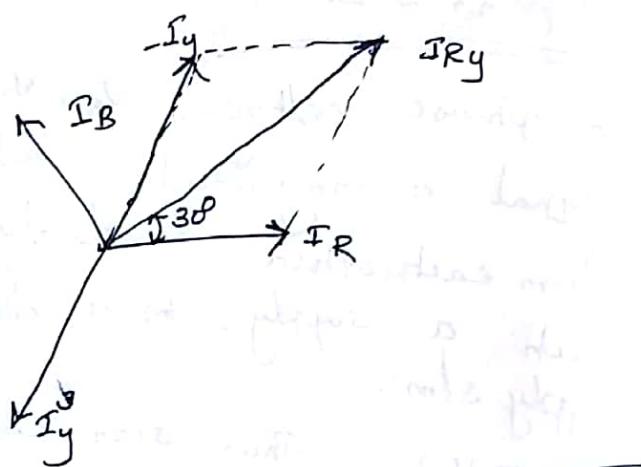
$$\overline{V_{RY}} = \overline{V_{RN}} - \overline{V_{YN}}$$

$V_{RN}$ ,  $V_{YN}$  phase voltage.

## Adv. of star

1.  $V_{ph} = V_L / \sqrt{3}$   
In an alternator induced emf  $\propto$  no. of turns.  
So star connected alternator has require lesser no. of turns.
2. For the same line voltage. alternator will require lesser amount of insulation.
3. Two levels of voltage
4. In star earthing of neutral permits use of relay.

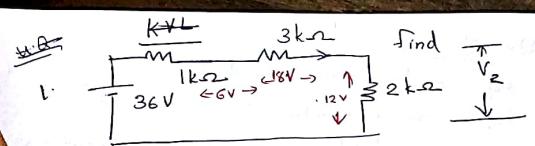
## Delta or Mesh



$$I_{RY} = \sqrt{I_R^2 + I_Y^2 + 2I_R I_Y \cos 60^\circ}$$

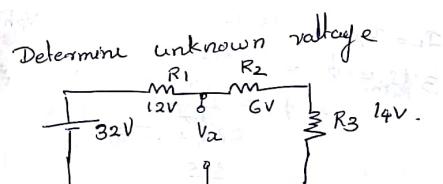
$$\boxed{\vec{I}_L = \sqrt{3} I_{ph}}$$

$$V_L = V_{ph}$$

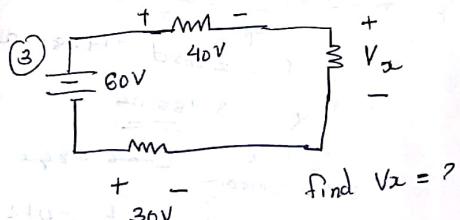


$$I = \frac{V}{R_{\text{Req}}} = \frac{36}{6k\Omega} = 6\text{mA}$$

$$V_2 = \underline{\underline{12V}}$$

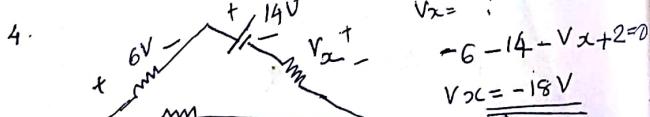


$$V_x = 6V + 14V = 20V$$



$$60 - 40 - V_x + 30 = 0$$

$$V_x = \underline{\underline{50V}}$$

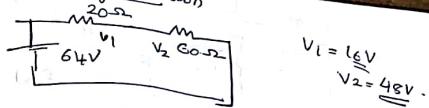


$$V_x = ?$$

$$-6 - 14 - V_x + 2 = 0$$

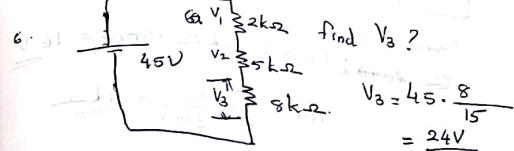
$$V_x = \underline{\underline{-18V}}$$

5. Voltage division



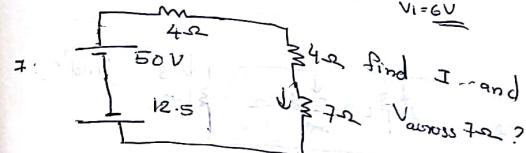
$$V_1 = \underline{\underline{16V}}$$

$$V_2 = \underline{\underline{48V}}$$



$$V_3 = 45 \cdot \frac{8}{15} = \underline{\underline{24V}}$$

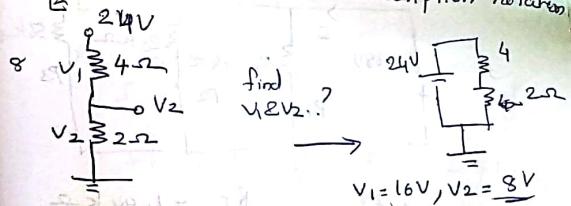
$$V_1 = \underline{\underline{6V}}$$



$$I = \frac{87.5}{15} = \underline{\underline{2.5A}}$$

$$V_{7\Omega} = (2.5 \times 7) = 1R = \underline{\underline{17.5V}}$$

Voltage  $V_{ab} = V_a - V_b$  In general double subscript notation means



$$V_1 = \underline{\underline{16V}}, V_2 = \underline{\underline{8V}}$$

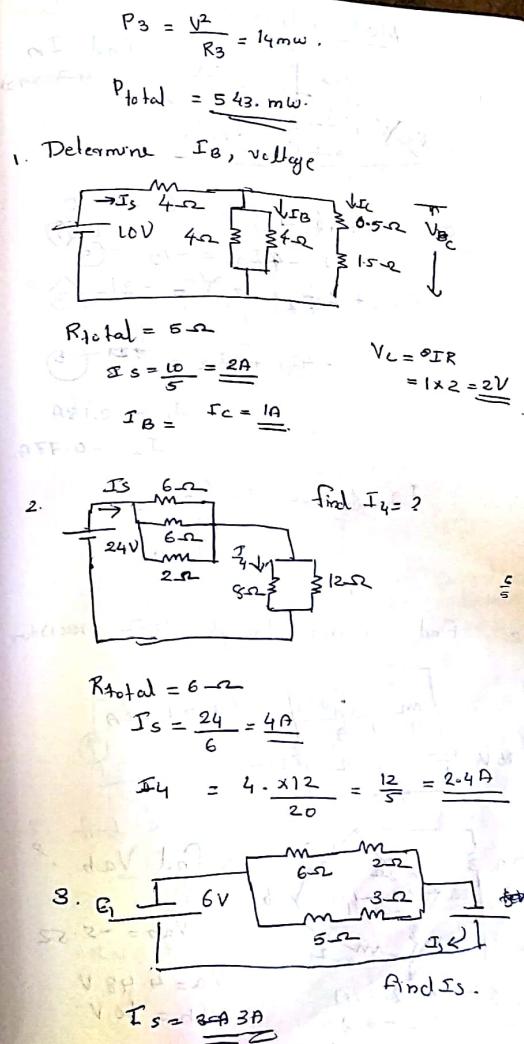
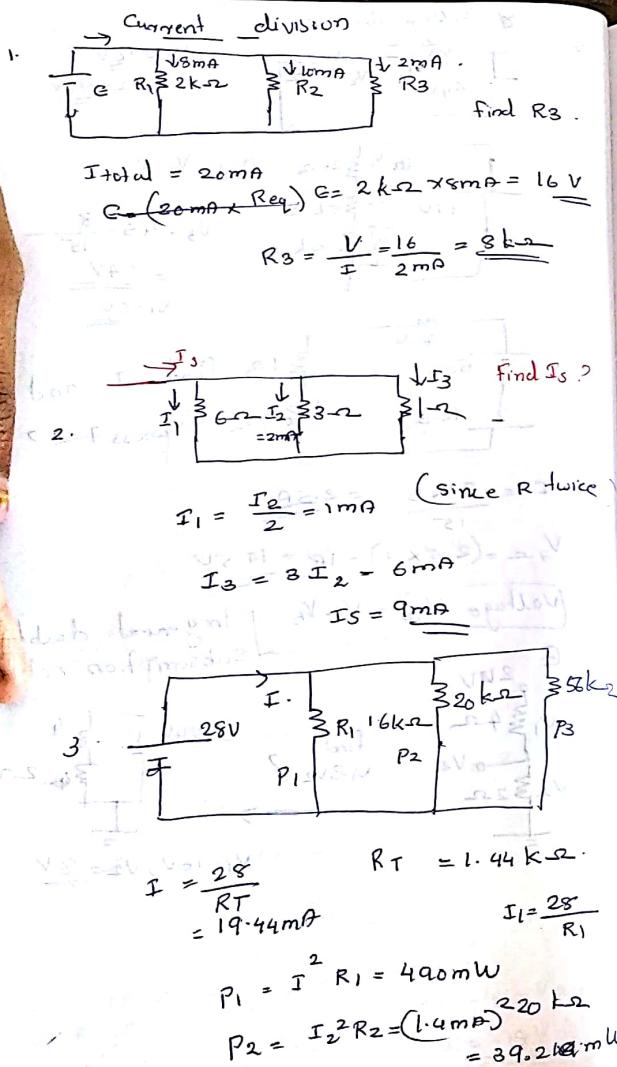
Wm P1 = 1 + 3 = 4

2 + 6.5 = 8.5

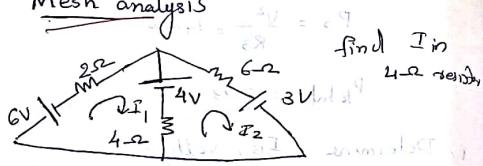
Wm P1 = 5 + 2 = 7

Wm P2 = 3 + 2 = 5

Wm P2 = 3 + 2 = 5



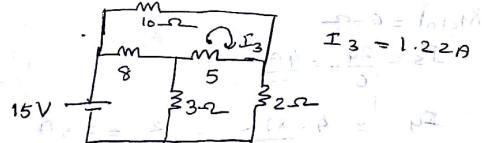
### Mesh analysis



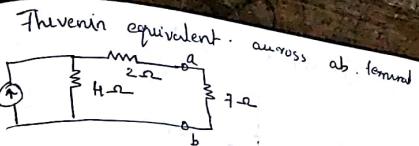
$$1. \quad \begin{aligned} 2I_1 + 4(I_1 - I_2) &= -6 - 4 = -10 \\ 6I_1 - 4I_2 &= -10 \quad (1) \\ 6I_2 + 4(I_2 - I_1) &= -21 \quad (2) \\ -4I_1 + 10I_2 &= -21 \quad (3) \\ I_1 &= -2.18A \\ I_2 &= -0.77A \end{aligned}$$

$$I \text{ through } 4\Omega = (I_2 - I_1) = -1.41A$$

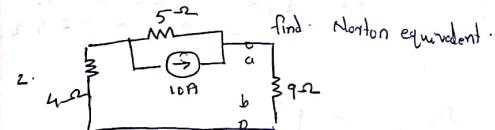
2. Find current through  $10\Omega$  resistor



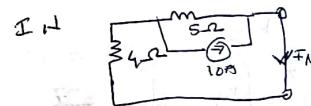
$$3. \quad \begin{aligned} V_{ab} &= -5.52V \\ V_a &= 4.48V \\ V_b &= 10V \\ F_1(R_w) &= 2.37\Omega \\ n \cdot F(w) &= -0.20 \quad \rightarrow 1.25n \end{aligned}$$



$$R_{Th} \rightarrow 13\Omega = 6\Omega \\ V_{Th} \rightarrow 12 \times 4 = 48V$$

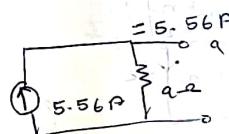


$$R_N \rightarrow 4 \Omega \rightarrow 9\Omega$$

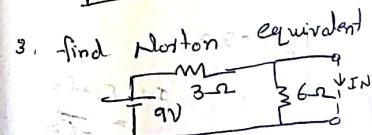


$$I_N = \frac{10 \cdot 5}{9} = \frac{50}{9}$$

Current division rule.



$$= 5.56\Omega$$

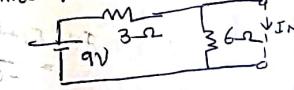


$$R_N = \frac{3 \times 6}{9} = 2\Omega$$

Current, voltage, since shorted.

$$I_N = \frac{9}{3} = 3A$$

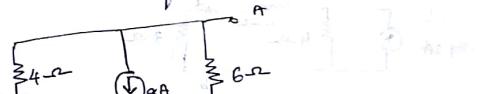
3. find Norton equivalent



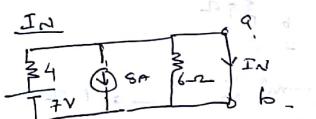
$$I_N$$

$$= 3A$$

1. Find Norton equivalent



$$R_N = \frac{24}{10} = 2.4\Omega$$



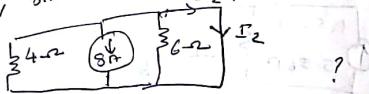
Use super position theorem

when 7V alone acting

$$I_1 = \frac{7}{4} = 1.75A$$

since 6Ω short circuited.

when, 8A alone acting



$$I_2 = -8A$$

$$I_N = I_1 + I_2$$

$$= -6.25A$$

and, unknown  $\rightarrow 2$

$$2 = \frac{2.4}{4} = 0.6A$$

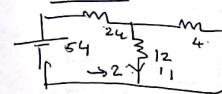
$$V_{oc} = 2.4V$$

$$V_{pc} = 0.6V$$

1. Use super position to find  $I$  through  $12\Omega$



when 54V alone



$$R_{eq} = \frac{12 \times 4}{16} = 3\Omega$$

$$I_s = \frac{54}{27} = 2A$$

$$I_1 = 2 \times \frac{4}{16} = 0.5A$$

when 48V alone



$$R_{eq} = 4 + 8 = 12\Omega$$

$$I_s = \frac{48}{12} = 4A$$

$$I_2 = \frac{4 \times 24}{36} = \frac{24}{9} = 2.67A$$

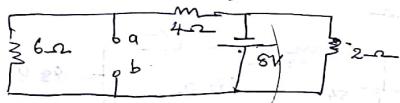
$$I_{12\Omega} = I_1 - I_2$$

$$\text{or } I_2 - I_1 = 2.67A \text{ upwards}$$

$$V_{oc} = ?$$

$$V_{pc} = ?$$

Q1. Find Thvenin equivalent.

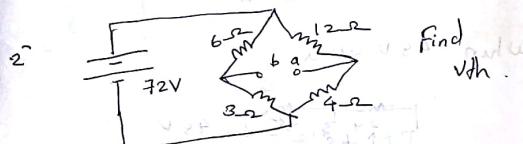


$$V_{th} = R_{th} \cdot I \rightarrow 6\Omega \parallel 4\Omega$$

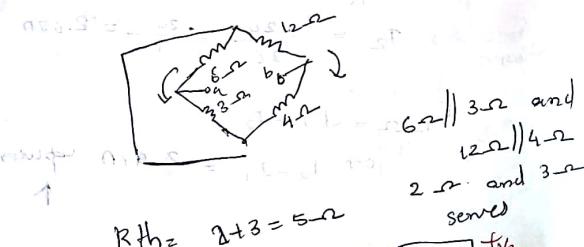
$$R_{th} = \underline{2.4\Omega}$$

$$V_{th} = \frac{8 \times 6}{10} = \underline{4.8V} = V_{ab}$$

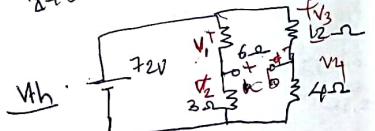
voltage division.



$$R_{th} = \underline{7.2\Omega \parallel 18\Omega} = 5.76\Omega$$



$$R_{th} = \underline{2 + 3} = 5\Omega$$



$$V_1 = \frac{72 \times 6}{9} = 48V$$

$$V_2 = \frac{72 \times 3}{9} = 24V$$

$$V_{th} = V_3 - V_2$$

Applying KCL,  $V_1 - V_3 + V_{th} = 0$

$$V_{th} = V_3 - V_1$$

$$\sqrt{3} = 72 \times \frac{12}{18} = \underline{54V}$$

$$V_{th} = \frac{3}{4} \times 54 = \underline{40.5V}$$

$$\therefore V_{th} = V_3 - V_1$$

$$= 54 - 48 = \underline{6V}$$

### 16. Magnetic Circuits.

- i. A magnetic field is produced by a coil of 300 turns which is wound on a closed iron ring. The ring has a cross-section of  $20\text{cm}^2$  & mean length of 120cm. Permeability of iron is  $800$ . If the current in the coil is  $10\text{A}$ . Find Energy stored in the magnetic field?

$$E = \frac{1}{2} L I^2$$

$$L = \frac{N^2}{S}$$

$$S = \frac{1}{\mu_0 M_r} = \frac{120 \times 10^{-2}}{4\pi \times 10^{-7} \times 800 \times 20 \times 10^{-4}}$$

$$L = \frac{N^2}{S}$$

$$= \frac{N^2}{I} \frac{NT}{S} = \frac{N^2}{S}$$

$$S = 596831 \text{ AT/wb}$$

$$L = \underline{0.15H}$$

$$E = \frac{1}{2} L I^2 = \underline{7.539J}$$

2017  
Ques. 1. Apparent power drawn by an ac circuit  $10\text{kVA}$ ,

active  $\rightarrow 8\text{kW}$  - find PF and  $\phi$ .

$$x = \sqrt{10^2 - 8^2} = 6\text{kVA}$$

$$PF = \cos \phi = \frac{8}{10} = \underline{-0.8}$$

Two impedances  $Z_1 = 6+j8 \Omega$  and  $Z_2 = 8-j6 \Omega$  are connected in parallel across 100V supply.

$$I = \frac{V}{R+j(X_L-X_C)} = \frac{100}{6+j8} = 6-j8$$

$$= 10 \angle -53^\circ$$

$$\text{PF} = \cos 53^\circ = 0.601 \text{ leading}$$

$$\text{Active power } VI \cos \phi = \frac{100}{100 \times 10 \times \cos 53^\circ} = 601 \text{ W}$$

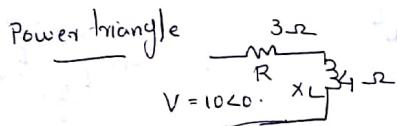
3. An electrical device is rated 5kVA, 200V at 0.6 PF lag. Find  $\text{Z} = ?$

$$I = \frac{5000}{100} = 50 \text{ A}$$

$$Z = \frac{E}{I} = \frac{100}{50 \angle -53^\circ} = 2 \angle 53^\circ$$

$$\phi = \cos^{-1}(0.6)$$

Ans so -ve,  $\phi$

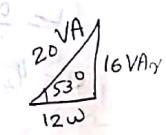


Find Active power?

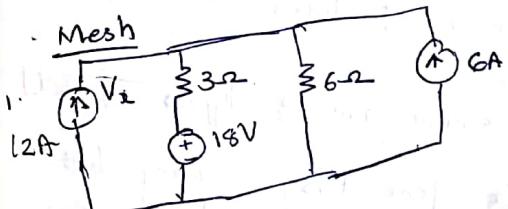
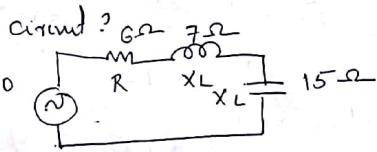
$$I = \frac{V}{Z} = \frac{10 \angle 0^\circ}{3+j4} = 1.2 - j1.6 = 2 \angle -53^\circ \text{ A}$$

$$P = VI \cos \phi$$

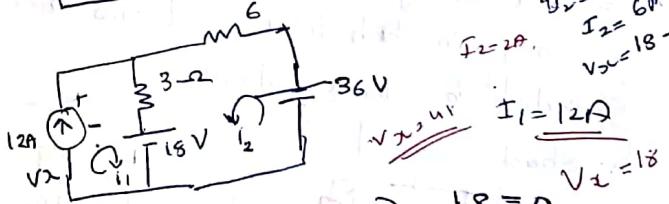
$$= 10 \times 2 \times \cos 53^\circ = 12 \text{ W}$$



2. Find Power factor of the given circuit?



Find  $V_x$  using mesh?



$$I_2 = 2 \text{ A}, \quad I_1 = 12 \text{ A}, \quad V_x = 18 \text{ V}$$

$$+V_x + 3(I_1 + I_2) - 18 = 0$$

$$V_x + 3I_2 - 18 = 0 \quad (1)$$

$$V_x + 3I_2 + 3I_1 - 36 = 0 \quad (2)$$

$$36 + 6I_2 + 3(I_2 + I_1) - 18 = 0 \quad (3)$$

$$36 + 9I_2 + 36 - 18 = 0 \quad (4)$$

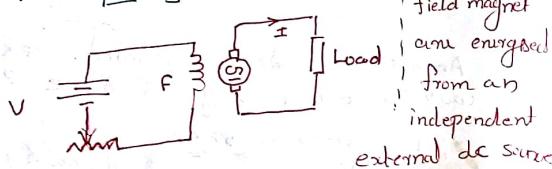
$$9I_2 = 18 - 72 \quad (5)$$

$$I_2 = 2 \text{ A} \quad (6)$$

$$V_x = -18 + 6$$

Types of generators classified according to fields are excited.

### 1. Separately excited



field magnet  
are energised  
from an  
independent  
external dc source

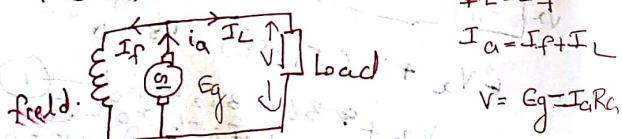
### 2. Self excited generators

field magnets are energised by the current produced by the generators themselves.

There are 3 types of self

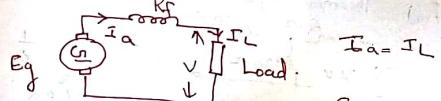
excited generators, according to the manner in which their field coils are connected to the armature.

#### a. shunt wound



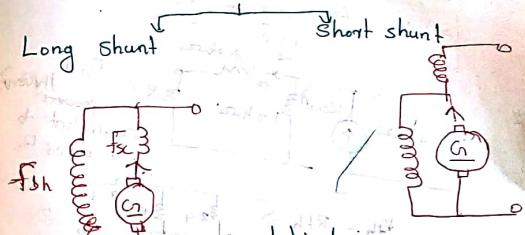
: used for charging batteries because their terminal voltages are almost constant

#### b. Series wound



Eg They are not used for power supply because of their rising characteristics. It is used as boosters in certain types of distribution systems particularly in railway services.

#### c. Compound wound



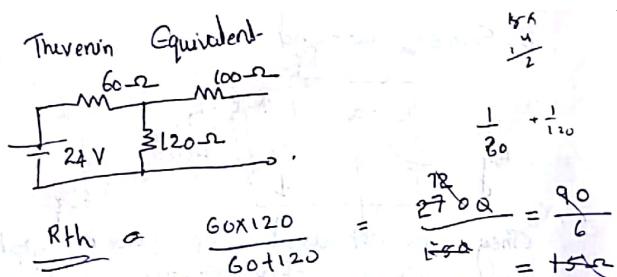
cumulatively compounded in Power stations  
diff. compounded in electric railways.  
and welding.

#### Motor applications

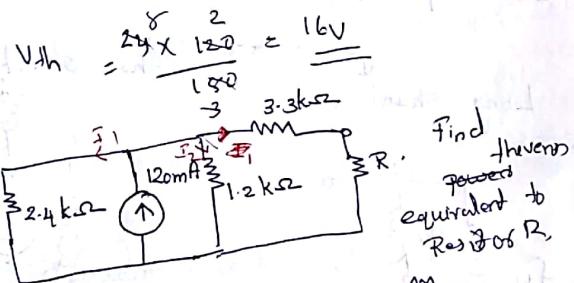
1. series - T  $\propto$  I. It is used where huge starting torque required for accelerating heavy masses.  
such as hoists and electric trains.

2. shunt motor ; Due to constant speed, suitable for driving shafts, machine tools, lathes, wood-working machine

Find Thévenin Equivalent



$$R_{Th} = 40 + \frac{60 \times 100}{60+100} = 140\Omega$$



$$R_{Th} = R_{Th} \Rightarrow 0.8k\Omega$$

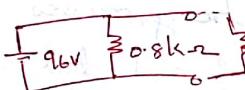
$$R_{Th} = 0.8 + 3.3 = 4.1k\Omega$$

$$V_{Th} = V_{\text{across } 1.2k\Omega}$$

$$= I_2 \times 1.2 \times 10^3$$

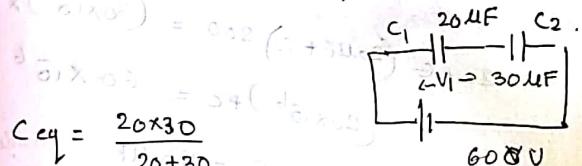
$$I_2 = \frac{3}{3.6 \times 10^3} = 0.833A$$

$$V_{Th} = 9.6V$$



### Capacitors

Two capacitors A, B having capacitance 20μF, 30μF are connected in series to a 600V DC supply. Determine potential difference across each capacitor.



$$Q = C V$$

Same as since C in series.

$$Q_{total} = C_{eq} \times 600 = 7200 \mu C$$

$$X_1 = Q_{total} \cdot \frac{C_2}{C_1 + C_2}$$

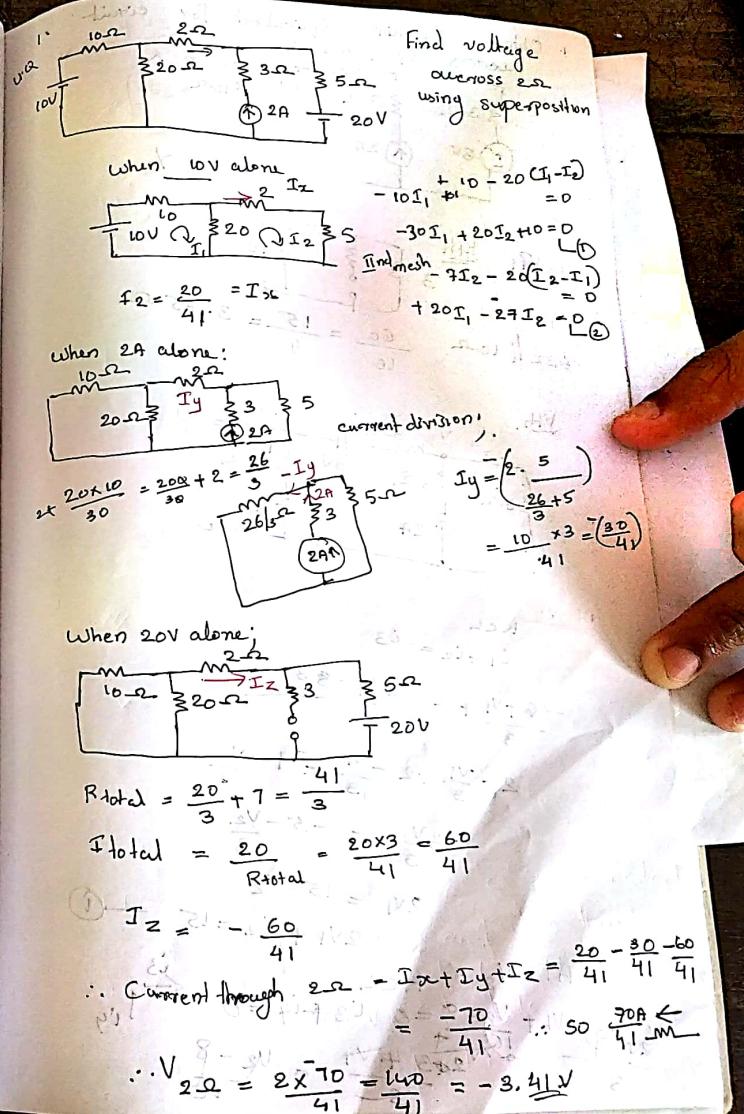
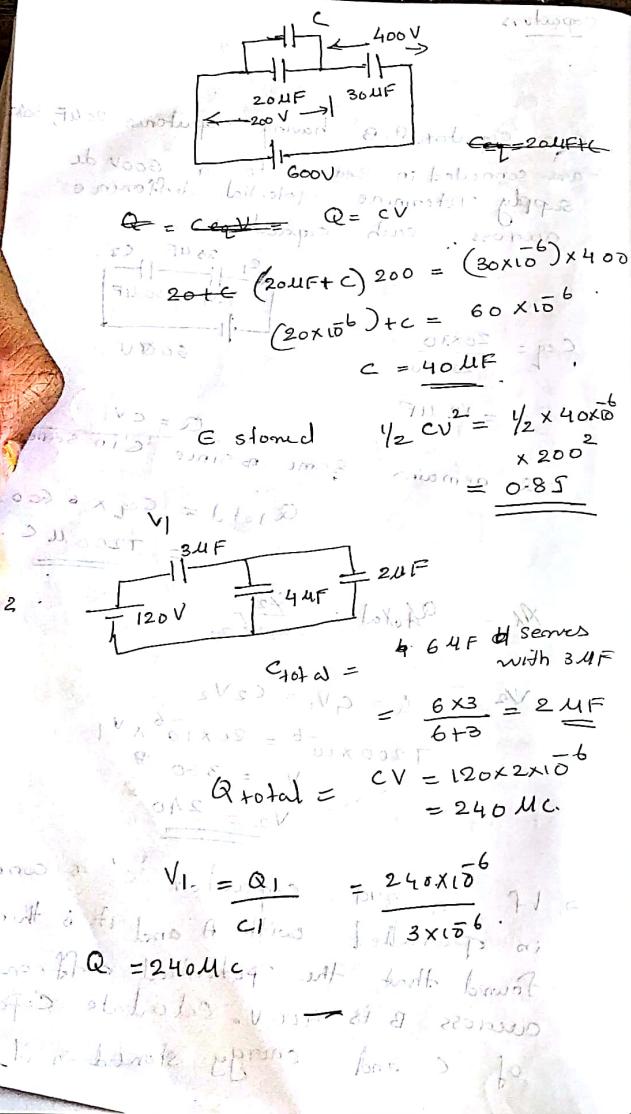
$$V_2 = Q = C_1 V_1 = C_2 V_2$$

$$7200 \times 10^{-6} = 20 \times 10^{-6} \times V_1$$

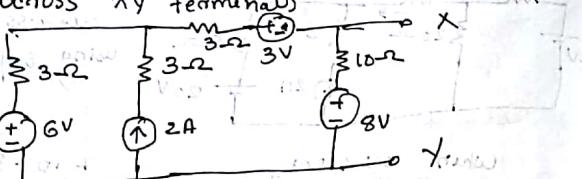
$$V_1 = 360$$

$$V_2 = 240$$

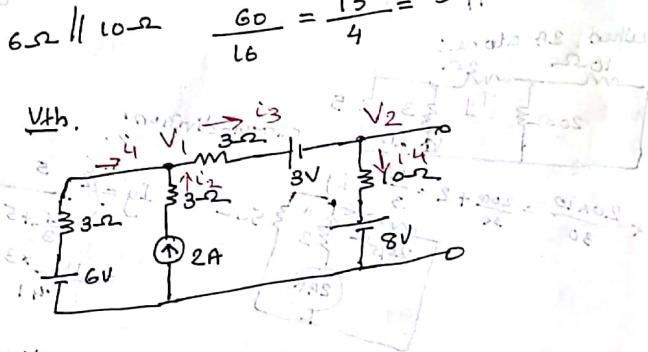
→ If a 3rd capacitor 'C' is connected in parallel with A and it is then found that the potential difference across B is 400V. Calculate capacitance of C and energy stored in it.



1. Obtain Thevenin's Equivalent for circuit across XY terminals



$$V_{th} = \frac{R_{th}}{R_{th}} = \frac{15}{15} = 3.75 \Omega$$



At.  $V_1$  KCL:  $i_1 + i_2 = i_3$

$$-\frac{6 + V_1}{3} + 2 = \frac{V_1 - 8}{3} - \frac{V_2}{3}$$

$$2 - \frac{V_1}{3} + 2 - \frac{V_1}{3} = 1 - \frac{V_2}{3}$$

$$\frac{-2V_1}{3} = -5 - \frac{V_2}{3}$$

$$2V_1 = 15 - V_2$$

$$2V_1 + V_2 = 15 \quad \text{--- (1)}$$

At.  $V_2$ : KCL:  $i_3 + i_4 = 0$

$$-\frac{V_1 + 8}{10} + \frac{V_2 + 8}{10} = 0$$

$$V_1 + V_2 = 16 \quad \text{--- (2)}$$

$$+\frac{V_2}{3} + 1 - \frac{V_1}{3} + \frac{V_2}{10} - \frac{8}{10} = 0$$

$$[+V_2 + 3 - V_1] 10 = 3V_2 + 24$$

$$+30 - 10V_1 = +3V_2 + 24$$

$$-10V_1 + 13V_2 = -6 \quad \text{--- (2)}$$

$$V_1 = 5.58 \text{ V}$$

$$V_2 = 3.83 \text{ V}$$

$$0.667V_1 - 0.33V_2 = 5$$

$$-0.33V_1 + 0.433V_2 = -0.2$$