

MODULE - III

AC FUNDAMENTALS

DC is has constant magnitude w.r.t to time

AC changes periodically both in magnitude & direction. The change in magnitude & direction is measured in terms of cycles or steps.

Instantaneous value: a value of an alternating quantity at a particular time.

Waveforms: graph of instantaneous values of an alternating quantity plotted against time.

Cycle: each repetition or a set of positive and negative instantaneous values of alternating quantity. Waveform which exhibit variations that occur after a regular time interval

is known as periodic waveforms.

1 cycle corresponds to 2π .

Time period: Time taken by an alternating quantity to complete one cycle. (T). After every T seconds, cycle of an alternating quantity repeats.

Frequency: No. of cycles completed by an alternating quantity per second (f).

$$f = \frac{1}{T} \text{ Hertz / cycles per sec.}$$

Time period and frequency are reciprocal of each other & the product of them is 1.

Amplitude: max value gained by an alternating quantity during positive / negative half cycle.

Angular frequency: Frequency expressed in electrical radians per second.

Frequency of AC wave = $\frac{1}{T}$ or $\frac{2\pi}{\omega}$ rps

Peak value: maximum instantaneous value measured

from zero potential to give a sinusoidal waveform

Peak to peak value: max variation b/w max

instaneous value & max -ve instantaneous value. It is $2V_m$.

For a sinusoidal wave form if it is $2V_m$.

peak amplitude: max instantaneous value measured from the mean value of a waveform.

Instantaneous value of a waveform: emf

$$e = E_m \sin \theta$$

$$e = E_m \sin \omega t$$

$$e = E_m \sin (2\pi f)t$$

$$e = E_m \sin \left(\frac{2\pi}{T} t \right)$$

RMS value / Effective value

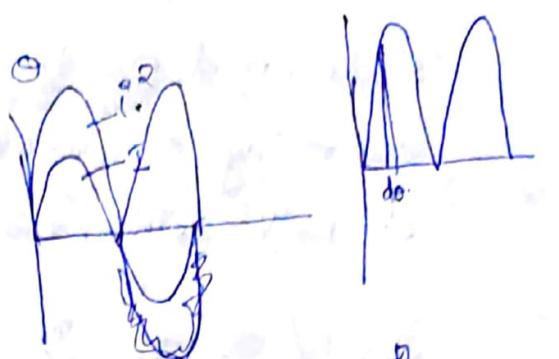
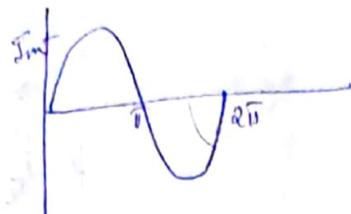
(Q) RMS value of an alternating current

is given by that steady state current which when

flowing through a given ckt for a given time produces the same amount of heat as produced

by ac & through the same circuit for the
same time.

$$i = I_m \sin \theta$$



$$\text{area} = i^2 d\theta$$

$$\text{Total area} = \int_0^\pi i^2 d\theta$$

$$\text{Mean} = \frac{1}{\pi} \int_0^\pi i^2 d\theta$$

$$\text{RMS Current} = \sqrt{\frac{1}{\pi} \int_0^\pi i^2 d\theta}$$

$$I_{\text{rms}}^2 = \frac{1}{\pi} \int_0^\pi I_m^2 \sin^2 \theta d\theta$$

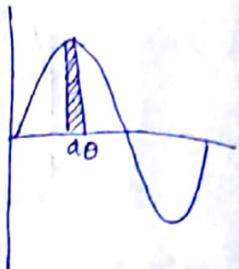
$$= I_m^2 \int_0^\pi \sin^2 \theta d\theta = \frac{I_m^2}{\pi} \int_0^\pi \frac{1 - \cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{\pi} \times \frac{1}{2} = \frac{I_m^2}{2\pi}$$

$$\boxed{R_{\text{rms}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m}$$

Average value

Average value of an alternating quantity is defined as that value which is obtained by averaging all instantaneous values over a period of half cycle.



$$I_{av} = \frac{1}{\pi} \int_0^{\pi} i d\theta$$

$$= \frac{1}{\pi} \int_0^{\pi} I_m \sin \theta d\theta$$

$$= \frac{I_m}{\pi} \int_0^{\pi} \sin \theta d\theta$$

$$= \frac{2 I_m}{\pi}$$

$$I_{av} = 0.637 I_m$$

Form Factor

$$\frac{I_{rms}}{I_{av}} = \frac{0.707 I_m}{0.637 I_m} = 1.1 \rightarrow \text{sinusoidal wave form.}$$

Peak Factor

$$k_p = \frac{\text{Max value}}{\text{RMS}} = \sqrt{2}$$

An alternating voltage has an equation

$v = 1414 \sin 377t$. What is the value of

- i) RMS voltage
- ii) Frequency
- iii) Instantaneous value

$$V = V_m \sin \omega t \\ = 141.4 \sin 377t$$

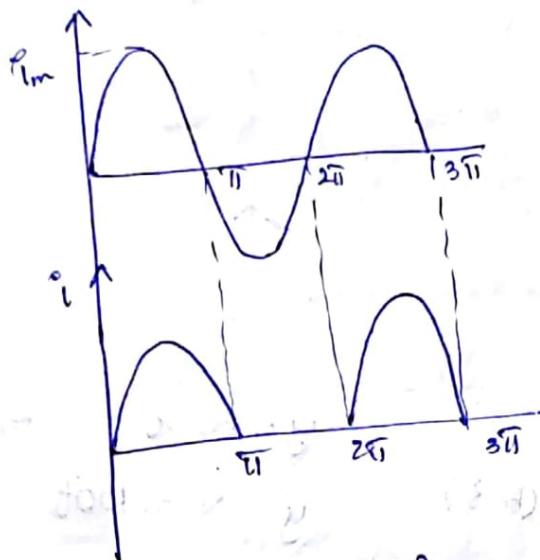
$$\omega = 377$$

$$i) V = \underline{127.9} = V_m \sin (377t + 3 \times 10^3)$$

$$ii) f = \frac{\omega}{2\pi} = \underline{50 \text{ Hz}}$$

Find the average and rms value.

RMS - half wave



$$= \frac{1}{2\pi} \int_0^{\pi} I_m^2 \sin^2 \theta d\theta \\ = \frac{I_m^2}{2\pi} \int_0^{\pi} \frac{1 - \cos 2\theta}{2} d\theta$$

$$\frac{I_m^2}{4\pi} \times \frac{\pi}{2} = \frac{I_m^2}{4}$$

$$\text{RMS value} = \underline{\frac{I_m}{2}}$$

Average of ~~full~~ half wave

$$\begin{aligned} & \frac{1}{2\pi} \int_0^{2\pi} I_m \sin \omega t dt \\ &= \frac{1}{2\pi} \left[\int_0^{\pi} I_m \sin \omega t dt + \int_0^{2\pi} 0 dt \right] \\ &= \frac{1}{2\pi} \int_0^{\pi} I_m \sin \omega t dt = \frac{1}{2\pi} \end{aligned}$$

$$\text{Avg} = \underline{\frac{I_m}{\pi}}$$

In case of full wave rectifier

$$\frac{1}{2\pi} \int_0^{2\pi} I_m \sin \omega t dt$$

$$\text{Avg} = \underline{\frac{2I_m}{\pi}}$$

$$\frac{1}{2\pi} \int_0^{2\pi} I_m^2 \sin^2 \theta d\theta$$

$$= \frac{I_m^2}{2\pi} \times \frac{2\pi}{2} = \underline{\frac{I_m^2}{2}}$$

$$\text{RMS value} = \underline{I_m / \sqrt{2}}$$

RMS

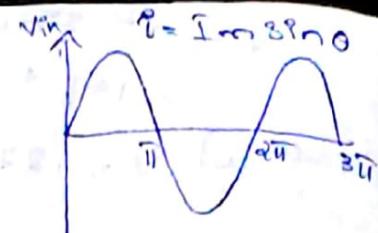
Mean

$$\text{rms value} = \frac{1}{2\pi} \int_0^T i^2 d\theta$$

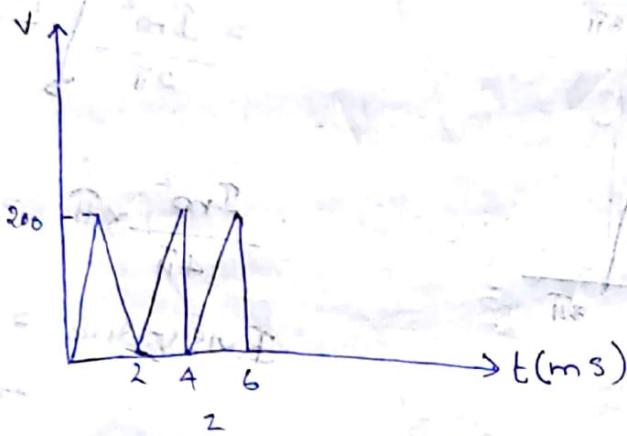
$$= \frac{1}{2\pi} \int_0^T I_m \sin^2 \theta d\theta$$

$$\text{Mean value} = \frac{I_m}{2}$$

$$\text{rms value} = \frac{I_m}{2}$$



Q)



$$y = mx \quad m = \frac{200}{2} = 100$$

$$y = V = 100t$$

$$\text{av. g value} = \frac{1}{2} \int_0^T 100t dt$$

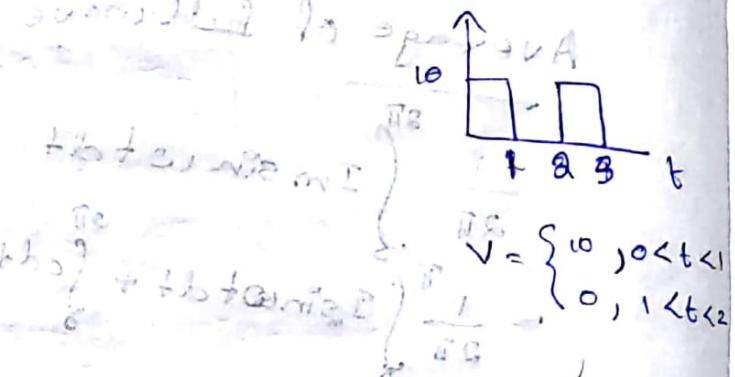
$$\text{av. g value} = \frac{1}{2} \left[100t^2 \right]_0^2$$

$$= \frac{50 \times 4 - 0}{2}$$

$$= \frac{100 \times 4}{2} = 100V$$

$$\text{R.M.S.} = \frac{1}{2} \int_0^T 100^2 t^2$$

$$= \frac{10000}{2} \int_0^T t^3 dt$$



$$= \frac{10000 \times 8^4}{2 \times 3}$$

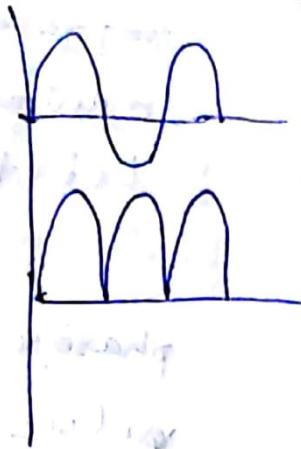
$$= 13,333.3$$

$$\text{rms value} = \underline{\underline{115.4V}}$$

Full wave avg. value =

$$= \frac{Im}{\pi} \int_0^{\pi} \sin \omega t dt$$

$$= \frac{Im}{\pi} \left[-\cos \omega t \right]_0^{\pi}$$



$$= \frac{Im}{\pi} (1 + 1) = \frac{2 Im}{\pi}$$

HWR

FWR

$$I_{avg} = \frac{Im}{\pi}$$

$$I_{avg} = \frac{2 Im}{\pi}$$

$$I_{rms} = \frac{Im}{\sqrt{2}}$$

$$I_{rms} = \frac{Im}{\sqrt{2}}$$

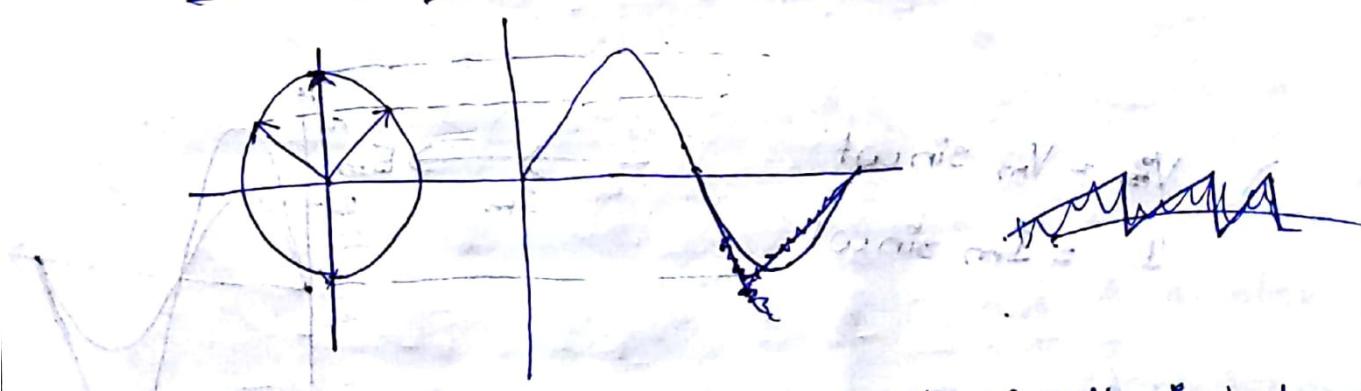
$$K_f = \pi/2 \quad \text{and short}$$

$$K_f = \frac{\pi}{2\sqrt{2}}$$

$$K_p = \frac{Im}{Im/2} = 2/\pi$$

$$K_p = \frac{2 Im}{Im/2} = \sqrt{2}/\pi$$

Phasor representation



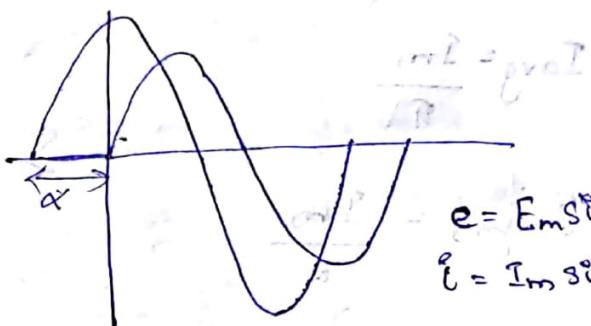
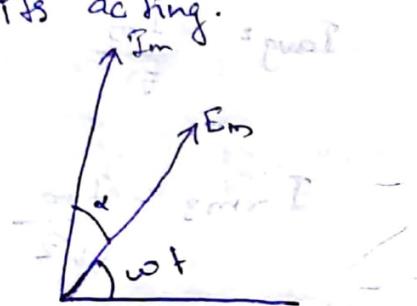
It is difficult to continuously handle instantaneous values in form of eqns like $e = Em \sin \omega t$. A conventional method is to employ vectors for

representing these sine waves. These vectors can be manipulated to achieve the desire result.

Let the alternating quantity equal to

$e = E_m \sin \omega t$, this represented by the phasor OP , length of the line gives the max value of the alternating quantity and inclination of the line with respect to any reference line gives the direction of the quantity and arrow head gives the direction of the alternating quantity.

Its acting.



$$e = E_m \sin \omega t$$

$$i = I_m \sin(\omega t + \alpha)$$

I_m leads E_m

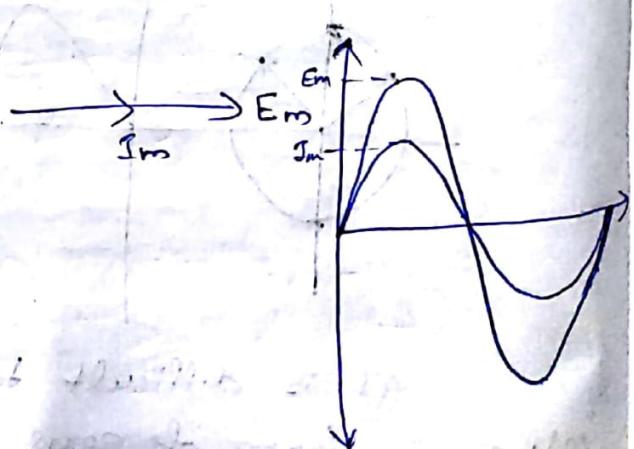
same freq \rightarrow alternating quantity - same phasor diagm.

difference b/w ~~two~~ phases of two alternating

~~currents~~ closed

$$V_B = V_m \sin \omega t$$

$$I = I_m \sin \omega t$$



phasor diagram of the two currents

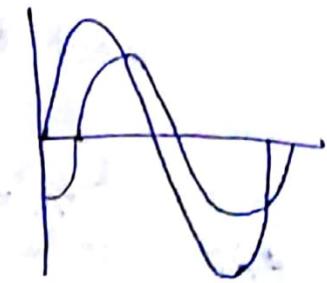
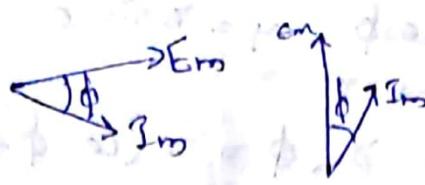
1. Resultant current will represent mean value of total current

of both currents in a given time

2)

$$e = E_m \sin \omega t$$

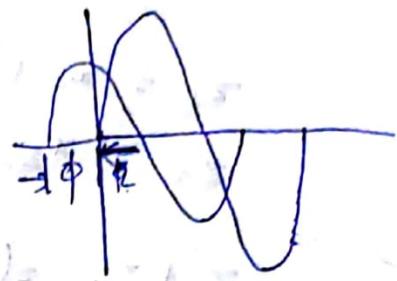
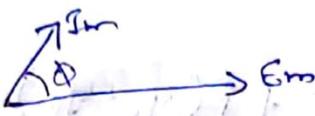
$$i = I_m \sin(\omega t - \phi)$$



3)

$$e = E_m \sin \omega t$$

$$i = I_m \sin(\omega t + \phi)$$

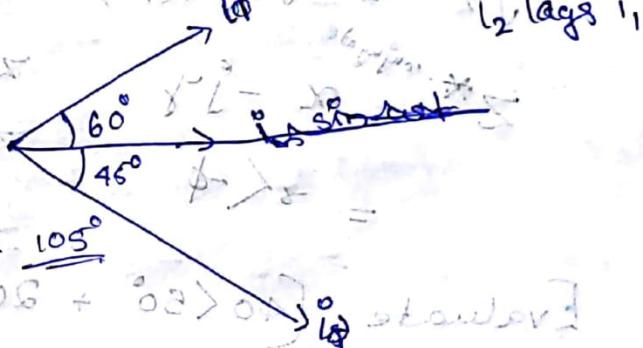


Two alternating quantities have same freq and they reach max & -ve value and zero values @ the same time.

@ the same time $\Rightarrow \phi = 0^\circ$

Consider two sinusoidal currents $i_1 = 10 \sin(\omega t + 15^\circ)$
 $i_2 = 15 \sin(\omega t - 15^\circ)$. Find the phase difference
 b/w them.

$$\text{phase difference} = 105^\circ$$



Mathematical representation

A phasor is a complex number that represents the amplitude and phase of a sinusoid. A complex number can be represented in rectangular form as $z = x + jy$, where $j = \sqrt{-1}$, or in polar form $z = r \angle \phi$.

$$r = \sqrt{x^2 + y^2}$$

and $\phi = \tan^{-1}(y/x)$



$$x = r \cos \phi, \quad y = r \sin \phi$$

$$z = x + jy = r \angle \phi = r(\cos \phi + j \sin \phi)$$

$$z_1 = x_1 + jy_1 = r_1 \angle \phi_1$$

$$z_2 = x_2 + jy_2 = r_2 \angle \phi_2$$

$$z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$$

$$z_1 z_2 = r_1 r_2 \angle (\phi_1 + \phi_2)$$

$$z_1/z_2 = r_1/r_2 \angle (\phi_1 - \phi_2)$$

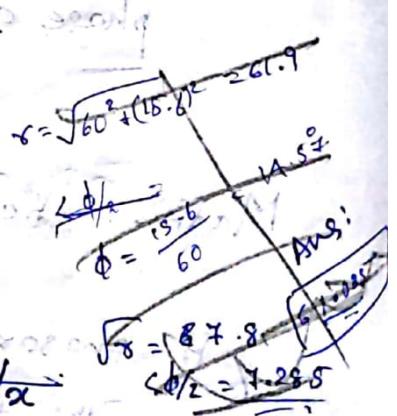
$$\frac{1}{z} = \frac{1}{r} \angle -\phi \quad \text{if } z = r \angle \phi$$

$$\sqrt{z} = \sqrt{r} \angle (\phi/2)$$

$$\begin{aligned} z^*_{\text{conjugate}} &= x - jy \\ &= r \angle -\phi \end{aligned}$$

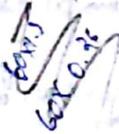
$$\text{Evaluate } [40 \angle 50^\circ + 20 \angle -30^\circ]^{1/2}$$

~~$$x_1^2 + y_1^2 = 1600$$~~



$$\tan^{-1}(y/x) = 50$$

$$\tan 50 = y/x$$



$$x_1 = 40 \cos 50, \quad y_1 = 40 \sin 50$$

$$x_2 = 20 \cos -30, \quad y_2 = 20 \sin -30$$

$$x_3 = 60 (\cos 50 + \cos -30), \quad y_3 = 60 (\sin 50 + \sin -30)$$

$$3+j4 \quad r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\tan^{-1}(y/x) = \phi = \tan^{-1}(4/3) = 53.13^\circ$$

$$x_3 = 4.303$$

$$y_3 = 20.64$$

$$r = \sqrt{47.7}$$

$$\underline{\underline{r}} = 6.91 \quad \underline{\underline{\phi}} = 12.8$$

$$8 < 30 \Rightarrow 8 \cos 30 + 8 \sin 30 = 6.92 + 4j$$

$$100 < -60 \Rightarrow 100 \cos 60 + 100 \sin 60 = 50 + 86.6j$$

$$20 < 240 \Rightarrow -10 + 17.32j$$

$$50 < -30 \Rightarrow 4.33 + 2.5j$$

$$(10 < -30) + (3 - j4)$$

$$(3 - j5)^* = 3 + j5$$

$$(2 + j4)(3 - j5)^*$$

$$\Rightarrow 5.83 < 59$$

$$10 < -30 = 8.66 + -5j + 3 - 4j$$

$$(2 + j4) \Rightarrow 4.472 < 63.4$$

$$\text{Numerator } 11.66 + -9j \Rightarrow 14.72 < -37.66$$

$$5.83 < 59 \times 4.472 < 63.4 \Rightarrow 36.8 < 122.47 \text{ Denominator}$$

$$= 0.56 < -160.13$$

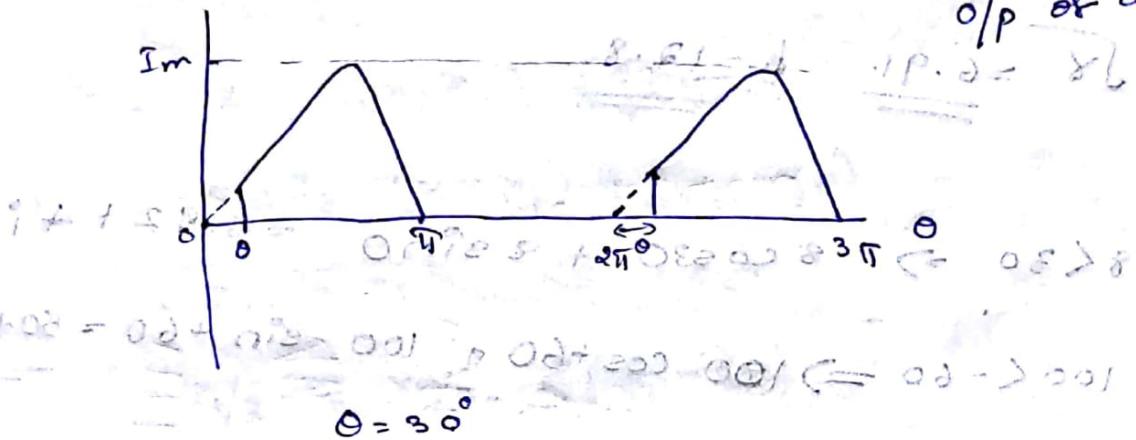
$$= 0.5266 < 0.1903^\circ //$$

$$\left[(5 + 2j)(-1 + j4) - 3 \angle 60^\circ \right]^\infty$$

~~5 + 2j~~

$$-13 + 18j = 2.5 + 4.3j$$

$$= -15.5 - 13.7j$$



O/p of a thyristor.

$$\begin{aligned} 0.43301 \\ 2 \\ 0.137 \\ = 0.208 \end{aligned}$$

$$\frac{1}{2} \text{ mean value} = \frac{I_m^2 \sin^2 \theta}{2\pi} = 0.43301$$

$$= \frac{I_m^2 \pi}{4\pi} \left(\frac{1}{2} \sin^2 \theta + (0.8 - 2\cos \theta) \right)$$

per sec. &

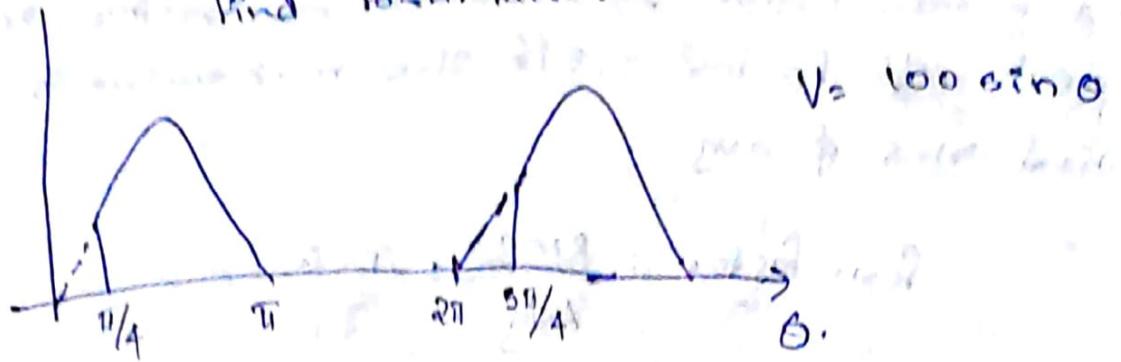
$$2SF + 1 = (2f + 6) + 10 = + 20 = \frac{I_m^2}{4\pi} \int_{\pi/6}^{5\pi/6} \frac{1 - 2\cos 2\theta}{2\pi} d\theta = 0.1$$

2f + 6 -> SF + 1 < 20 -> 20.11 reasonable

$$\text{max. p.f. } S = \frac{I_m^2}{4\pi} \times \left(\frac{5\pi}{6} - 0.43 \right) = \frac{I_m^2}{4\pi} \times \frac{\pi}{6} = [0.43]$$

$$m_s = \frac{I_m}{2} = \frac{0.439 I_m}{2} = \frac{I_m^2 \times 5\pi}{4\pi \times 6} = 0.43$$

Find form factors



$$V = 100 \sin \theta$$

$$I_{\text{mean}} = \frac{1}{2\pi} \int_{\pi/4}^{\pi} 100 \sin \theta d\theta = \frac{1}{2\pi} [-\cos \theta]_{\pi/4}^{\pi} = \frac{1}{2\pi} [-\cos \pi + \cos \pi/4] = \frac{1}{2\pi} [1 + \sqrt{2}] = \frac{100}{2\pi} + 1.7071$$

$$= +27.16 V$$

$$I_{\text{rms}} = \sqrt{\frac{100^2}{2\pi} \int_{\pi/4}^{\pi} \sin^2 \theta d\theta}$$

$$= \sqrt{\frac{100^2}{4\pi} \int_0^{\pi/2} 1 - \cos 2\theta d\theta}$$

$$= \frac{100}{2} \sqrt{\frac{1}{\pi} \times \left(\frac{\pi}{2} - \frac{1}{4}\right)} = \left(0 + \frac{1}{\sqrt{2}}\right)$$

$$= \frac{100}{2} \sqrt{\frac{3\pi}{4} \times \frac{1}{\sqrt{2}}} = \frac{100}{2} \sqrt{\frac{3\pi}{4} \times \frac{1}{\sqrt{2}}} = 47.67$$

$$= \frac{100}{4} \sqrt{\frac{3\pi}{2}}$$

$$= \frac{100}{4} \times 1.4 = 47.67$$

$$K_F = \frac{\text{RMS}}{\text{Avg.}} = \frac{47.67}{27.16} =$$

A certain wave form has a form factor 1.2 and peak ~~volt~~ factor 1.5 if the max value is 100V
Find rms & avg.

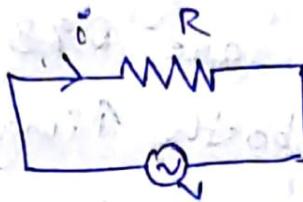
$$\text{Form factor} = \frac{\text{RMS}}{\text{Avg}} = 1.2$$

$$\text{peak factor} = \frac{\text{Max}}{\text{rms}} = 1.5$$

$$\text{rms} = \frac{100}{1.5} = \underline{\underline{66.67\text{V}}}$$

$$\text{Avg} = \underline{\underline{79.9\text{V}}}$$

AC through pure resistance



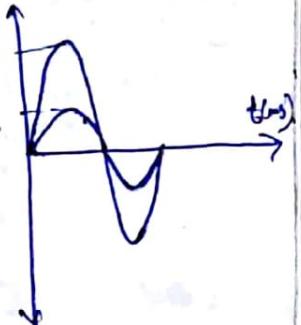
Applied voltage has to supply ohmic voltage drop only.

$$V = IR \quad V_{\text{drop}} = IR \quad V_{\text{drop}} = I^2 R$$

$$V_m \sin \omega t = I^2 R$$

$$I = \frac{V_m}{R} \sin \omega t$$

$$P = I_m^2 R \sin^2 \omega t$$



$$\text{Average of } P = \frac{1}{T} \int_0^T P \sin^2 \omega t dt$$

$$= V_m I_m \sin^2 \omega t$$

$$= \frac{1}{2} I_m^2 R$$

$$= I_m V_m \left(1 - \cos 2\theta \omega t \right)$$

$$\theta = \phi - \frac{\pi}{2}$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m \cos 2\theta \omega t}{2}$$

Average value of fluctuating component of the power in a full cycle is zero. Therefore avg power consumption over 1 cycle is equal to constant component

$$P_{\text{avg}} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$$

$$= V_{\text{rms}} \cdot I_{\text{rms}}$$

The max values of alternating V and I are $400V$ & $20A$ respectively in a coil connected to a $50Hz$ supply. And the quantities are sinusoidal. Instantaneous values of V & I are $283V$ & $10A$ respectively.

@ time $t=0$, both being 0 .

- Find the expression for V & I @ time t
- Determine the power consumed.

$$V_{rms} = 400V \quad I_{rms} = 20A \quad f = 50Hz \quad 2\pi f = \omega$$

$$V = V_{rms} \sin \omega t + \phi \quad I = I_{rms} \sin \omega t + \phi = 0$$

$$\begin{aligned} V &= V_{rms} \sin \omega t + \phi \\ &= 400 \sin 100\pi t + \frac{\pi}{4} \quad \phi = 45^\circ \end{aligned}$$

$$I_{rms} = ?$$

$$I = I_{rms} \sin (\omega t + \phi) = 10 \sin (\omega t + \phi)$$

$$\sin (\omega t + \phi) = \frac{10}{20} = \frac{1}{2}$$

$$\sin(\omega t + \phi) = \frac{1}{2}$$

$$\sin \phi = \frac{1}{2}$$

$$\tan \phi = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{\sqrt{3}}$$

$$I = \frac{1}{2} I_{rms} \sin 100\pi t + \frac{\pi}{6}$$

According to Lenz's law, $\phi = 30^\circ$

From $\cos \phi = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2}$

~~$$P = V_{rms} I_{rms} \cos \phi$$~~

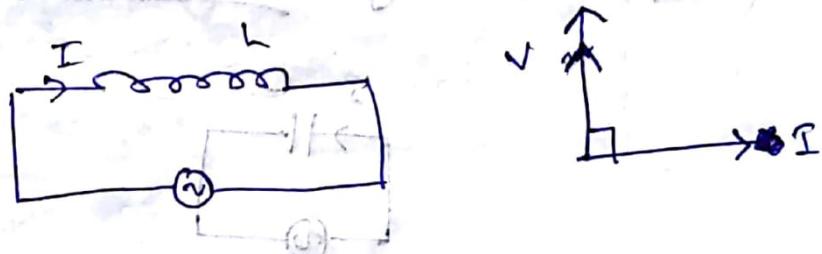
~~$$P = 386.9W$$~~

~~$$P = 386.9W$$~~

AC through pure inductance

pure inductance has zero ohmic resistance
when an ac is applied through a purely inductive coil, an back emf is produced due to the self inductance of the coil.

This back emf for every instant opposes the rise or fall of current through the current. Since, there is no ohmic drop resistance, the applied voltage opposes the applied voltage. self induced emf



$$e_L = -L \frac{di}{dt}$$

$$V = -e_L = L \frac{di}{dt} = L \frac{d}{dt} Im \sin \omega t$$

$$= L I_m \omega \cos \omega t$$

$$\therefore V = L I_m \omega (\sin \omega t + 90^\circ)$$

$$V = V_m \sin(\omega t + 90^\circ)$$

$$V_m = L I_m \omega$$

$$\sqrt{2} V_{rms} = L \sqrt{2} I_{rms} \omega$$

$$WL = \frac{V}{I} \quad X_L = 2\pi f L$$

• Effect of inductor in a circuit

• Inductor oppose change in current
• If current changes from zero to maximum
induced emf is $-V_m$

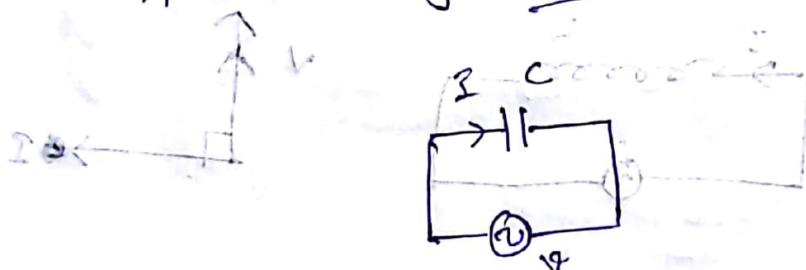
• In order to neglect effect of inductor
 $25 \text{ mH} = 50 \text{ H}$

$$\text{Reactance of inductor} X_L = 2\pi f L = 2\pi \times 50 \times 25 \times 10^{-3} = 7.85 \Omega$$

• Inductance becomes negligible when inductor value is small
• Inductance of $\frac{1}{2}$ henry is $3.14 \times 10^8 \text{ mH}$

• Inductance of 1 mH is $3.14 \times 10^{-8} \text{ henry}$

• AC through pure Capacitance circuit



$$I = \frac{dQ}{dt}$$

$$\text{From Faraday's Law} \frac{b}{ab} I = \frac{dq}{dt} = \frac{1}{C} V = V$$

$$I = \frac{d(CV)}{dt} = C \frac{dV}{dt}$$

$$(OP + i \omega t) \frac{dV}{dt} = V dt$$

$$I = C \frac{d}{dt} (V_m \sin \omega t)$$

$$(OP + \text{term}) \frac{d}{dt} (\omega t) = V$$

$$= C \frac{d}{dt} V_m \omega \cos \omega t$$

$$= C V_m \omega \sin(\omega t + 90^\circ)$$

$$I = C V_m \omega \sin(\omega t + 90^\circ)$$

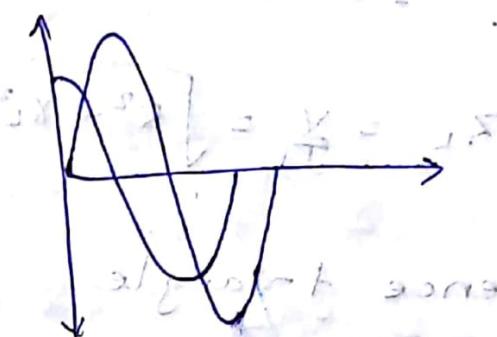
$$I = I_m \sin(\omega t + 90^\circ)$$

$\text{current } I = V / Z$
 $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$
 $I = V / \sqrt{R^2 + (\omega L)^2} = V / \sqrt{R^2 + (2\pi f L)^2}$

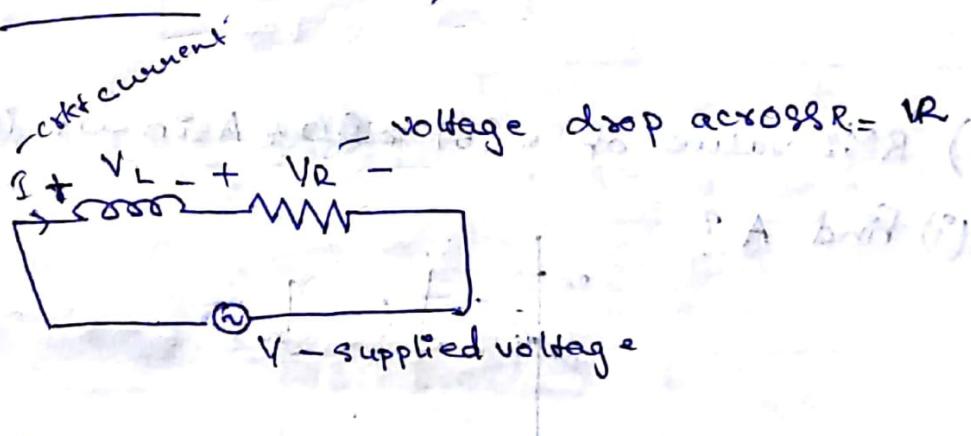


$$\frac{V}{I} = \sqrt{\frac{1}{R^2} + \left(\frac{2\pi f L}{R}\right)^2}$$

$$\frac{1}{\omega C} = X_C = \frac{1}{2\pi f C}$$

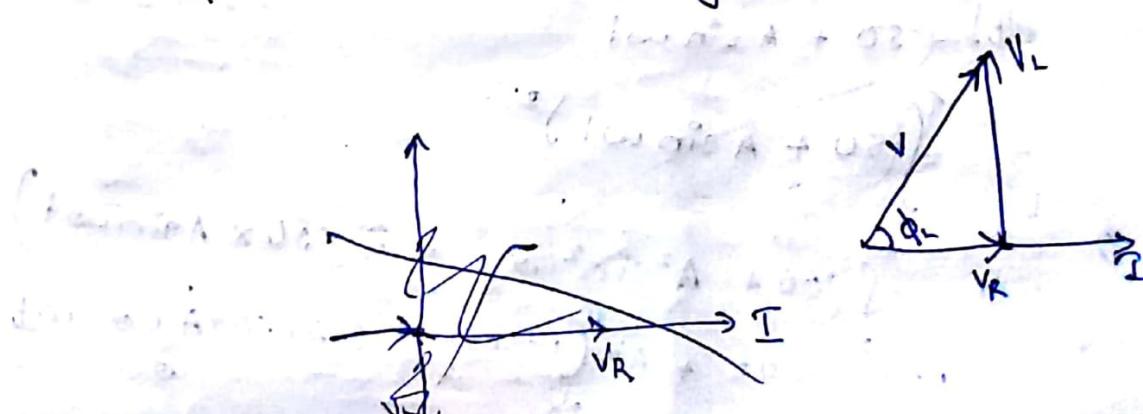


Series R, L circuit



$$\text{Voltage drop across } L = I X_L \neq V$$

ϕ is the phase angle b/w I & V .



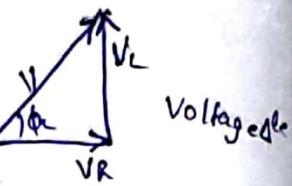
I is same through the V_L, V_R , thus
used as reference phasor V_R is in phase
with I and V_L leads by 90° .

$$V^2 = V_R^2 + V_L^2$$

$$\Rightarrow (IR)^2 + (I \times L)^2 \rightarrow X = \frac{1}{2\pi f} L$$

$$\frac{V^2}{I^2} = R^2 + X_L^2$$

$$Z_L = \frac{V}{I} = \sqrt{R^2 + X_L^2}$$

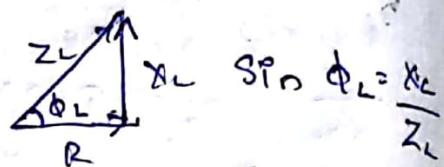


$$\sin \phi_L = \frac{V_L}{V}$$

$$\cos \phi_L = \frac{V_R}{V}$$

$$\tan \phi_L = \frac{V_L}{V_R}$$

Impedance triangle

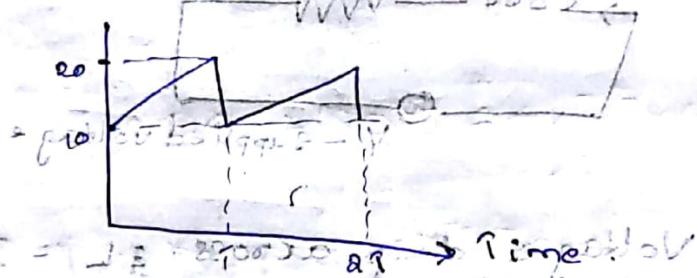


$$\cos \phi_L = \frac{R}{Z_L}$$

$$\tan \phi_L = \frac{X_L}{R}$$

1) RMS value of $e(t) = 50 + A \sin \omega t$ is 656 V

(i) Find A?



$$f = \frac{1}{T}$$

$$2\pi f = \frac{2\pi}{T}$$

$\therefore V \propto I$ and $3/4\pi$ second - cut off ϕ

$$e(t) = 50 + A \sin \omega t$$

$$= \sqrt{(50 + A \sin \omega t)^2}$$

$$= \sqrt{(50 + A^2 \sin^2 \omega t + 2 \times 50 \times A \sin \omega t)}$$

$$= \sqrt{50^2 + A^2 \left(1 - \cos 2\omega t + 2 \times 50 \times A \cos \omega t \right)}$$

$$I_{dc} + \frac{A^2}{2} T \sin 2\omega F \times \frac{\pi A^2}{2} - \frac{100A \cos \omega F}{\omega}$$

$$I_{rms}^2 = I_{dc}^2 + \left(\frac{I_{m1}}{\sqrt{2}}\right)^2 - \text{dc offset.}$$

$$I_{rms}^2 = \left(\frac{I_{m1}}{\sqrt{2}}\right)^2 + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 + \left(\frac{A^2}{2}\right)^2 - \text{AC component}$$

$$1) \quad \left(\frac{SD}{\sqrt{2}}\right)^2 + \frac{A^2}{2} = (65.6)^2$$

$$A = \underline{60.05 V}$$

$$2) \quad y = mx + c + 10$$

$$V = \frac{10t}{T} + 10$$

$$V_{rms} = \sqrt{\int_0^T \left(\frac{10t}{T} + 10\right)^2 dt} = \sqrt{\int_0^T \frac{100t^2}{T^2} + \int_0^T 100 + \int_0^T \frac{200t}{T}}$$

$$= \sqrt{\frac{100 \times \frac{1}{3}}{3T^2} + 100T + \frac{200T^2}{2T}} = \sqrt{\frac{100}{3} + 200} = \underline{15.27 V}$$

$$V_{avg} = \frac{1}{T} \int_0^T$$

A choke coil takes the current of 2 A lagging behind the applied voltage 200 V @ 50 Hz. Calculate Resistance, Inductance & Z. Also determine power consumed when it is connected across 100 V 25 Hz supply.

$$I = 2 \text{ A}$$

$$\phi = 60^\circ$$

$$V = 200 \text{ V} \quad f = 50 \text{ Hz.}$$

$$Z = \frac{V}{I} = \frac{200}{2} = \frac{100 \Omega}{\cancel{2}} = \sqrt{R^2 + (2\pi f L)^2}$$

$$L = \cancel{\frac{200}{2}} \times X_L \times I \quad \Rightarrow \cos \phi = \frac{1}{2} = \frac{R}{100}$$

~~$$Z = \frac{\sqrt{R^2 + (2\pi f L)^2}}{2}$$~~

$$10000 = (50)^2 + 4 \times \pi^2 \times (50)^2 + L^2$$

$$4 = (1 + 4\pi^2 L^2)$$

$$L = \sqrt{\frac{10000 - (50)^2}{4\pi^2 \times 50^2}} = 0.275 \text{ H}$$

Now if $f = 25 \text{ Hz}$

$$X_L = 2\pi f \times L \\ = 43.19 \Omega$$

43.1

$$Z = \sqrt{(50)^2 + (43.19)^2}$$

$$= \underline{66.1\Omega}$$

$$I = \frac{V}{Z} = \frac{100}{66.1} A$$

$$\cos\phi = \frac{R}{Z} = \frac{50}{66.1} = 0.75$$

$$\underline{P = VI \cos\phi}$$

$$= 100 \times \frac{100}{66.1} \times \frac{50}{66.1} = \underline{\underline{112.7 W}}$$

When a certain inductive coil is connected to a dc supply at 240 V. The current in the coil 16A. When the same coil is connected to AC 240 V 50 Hz current is 12.27 A. Calculate R, Z, X_L of the coil
 b) If supply freq is reduced to 60 Hz would the current be greater or lesser than the value given above.

$$R = \frac{240}{16} = \underline{\underline{15\Omega}}$$

$$X_L = \frac{2 \times \pi \times 60 \times 15}{=}$$

$$X_L = 2 \times \pi \times$$

$$X_L \propto f$$

$$Z = \frac{240}{12.27} = \underline{\underline{19.6\Omega}}$$

$$Z^2 = X_L^2 + R^2$$

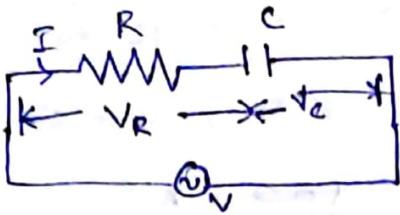
Z↑

$$X_L = \underline{\underline{12.6\Omega}}$$

$$I = \frac{V}{Z}$$

$$L = \frac{X_L}{2\pi f} = \frac{12.6}{2\pi \times 50} = 0.04 H \quad I \downarrow \text{es.}$$

Series R C circuit

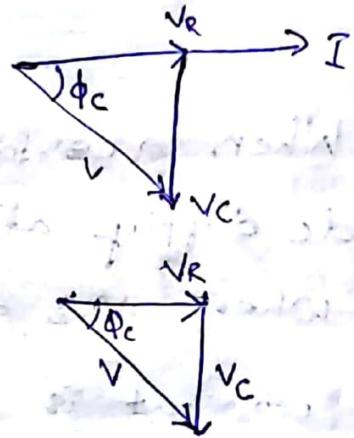


V is the supplied voltage, I is circuit current

V_R is the voltage across the $R = IR$

V_C is the voltage across the $C = I X_C$

$$= \frac{I}{2\pi f C}$$



$$V^2 = V_R^2 + V_C^2$$

$$= (IR)^2 + (I X_C)^2$$

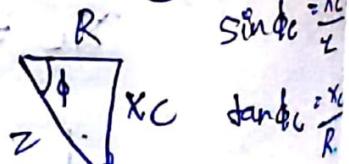
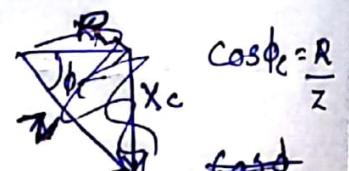
$$\cos \phi_C = \frac{V_R}{V}$$

$$Z^2 = R^2 + X_C^2$$

$$\sin \phi_C = \frac{V_C}{V}$$

$$Z = \frac{V}{I} = \sqrt{R^2 + X_C^2}$$

$$\tan \phi_C = \frac{V_C}{V_R}$$



Q) A capacitor of capacitance $795\mu F$ is connected in series with a non-inductive resistance of 30Ω across $100V, 50\text{ Hz}$ supply. Find Z, I, ϕ , and eqn instantaneous value of current.

$$C = 795\mu F$$

$$R = 30\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 79.5 \times 10^{-6}} = 40.03\Omega$$

$$Z = \sqrt{R^2 + (X_C)^2} = \sqrt{30^2 + (40.03)^2} = 50.02\Omega$$

$$\cos \phi = \frac{R}{Z} = \frac{30}{50.02} = 0.599$$

$$\phi = \underline{\underline{53.2^\circ}}$$

$$I = \frac{V}{Z} = \frac{100}{50.02} = 1.99A = \underline{\underline{2A}}, \quad I_m = \sqrt{2} \times 2 = 2.82A$$

$$I = I_m \sin(\omega t + 53.2^\circ)$$

$$I = \underline{\underline{2.82 \sin(\omega t + 53.2^\circ)}}$$

Q) A 10Ω resistor & $400\mu F$ capacitor are connected in series to a $60V$ sinusoidal supply, current I is $5A$.

Calculate R, ϕ, I_m

$$R = 10$$

$$C = 400\mu F$$

$$V = 60$$

$$I = 5A$$

$$Z = \frac{V}{I} = \underline{\underline{12\Omega}}$$

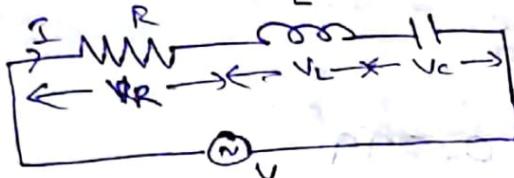
$$X_C = \sqrt{Z^2 - R^2} = \sqrt{144 - 100} = \underline{\underline{6.6 \Omega}}$$

$$6.6 = \frac{1}{2\pi f \times 400 \times 10^6} \Rightarrow f = \underline{\underline{60.2 \text{ Hz}}}$$

$$\cos \phi = \frac{R}{Z} = \frac{10}{12} = \frac{5}{6} = 0.83$$

$$\phi = \underline{\underline{33.5^\circ}}$$

RLC circuit

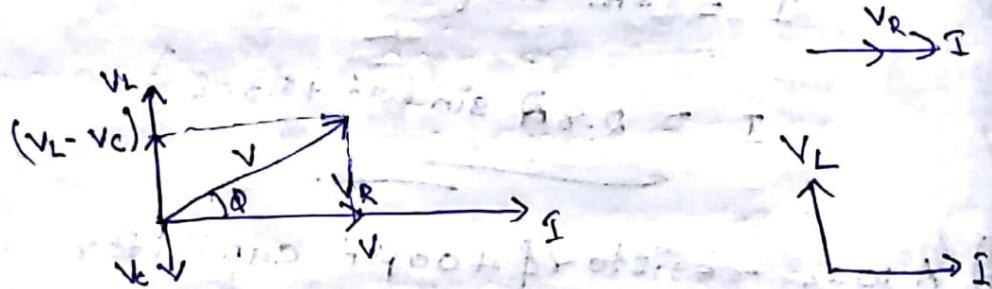


V is the applied voltage I is the current

V_R is the voltage drop across $R = IR$

V_L is the voltage drop across $L = IX_L = I \times 2\pi f L$

V_C is the voltage drop across $C = I \times X_C = \frac{I}{2\pi f C}$



i) $V_L > V_C (X_L > X_C)$

circuit is predominantly inductive.

I lags voltage V by ϕ

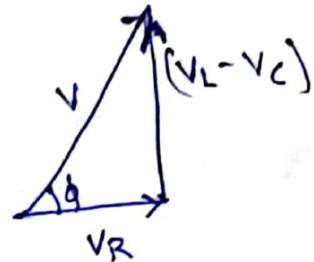
Applied voltage is the phasor sum of V_R , V_L & V_C .

To get this, find the phasor sum of V_L & V_C (Arithmetical difference $V_L - V_C$). Then phasor sum of $V_L - V_C$ and V_R by parallelogram law.

$$V^2 = V_R^2 + (V_L - V_C)^2$$

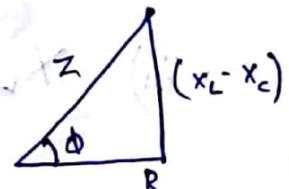
$$= (IR)^2 + (IX_L - IX_C)^2$$

$$\frac{V^2}{R^2} = R^2 + (X_L - X_C)^2$$



Z is the impedance of the series RLC ckt.

It is the total opposition offered to the current flow ~~offered by~~ by the resistance, inductive reactance & capacitive reactance of the ckt.

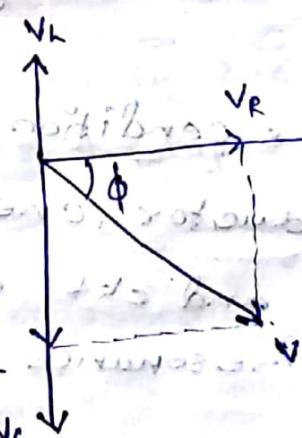


$$\text{ii) } X_L < X_C \quad (V_L < V_C)$$

$$\sin \phi = \frac{X_L - X_C}{Z}$$

$$\cos \phi = \frac{R}{Z}$$

$$\tan \phi = \frac{X_L - X_C}{R}$$

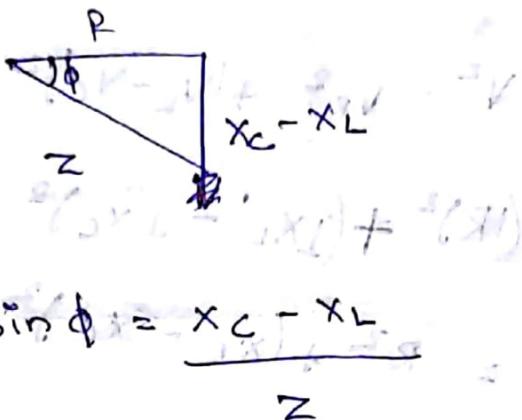


circuit is predominantly capacitive

I leads voltage by ϕ

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$V^2 = V_R^2 + (V_C - V_L)^2$$

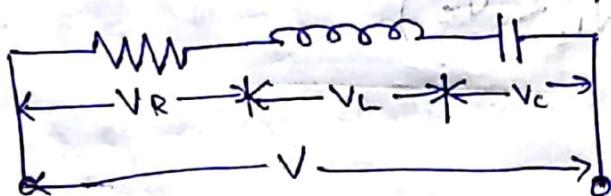


$$\sin \phi = \frac{X_C - X_L}{Z}$$

$$\cos \phi = \frac{R}{Z}$$

$$\tan \phi = \frac{X_C - X_L}{R}$$

(iii) $X_L = X_C$ (Resonance)



Resonance is the condition in a circuit containing at least one inductor, one capacitor when supplied voltage and circuit current are in phase.

At resonance equivalent impedance of the circuit is purely resistive

Power factor for a resonance circuit is unity.

$$X_L = 2\pi f L \quad X_C = 1/2\pi f C$$

$$X_{L_0} = X_{C_0}$$

Let X_{L_0} be inductive reactance $\approx X_C$
be capacitive reactance and Z_0 be the
impedance \neq @ resonance.

$$Z_0 = X_{L_0} - X_{C_0} = 0$$

$$\begin{aligned} X_{L_0} &= X_{C_0} \\ Z_0 &= R \end{aligned}$$

Impedance @ resonance is known as
dynamic impedance.

$$\text{Voltage across } L = V_L = I_0 X_{L_0}$$

$$\text{Voltage across } C = V_C = I_0 X_{C_0}$$

$$\Rightarrow V_L = V_C$$

$$\Rightarrow V_L - V_C = 0$$

at resonance the voltage across inductor is
equal in magnitude and opposite in phase to the
voltage across the capacitor.

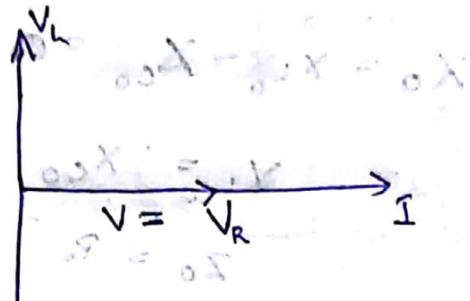
$$V = \sqrt{V_R^2 + (V_{L_0} - V_{C_0})^2} = \sqrt{V_R^2} = V_R = I_0 R$$

Thus @ resonance the entire supply voltage is
used to develop the voltage drop across R
although total reactance voltage is equal to
zero V_{L_0} and V_{C_0} may reach values greater than the

supplied voltage causing the crkt insulation to be damaged.

@resonance $\phi_0 = 0$, $\cos\phi_0 = 1$

ratio of loss resistance with regard to reactance is at resonance



so ratio of V_L to V_R is $\cos\phi_0 = 1$

$$X_L = X_C \quad \text{at resonance frequency}$$

$$\omega X_0 L = \omega V = \frac{1}{\omega C} \quad \text{from equation}$$

$$2\pi f_0 L = 3V = \frac{1}{2\pi f_0 C}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \sqrt{\frac{V}{LC}}$$

General results:

if $Z = R + jX$ then X is positive if $X > 0$

$X = (X_L - X_C)$ is the total reactance

$X_L > X_C \rightarrow X$ is positive
crkt is inductive

$X_L < X_C \rightarrow X$ is negative

if $X < 0$ then $X_C > X_L$ crkt is capacitive

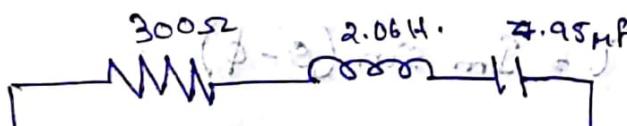
if $X_L = X_C \rightarrow X = 0$ crkt is pure resistive.

$$V \cdot \cos \phi = \frac{R}{Z}$$

$$\tan \phi = \frac{X_L - X_C}{R}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

- Q) A series circuit consists of a 300Ω ^{non-inductive} resistor, a $4.95\mu F$ capacitor and $2.06H$ inductor, of negligible resistance if the supply voltage is $250V$ @ $50Hz$. Calculate i) circuit current ii) phase angle 3) Voltage drop across each element.



Using superposition principle

($\phi - 90^\circ$) $X_L = 2\pi f L$ and $X_C = \frac{1}{2\pi f C}$

$$((\phi - 90^\circ) X_L - (\phi - 90^\circ) X_C) = \frac{1}{2\pi f C}$$

$$26((\phi - 90^\circ) = 647.16 \Omega = 400.3 \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad X = 246.86$$

$$= 388.5 \Omega$$

$$I = \frac{V}{Z} = \frac{250}{388.5} = 0.64A$$

$$\cos \phi = \frac{R}{Z} = \frac{300}{388.5} = 0.77$$

$$\phi = 39.4^\circ \text{ (lag)}$$

$$V_R = I \times R = 0.64 \times 300 = 192 V$$

$$V_L = I \times X_L = 0.64 \times 647.16 = 414.19 V$$

$$V_C = I \times X_C = 0.64 \times 400.35 = 256.19 V$$

V.Vin Power in an AC circuit

Power in a general series ckt.

Let $\vec{V} = V_m \sin \theta$

$$\vec{I} = I_m \sin(\theta - \phi)$$

Instantaneous power

$$P = VI = V_m I_m \sin \theta \sin(\theta - \phi)$$

$$= \frac{V_m I_m}{2} [\cos(2\theta - \phi) - \cos(2\theta + \phi)]$$

$$P_{avg} = \frac{1}{2\pi} \int_0^{\pi} \left[\frac{V_m I_m}{2} [\cos \phi - \cos(2\theta - \phi)] \right] d\theta$$

~~$$= \frac{V_m I_m \cos \phi}{4\pi} \int_0^{\pi} [\cos(2\theta - \phi)] d\theta$$~~

~~$$= \frac{V_m I_m \cos \phi}{2\pi} \int_0^{\pi} [\cos 2\theta \cos \phi - \sin 2\theta \sin \phi] d\theta$$~~

~~$$= \frac{V_m I_m \cos \phi}{2\pi} \left[\cos \phi \left[\frac{\sin 2\theta}{2} \right]_0^\pi + \sin \phi \left[\frac{\cos 2\theta}{2} \right]_0^\pi \right]$$~~

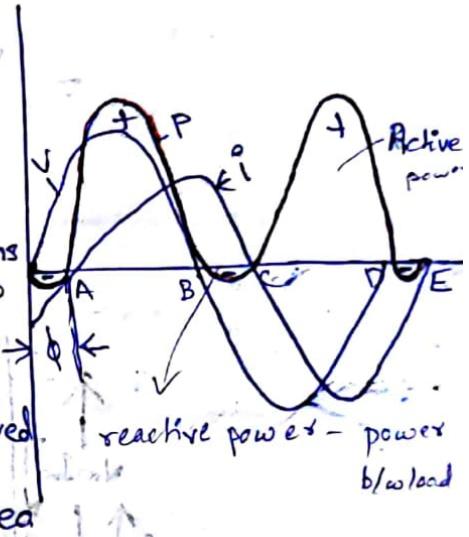
~~$$= \frac{V_m I_m \cos \phi}{2\pi} [\cos \phi \times 0 + \sin \phi]$$~~

$\cos \phi$ is constant for a given ckt

$$= \frac{V_m I_{ms}}{4\pi} \left\{ \cos \phi [0]^{2\pi} - \frac{1}{2} [\sin 2\theta - \phi]_0^{2\pi} \right\}$$

$$= \frac{V_m I_{ms}}{4\pi} \cos \phi (2\pi) - \frac{1}{2} [\sin (4\pi - \phi) - \sin (-\phi)]$$

$$= \frac{\sqrt{2} V_m I_{ms}}{4\pi} \cos \phi = VI \cos \phi$$



The voltage, current & power waveforms in general series circuit are as shown. From the power waveform, it is observed during the interval OA, power is -ve. The area represents the energy returned to the source from the circuit. During interval AB, power is positive, Area represents the energy supplied from the source to load. Intervals BC and CD are repetition of OA and AB respectively. Hence it is observed that during each current (voltage) cycle, a part of energy is called Active energy is consumed while the other part called reactive energy is interchanged b/w source & load.

Power through pure resistance.

$$V = V_m \sin \theta$$

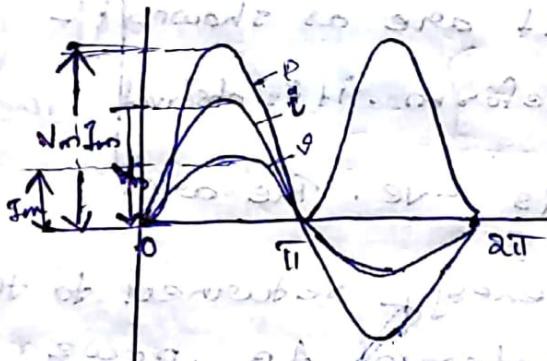
$$I = I_m \sin \theta$$

$$P = \text{instantaneous power} = V_i = V_m I_m \sin^2 \theta$$

$$P_{avg} = \frac{V_m I_m}{2} (1 - \cos 2\theta)$$

$$P_{avg} = \frac{V_m I_m}{2\pi \times 2} \int_0^{2\pi} (1 - \cos 2\theta) d\theta$$

$$= \frac{V_m I_m}{2\pi \times 2} \times 2\pi = 0 = \frac{V_m I_m}{2}$$



Instantaneous power P remains ~~constant~~ positive

irrespective of direction of current. This is because

Voltage & current are in phase. \Rightarrow

Even when V & I are -ve, ϕ is positive

It shows that the power flow is only in the direction of source to load. The energy

received by the resistor is called active energy. It is consumed in R and appears

In the form of heat the rate of this energy consumption is called active power.

Power through pure inductance

$$V = V_m \sin(\theta + 90^\circ)$$

$$I = I_m \sin \theta$$

$$\text{Inst power } P = VI = V_m I_m \frac{\sin(\theta+90^\circ) \cos \theta + \cos(\theta+90^\circ) \sin \theta}{\sin \theta}$$

$$\sin \theta \cos \theta$$

$$\sin(\theta+90^\circ) \cos \theta + \cos(\theta+90^\circ) \sin \theta$$

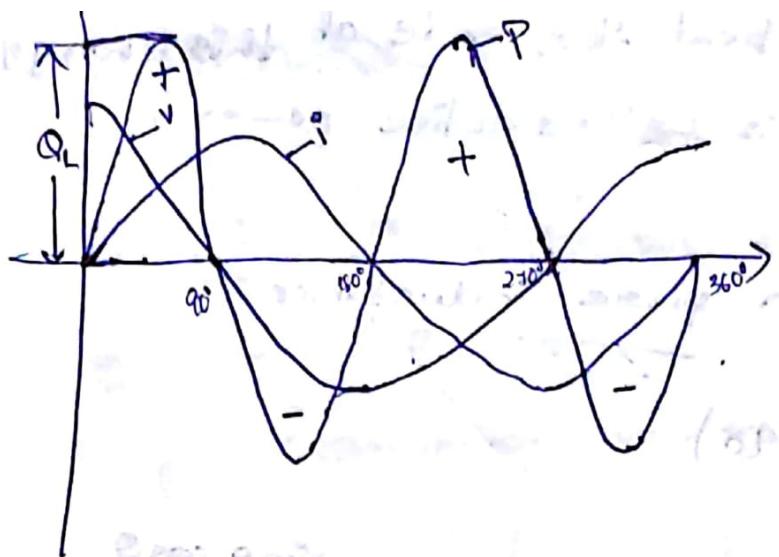
$$= V_m I_m \frac{\sin 2\theta}{2}$$

$$P_{avg} = \frac{V_m I_m \int_0^{\frac{\pi}{2}} \cos 2\theta}{2\pi} \cdot \frac{2\pi}{2}$$

$$= \frac{V_m I_m}{4\pi} [\sin 2\theta]_0^{\frac{\pi}{2}}$$

$$\text{Net average power} = \frac{V_m I_m}{4\pi} \times 0.5 = 0\%$$

and we observe ~~that~~ ~~it~~ ~~is~~ ~~zero~~ ~~because~~ ~~current~~ ~~and~~ ~~voltage~~ ~~are~~ ~~90°~~ ~~out~~ ~~of~~ ~~phase~~



Electric axis
current + cos($\theta + \phi$)
voltage power curve is a sine wave of twice the freq.
of current or voltage wave (power wave
completes two cycles when the current or
voltage wave completes one cycle.)

for when $\theta = 0$ to $\theta = 90^\circ$ power curve
is above the horizontal axis, power is positive
that draws energy from the source.

This energy is stored in the magnetic field
and represented by +ve shaded area b/w
power curve and axis

for $\theta = 90^\circ$ to $\theta = 180^\circ$, power is negative
previously stored energy is returned to
source. This is represented by the -ve
shaded area b/w power curve and horizontal
axis. Thus it is seen that energy stored
in the circuit during the first quarter cycle
is equal to the energy returned to the source

during the second quarter cycle. ~~The 3rd & 4th~~
 quarter cycle is the repetition of ~~1st & 2nd~~
 quarter cycle. Thus total energy dissipated
 is zero. The avg power over a complete
 cycle of current in a purely inductive
 ckt is therefore zero.

Power through pure capacitance

At time $t = 0$, voltage across circuit
 becomes $v = V_m \sin \theta$.
 At time $t = i = I_m \sin(\theta + 90^\circ)$

$$\text{Inst. power, } p = VI = V_m I_m \sin \theta \sin(\theta + 90^\circ)$$

$$= \frac{V_m I_m}{2} \sin 2\theta$$

Integrating over one cycle, we get

$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\theta d\theta$$

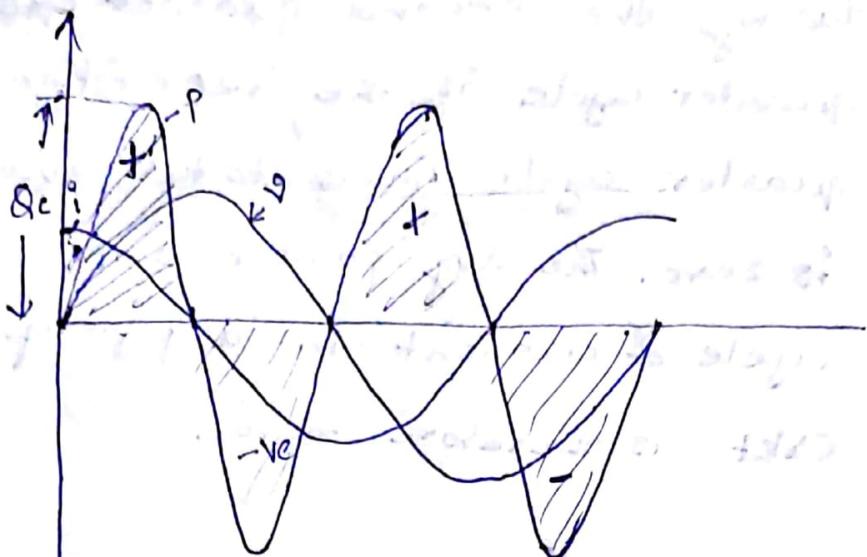
Integrating by parts, we get

$$P_{avg} = \frac{V_m I_m}{2\pi} \left[\frac{\cos 2\theta}{2} \right]_0^{2\pi}$$

Integrating over one cycle, we get

$$P_{avg} = \frac{V_m I_m}{2\pi} \times 0 = 0 \text{ W}$$

Thus power delivered by source is zero.



~~discharges every quarter wave~~

Power wave is a sine wave with freq twice that of voltage or current wave. For $0 \rightarrow 90^\circ$, power wave is above the horizontal axis and is +ve. The applied voltage gradually increases from 0 to its maximum value, i.e. it draws energy from the source and capacitor is charged. Energy is stored in the electric field of the capacitor. During the $\frac{1}{4}$ th quarter cycle, power curve is below horizontal axis and is -ve. voltage gradually decreases max to zero. Capacitor discharges and energy is returned to the source.

Defn: \rightarrow (Var) :
Reactive voltampères (Q_W)

$$\text{peak value of } P, Q_L = \frac{V_m I_m}{2} = VI$$

In a purely inductive crkt, $V = V_L = IX_L$

$$\text{and } Q_L = V_L I = \frac{V_L^2}{X_L} = I^2 X_L$$

Q_L is the reactive voltampere for an inductive crkt, energy which is continuously exchanged b/w the source and reactive load is called reactive energy. By convention Q_L is +ve.

Reactive voltampere (Q_C)

$$\text{peak value of } P, Q_C = \frac{V_m I_m}{2} = VI$$

In a purely capacitive crkt, $V = V_C = IX_C$

$$Q_C = V_C I = \frac{V_C^2}{X_C} = I^2 X_C$$

Q_C is the reactive voltampere for a capacitive crkt. It is measured in Var. It is the rate of interchange of energy b/w capacitor & load & source.

By convention Q_C is taken -ve.

Voltamperes: product of RMS values of voltage and current in a circuit is called circuit voltamperes. It is also called apparent power denoted by S .

$$S = V I$$

$$V = I Z$$

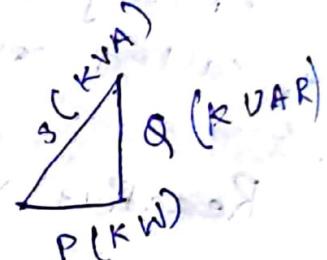
Power Triangle

1) Resistance circuit

$$P_R = I^2 R = \frac{V^2}{Z} = I^2 Z \cos \phi = \frac{R}{Z} (S)$$

$$\Phi_R = 0$$

Apparent power



2) Inductive circuit (purely)

$$P_L = 0$$

$$Q_L = V_L \cdot I - +ve \\ = I^2 X_L = \frac{V_L^2}{X_L}$$

$$\cos \phi = P/S$$

$$\sin \phi = Q/S$$

3) Purely capacitive circuit

$$P_C = 0$$

$$Q_C = V_C I = I^2 X_C = \frac{V_C^2}{X_C} (-ve)$$

$$S = V I$$

$$Q = V I \sin \phi$$

$$P^2 + Q^2 = S^2$$

$$S = P \pm j Q$$

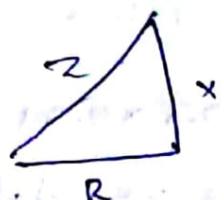
Net reactive voltamperes

$$Q = Q_L - Q_C$$

$$= I^2 X_L - I^2 X_C$$

$$= I^2 (X_L - X_C)$$

$$Q_r = I^2 X$$



$$\cos \phi = R/Z$$

$$\sin \phi = X/Z$$

$$V = IZ \quad X = Z \sin \phi$$

$$Q = VI \sin \phi = I^2 X = \frac{V^2}{Z} = I^2 Z \sin^2 \phi \text{ (VAR)}$$

A voltage of $250 \angle 0^\circ$ is applied to an inductive circuit of impedance $(5+11j) \Omega$. Calculate @ circuit current (a) P.F (b) Power consumption (c) Apparent power (d) Reactive power.

a) $Z = (5+11j)$

$$V = 250 \angle 0^\circ \\ = 250 + 0j$$

$$I = \frac{V}{Z} = \frac{250 \angle 0^\circ}{12.08 \angle 65.56^\circ} = 20.70 \angle -65.56^\circ \\ = 8.08 - 18.84j$$

b) P.F = $\cos(-65.56^\circ)$

$$= 0.4137$$

$$R = Z \cos \phi = 0.91 \Omega$$

$$P = VI \cos \phi = 0.4137 \times 12.08 \angle 65.56^\circ = -0.91 \times 12.08 \angle 65.56^\circ$$

$$= 2.667 \Omega$$

$$= -4.54 - 10.0j \Omega$$

Power consumption = $V I \cos \phi$

$$= 250 \times 20.70 \times 0.4137 = 2140.45 \text{ W}$$

Apparent power (S) = $VI = (250+0j)(8.08-18.84j) = 250 \times 20.70 = 5175 \text{ VA}$

Reactive power = $\Theta = VI \sin \phi$

$$(\text{lagging}) = 250 \times 20.7 \times 0.91 \\ = -4709.25 \text{ Watts (Volts)}$$

An alternative voltage $(80 + j60)$ is applied & current $(-4 + 10j) A$, find Z, P, ϕ

$$V = 100 \angle 36.87^\circ \\ I = 10.77 \angle 111.8^\circ \\ Z = \frac{V}{I} = \frac{100 \angle 36.87^\circ}{10.77 \angle 111.8^\circ} = 9.28 \angle -74.93^\circ \\ = 2.413 - j8.96$$

$$R = \underline{\underline{2.413 \Omega}}$$

$$P = \underline{\underline{0.89 R}} = (10.77)^2 \times (2.413) = 280.89 \text{ W}$$

$$P = VI \cos \phi$$

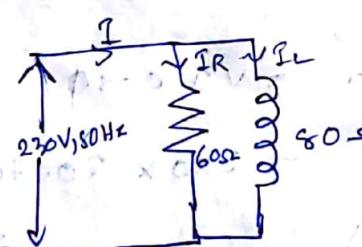
$$\cos \phi = \frac{280.89}{100 \times 10.77} = 0.26 \Rightarrow \phi = \underline{\underline{74.9^\circ}}$$

A 60Ω resistance is connected in parallel

with an inductive resistance of 80Ω to a

250V 50Hz, calculate ① current through R & X_L .

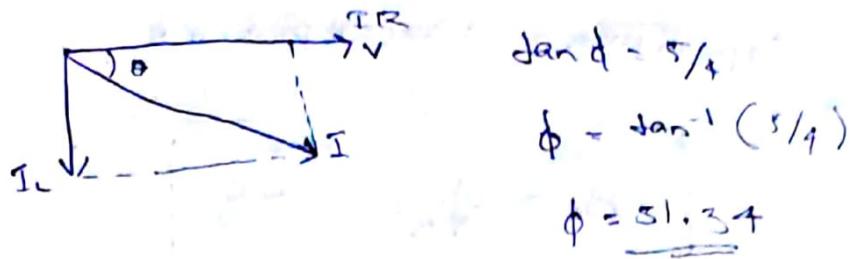
② Supply current ③ phase current



$$I_R = \underline{\underline{4A}}$$

$$I_L = \underline{\underline{3A}}$$

$$\therefore I = \sqrt{I_R^2 + (I_C)^2} \\ = \sqrt{16+9} = \underline{\underline{5A}}$$



$$Z = \frac{60 + j80}{60 + 80j} = \underline{\underline{38.4 - j52}}$$

Admittance

It is the reciprocal of impedance. It is denoted by (Y). Its S.I. units are siemens.

(3)

$$Y = \frac{1}{Z} = \frac{I}{V}$$

$$Z = R + jX$$

For pure resistor

$Y_R = \frac{1}{Z_R}$	$\frac{1}{Z_R}$
$Y_R = G$	

For pure inductance circuit.

$$Y_L = \frac{1}{Z_L} = \frac{1}{jX_L} = \frac{j}{X_L} = -jB_L$$

$$B_L = \frac{1}{X_L} = \frac{1}{\omega L}$$

For purely capacitor

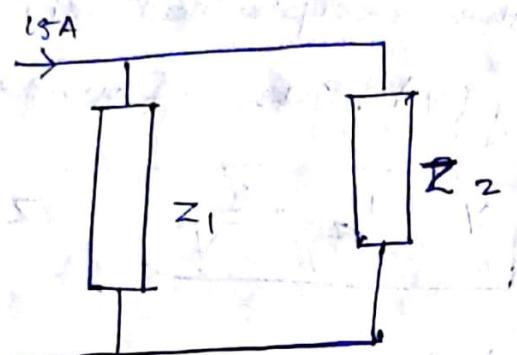
$$Y_C = \frac{1}{Z_C} = \frac{1}{-jX_C} = \frac{j}{X_C} + jB_C$$

Capacitive susceptance

$$B_C = \frac{1}{X_C} = \omega C$$

Two circuit impedances given by $Z_1 = (10 + j10)$
 ω or $Z_2 = (6 - j8) \Omega$ are connected in parallel.

If total current is $15A$. What is the power consumed by each branch?



$$Z_1 = 18.03 \angle 56.31^\circ \quad Z_2 = 10 \angle -53.13^\circ$$

$$Z_{eq} = \frac{Z_1 \times Z_2}{Z_1 + Z_2} = \frac{18.03 \angle 56.31^\circ \times 10 \angle -53.13^\circ}{10 + j15 + 6 - j8}$$

$$= \left(\frac{180.3 \times 3.18}{16 - 7j} \right) = \frac{180.3 \times 3.18}{17.46 \times 23.63}$$

$$10.33 \angle 26.81^\circ = 9.23$$

$$V = iZ = 15 \times 10.33 = 154.95 V$$

$$I \text{ through } Z_1 (I_1) = \frac{154.95}{180.3} = 8.60$$

$$I \text{ through } Z_2 (I_2) = \frac{154.95}{10} = 15.495$$

$$P_1 = I^2 R = 10 \times (8.60)^2 = 737.8 W$$

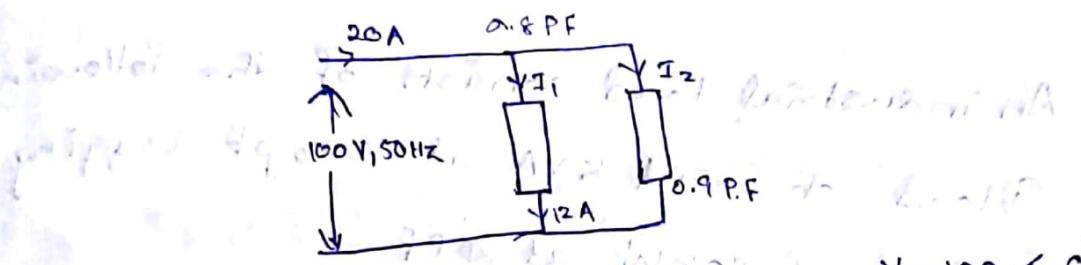
$$P_2 = (15.495)^2 \times 6 = 1437.78 W$$

Two inductive coils A & B are in parallel across a 100 V 50Hz supply. coil A takes 12A at 0.9 power factor and the total current for both coils is 20A at 0.8 power factor determine.

① Equivalent resistance & reactance

② Individual resistance & reactance

③ Power factor of coil B.



$$I = 20 \angle -\cos^{-1}(0.8)$$

$$= 20 \angle 36.86^\circ$$

~~$$I = I_1 + I_2$$~~

$$(W.A) R I_1 = 12 \angle -\cos^{-1}(0.9)$$

$$= 12 \angle -25.84^\circ$$

~~$$I_2 = 5.2 - 6.8^\circ (I_1 - I_2)$$~~

Equivalent Impedance

$$Z = \frac{V}{I} = \frac{100 \angle 0^\circ}{20 \angle -36.86} = 5 \angle 36.86^\circ$$

$$Z_{eq} = R_{eq} + jX_{eq} = 4 + 2.99j$$

$$Z_1 = \frac{V}{I_1} = \frac{100 \angle 0^\circ}{12 \angle -25.84} = 8.33 \angle 25.84^\circ$$

$$I_2 = I - I_1 = 5.2 - 6.79j = 8.552 \angle -52.55^\circ$$

$$Z_2 = \frac{100 \angle 0^\circ}{8.55 \angle -52.55^\circ} = 11.695 \angle 52.55^\circ \\ = 7.11 + j9.8j$$

$$\text{Power factor of coil B} = \cos(-52.55^\circ) \\ = 0.6080 \underline{\text{W}}$$

An industrial load consists of the following

① load of 2000 KVA at 0.8 pf lagging

② load of 50kW at 0PF

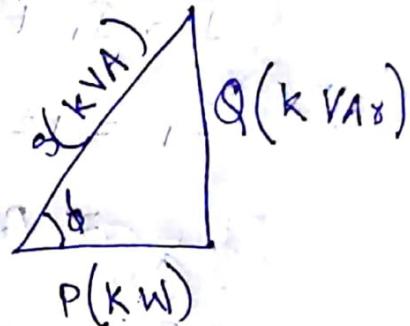
③ 48 kW at 0.6 PF leading

Calculate KW, KVar, KVA, overall PF.

~~$\text{pf cos}(\cos) = \phi = 36.86^\circ$~~

~~$\phi = 36.86^\circ$~~

$$\cos\phi = \frac{\text{KW}}{\text{KVA}}$$



$$P_{\text{1}}(\text{KW}) = \underline{160 \text{ KW} \pm j20}$$

$$\cos\phi = \frac{\text{KW}}{\text{KVA}}$$

$$Q_{\text{1}} = \sin\phi * 200 = 120 \quad \sin\phi = \frac{\text{KVar}}{\text{KVA}}$$

$$S_{\text{1}} = \underline{160 + j120}$$

+ — lag
- - leading

$$S_{\text{2}} = 50 \pm j0$$

$$S = P \pm j Q$$

S_{total}

48 kW

$$\cos \phi = 0.6$$

$$\phi = 53.13^\circ$$

KVA

$$Q = 63.99 \approx \underline{64}$$

$$S_3 = 48 + j64$$

$$S_{\text{total}} = \underline{258 + j56}$$

$$P_{\text{rated}} = \underline{258 \text{ W}}$$

$$PQ_{\text{total}} = 56 \text{ KVA}$$

$$P_{\text{overall}} \cdot \phi = \tan^{-1} \left(\frac{56}{258} \right) = \underline{\underline{12.24^\circ}}$$

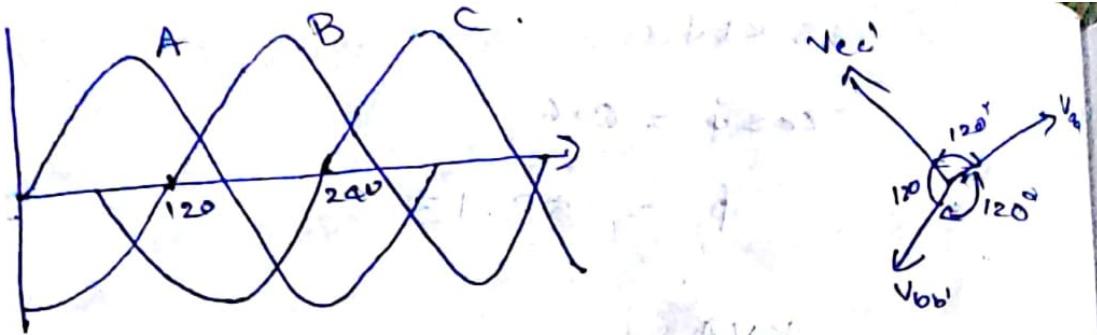
$$\cos \phi = \underline{\underline{0.97 \text{ lagging}}}$$

3 phase Systems Advantages of 3 phase

$$V_{aa'} = V_m \sin \omega t \quad V_{cc'} = V_m \sin(\omega t + 120^\circ)$$

$$V_{bb'} = V_m \sin(\omega t - 120^\circ) \quad V_{cl} = \underline{\underline{V \angle +120^\circ}}$$

When 3 identical coils are placed with axis 120° apart from each other & rotated in a uniform manner, a sinusoidal wave is formed.



Order in which the phase voltage reach the max voltage is called phase sequence.

End of the coil where the current leaves or simply is starting end / start is end of the coil where the current enters is ending finishing end / finish.

No. of wires can be reduced by

Inter connecting phases

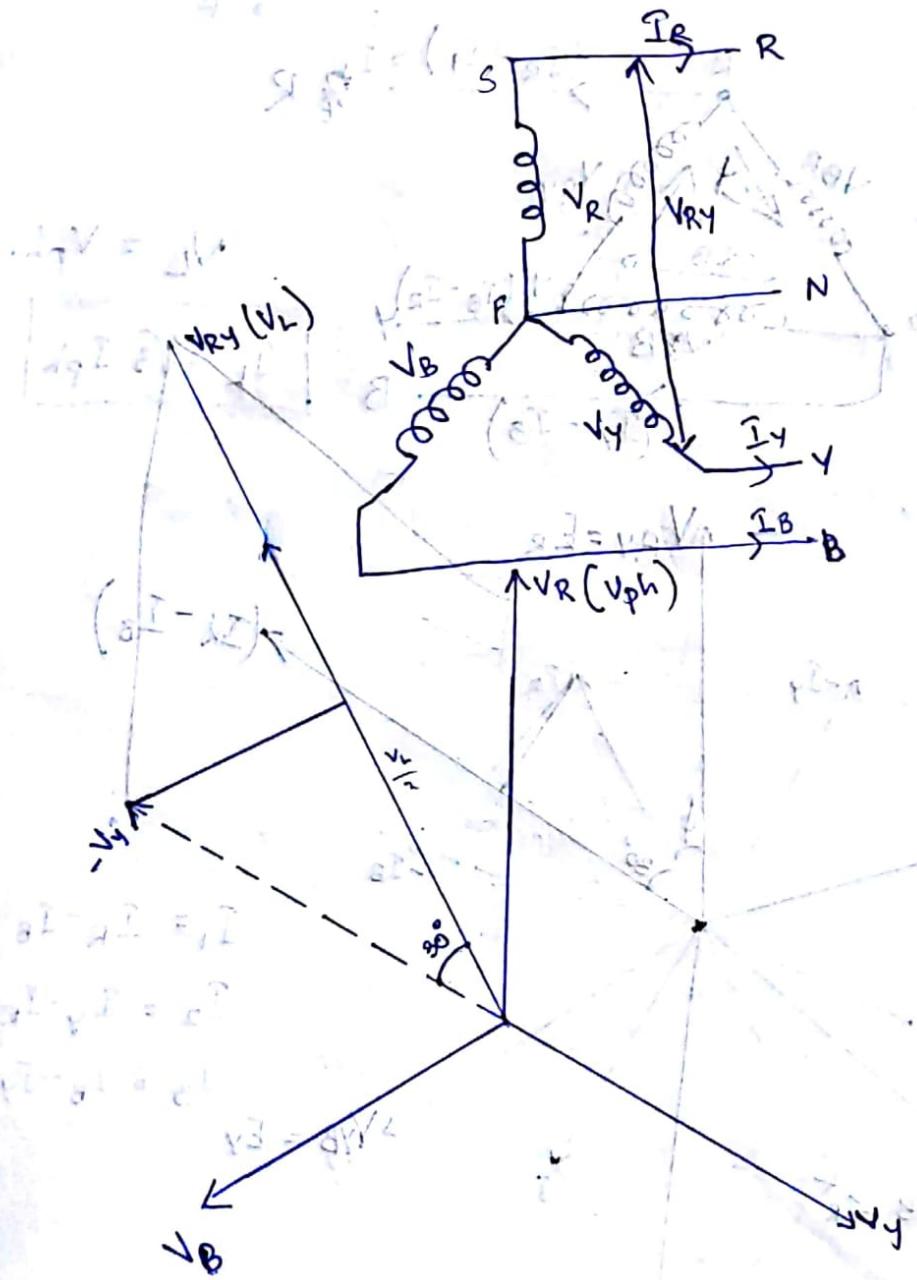
Advantages of 3 phase systems

- * No. of wires will be less
- * Power delivered is constant. In single phase crkt, the power is pulsating & objectionable for many applications.
- * For a given frame size a 3 phase machine gives a higher O/P than single phase machine.
- * For transmitting the same amount of power at the same voltage, a 3 phase transmission line requires less conductor material than a single phase.

line. 3 phase transmission system is so cheaper.

- * Three phase motors have uniform torque whereas most of the single motors have pulsating torque
- * Comparing with single phase motor, three phase induction motor has high power factor & efficiency. Also they are self starting.

3 φ 4 Wire Systems (Star Connection)



$$I_L = I_{ph}$$

$$\begin{aligned}V_L &= V_{RY} \\&= V_{YB} \\&= V_{BR}\end{aligned}$$

$$\cos 30^\circ = \frac{N_L}{2 V_{ph}}$$

$$V_L = \sqrt{3} V_{ph}$$

$$\begin{aligned}&\approx V_L I_L \text{ cost.} \\&P_L = V_{ph} I_{ph} \cos 30^\circ\end{aligned}$$

$$\frac{2}{3} \frac{9}{49}$$

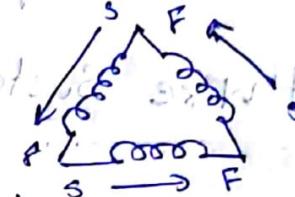
Total power: $3 \times \text{Power}$

$$= 3 \times V_{ph} I_{ph} \cos \phi$$

$$\text{Input power and work} = 3 \times \frac{V_L I_L \cos \phi}{\sqrt{3}}$$

$$\boxed{P_{ph} = \sqrt{3} V_L I_L \cos \phi}$$

Delta Connection

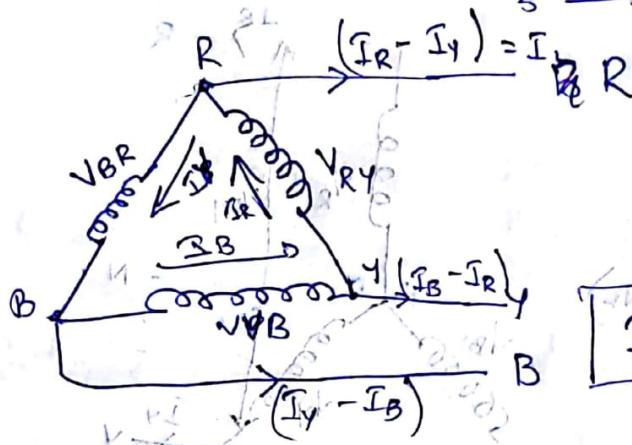


$$\delta q I = \omega L$$

$$\delta V = V$$

$$\delta V = V$$

$$\delta V =$$



$$V_L = V_{ph}$$

$$\boxed{I_L = \sqrt{3} I_{ph}}$$

$$V_{RY} = E_R$$

$$(V_R - V_Y)$$

$$(I_R - I_B)$$

$$\delta q V = V$$

$$\delta q V = V$$

$$\delta q V = V$$

$$I_1 = I_R - I_B$$

$$I_2 = I_Y - I_R$$

$$I_3 = I_B - I_Y$$

$$V_{BR} = E_B - I_R$$

$$V_{YB} = E_Y$$

$$(I_Y - I_R)$$

$$\cos 30^\circ = \frac{I_R - I_B}{2 \times I_R} = \frac{I_2}{2 I_R} = \frac{I_L}{2 I_{ph}}$$

$$I_{ph} I_L = \frac{\sqrt{3}}{2} \times 2 \times I_{ph} \quad I_L = \sqrt{3} I_{ph}$$

$$\text{Power (phase)} = V_{ph} I_{ph} \cos \phi$$

$$\text{Total power} = 3 \times V_{ph} I_{ph} \cos \phi = \sqrt{3} V_L I_L \cos \phi$$

A ^{3 phase} star connected alternator supplies a delta connected load impedance of each branch is $(8 + j6) \Omega/\text{ph}$.

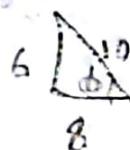
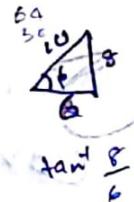
Line voltage is 230V. Determine ① I in load branch ② Power consumed ③ Power factor ④ Reactive power of the load.

a) $I_{ph} = \frac{230}{10} = \underline{23A}$ $Z = 10\Omega$

b) $P_{active} = \underline{V_L I_L \cos \phi} = 3 V_{ph} I_{ph} \cos \phi$

$P_{reactive} = \underline{\sqrt{3} V_L I_L \sin \phi}$

3) $\cos \phi = \frac{8}{10} = \underline{0.8}$



Comparison b/w

Star & delta

↳ Applications -

* Comparison b/w Star and Delta connection.

Star connection	Delta connection
i, A star connection is a 4 wire connection	(i), A Delta connection is a 3 wire connection .
(ii) Two types of star connection systems are possible : 4-wire, 3-phase system and 3-wire 3 phase system	(ii), In Delta connection, only 3-wire 3 phase system is possible .
(iii), Out of the 4 wires, 3 wires are the phases and 1 wire is the neutral.	(iii), All the 3 wires are phases in a Delta connection .
(iv), The common pt. of the star connection is called Neutral or star point.	(iv), There is no neutral in Delta connection .
(v), line voltage & phase voltage is different $V_L = \sqrt{3} V_P$	(v), line voltage and phase voltage are same .
(vi), line current & phase current is same .	(vi), line current & phase current is different . $I_L = \sqrt{3} I_P$