3.4. THREE PHASE SYSTEMS

Introduction

The a.c. circuits discussed so far are termed as single phase circuits because they contain a single alternating current and voltage wave. A single phase generator producing a single phase supply has only one armature winding. A two phase generator producing two phase voltage has two windings displaced by 90^0 and a three phase generator has three winding displaced by 120^0. In general we can say that a poly phase system has many phases or circuits, each phase having a single alternating voltage of equal magnitude and frequency but displaced from one another by equal electrical angles. Although several polyphase systems are possible the three phase system is the most popular one.

3.4.1. Reasons for the use of Three Phase System

Electric power is generated, transmitted and distributed in the form of 3 phase power. Homes and small establishments have single phase power but this merely represents a tap-off from the basic three phase system.

Advantages of three phase power over single phase power are

1. For a given size of frame, a three phase generator or motor has greater output than that of a single phase genertor.
2. Three phase generators work in parallel without any difficulty.
3. Three phase transmission line requires lesser amount of conductor material for transmitting the same amount of power over a single phase line.
4. Three phase motors possess uniform torque whereas single phase motors possess a pulsating torque.
5. Poly phase induction motors are self starting whereas single phase a.c. motors are not self starting.

3.4.2. Elementary three phase alternator

Figure shows an elementary three phase alternator. The three identical coils A, B, and C are symmetrically placed in such a way that emf s induced in them are displaced by 120^0 (elect) from one another. Since the coils are identical and are subjected to the same rotating field the emf s induced in them will be of same magnitude and frequency.

The equations of the three emf s are

$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin (\omega t - 120^0)$$

$$e_C = E_m \sin (\omega t - 240^0)$$

Figure shows the wave diagram of the three emf s and also the phasor diagram.

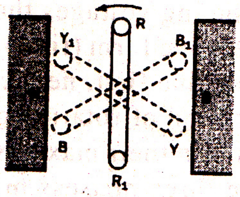

Fig. 3.19a

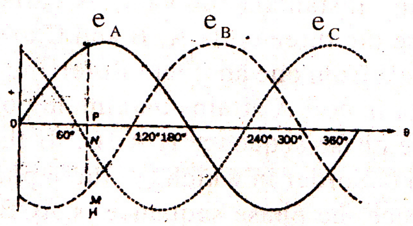

Fig. 3.19b

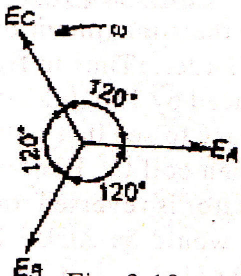

Fig. 3.19c

It can be proved that the sum of the three emfs at any instant is zero.

1. $$e = e_A + e_B + e_C$$
$$= E_m\,[\mathrm{Sin}\ \omega t + \mathrm{Sin}\ (\omega t - 120^0) + \mathrm{Sin}\ (\omega t - 240^0)]$$
$$= E_m\,[\mathrm{Sin}\ \omega t + 2\ \mathrm{Sin}\ (\omega t - 180^0)\ \mathrm{Cos}\ 60^0]$$
$$= E_m\,[\mathrm{Sin}\ \omega t - 2\ \mathrm{Sin}\ \omega t\ \mathrm{Cos}\ 60^0] = 0$$

2. Referring to the wave diagram the sum of the three emfs at any instant is zero. For example at the instant P, ordinate PL is positive while the ordinates PN and PH are negative. If actual measurements are made it will be seen that

$$PL+(-PN)+(-PH) = 0$$

3. Since the three windings or coils are identical, $E_A = E_B = E_C = E$ in magnitude As shown in figure 3.19 resultant of E_A and E_B is E_r and to its magnitude is 2E Cos $60^0 = E$. This resultant is equal and opposite to E_C. Hence the resultant of the three emfs is zero.

4. Using complex algebra we can again prove that the sum of the three emf s is zero. Thus taking E_A as reference phasor we have

$$E_A = E\underline{/0^0} = E + j0$$
$$E_B = E\underline{/\bar{1}20^0} = E(-0.5 - j0.866)$$
$$E_C = E\underline{/-240^0} = E(-0.5 + j0.866)$$
$$E_A + E_B + E_C = (E + j0) + E(-0.5 - j0.886) + E(-0.5 + j0.866) = 0$$

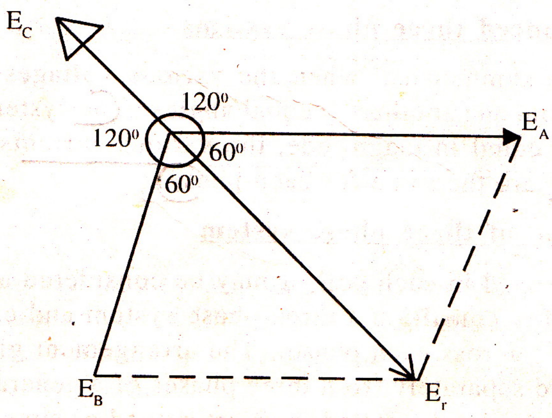

Fig. 3.20

3.4.3 **Phase sequence:**

The order in which the voltages in the three phases or coils reach their maximum or any instantaneous value is called the phase sequence or the phase order. Thus in figure the three coils A, B and C are producing voltages that are displaced by 120^0 (electrical) from one another. Referring to the wave form (fig. 3.19b) it is easy to see that voltage in coil A attains maximum positive value first, next coil B and then coil C. Hence the phase sequence is ABC. If the direction of rotation of the alternator is reversed, then the order in which the three phases attain their max positive value would be ACB. Hence the phase sequence is ACB. The three phasess may be numbered (1,2,3) or lettered (A,B,C) or the three phases may be named after the three natural colours that is Red R, Yellow Y and blue B. In this case phase sequence is RYB or RBY.

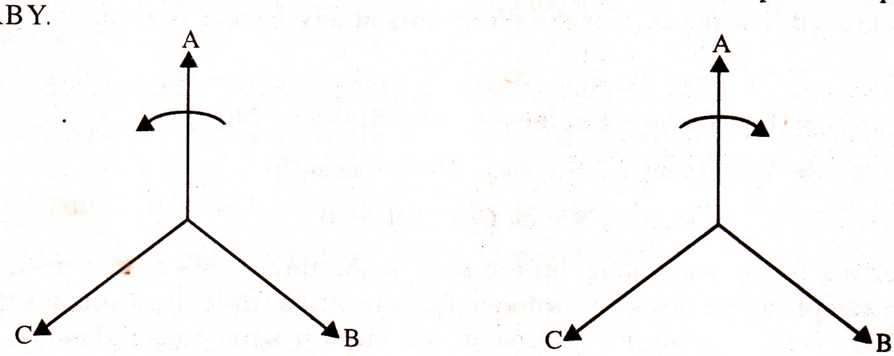

Fig. 3.21

3.4.4 **Double subscript notation:**

The double subscript notation is a very useful concept and may be found advantageous in the analysis of three phase systems. In this notation two letters are placed at the foot of the symbol for voltage or current. The two letters indicate the two points between which voltage or current exists and the order of the letters indicates the relative polarity of voltage or current during positive half cycle. Thus V_{RY} indicates a voltage V between points R and Y with point R being positive w.r.t. Y during its positive half cycle.

ie. $V_{RY} = -V_{YR}$

Again I_{RY} indicates a current I between points R and Y and that its direction is from R to Y during its positive half cycle.

Advantage of using double subscript notation is that the subscripts and the order of the subscripts describe the quantity completely.

3.4.5 **Symmetrical and balanced three phase systems**

A system is said to be symmetrical when the various voltages are equal in magnitude and are displaced from one another by equal angles. The system is balanced when the various voltages are equal in magnitude, the various currents are equal in magnitude and the phase angle are the sàme for each phase.

3.4.6 **Methods of connection of three phase system**

Since a voltage is generated in each coil, it may be considered as a source of voltage. The three coils together constitute a three phase system and each coils is a phase. Let a load be connected across each phase. The arrangement given in figure 3.27 shows three loads supplied separately from three phases of a generator. The end of a coil where the current leaves may be called the starting end or simply the start.

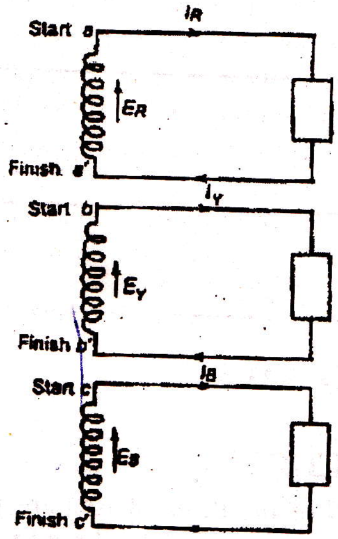

Fig. 3.22

The other end where the current enters the coil is called the finishing end or simply the finish. The ends a,b,c are the starting ends while a′,b′,c′ are the finishing ends. The arrangement thus shown requires six wires to connect the loads. This is equivalent to three separate single phase systems. Such a system is called a three phase, six wire system. The number of connecting wires may be reduced by the interconnection of the phases to form a single three phase a.c. system. There are two methods of interconnecting the three phases. These are called (1) Star (Y) and (2) Delta (Δ) connections.

1. In Y connection, similar ends (start or finish) of the three phases are joined together within the alternator and the three lines are run from the other free ends as shown in figure (Fig. 3.23a). The common point N may or may not be brought out. If a neutral conductor is present it is called a three phase four wire system.

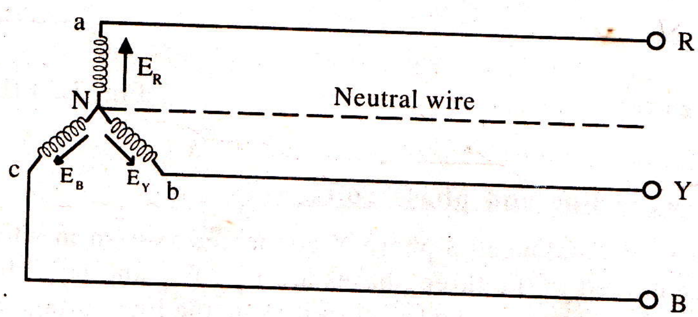

Fig. 3.23(a)

2. In Δ connection, dissimilar ends (start or finish) of the phases are joined to form a closed mesh and the three lines are run from the junction points as shown in figure (Fig. 3.23b). In Δ connection no neutral point exists and hence only a three phase three wire system can be formed.

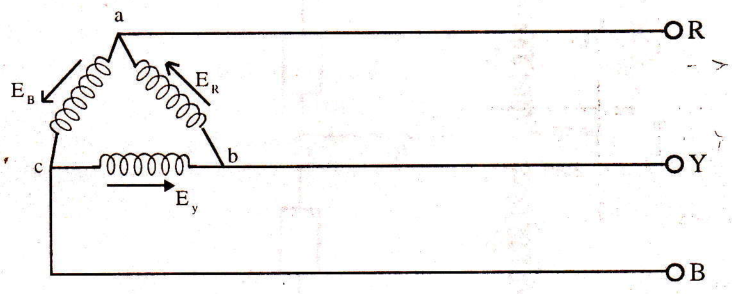

Fig. 3.23(b)

3.5. STAR OR WYE CONNECTION

In this method, similar ends of the three phases are joined together to form a common junction N called star or neutral point. The three line conductors are run from the three free ends and are designated as R,Y and B. The voltage between any line and the neutral point ie. voltage across each winding is called phase voltage while the voltage between any two lines is called the line voltage.

The currents flowing in the phase are called phase currents and those flowing in the lines are called the line currents. Phase sequence is RYB.

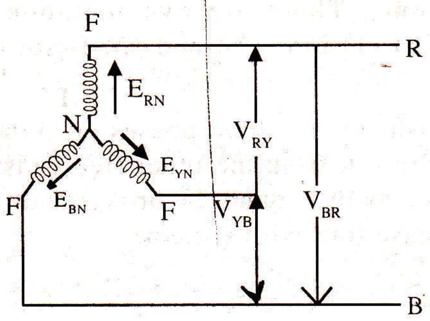

Fig. 3.24 (a)

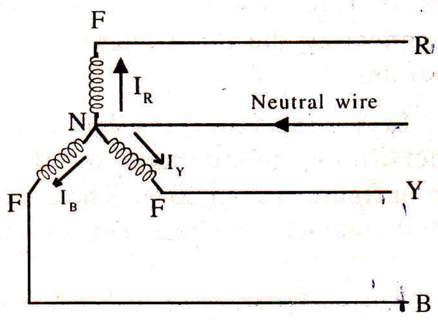

Fig. 3.24 (b)

3.5.1 Relation between line and phase voltages

Figure 3.24 shows a balanced 3 phase Y connected system in which the r.m.s. values of the emfs generated in the three phases are E_{RN}, E_{YN} and E_{BN}. It is clear that the potential difference between any two line terminals ie the line voltage is the phasosr difference between the potential of these terminals w.r.t. neutral point ie

$$V_{RY} = E_{RN} - E_{YN}$$

$$V_{YB} = E_{YN} - E_{BN}$$

$$V_{BR} = E_{BN} - E_{RN}$$

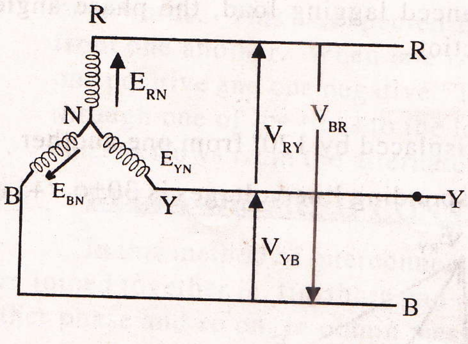

Fig. 3.25

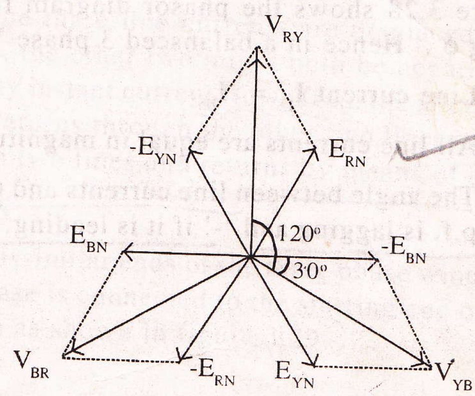

Fig. 3.26

Considering lines R and Y the line voltage V_{RY} is equal to the phasor difference of E_{RN} and E_{YN}. To subtract E_{YN} from E_{RN} reverse the phasor E_{YN} and find its phasor sum with E_{RN} as shown in the phasor diagram. The two phasors E_{RN} and $-E_{YN}$ are equal in magnitude and equal to Eph and 60^0 apart.

$$V_{RY} = 2Eph \cos 60^0/2 = 2Eph \cos 30^0$$

$$= \sqrt{3}\ Eph$$

Similarly $V_{YB} = \sqrt{3}\ Eph$

$V_{BR} = \sqrt{3}\ Eph$

Hence in a balanced three phase Y connection

1. Line voltage = $\sqrt{3}$ Eph .

   All line voltages are equal in magnitude but displaced by 120 degree from one another.

2. Line voltages are 30^0 ahead of their respective phase voltages.

3.5.2. Relation between line and phase currents

In Y connection each line conductor is connected in series to a separate phase as shown in figure 3.27. Therefore current in a line conductor is the same as that in the phase to which the line conductor is connected

Line current I_L = Iph.

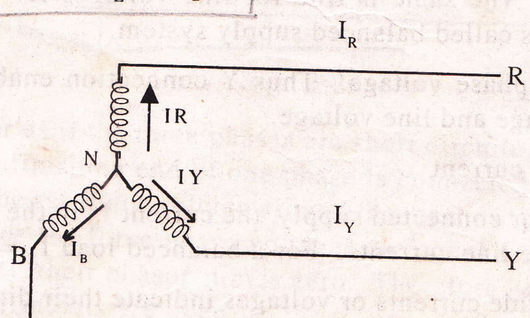

Fig. 3.27

Figure 3.28 shows the phasor diagram for a balanced lagging load, the phase angle being ϕ . Hence in a balansced 3 phase Y connection.

1. Line current $I_L = I_{ph}$.
2. All line currents are equal in magnitude but displaced by 120^0 from one another.
3. The angle between line currents and the corresponding line voltages is $30 \pm \phi$, '+' if p.f. is lagging and '-' if it is leading.

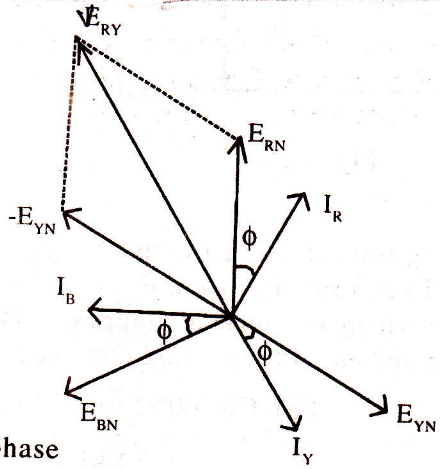

Fig. 3.28

Power

Total power P = 3 x power in each phase

$$= 3 \times E_{ph} \times I_{ph} \times \cos\phi$$

$$\boxed{= 3\, E_{ph}\, I_{ph} \cos\phi}$$

For Y connection $E_{ph} = \dfrac{V_L}{\sqrt{3}}$ and $I_{ph} = I_L$

$$P = 3\frac{V_L}{\sqrt{3}} \times I_L \times \cos\phi$$

$$\boxed{P = \sqrt{3}\, V_L\, I_L \cos\phi}$$

$\cos\phi$ is the power factor and is the phase difference between phase voltage and the phase current.

3.5.3 Points to remember

1. The three phase voltage [ie. E_{RN}, E_{YN}, E_{BN}] are equal in magnitude but displaced 120^0 from each other. The same is true for line voltages ie. V_{RY}, V_{YB} and V_{BR}. Such a supply system is called balanced supply system.
2. Line voltage = $\sqrt{3}$ x phase voltage. Thus Y connection enables us to use two voltages ie. phase voltage and line voltage.
3. Line current = Phase current
4. For a 3 phase 4 wire star connected supply, the current I_N in the neutral wire is the phasor sum of the three line currents. For a balanced load $I_N = 0$
5. The arrowheads alongside currents or voltages indicate their directions when they are assumed to be positive and not their actual direction at a particular instant. At no instant will all the three line currents flow in the same direction either outwards

or inwards. This is expected because the three line currents are displaced 120° from one another. When one is positive, the other two might both be negative or one positive and one negative. Thus at any instant current flows from the alternator through one of the lines to the load and returns through the other two lines or else current flows from the alternator through two lines and returns by means of third.

3.6 DELTA OR MESH CONNECTION

In this method of interconnection the dissimilar ends of the three phase windings are joined together ie. finishing end of one phase is connected to the starting end of the other phase and so on, to obtain mesh or delta as shown in figure 3.29.

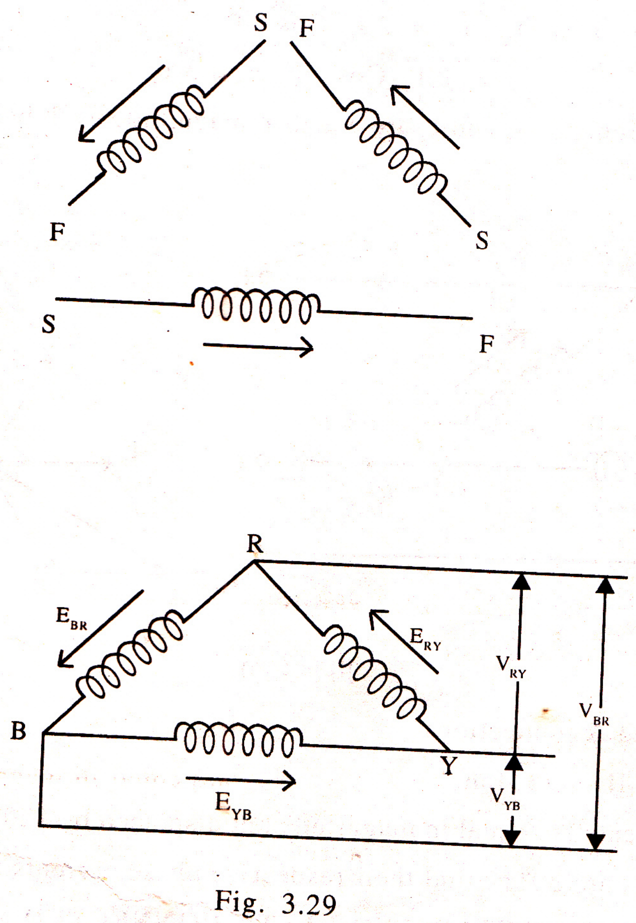

Fig. 3.29

It may appear as if the three phases are short circuited on themselves. But that is not the case. The finishing end of one phase is connected to the starting end of the other phase so that the resultant voltage around the mesh is the phasor sum of the three phase voltages. Since the three phase voltages are equal in magnitude and displaced 120° from one another, their phasor sum is zero. Therefore no current can flow around the mesh when the terminals are open

3.6.1 Relation between line and phase voltages

Since the system is balanced the three phase voltages are equal in magnitude but displaced by 120^0 from each other. From Figure 2.31 it is clear that only one phase winding is included between any pair of lines. Hence in delta connection $\boxed{V_L = V_{ph}.}$

3.6.2 Relation between line and phase currents

Since the system is balanced, the three phase currents I_R, I_Y and I_B are equal in magnitude and displaced from each other by 120^0. An examination of current shows that current in any line is equal to the phasor difference of the currents in the two phasors attached to that line.

Current $I_L = I_R - I_B$. The current in the line is the phasor difference of I_R and I_B.

$$I_L = I_1 = I_R - I_B = 2\, I_{ph} \cos 60^0/2$$

$$= 2\, I_{ph} \cos 30^0 = \sqrt{3}\, I_{ph.}$$

The three line currents I_1, I_2 and I_3 are equal in magnitude each being equal to $\sqrt{3}\, I_{ph.}$

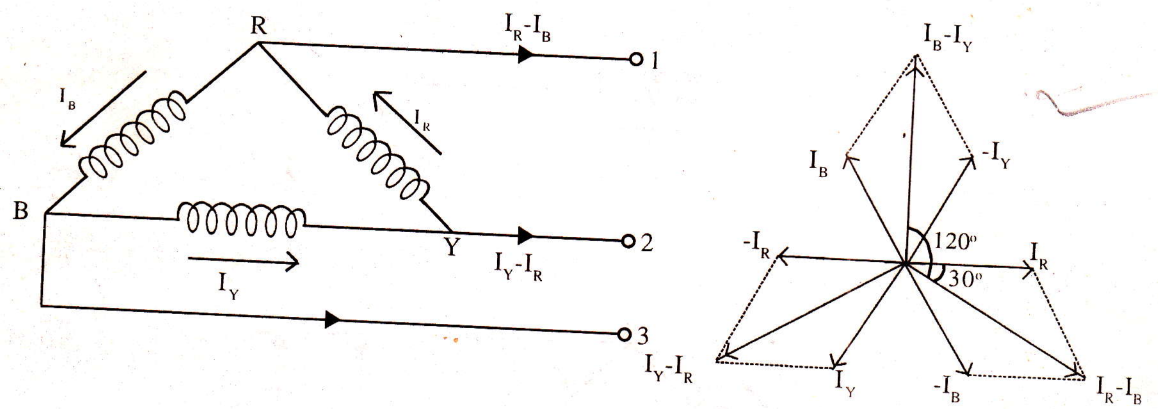

Fig. 3.30

Hence in balanced Δ connection

1. Line current $\boxed{IL = \sqrt{3}\ Iph.}$
2. All line currents are equal in magnitude but displaced by 120^0 from one another.
3. Line currents are 30^0 behind their respective phase currents.

3.7. ADVANTAGES OF STAR AND DELTA CONNECTED SYSTEMS

In three phase system, the alternators may be star or delta connected. Also three phase loads may be star or delta connected. Following are the advantages:

3.7.1. Star Connection

1. In star connection $Vph = V_L / \sqrt{3}$. Since the induced emf in the phase winding of an alternator is directly proportional to the number of turns, a star connected alternator will require lesser number of turns than a delta connected alternator for the same line voltage.

2. For the same line voltage, a star connected alternator will require lesser amount of insulation. Due to the above reasons, three phase alternators are generally star connected.

3. With star connection, it is possible to use two levels of voltages, that is phase voltage and line voltage.

4. In star connection, neutral point may be earthed. Earthing of neutral permits the use of relays.

3.7.2. Delta Connection

1. This type of connection is most suitable for rotary converters.
2. Most of the three phase induction motors are delta connected.
3. Three phase loads are generally delta connected. This is because of the flexibility with which load may be added or removed on a single phase which is more difficult with three phase star connected load.

3.8. EXAMPLES

3.8.1. A balanced star connected load of impedance (6+j8) ohms per phase is connected to a three phase 230V, 50Hz. supply. Find the line current and power absorbed by each phase.

Solution

$$Z_{ph} = \sqrt{6^2 + 8^2} = 10\ \Omega$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 133\ \text{V}$$

$$\text{Cos}\phi = \frac{R}{Z} = \frac{6}{10} = 0.6\ \text{lag}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{133}{10} = 13.3\ \text{A}$$

$$I_L = I_{ph} = 13.3\ \text{A}$$

Power absorbed per phase

$$= \text{Vph Iph Cos}\phi_A$$

$$= 133 \times 13.3 \times 0.6 = 1061\ \text{W}$$

3.8.2 The load to a 3 phase supply comprises of three similar coils connected in star. The line currents are 25 A and kVA and kW inputs are 20 and 11 respectively. Find (1) the phase and line voltage (2) the kVAr input (3) resistance and reactance of each coil.