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Programming Assignment

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1) Write a program to decide diagonal dominance (or) otherwise of given square matrix. Also find magnitude of diagonal dominance (if so)

```
//*****  
  
#include<iostream>  
#include<math.h>  
#define N 4  
using namespace std;  
int main()  
{  
    int n = 4;  
    int m[N][N] = { { 8, 0, -2, -3 },  
                    { -2, -10, 2, 2 },  
                    { 1, -6, 12, -3 },  
                    { -3, 2, 3, -15 } };  
  
    bool status = true;  
    int diff_array[n] , delta;  
  
    // for each row  
    for (int i = 0; i < n; i++)  
    {  
        // for each column, finding sum of each row.  
        int sum = 0;  
        for (int j = 0; j < n; j++)  
        {  
            sum = sum + abs(m[i][j]);  
        }  
  
        // removing the diagonal element.  
        sum = sum - abs(m[i][i]);  
  
        // checking if diagonal element is less  
        // than sum of non-diagonal element.  
        if (abs(m[i][i]) < sum)  
        {  
            status = false;  
        }  
        else  
        {  
            diff_array[i] = abs(m[i][i]) - sum;  
            //cout<<diff_array[i]<<"\n";  
        }  
    }  
  
    //check for diagonal dominant or not  
    if(status == true)  
    {  
        //let delta=diff_array[0]  
        delta = diff_array[0];  
        for(int j=1;j<n;j++)  
        {  
            if(delta>diff_array[j])  
            {
```

```

        delta = diff_array[j];
    }
}

cout<<"Given Square matrix is diagonal dominant"<<"\n"<<"Magnitude of diagonal dominance is delta = "<<delta;

}
else
{
    cout<<"Given Square matrix is NOT diagonal dominant";
}

return 0;
}
//*****

```

Input:

8	0	-2	-3
-2	-10	2	2
1	-6	12	-3
-3	2	3	-15

Output: Given Square matrix is diagonal dominant
Magnitude of diagonal dominance is delta = 2

2) Write a program for finding an estimate of $\text{cond}_{\infty}(A)$ for a diagonally dominant matrix

```
//*****
#include<iostream>
#include<math.h>
#define N 4
using namespace std;
int main()
{
    int m[N][N] = { { 8, 0, -2, -3 },
                    { -2, -10, 2, 2},
                    { 1, -6, 12, -3},
                    { -3, 2, 3, -15} };

    bool status = true;
    int diff_array[N] , delta;
    int row_sum[N], max_row_sum = 0;

    // for each row
    for (int i = 0; i < N; i++)
    {

        // for each column, finding sum of each row.
        int sum = 0;
        for (int j = 0; j < N; j++)
        {
            sum = sum + abs(m[i][j]);
            row_sum[i] = sum;
            if(max_row_sum<row_sum[i])
            {
                max_row_sum = row_sum[i];
            }
        }

        // removing the diagonal element.
        sum = sum - abs(m[i][i]);
        // checking if diagonal element is less
        // than sum of non-diagonal element.
        if (abs(m[i][i]) < sum)
        {
            status = false;
        }
        else
        {
            diff_array[i] = abs(m[i][i]) - sum;
            //cout<<diff_array[i]<<"\n";
        }
    }

    //check for diagonal dominant or not
    if(status == true)
    {
        //let delta=diff_array[0]
```

```

delta = diff_array[0];
for(int j=1;j<N;j++)
{
    if(delta>diff_array[j])
    {
        delta = diff_array[j];
    }
}
cout<<"Given Square matrix is diagonal dominant"<<"\n"<<"Magnitude of diagonal dominance is delta = "
<<delta<<"\n";
//To find infinity norm of matrix
float condition_no;
condition_no = (float)max_row_sum/(float)delta;
cout<<"Condition number of matrix is <= "<<condition_no;

}
else
{
    cout<<"Given Square matrix is NOT diagonal dominant";
}

return 0;
}
//*****

```

Input:

8	0	-2	-3
-2	-10	2	2
1	-6	12	-3
-3	2	3	-15

Output: Given Square matrix is diagonal dominant
Magnitude of diagonal dominance is delta = 2
Condition number of matrix is <= 11.5

3) Write a program which gives solution of diagonal matrix, lower triangular, upper triangular, unitary matrix

//Part 1: A is diagonal matrix

```
//*****  
  
#include<iostream>  
#include<math.h>  
#define N 3  
using namespace std;  
  
int main()  
{  
    float A[N][N] = { {2,0,0},  
                      { 0,3,0},  
                      { 0,0,4} };  
  
    float b[N] = {2,9,8};  
    float x[N];  
    int i,j,k;  
    for(i=0;i<N;i++)  
    {  
        x[i] = b[i]/A[i][i];  
    }  
  
    cout<<"Solution of Ax=b is x = ";  
    for(j=0;j<N;j++)  
    {  
        cout<<x[j]<<" ";  
    }  
  
    return 0;  
}  
//*****
```

Input: $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ $b = [2 \ 9 \ 8]$

Output: Solution of Ax=b is x = [1 3 2]

//Part 2: A is lower triangular

```
//*****  
  
#include<iostream>  
#include<math.h>  
#define N 3  
  
using namespace std;
```

```

int main()
{
    float A[N][N] = { {10,0,0 },
                      { 2,-10.6,0},
                      { 3, 1.1, (-1163/106)} };

    float b[N] = {15,37,-10};
    float x[N], sum;

    int k,j;
    for(int i=0;i<N;i++)
    {
        sum=0;
        for(j=0;j<i;j++)
        {
            sum = sum + (A[i][j]*x[j]);
        }
        x[i] = (b[i] - sum)/A[i][i];
    }

    cout<<"Solution of Ax=b is x = ";
    for(j=0;j<N;j++)
    {
        cout<<x[j]<<" ";
    }

    return 0;
}
//*****

```

Input: $A = \begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.6 & 0 \\ 3 & 1.1 & (-1163/106) \end{bmatrix}$ $b = [15 \ 37 \ -10]$

Output: Solution of Ax=b is x = [1.5 -3.20755 1.09717]

//Part 3: A is upper triangular

```

//*****

#include<iostream>
#include<math.h>
#define N 3

using namespace std;

int main()
{
    float A[N][N] = { {1,0.3,0.4 },
                      { 0,1,(-11/53)},
                      { 0,0,1} };

    float b[N] = {1.5,(-170/53),1};
    float x[N], sum;

```

```

int k,j;
for(int i=N-1;i>=0;i--)
{
    sum=0;
    for(j=i+1;j<N;j++)
    {
        sum = sum + (A[i][j]*x[j]);
    }
    x[i] = (b[i] - sum)/A[i][i];
}

cout<<"Solution of Ax=b is x = ";
for(j=0;j<N;j++)
{
    cout<<x[j]<<" ";
}

return 0;
}
//*****

```

Input:

$$A = \begin{bmatrix} 1 & 0.3 & 0.4 \\ 0 & 1 & (-11/53) \\ 0 & 0 & 1 \end{bmatrix}$$

$$b = [1.5 \ (-170/53) \ 1]$$

Output: Solution of Ax=b is x = [2 -3 1]

//Part 4: A is unitary matrix

```

//*****

#include<iostream>
#include<math.h>
#define N 4

using namespace std;

int main()
{
    int A[N][N] = { {5, -2, 2, 7},
                    {1, 0, 0, 3},
                    {-3, 1, 5, 0},
                    {3, -1, -9, 4}};

    int x[N];
    int b[N] = {1,2,3,4};

    //A* = inverse(A), A* = conj(transpose(A))
    //transpose of A
    int i,j,k;
    for(i=0;i<N;i++)
    {
        for(j=i+1;j<N;j++)
        {
            k = A[i][j];

```



```

        A[i][j] = A[j][i];
        A[j][i] = k;
    }
}

int sum;

for(i=0;i<N;i++)
{
    sum = 0;
    for(j=0;j<N;j++)
    {
        sum = sum + (A[i][j] * b[j]);
    }
    x[i] = sum;
}

cout<<"Solution of Ax=b is x = ";
for(j=0;j<N;j++)
{
    cout<<x[j]<<" ";
}
return 0;
}
//*****

```

Input:

$$A = \begin{bmatrix} 5 & -2 & 2 & 7 \\ 1 & 0 & 0 & 3 \\ -3 & 1 & 5 & 0 \\ 3 & -1 & -9 & 4 \end{bmatrix} \quad b = [1 \ 2 \ 3 \ 4]$$

Output: Solution of Ax=b is x = [10 -3 -19 29]

4) Write a program to find solution of $Ax=b$ using gauss elimination

```
/**
*****
#include<iostream>
#include<math.h>

#define N 3 //order of matrix A
using namespace std;

int main()
{
    int i,j,k,p,q,r;
    float c,x[N],sum=0;

    //Augmented matrix A
    float A[N+1][N+1] = { { 0.143,0.357,2.01,-5.17 },
        { -1.31,0.911,1.99,-5.46},
        { 11.2,-4.3,0.605,4.42} };

    printf("\n Elements are inserted in augmented matrix form:\n\n");

    for(j=0; j<N; j++) /* loop for the generation of upper triangular matrix*/
    {
        for(i=0; i<N; i++)
        {
            if(i>j)
            {
                c=A[i][j]/A[j][j];
                for(k=0; k<N+1; k++)
                {
                    A[i][k]=A[i][k]-c*A[j][k];
                }
            }
        }
    }
    x[N-1]=A[N-1][N]/A[N-1][N-1];
    /* this loop is for backward substitution*/

    for(i=0;i<N;i++)
    {
        for(j=0;j<N+1;j++)
        {
            cout<<A[i][j]<<" ";
        }
        cout<<"\n";
    }
}
```

```

        cout<<"....."<<"\n";

for(int i=N-1;i>=0;i--)
{
    sum=0;
    for(j=i+1;j<N;j++)
    {
        sum = sum + (A[i][j]*x[j]);
    }
    x[i] = (A[i][N] - sum)/A[i][i];
}

cout<<"Solution of Ax=b is x = ";
for(j=0;j<N;j++)
{
    cout<<x[j]<<" ";
}

return 0;
}
//*****

```

Input: Elements are inserted in augmented matrix form

$$A|b = \begin{bmatrix} 0.143 & 0.357 & 2.01 & -5.17 \\ -1.31 & 0.911 & 1.99 & -5.46 \\ 11.2 & -4.3 & 0.605 & 4.42 \end{bmatrix}$$

Output: Solution of Ax=b is x = [-10.3127 -27.4614 3.03902]

5) Write a program to find solution of $Ax=b$ using gauss elimination with trivial pivoting

```
//*****  
  
#include<iostream>  
#include<math.h>  
  
#define N 4 //order of matrix A  
using namespace std;  
  
int main()  
{  
    int i,j,k,p,q,r;  
    float c,x[N],sum=0;  
  
    //Augmented matrix A  
    float A[N][N+1] = { {1,-1,2,-1,-8 },  
                        { 2,-2,3,-3,-20},  
                        { 1,1,1,0,-2},  
                        {1,-1,4,3,4} };  
  
    float b[N] = {-8,-20,-2,4};  
  
    printf("\n Elements are inserted in augmented matrix form:\n\n");  
  
    for(j=0; j<N; j++) /* loop for the generation of upper triangular matrix*/  
    {  
        //check that A[j][j] is non zero otherwise interchange  
        if(A[j][j]==0)  
        {  
            for(k=j+1;k<N;k++)  
            {  
                if(A[k][j]!=0)  
                {  
                    p=k;  
                    break;  
                }  
            }  
  
            for(k=0;k<N+1;k++)  
            {  
                q=A[j][k];  
                A[j][k] = A[p][k];  
                A[p][k] = q;  
            }  
        }  
  
        for(i=0; i<N; i++)  
        {  
            if(i>j)  
            {  
                c=A[i][j]/A[j][j];  

```

```

        for(k=0; k<N+1; k++)
        {
            A[i][k]=A[i][k]-c*A[j][k];
        }
    }
}
x[N-1]=A[N-1][N]/A[N-1][N-1];
/* this loop is for backward substitution*/

for(i=0;i<N;i++)
{
    for(j=0;j<N+1;j++)
    {
        cout<<A[i][j]<<" ";
    }
    cout<<"\n";
}
cout<<"....."<<"\n";

for(int i=N-1;i>=0;i--)
{
    sum=0;
    for(j=i+1;j<N;j++)
    {
        sum = sum + (A[i][j]*x[j]);
    }
    x[i] = (A[i][N] - sum)/A[i][i];
}

cout<<"Solution of Ax=b is x = ";
for(j=0;j<N;j++)
{
    cout<<x[j]<<" ";
}

return 0;
}
//*****

```

Input: Elements are inserted in augmented matrix form

$$A|b = \begin{bmatrix} 1 & -1 & 2 & -1 & -8 \\ 2 & -2 & 3 & -3 & -20 \\ 1 & 1 & 1 & 0 & -2 \\ 1 & -1 & 4 & 3 & 4 \end{bmatrix}$$

Output: Solution of Ax=b is x = [-7 3 2 2]

6) Write a program to find solution of $Ax=b$ using gauss elimination with partial pivoting

```
//*****

#include<iostream>
#include<math.h>

#define N 3 //order of matrix A
using namespace std;

int main()
{
    int i,j,k,p,q,r;
    float c,x[N],sum=0,max;

    //Augmented matrix A
    float A[N+1][N+1] = { {10,-7,0,7},
        { -3,2.099,6,3.901},
        { 5,-1,5,6} };

    printf("\n Elements are inserted in augmented matrix form:\n\n");

    for(j=0; j<N; j++) /* loop for the generation of upper triangular matrix*/
    {

        //check that A[j][j] is max among row otherwise interchange with max
        max = abs(A[j][j]);
        p=j;
        for(k=j+1;k<N;k++)
        {
            if(abs(A[k][j])>max)
            {
                max = A[k][j];
                p=k;
            }

            for(k=0;k<N+1;k++)
            {
                q=A[j][k];
                A[j][k] = A[p][k];
                A[p][k] = q;
            }
        }

        for(i=0; i<N; i++)
        {
            if(i>j)
            {
                c=A[i][j]/A[j][j];
                for(k=0; k<N+1; k++)
```

```

        {
            A[i][k]=A[i][k]-c*A[j][k];
        }
    }
}
for(p=0;p<N;p++)
{
    for(q=0;q<N+1;q++)
    {
        cout<<A[p][q]<<" ";
    }
    cout<<"\n";
}
cout<<"....."<<"\n";

}
x[N-1]=A[N-1][N]/A[N-1][N-1];
/* this loop is for backward substitution*/

for(i=0;i<N;i++)
{
    for(j=0;j<N+1;j++)
    {
        cout<<A[i][j]<<" ";
    }
    cout<<"\n";
}
cout<<"....."<<"\n";

for(int i=N-1;i>=0;i--)
{
    sum=0;
    for(j=i+1;j<N;j++)
    {
        sum = sum + (A[i][j]*x[j]);
    }
    x[i] = (A[i][N] - sum)/A[i][i];
}

cout<<"Solution of Ax=b is x = ";
for(j=0;j<N;j++)
{
    cout<<x[j]<<" ";
}
return 0;
}
//*****

```

Input: Elements are inserted in augmented matrix form

$$A|b = \begin{bmatrix} 10 & -7 & 0 & 7 \\ -3 & 2.099 & 6 & 3.901 \\ 5 & -1 & 5 & 6 \end{bmatrix}$$

Output: Solution of Ax=b is x = [0 -1 1]

7) Write a program to solve a linear system $Ax=b$ using Gauss Jordan elimination

```
//*****  
  
#include<iostream>  
#include<math.h>  
#define N 3  
using namespace std;  
  
int main()  
{  
    int i,j,k,p,q,r;  
    float c,x[N],sum=0;  
  
    //Augmented matrix A  
    float A[N][N+1] = { {0,2,1,4 },  
                        { 1,1,2,6},  
                        { 2,1,1,7} };  
  
    printf("\n Elements are inserted in augmented matrix form:\n\n");  
  
    for(j=0; j<N; j++) /* loop for the generation of upper triangular matrix*/  
    {  
  
        //check that A[j][j] is non zero otherwise interchange  
        if(A[j][j]==0)  
        {  
            for(k=j+1;k<N;k++)  
            {  
                if(A[k][j]!=0)  
                {  
                    p=k;  
                    break;  
                }  
            }  
  
            for(k=0;k<N+1;k++)  
            {  
                q=A[j][k];  
                A[j][k] = A[p][k];  
                A[p][k] = q;  
            }  
        }  
  
        for(i=0; i<N; i++)  
        {  
            if(i != j)  
            {  
                c=A[i][j]/A[j][j];  
                for(k=0; k<N+1; k++)
```



```

        {
            A[i][k]=A[i][k]-c*A[j][k];
        }
    }
}
}
x[N-1]=A[N-1][N]/A[N-1][N-1];
/* this loop is for backward substitution*/

for(i=0;i<N;i++)
{
    for(j=0;j<N+1;j++)
    {
        cout<<A[i][j]<<" ";
    }
    cout<<"\n";
}
cout<<"....."<<"\n";

for(int i=N-1;i>=0;i--)
{
    sum=0;
    for(j=i+1;j<N;j++)
    {
        sum = sum + (A[i][j]*x[j]);
    }
    x[i] = (A[i][N] - sum)/A[i][i];
}

cout<<"Solution of Ax=b is x = ";
for(j=0;j<N;j++)
{
    cout<<x[j]<<" ";
}
return 0;
}
//*****

```

Input: Elements are inserted in augmented matrix form

$$A|b = \begin{bmatrix} 0 & 2 & 1 & 4 \\ 1 & 1 & 2 & 6 \\ 2 & 1 & 1 & 7 \end{bmatrix}$$

Output: Solution of Ax=b is x = [2.2 1.4 1.2]

8) Write a program to find inverse of non-singular matrix using Gauss Jordan elimination

```
//*****  
  
#include<iostream>  
#include<math.h>  
#define N 3  
using namespace std;  
  
int main()  
{  
    int i,j,k,p;  
    float q,r;  
    float c,sum=0;  
  
    //Augmented matrix A|I  
    float A[N][N+N] = { {5,7,9,1,0,0 },  
        { 4,3,8,0,1,0},  
        { 7,5,6,0,0,1} };  
  
    float A_inverse[N][N];  
  
    printf("\n Elements are inserted in augmented matrix form, A|I \n\n");  
  
    for(j=0; j<N; j++) /* loop for the generation of upper triangular matrix*/  
    {  
  
        //check that A[j][j] is non zero otherwise interchange  
        if(A[j][j]==0)  
        {  
            for(k=j+1;k<N;k++)  
            {  
                if(A[k][j]!=0)  
                {  
                    p=k;  
                    break;  
                }  
            }  
  
            for(k=0;k<N+N;k++)  
            {  
                q=A[j][k];  
                A[j][k] = A[p][k];  
                A[p][k] = q;  
            }  
        }  
  
        for(i=0; i<N; i++)  
        {  
            if(i != j)  
            {  
                c=A[i][j]/A[j][j];  
                for(k=0; k<N+N; k++)
```

```

        {
            A[i][k]=A[i][k]-c*A[j][k];
        }
    }
}

//reduce diagonal elements to 1
for(i=0;i<N;i++)
{
    q = A[i][i];
    //cout<<"p"<<p<<"P";
    for(j=0;j<N+N;j++)
    {
        A[i][j] = A[i][j]/q;

        if(j>=N)
        {
            A_inverse[i][j-N] = A[i][j];
        }
    }
}

cout<<"Inverse of matrix is"<<"\n";
for(i=0;i<N;i++)
{
    for(j=0;j<N;j++)
    {
        cout<<A_inverse[i][j]<<" ";
    }
    cout<<"\n";
}
cout<<"....."<<"\n";

return 0;
}
//*****

```

Input: Elements are inserted in augmented matrix form

$$A|I = \begin{bmatrix} 5 & 7 & 9 & 1 & 0 & 0 \\ 4 & 3 & 8 & 0 & 1 & 0 \\ 7 & 5 & 6 & 0 & 0 & 1 \end{bmatrix}$$

Output: Inverse of matrix A is

$$\begin{bmatrix} -0.209524 & 0.0285714 & 0.27619 \\ 0.304762 & -0.314286 & -0.0380952 \\ -0.00952383 & 0.228571 & -0.12381 \end{bmatrix}$$

9) Write a program to decompose non-singular matrix A as product LU of lower & upper triangular matrices

i) with assumption $L_{ii} = 1$ (Doolittle method)

ii) with assumption $U_{ii} = 1$ (Crouts method)

// Doolittle method

```
//*****

#include<iostream>
#include <bits/stdc++.h>
#include<math.h>
#define N 3

//lii = 1 doolittle method
using namespace std;

int main()
{
    int l[N][N], u[N][N];
    memset(l, 0, sizeof(l));
    memset(u, 0, sizeof(u));

    int a[N][N] = { {2,-1,-2},
                    {-4,6,3},
                    {-4,-2,8}};

    for (int i = 0; i < N; i++) {

        // Upper Triangular
        for (int k = i; k < N; k++) {

            // Summation of L(i, j) * U(j, k)
            int sum = 0;
            for (int j = 0; j < i; j++)
            {
                sum += (l[i][j] * u[j][k]);
            }
            // Evaluating U(i, k)
            u[i][k] = a[i][k] - sum;
        }

        // Lower Triangular
        for (int k = i; k < N; k++) {
            if (i == k)
                l[i][i] = 1; // Diagonal as 1
            else {

                // Summation of L(k, j) * U(j, i)
                int sum = 0;
                for (int j = 0; j < i; j++)
                    sum += (l[k][j] * u[j][i]);
            }
        }
    }
}
```

```

        // Evaluating L(k, i)
        l[k][i] = (a[k][i] - sum) / u[i][i];
    }
}

cout<<"A = "<<"\n";
for(int i=0;i<N;i++)
{
    for(int j=0;j<N;j++)
    {
        cout<<a[i][j]<<" ";
    }
    cout<<"\n";
}

cout<<"\nL = "<<"\n";

for(int i=0;i<N;i++)
{
    for(int j=0;j<N;j++)
    {
        cout<<l[i][j]<<" ";
    }
    cout<<"\n";
}

cout<<"\nU = "<<"\n";

for(int i=0;i<N;i++)
{
    for(int j=0;j<N;j++)
    {
        cout<<u[i][j]<<" ";
    }
    cout<<"\n";
}
return 0;
}
//*****

```

Input:

$$A = \begin{bmatrix} 2 & -1 & -2 \\ -4 & 6 & 3 \\ -4 & -2 & 8 \end{bmatrix}$$

Output: $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix}$ $U = \begin{bmatrix} 2 & -1 & -2 \\ 0 & 4 & -1 \\ 0 & 0 & 3 \end{bmatrix}$

// Crouts method

```
//*****

#include<iostream>
#include <bits/stdc++.h>
#include<math.h>
#define N 3

//u11 = 1 crouts method
using namespace std;

int main()
{
    float l[N][N], u[N][N];
    memset(l, 0, sizeof(l));
    memset(u, 0, sizeof(u));

    float a[N][N] = { {10,3,4},
                      {2,-10,3},
                      {3,2,-10}};

    for (int i = 0; i < N; i++) {

        // Lower Triangular
        for (int k = i; k < N; k++) {

            // Summation of L(k, j) * U(j, i)
            float sum = 0;
            for (int j = 0; j < i; j++)
            {
                sum += (l[k][j] * u[j][i]);
            }

            // Evaluating L(k, i)
            l[k][i] = a[k][i] - sum;

            // Upper Triangular
            for (int k = i; k < N; k++) {

                if (i == k)
                {
                    u[i][i] = 1; // Diagonal as 1
                }
                else {
                    // Summation of L(i, j) * U(j, k)
                    int sum = 0;
                    for (int j = 0; j < i; j++)
                    {
                        sum += (l[i][j] * u[j][k]);
                    }
                    // Evaluating U(i, k)
                    u[i][k] = (a[i][k] - sum)/l[i][i];
                }
            }
        }
    }
}
```

```

    }

    }
}

cout<<"A = "<<"\n";
for(int i=0;i<N;i++)
{
    for(int j=0;j<N;j++)
    {
        cout<<a[i][j]<<" ";
    }
    cout<<"\n";
}

cout<<"\nL = "<<"\n";

for(int i=0;i<N;i++)
{
    for(int j=0;j<N;j++)
    {
        cout<<l[i][j]<<" ";
    }
    cout<<"\n";
}

cout<<"\nU = "<<"\n";

for(int i=0;i<N;i++)
{
    for(int j=0;j<N;j++)
    {
        cout<<u[i][j]<<" ";
    }
    cout<<"\n";
}

return 0;
}
//*****

```

Input:

$$A = \begin{bmatrix} 10 & 3 & 4 \\ 2 & -10 & 3 \\ 3 & 2 & -10 \end{bmatrix}$$

Output: L =

$$\begin{bmatrix} 10 & 0 & 0 \\ 2 & -10.6 & 0 \\ 3 & 1.1 & -10.8887 \end{bmatrix}$$

U =

$$\begin{bmatrix} 1 & 0.3 & 0.4 \\ 0 & 1 & -0.283019 \\ 0 & 0 & 1 \end{bmatrix}$$

10) Find Approximation of $\|A\|_2$ when A is lower triangular matrix

```
/**
*****

#include<iostream>
#include<math.h>
#define N 4
using namespace std;
int main()
{
    int i,j,k,sum;
    //A is lower triangular matrix
    float A[N][N] = {{2,0,0,0},{4,-2,0,0},{-5,6,1,0},{1,5,3,3}};
    float x[N];
    float y[N];

    //step 1
    x[0] = 1;
    y[0] = A[0][0]*x[0]
    //step 2
    for(i=1;i<N;i++)
    {
        sum=0;
        for(j=0;j<i;j++)
        {
            sum = sum + A[i][j]*x[j];
        }
        if(abs(A[i][i] + sum) > abs(A[i][i] - sum))
        {
            x[i] = 1;
        }
        else
        {
            x[i] = -1;
        }
        y[i] = (A[i][i]*x[i]) + sum;
    }
    //step 3
    float y_2_norm=0;
    for(i=0;i<N;i++)
    {
        y_2_norm = y_2_norm + (y[i]*y[i]);
    }
    y_2_norm = sqrt(y_2_norm);
    cout<<"2 norm of vector y is found out to be "<<y_2_norm<<"\n";
    cout<<"Approximation of 2 norm of matrix A estimates to "<<y_2_norm/sqrt(N)<<"\n";
    return 0;
}
/**
*****
```

Input:

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ -5 & 6 & 1 & 0 \\ 1 & 5 & 3 & 3 \end{bmatrix}$$

Output: Approximation of 2 norm of matrix A estimates to 8.42615

11) Find Approximation of $\|A^{-1}\|_2$ when A is lower triangular matrix

```
/**
*****

#include<iostream>
#include<math.h>
#define N 4
using namespace std;

int main()
{
    int i,j,k,sum;
    //A is lower triangular matrix
    float A[N][N] = {{2,0,0,0},{4,-2,0,0},{-5,6,1,0},{1,5,3,3}};
    float x[N];
    float y[N];
    int sign;

    //step 1
    x[0] = 1;
    y[0] = 1/(A[0][0]);

    //step 2
    for(i=1;i<N;i++)
    {
        sum=0;
        for(j=0;j<i;j++)
        {
            sum = sum + A[i][j]*y[j];
        }

        if(sum>0)
        {
            sign = 1;
        }
        else
        {
            sign = -1;
        }

        y[i] = (-1/A[i][i])*(sum + sign);
    }

    //step 3
    float y_2_norm=0;
    for(i=0;i<N;i++)
    {
        y_2_norm = y_2_norm + (y[i]*y[i]);
    }

    y_2_norm = sqrt(y_2_norm);

    cout<<"2 norm of vector y is found out to be "<<y_2_norm<<"\n";
}
```

```

        cout<<"Approximation of 2 norm of matrix A inverse estimates to "<<y_2_norm/sqrt(N)<<"\n";

        return 0;
    }
//*****

```

Input:

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ -5 & 6 & 1 & 0 \\ 1 & 5 & 3 & 3 \end{bmatrix}$$

Output: Approximation of 2 norm of matrix A inverse estimates to 5.06211

12) For lower matrix A, find approximation of $\text{cond}_2(A)$

```

//*****

#include<iostream>
#include<math.h>
#define N 4

using namespace std;

float approximation_of_2_norm(float A[N][N])
{
    int i,j,k,sum;
    float x[N];
    float y[N];

    //step 1
    x[0] = 1;
    y[0] = A[0][0]*x[0];

    //step 2
    for(i=1;i<N;i++)
    {
        sum=0;
        for(j=0;j<i;j++)
        {
            sum = sum + A[i][j]*x[j];

            if(abs(A[i][i] + sum) > abs(A[i][i] - sum))
            {

```

```

        x[i] = 1;
    }
    else
    {
        x[i] = -1;
    }
    y[i] = (A[i][i]*x[i]) + sum;
}

//step 3
float y_2_norm=0;
for(i=0;i<N;i++)
{
    y_2_norm = y_2_norm + (y[i]*y[i]);
}

y_2_norm = sqrt(y_2_norm);

//cout<<"2 norm of vector y is found out to be "<<y_2_norm<<"\n";

return y_2_norm/sqrt(N);
}

```

```

float approximation_of_2_norm_inverse(float A[N][N])
{

```

```

    int i,j,k,sum;
    //A is lower triangular matrix
    float x[N];
    float y[N];
    int sign;

    //step 1
    x[0] = 1;
    y[0] = 1/(A[0][0]);

    //step 2
    for(i=1;i<N;i++)
    {
        sum=0;
        for(j=0;j<i;j++)
        {
            sum = sum + A[i][j]*y[j];
        }

        if(sum>0)
        {
            sign = 1;
        }
        else
        {
            sign = -1;
        }

```

```

        y[i] = (-1/A[i][i])*(sum + sign);
    }
}

```

```

//step 3
float y_2_norm=0;
for(i=0;i<N;i++)
{
    y_2_norm = y_2_norm + (y[i]*y[i]);
}

y_2_norm = sqrt(y_2_norm);

//cout<<"2 norm of vector y is found out to be "<<y_2_norm<<"\n";

return y_2_norm/sqrt(N);
}

int main()
{
    float A[N][N] = {{2,0,0,0},{4,-2,0,0},{-5,6,1,0},{1,5,3,3}};
    float norm_A, norm_A_inverse;
    norm_A = approximation_of_2_norm(A);
    norm_A_inverse = approximation_of_2_norm_inverse(A);

    cout<<"Approximation of condition number A is : "<<norm_A*norm_A_inverse;

    return 0;
}
//*****

```

Input:

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & -2 & 0 & 0 \\ -5 & 6 & 1 & 0 \\ 1 & 5 & 3 & 3 \end{bmatrix}$$

Output: Approximation of condition number A is : 42.6541