Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

   Ans : 0.2676
   from scipy import stats
   1 – stats.norm.cdf(50,45,8)

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ =6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.

      Ans : True
      P(age >=44) = P1 = 0.15865525393145707
      P(38<=age <= 44) = P2 = 0.3413447460685429
      P1 * 400 > P2 * 400
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

      Ans : True
      P(age <= 30) = 0.09121121972586788
      P * 400 = 36 (Approx.)

3. If X_1~ $N(\mu, \sigma^2)$ and X_2 ~ $N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.
   Ans : If X ~ N(μ1, σ1^2), and Y ~ N(μ2, σ2^2) are two independent random variables then
   X + Y ~ N(μ1 + μ2, σ1^2 + σ2^2) , and   X – Y ~ N(μ1 – μ2, σ1^2 + σ2^2) .
   Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z ~ N(aμ1 + bμ2, a^2σ1^2 + b^2σ2^2).

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9

B. 80.2, 119.8

C. 22, 78

D. 48.5, 151.5

E. 90.1, 109.9

Ans : 48.5,151.5

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans : Range is Rs. 99.0081034, 980.991896

B. Specify the 5th percentile of profit (in Rupees) for the company

Ans : 169.875

C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans : stats.norm.cdf(0.5,3)

0.0477903

Stats.norm.cdf(0,7,4)

0.0400591