5)

A)

```
>> A=[0 1/2 0;1 0 0;0 1/2 1]

A =

      0          1/2          0
      1          0            0
      0          1/2          1

>> A^5

ans =

      0          1/8          0
      1/4        0            0
      3/4        7/8          1

>> A^100

ans =

      *          0            0
      0          *            0
      1          1            1

>> A^101

ans =

      0          *            0
      *          0            0
      1          1            1

>> A^10000

ans =

      0          0            0
      0          0            0
      1          1            1
```

B)

Theorem 4:

```
>> eigs(A)

ans =

      1
    985/1393
   -985/1393

>> rref(A-eye(3))

ans =

      1         0         0
      0         1         0
      0         0         0
```

Although they hypothesis is false, we still arrive at the conclusion of theorem 4, as we eigenvalue = 1 is a distinct eigenvalue and therefore corresponds to a unique eigenvector

Theorem 6:

```
>> abs(985/1393 )

ans =

    985/1393

>> abs(-985/1393 )

ans =

    985/1393
```

The eigenvalues of A which are not equal to 1 satisfy being less than 1

Theorem 8:

```
>> A^10000000

ans =

      0         0         0
      0         0         0
      1         1         1

>> A^100000000
```

ans =

```
     0     0     0
     0     0     0
     1     1     1
```

```
>> A^1000000001
```

ans =

```
     0     0     0
     0     0     0
     1     1     1
```

Despite the matrix failing to be regular, for very large exponential values, k, A^k reaches a clear limit where its columns are identical and each column is the same steady state probability vector for A.

6)

A)

```
>> A = [0 1 0;1 0 0;0 0 1]
```

A =

```
     0     1     0
     1     0     0
     0     0     1
```

```
>> A^100
```

ans =

```
     1     0     0
     0     1     0
     0     0     1
```

```
>> A^532
```

ans =

```
     1     0     0
     0     1     0
```

```
     0     0     1

>> A^10234

ans =

     1     0     0
     0     1     0
     0     0     1
```

For very large exponential values k, A^k is not a positive matrix
B)

Theorem 4:

```
>> eigs(A)

ans =

    -1
     1
     1

>> rref(A-eye(3))

ans =

     1    -1     0
     0     0     0
     0     0     0
```

This theorem cannot be satisfied, as eigenvalue = 1 has an algebraic multiplicity of greater than 1, therefore we cannot obtain a unique probability eigenvector

Theorem 6:

```
>> abs(-1)

ans =

     1
```

This theorem fails to be satisfied as 1 <1, is false

Theorem 8:

```
>> A^1000000001
```

ans =

```
     0     1     0
     1     0     0
     0     0     1
```

```
>> A^10000000019
```

ans =

```
     0     1     0
     1     0     0
     0     0     1
```

For very large exponential values, k, A^k, or M's columns do not reach a clear limit where its columns are identical and each column is the same steady state probability vector for A

7)

A)

```
>> A = [0.2 0.4 0.1 0;0.8 0.3 0.5 0;0 0 0.2 0.4;0 0.3 0.2 0.6]
```

A =

```
     1/5        2/5        1/10       0
     4/5        3/10       1/2        0
     0          0          1/5        2/5
     0          3/10       1/5        3/5
```

```
>> A^2
```

ans =

```
     9/25       1/5        6/25       1/25
     2/5        41/100     33/100     1/5
     0          3/25       3/25       8/25
     6/25       27/100     31/100     11/25
```

```
>> A^3
```

ans =

```
    29/125     27/125     24/125     3/25
    51/125     343/1000   351/1000   63/250
    12/125     33/250     37/250     6/25
```

```
   33/125       309/1000      309/1000       97/250
```

A is regular because A^k, where k is some positive power, is a positive matrix

B)
```
>> eigs(A)

ans =

      1
    597/1180
   -527/1985
    181/3039
```

Yes, as eigenvalue = 1 has an algebraic multiplicity of 1, or in other words, it is not a repeated eigenvalue. This means the eigenvector corresponding to eigenvalue = 1 is unique

C)
Theorem 6:

```
>> abs(597/1180)

ans =

    597/1180

>> abs(-597/1180)

ans =

    597/1180

>> abs(181/3039)

ans =

    181/3039
```

Since A is regular, theorem 6 holds, as the absolute value of all eigenvalues of A that are not equal to 1 are less than 1

Theorem 8:

```
>> A^10001

ans =
```

```
      9/49          9/49          9/49          9/49
     16/49         16/49         16/49         16/49
      8/49          8/49          8/49          8/49
     16/49         16/49         16/49         16/49

>> A^100000

ans =

      9/49          9/49          9/49          9/49
     16/49         16/49         16/49         16/49
      8/49          8/49          8/49          8/49
     16/49         16/49         16/49         16/49

>> A^100000000

ans =

      9/49          9/49          9/49          9/49
     16/49         16/49         16/49         16/49
      8/49          8/49          8/49          8/49
     16/49         16/49         16/49         16/49
```

Theorem 8 holds, because A is a regular, diagonalizable matrix

8)