

⑤ a) For very large powers of R , A^k is not a positive matrix

b) Theorems 4, 6, and 8 rely on the initial assumption that A is either a positive or regular matrix.

$$⑧ G_1 = \alpha A + (1-\alpha) S$$

$\sum_{j=1}^n G_{1,j} = \sum_{i=1}^n \alpha A_{ij} + (1-\alpha) S_{ij}$ Assuming G_1 is stochastic, the sum of the j th column of G_1 and $\alpha A + (1-\alpha) S$ must be 1

$$\sum_{i=1}^n G_{1,i} = \sum_{i=1}^n \alpha A_{ij} + \sum_{i=1}^n (1-\alpha) S_{ij}$$
 Distribute summation to each term

$$\sum_{i=1}^n G_{1,i} = \alpha \sum_{i=1}^n A_{ij} + (1-\alpha) \sum_{i=1}^n S_{ij}$$
 Constants can be factored out

$$1 = \alpha(1) + (1-\alpha)(1)$$
 The sum of the j th column of A and S equals 1

$$1 = \alpha + (1-\alpha) \rightarrow 1 = 1 \checkmark \therefore \alpha A + (1-\alpha) S \text{ is a stochastic matrix}$$