7)

a)

```
>> [U,S,V] = svd(A)

U =

    -1/2        1/2        797/1128     245/8816
    -1/2       -1/2        245/8816    -797/1128
    -1/2        1/2       -797/1128    -245/8816
    -1/2       -1/2       -245/8816     797/1128


S =

  1351/195        0            0
     0         3363/1189       0
     0            0            *
     0            0            0


V =

  -780/1351    -985/1393    -881/2158
  -780/1351     985/1393    -881/2158
  -780/1351       *          881/1079

>> U = U'

U =

    -1/2        -1/2        -1/2        -1/2
     1/2        -1/2         1/2        -1/2
  797/1128     245/8816    -797/1128    -245/8816
  245/8816    -797/1128    -245/8816     797/1128

>> U_pinv = U(1:2,:)

U_pinv =

    -1/2        -1/2        -1/2        -1/2
     1/2        -1/2         1/2        -1/2

>> V_pinv = V(:,1:2)

V_pinv =
```

```
   -780/1351      -985/1393
   -780/1351       985/1393
   -780/1351          *

>> S_pinv =  inv(S(1:2,1:2))

S_pinv =

    195/1351         0
      0          1189/3363

>> A_pinv = V_pinv*S_pinv*U_pinv

A_pinv =

     -1/12          1/6          -1/12          1/6
      1/6          -1/12          1/6          -1/12
      1/24          1/24          1/24          1/24
```

b)

```
>> x_ls = A_pinv*b

x_ls =

     -5/6
     11/12
      1/24
```

c)

```
>> norm(b-A*x_ls)

ans =

   1.581138830084190
```

Yes, this residual is consistent with the other residuals obtained for the other least squares solutions

d)

```
>> norm([-5/6; 11/12; 1/24])

ans =

   1.239539565053626
```

```
>> norm([-21/16; 7/16; 1])

ans =

   1.707062535468458
>> norm([-29/16; -1/16; 2])

ans =

   2.699826383306897

>> norm([-45/16; -17/16; 4])

ans =

   5.003904725312024
```

The norm of the least square solution obtained corresponding to the pseudoinverse is the minimum norm. This should be expected, because the particular solution is the unique solution among all possible least square solutions. Additionally, according to theorem 6, the pseudo-inverse yields the least squares solution with the smallest Euclidean norm, which minimizes the length of the solution vector while still fitting the data as closely as possible.

8)

a)

```
>> A = [-1;3;-2]

A =

    -1
     3
    -2

>> inv(A'*A)*A'

ans =

     -1/14          3/14          -1/7
```

b)

```
>> A = [3 1 -2]

A =
```

```
     3          1          -2
```

```
>> A_pinv = A'*inv(A*A')

A_pinv =

     3/14
     1/14
    -1/7
```

c)

```
>> A= [1 2 2; 1 3 1]

A =

     1          2          2
     1          3          1

>> A_pinv = A'*inv(A*A')

A_pinv =

     1/9        0
    -5/18       1/2
    13/18      -1/2
```