1)

A)

```
>> A = [0 0 1/3 1/3 0 1/3 0;0 0 0 0 1 0 0;1/2 0 0 0 1/2 0 0;0 0 1/2 0 0 1/2 0;1/3 0 1/3 0 0
0 1/3;1/4 1/4 0 1/4 1/4 0 0;1 0 0 0 0 0 0]'

A =

         0         0    0.5000         0    0.3333    0.2500    1.0000
         0         0         0         0         0    0.2500         0
    0.3333         0         0    0.5000    0.3333         0         0
    0.3333         0         0         0         0    0.2500         0
         0    1.0000    0.5000         0         0    0.2500         0
    0.3333         0         0    0.5000         0         0         0
         0         0         0         0    0.3333         0         0

>> A(:,2) = [1/2 0 0 0 1/2 0 0]

A =

         0    0.5000    0.5000         0    0.3333    0.2500    1.0000
         0         0         0         0         0    0.2500         0
    0.3333         0         0    0.5000    0.3333         0         0
    0.3333         0         0         0         0    0.2500         0
         0    0.5000    0.5000         0         0    0.2500         0
    0.3333         0         0    0.5000         0         0         0
         0         0         0         0    0.3333         0         0

>> A(:,3) = [1/3 1/3 0 0 1/3 0 0]

A =

         0    0.5000    0.3333         0    0.3333    0.2500    1.0000
         0         0    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000    0.3333         0         0
    0.3333         0         0         0         0    0.2500         0
         0    0.5000    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000         0         0         0
         0         0         0         0    0.3333         0         0

>> A

A =

         0    0.5000    0.3333         0    0.3333    0.2500    1.0000
         0         0    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000    0.3333         0         0
```

```
    0.3333         0         0         0         0    0.2500         0
         0    0.5000    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000         0         0         0
         0         0         0         0    0.3333         0         0
```

B)

>> A^4

ans =

```
    0.2620    0.2083    0.2454    0.2714    0.2407    0.2448    0.2593
    0.0837    0.0810    0.1049    0.1256    0.1312    0.0787    0.0972
    0.2045    0.2060    0.1836    0.1782    0.1806    0.2020    0.1713
    0.1130    0.1412    0.1265    0.1209    0.1034    0.1192    0.1065
    0.1323    0.1574    0.1559    0.1256    0.1636    0.1534    0.1944
    0.1397    0.1551    0.1497    0.1296    0.1404    0.1522    0.1065
    0.0648    0.0509    0.0340    0.0486    0.0401    0.0498    0.0648
```

A is not positive, however it is regular

C)

Theorem 1:

>> eigs(A)

ans =

```
   1.0000 + 0.0000i
   0.0529 + 0.4495i
   0.0529 - 0.4495i
  -0.3570 + 0.2729i
  -0.3570 - 0.2729i
  -0.3919 + 0.0000i
```

Theorem 6:

Every eigenvalue not equal to 1 has magnitude less than 1

D)

>> [S,D] = eig(A)

S =

Columns 1 through 5

```
  -0.6076 + 0.0000i   0.5593 + 0.0000i   0.5593 + 0.0000i  -0.4300 + 0.2164i  -0.4300 -
0.2164i
  -0.2431 + 0.0000i  -0.2715 + 0.1080i  -0.2715 - 0.1080i  -0.4942 + 0.0000i  -0.4942 +
0.0000i
  -0.4687 + 0.0000i   0.0168 - 0.1249i   0.0168 + 0.1249i   0.3813 - 0.3045i   0.3813 +
0.3045i
  -0.2894 + 0.0000i  -0.1334 - 0.2780i  -0.1334 + 0.2780i   0.2187 + 0.0585i   0.2187 -
0.0585i
  -0.3646 + 0.0000i  -0.1880 + 0.4199i  -0.1880 - 0.4199i  -0.0573 + 0.3340i  -0.0573 -
0.3340i
  -0.3472 + 0.0000i  -0.2741 - 0.2987i  -0.2741 + 0.2987i   0.1971 - 0.1334i   0.1971 +
0.1334i
  -0.1215 + 0.0000i   0.2909 + 0.1737i   0.2909 - 0.1737i   0.1842 - 0.1711i   0.1842 +
0.1711i

  Columns 6 through 7

   0.3355 + 0.0000i   0.4867 + 0.0000i
  -0.5244 + 0.0000i   0.0000 + 0.0000i
   0.2996 + 0.0000i   0.4867 + 0.0000i
  -0.5549 + 0.0000i  -0.3244 + 0.0000i
   0.1446 + 0.0000i  -0.0000 + 0.0000i
   0.4226 + 0.0000i  -0.6489 + 0.0000i
  -0.1230 + 0.0000i  -0.0000 + 0.0000i


D =

  Columns 1 through 5

   1.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 +
0.0000i
   0.0000 + 0.0000i   0.0529 + 0.4495i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 +
0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0529 - 0.4495i   0.0000 + 0.0000i   0.0000 +
0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3570 + 0.2729i   0.0000 +
0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.3570 -
0.2729i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 +
0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 +
0.0000i

  Columns 6 through 7
```

```
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i
  -0.3919 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.0000 + 0.0000i

>> steady_state = S(:,1)

steady_state =

   -0.6076
   -0.2431
   -0.4687
   -0.2894
   -0.3646
   -0.3472
   -0.1215

>> steady_state/sum(steady_state)

ans =

    Page 1: 0.2488
    Page 2: 0.0995
    Page 3: 0.1919
    Page 4: 0.1185
    Page 5: 0.1493
    Page 6: 0.1422
    Page 7: 0.0498
```

Importance Score Rankings(Most important to least important): 1,3,5,6,4,2,7

E)

```
>> G = ones(7)*(1/7)

G =

    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
```

```
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
```

m = 0.1

```
>> M=(1-0.1)*A+0.1*G

M =

    0.0143    0.4643    0.3143    0.0143    0.3143    0.2393    0.9143
    0.0143    0.0143    0.3143    0.0143    0.0143    0.2393    0.0143
    0.3143    0.0143    0.0143    0.4643    0.3143    0.0143    0.0143
    0.3143    0.0143    0.0143    0.0143    0.0143    0.2393    0.0143
    0.0143    0.4643    0.3143    0.0143    0.0143    0.2393    0.0143
    0.3143    0.0143    0.0143    0.4643    0.0143    0.0143    0.0143
    0.0143    0.0143    0.0143    0.0143    0.3143    0.0143    0.0143

>> [S,D] = eig(M);
>> steady_state = S(:,1)/sum(S(:,1))

steady_state =

   Page 1:  0.2448
    Page 2: 0.1019
    Page 3: 0.1859
    Page 4: 0.1196
    Page 5: 0.1477
    Page 6: 0.1415
    Page 7: 0.0586
```

Importance Score Rankings(Most important to least important): 1,3,5,6,4,2,7

m = 0.15

```
>> M=(1-0.15)*A+0.15*G;
>> [S,D] = eig(M);
>> steady_state = S(:,1)/sum(S(:,1))

steady_state =

    Page 1: 0.2424
    Page 2: 0.1032
    Page 3: 0.1828
    Page 4: 0.1201
    Page 5: 0.1471
    Page 6: 0.1412
    Page 7: 0.0631
```

Importance Score Rankings(Most important to least important): 1,3,5,6,4,2,7

m = 0.2

```
>> M=(1-0.2)*A+0.2*G;
>> [S,D] = eig(M);
>> steady_state = S(:,1)/sum(S(:,1))

steady_state =

   Page 1: 0.2398
   Page 2: 0.1047
   Page 3: 0.1799
   Page 4: 0.1207
   Page 5: 0.1466
   Page 6: 0.1408
   Page 7: 0.0677
```

Importance Score Rankings(Most important to least important): 1,3,5,6,4,2,7

Yes, it appears all the ranking orders are consistent

2)

A)

```
>> A(:,7) = [0;0;0;0;0;0;0]

A =

         0    0.5000    0.3333         0    0.3333    0.2500         0
         0         0    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000    0.3333         0         0
    0.3333         0         0         0         0    0.2500         0
         0    0.5000    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000         0         0         0
         0         0         0         0    0.3333         0         0

>> B = A

B =

         0    0.5000    0.3333         0    0.3333    0.2500         0
         0         0    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000    0.3333         0         0
    0.3333         0         0         0         0    0.2500         0
```

```
         0    0.5000    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000         0         0         0
         0         0         0         0    0.3333         0         0
```

B)

```
>> B^4

ans =

    0.1971    0.2083    0.1898    0.1742    0.1389    0.2031         0
    0.0837    0.0810    0.1049    0.1256    0.0664    0.0787         0
    0.2045    0.1505    0.1466    0.1782    0.1250    0.1742         0
    0.1130    0.0856    0.0895    0.1209    0.0756    0.0914         0
    0.1323    0.1574    0.1559    0.1256    0.0988    0.1534         0
    0.1397    0.0995    0.1127    0.1296    0.0849    0.1244         0
    0.0648    0.0509    0.0340    0.0486    0.0401    0.0498         0

>> B^100

ans =

   1.0e-03 *

    0.6802    0.6044    0.6168    0.6666    0.4588    0.6394         0
    0.3283    0.2917    0.2977    0.3217    0.2214    0.3086         0
    0.6064    0.5389    0.5499    0.5943    0.4091    0.5701         0
    0.3544    0.3149    0.3213    0.3473    0.2390    0.3331         0
    0.5025    0.4465    0.4556    0.4924    0.3389    0.4723         0
    0.4287    0.3809    0.3887    0.4201    0.2892    0.4030         0
    0.1778    0.1579    0.1612    0.1742    0.1199    0.1671         0
```

The matrix B is not regular

C)

```
>> eigs(B)

ans =

   0.9423 + 0.0000i
  -0.2178 + 0.3680i
  -0.2178 - 0.3680i
  -0.3556 + 0.0000i
  -0.1511 + 0.0000i
  -0.0000 + 0.0000i
```

D)

Theorem 1:
B does not have an eigenvalue of one

Theorem 6:
Every eigenvalue not equal to 1 has absolute value less than 1. Therefore, B satisfies the conclusion of theorem 6

F)

```
>> C = B

C =

         0    0.5000    0.3333         0    0.3333    0.2500         0
         0         0    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000    0.3333         0         0
    0.3333         0         0         0         0    0.2500         0
         0    0.5000    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000         0         0         0
         0         0         0         0    0.3333         0         0

>> C(:,7) = [1/7;1/7;1/7;1/7;1/7;1/7;1/7]

C =

         0    0.5000    0.3333         0    0.3333    0.2500    0.1429
         0         0    0.3333         0         0    0.2500    0.1429
    0.3333         0         0    0.5000    0.3333         0    0.1429
    0.3333         0         0         0         0    0.2500    0.1429
         0    0.5000    0.3333         0         0    0.2500    0.1429
    0.3333         0         0    0.5000         0         0    0.1429
         0         0         0         0    0.3333         0    0.1429
```

G)

```
>> C^4

ans =

    0.2064    0.2455    0.2225    0.1881    0.2166    0.2276    0.2148
    0.0930    0.0983    0.1244    0.1395    0.1073    0.0933    0.1084
    0.2137    0.1816    0.1753    0.1921    0.1950    0.1957    0.1945
    0.1223    0.1029    0.1090    0.1348    0.1205    0.1060    0.1161
    0.1416    0.1866    0.1833    0.1395    0.1570    0.1739    0.1619
```

```
    0.1489    0.1228    0.1361    0.1435    0.1355    0.1420    0.1416
    0.0741    0.0623    0.0494    0.0625    0.0681    0.0614    0.0627
```

C is regular

H)

```
>> eigs(C)

ans =

   1.0000 + 0.0000i
  -0.2218 + 0.3774i
  -0.2218 - 0.3774i
  -0.3438 + 0.0000i
  -0.1808 + 0.0000i
   0.1111 + 0.0000i
```

I)

```
>> [S,D] = eig(C);
>> steady_state = S(:,1)/sum(S(:,1))

steady_state =

    Page 1: 0.2168
    Page 2: 0.1084
    Page 3: 0.1935
    Page 4: 0.1161
    Page 5: 0.1626
    Page 6: 0.1394
    Page 7: 0.0632
```

Importance Score Rankings(Most important to least important): 1,3,5,6,4,2,7

J)

```
>> M=(1-0.15)*C+0.15*G;
>> [S,D] = eig(M);
>> steady_state = S(:,1)/sum(S(:,1))

steady_state =

    Page 1: 0.2064
    Page 2: 0.1128
    Page 3: 0.1852
    Page 4: 0.1188
```

Page 5: 0.1608
Page 6: 0.1397
Page 7: 0.0762

Importance Score Rankings(Most important to least important): 1,3,5,6,4,2,7

3)

A)

>> A = [0 1/2 1/2 0 0 0 0;0 0 0 1 0 0 0;1/3 1/3 0 1/3 0 0 0;1/2 0 1/2 0 0 0 0;0 0 0 0 0 1/2 1/2;0 0 0 0 0 0 1;0 0 0 0 1 0 0]'

A =

0	0	1/3	1/2	0	0	0
1/2	0	1/3	0	0	0	0
1/2	0	0	1/2	0	0	0
0	1	1/3	0	0	0	0
0	0	0	0	0	0	1
0	0	0	0	1/2	0	0
0	0	0	0	1/2	1	0

B)

>> A^10000

ans =

4/17	4/17	4/17	4/17	0	0	0
7/34	7/34	7/34	7/34	0	0	0
9/34	9/34	9/34	9/34	0	0	0
5/17	5/17	5/17	5/17	0	0	0
0	0	0	0	2/5	2/5	2/5
0	0	0	0	1/5	1/5	1/5
0	0	0	0	2/5	2/5	2/5

For very large powers k, A^k is not a positive matrix, so it is not regular

C)

>> eigs(A)

ans =

1 + 0i
1 + 0i

-1/2 + 1/2i
-1/2 - 1/2i
-611/1998 + 745/1286i
-611/1998 - 745/1286i

D)

Theorem 1:
A has an eigenvalue of 1, so A satisfies the conclusion of theorem 1

Theorem 6:
The absolute value of all eigenvalues of A not equal to 1 is less than 1

Corollary 4:
No, the conclusion to this statement is not satisfied, as the dimension of the eigenspace corresponding to eigenvalue = 1 is 2 for matrix A

E)

```
>> [S,D] = eig(A);
>> D

D =

  Columns 1 through 5

     1     +  0i          0     +  0i          0     +  0i          0     +  0i          0
+  0i
     0     +  0i       -611/1998  +  745/1286i       0     +  0i          0     +  0i
0     +  0i
     0     +  0i          0     +  0i       -611/1998  -  745/1286i       0     +  0i
0     +  0i
     0     +  0i          0     +  0i          0     +  0i       -388/999  +  0i
0     +  0i
     0     +  0i          0     +  0i          0     +  0i          0     +  0i          1
+  0i
     0     +  0i          0     +  0i          0     +  0i          0     +  0i          0
+  0i
     0     +  0i          0     +  0i          0     +  0i          0     +  0i          0
+  0i

  Columns 6 through 7

     0     +  0i          0     +  0i
     0     +  0i          0     +  0i
     0     +  0i          0     +  0i
     0     +  0i          0     +  0i
```

```
      0        +    0i            0        +    0i
     -1/2      +    1/2i          0        +    0i
      0        +    0i           -1/2      -    1/2i
```

```
>> steady_state1 = S(:,1)/sum(S(:,1))

steady_state1 =

    0.2353
    0.2059
    0.2647
    0.2941
         0
         0
         0
```

Importance ranking(Most important to least): 4,3,1,2,5,6,7

```
>> steady_state2 = S(:,5)/sum(S(:,5))

steady_state2 =

         0
         0
         0
         0
    0.4000
    0.2000
    0.4000
```

Importance ranking(Most important to least): 5,7,6,4,3,2,1

m = 0.1

```
>> M=(1-0.1)*A+0.1*G;
>> [S,D] = eig(M);
>> steady_state = S(:,1)/sum(S(:,1))

steady_state =

    0.1345
    0.1198
    0.1500
    0.1671
    0.1680
    0.0899
    0.1707
```

Importance Rankings(Most important to least)**: 7,5,4,3,1,2,6**

m = 0.15

```
>> M=(1-0.15)*A+0.15*G;
>> [S,D] = eig(M);
>> steady_state = S(:,1)/sum(S(:,1))

steady_state =

    0.1345
    0.1209
    0.1494
    0.1666
    0.1662
    0.0921
    0.1703
```

Importance Rankings(Most important to least)**: 7,4,5,3,1,2,6**

m = 0.2

```
>> M=(1-0.2)*A+0.2*G;
>> [S,D] = eig(M);
>> steady_state = S(:,1)/sum(S(:,1))

steady_state =

    0.1346
    0.1221
    0.1488
    0.1659
    0.1644
    0.0943
    0.1698
```

Importance Rankings(Most important to least)**: 7,4,5,3,1,2,6**

Though the page rankings may slightly differ based on the value of m, we now only obtain a unique steady state vector which corresponds to eigenvalue = 1 for each Google Matrix M.

4)

A)

```
>> x0 = [1;0;0;0;0;0;0]

x0 =

     1
     0
     0
     0
     0
     0
     0

>> A

A =

         0    0.5000    0.3333         0    0.3333    0.2500    1.0000
         0         0    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000    0.3333         0         0
    0.3333         0         0         0         0    0.2500         0
         0    0.5000    0.3333         0         0    0.2500         0
    0.3333         0         0    0.5000         0         0         0
         0         0         0         0    0.3333         0         0

>> G

G =

    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429
    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429    0.1429

>> M=(1-0.15)*A+0.15*G;
>> M^10*x0

ans =

    0.2424
    0.1033
    0.1828
    0.1201
    0.1471
    0.1412
```

0.0631

Importance Rankings(Most important to least)**:** 1,3,5,6,4,2,7

Yes, the rankings obtained at the end of the iterations match the previous rankings in 1e

B)

>> x0 = [1/2;1/2;0;0;0;0;0]

x0 =

0.5000
0.5000
0
0
0
0
0

>> M^10*x0

ans =

0.2424
0.1033
0.1828
0.1201
0.1471
0.1412
0.0631

Importance Rankings(Most important to least)**:** 1,3,5,6,4,2,7

Yes, the rankings obtained at the end of the iterations match the previous rankings in 1e