Digital Image Processing Laboratory 2

2-D Random Processes
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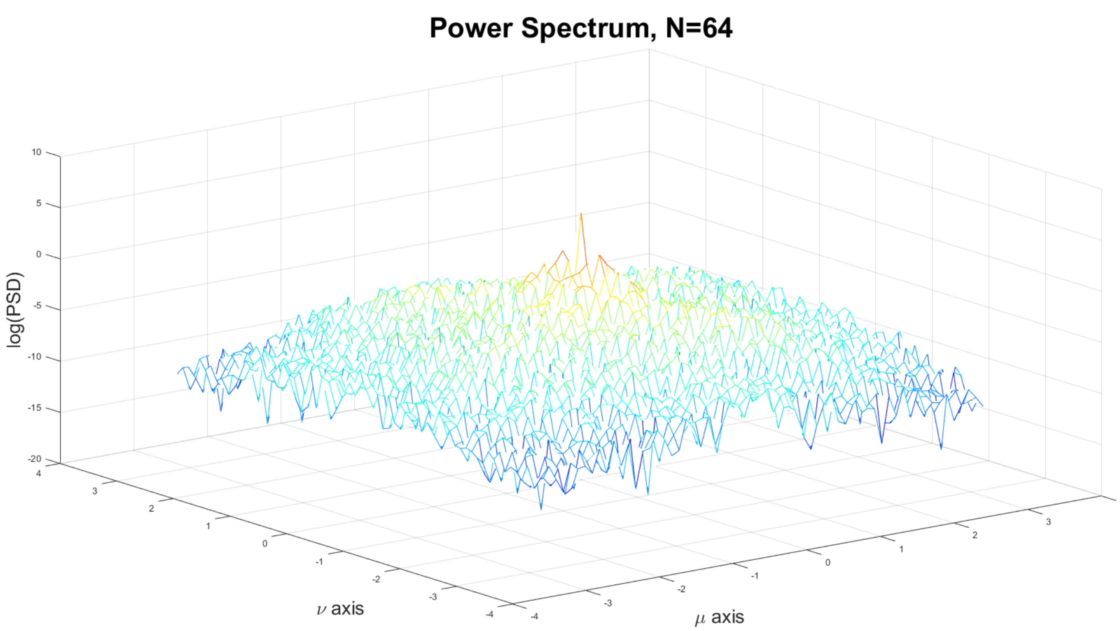
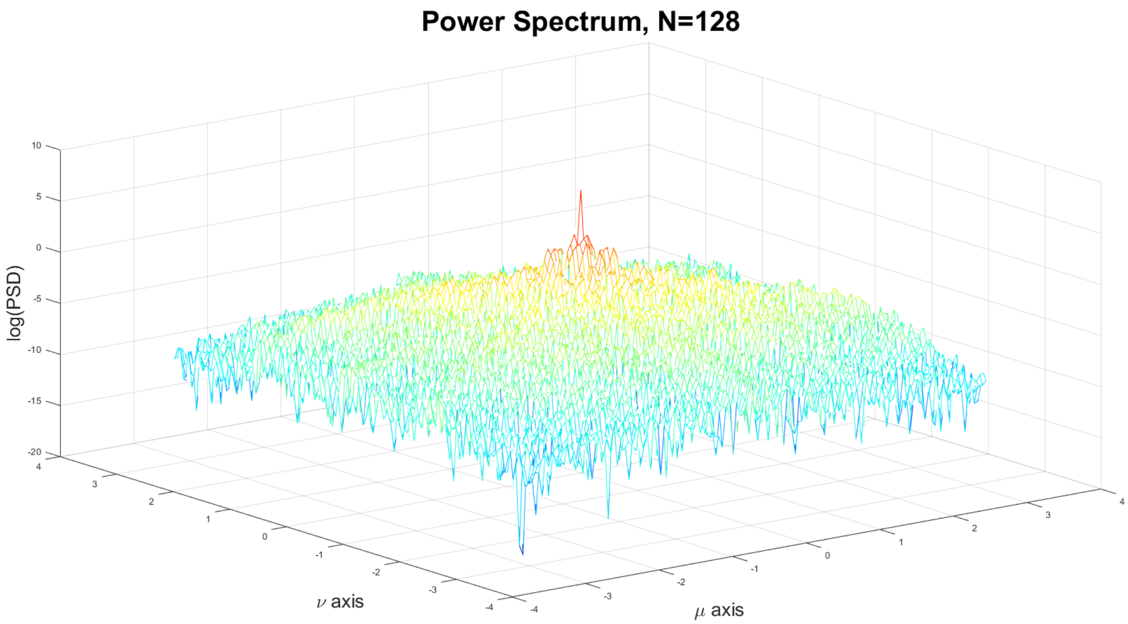
1 Power Spectral Density of Image

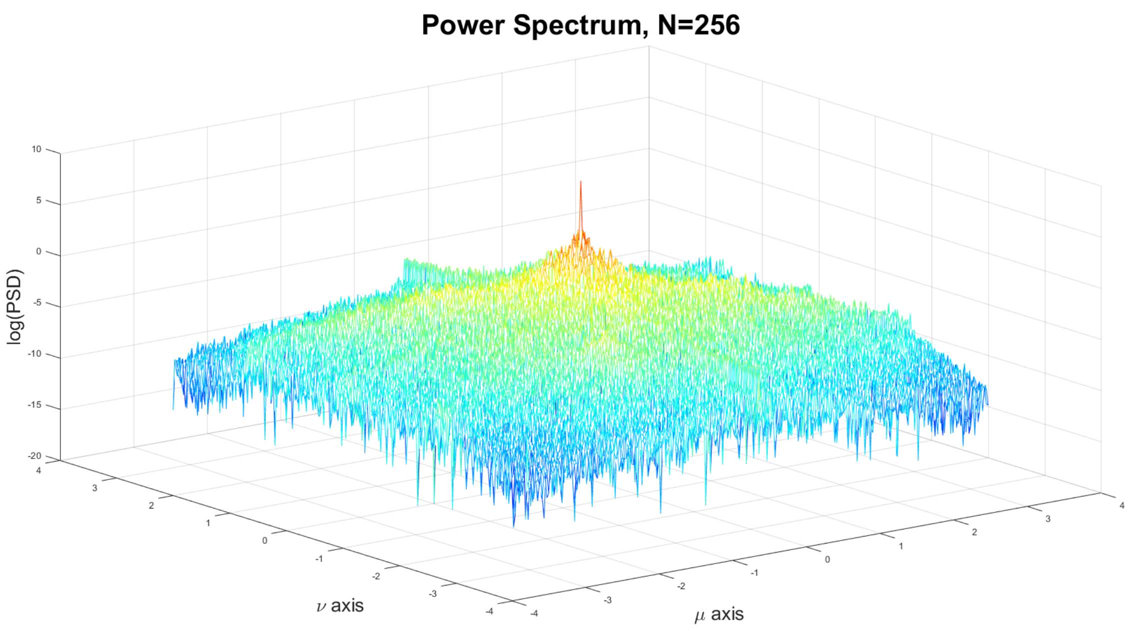
In this problem, we use Matlab to read and analyze the gray scale image img04g.tif.

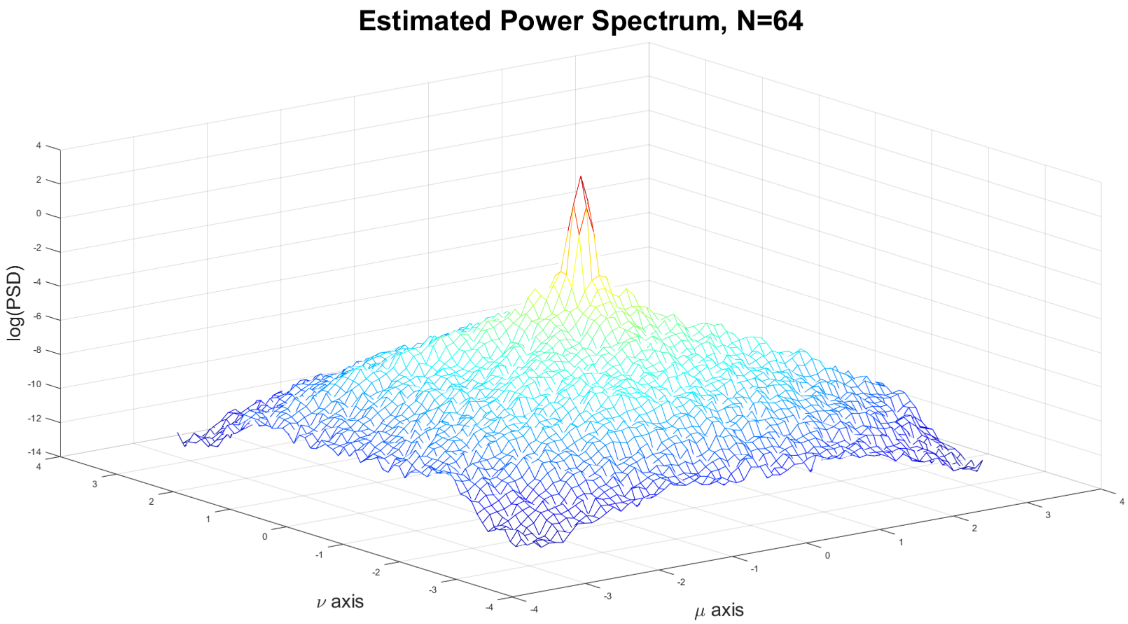
1.1 Gray Scale Image $img04g.tif$

1.2 Power Spectral Density Plots




Power Spectrum, N=128
log(PSD)
10
5
0
-5
-10
-15
-20
ν axis
4
3
2
1
0
-1
-2
-3
-4
μ axis
-4
-3
-2
-1
0
1
2
3
4


Power Spectrum, N=256
log(PSD)
10
5
0
-5
-10
-15
-20
ν axis
4
3
2
1
0
-1
-2
-3
-4
μ axis
-4
-3
-2
-1
0
1
2
3
4



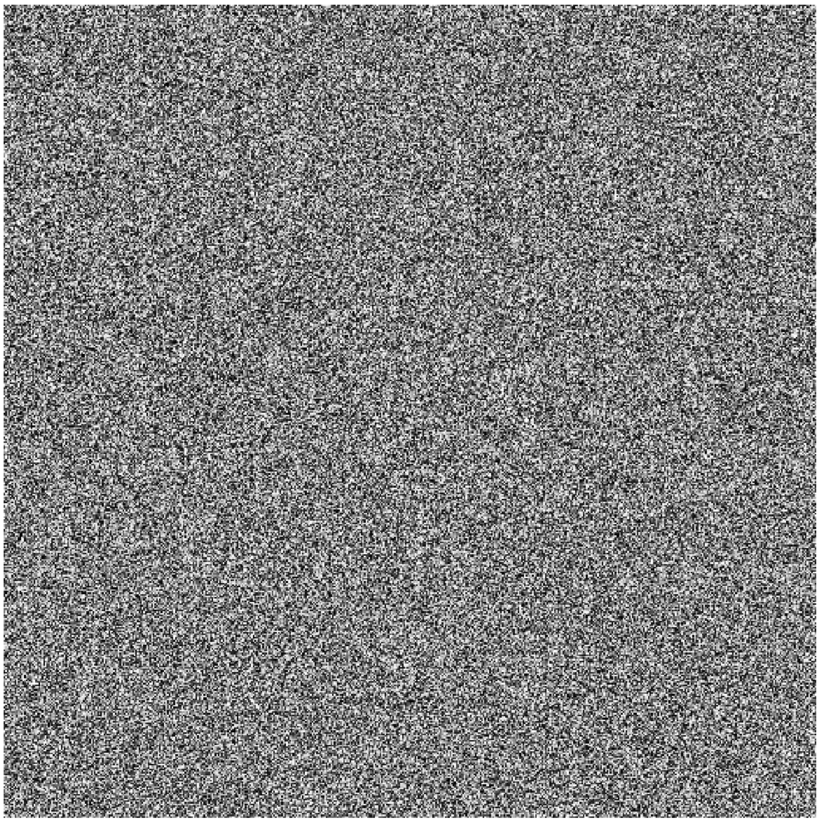
1.3 MATLAB Code

```
function res=BetterSpecAnal(X)
   n=64;
   x = 2*pi*((0:(n-1)) - n/2)/n;
   y = 2*pi*((0:(n-1)) - n/2)/n;
   window=hamming(n)*hamming(n)';
   img_res=size(X);
   img_w=img_res(2);
   img_h=img_res(1);
   % To ensure we create 25 non-overlapping regions
    start_w=(img_w-(5*n))/2;
    start_h=(img_h-(5*n))/2;
    res=zeros(n,n);
    for w=1:5
        for h=1:5
            z=X(start_h+(h-1)*n:start_h+h*n-1,start_w+(w-1)*n:start_w+w*n-1);
            z=fftshift(fft2(z.*window)).^2;
            res=res+log((1/n^2)*abs(z));
        end
    end
    res=res/25;
    figure
    mesh(x,y,res)
    colormap jet
    xlabel('\mu axis','FontSize',20)
    ylabel('\nu axis','FontSize',20)
    zlabel('log(PSD)','FontSize',20)
    title('Estimated Power Spectrum, N=64','FontSize',30)
```

2 Power Spectral Density of a 2-D AR Process

In this problem, we will generate a synthetic 2-D autoregressive (AR) process using Matlab, and analyze its power spectral density.

2.1 The image $255 + (x + 0.5)$



2.2 Difference Equation

In order to implement the filter, we need to use the difference equation.

Let's consider the given equation,

$$H(z_1, z_2) = \frac{3}{1 - 0.99z_1^{-1} - 0.99z_2^{-1} + 0.9801z_1{-}1z_2^{-1}} \tag{1}$$

But we know that,

$$H(z_1, z_2) = \frac{Y(z_1, z_2)}{X(z_1, z_2)}$$

Thus 1 becomes,

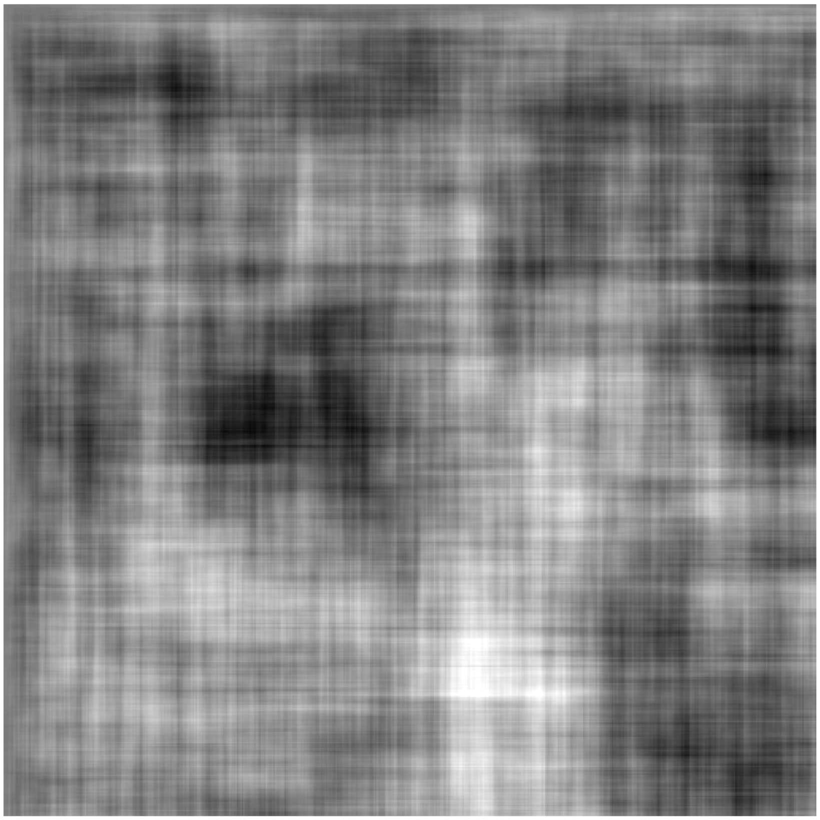
$$\frac{Y(z_1, z_2)}{X(z_1, z_2)} = \frac{3}{1 - 0.99z_1^{-1} - 0.99z_2^{-1} + 0.9801z_1{-}1z_2^{-1}}$$

$$Y(z_1, z_2)1 - 0.99z_1^{-1} - 0.99z_2^{-1} + 0.9801z_1{-}1z_2^{-1} = 3 * X(z_1, z_2)$$

Taking Inverse Z-Transform and simplifying gives us the required difference equation,

$$y(m, n) = 0.99(y(m - 1, n) + y(m, n - 1)) - 0.9801y(m - 1, n - 1) + 3x(m, n)$$

2.3 The Filtered image $y + 127$



2.4 Derivation of Power Spectral Density

We know that $S_y(u, v)$ is given as,

$$S_y(u, v) = |H(u, v)|^2 S_x(u, v) \tag{2}$$

Let us first find the PSD of x. We know that,

$$S_x(u, v) = \lim_{N->\infty} \frac{E(|X_N(u, v)|^2)}{N} \tag{3}$$

We know that X is uniformly distributed i.e $U[-0.5, 0.5]$. Hence,

$$\mu(X) = E[X] = (-0.5 + 0.5)/2 = 0$$
$$\sigma^2 = Var[X] = (0.5 + 0.5)^2/12 = 1/12$$

But,

$$Var[X] = E[X^2] - (E[X])^2$$
$$Var[X] = E[X^2] = 1/12$$

Thus using these values, 3 becomes,

$$S_x(u, v) = \lim_{N->\infty} \frac{N * Var[X]}{N}$$
$$= Var[X]$$
$$S_x(u, v) = 1/12$$

Using the above value in 2, we get,

$$S_y(u,v) = \frac{|H(u,v)|^2}{12}$$

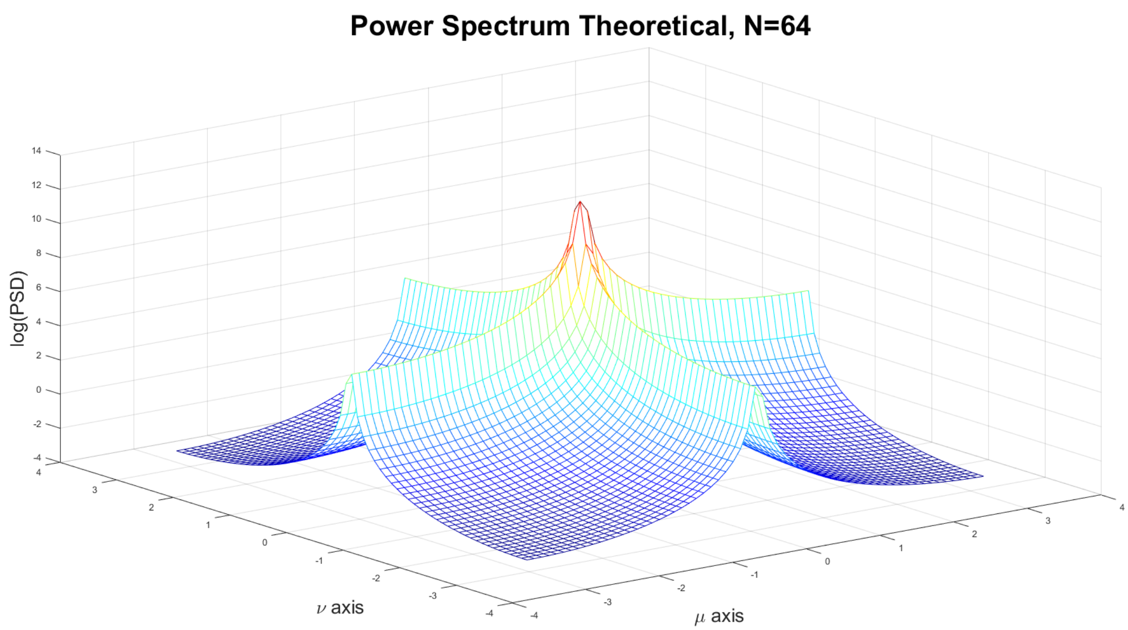
From 1, the 2-D Fourier Transform can be obtained by substituting $z_1 = e^{ju}$ & $z_2 = e^{jv}$

$$H(u,v) = \frac{3}{1 - 0.99e^{-ju} - 0.99e^{-jv} + 0.9801e^{-j(u+v)}} \tag{4}$$

Thus,

$$S_y(u,v) = \frac{1}{12} * \left| \frac{3}{1 - 0.99e^{-ju} - 0.99e^{-jv} + 0.9801e^{-j(u+v)}} \right|^2$$

2.5 A mesh plot of the function $log(S_y(u,v)))$



2.6 A mesh plot of the function $log(S_y(u, v)))$ generated using BetterSpecAnal.m