ECE661: Computer Vision

Homework 1
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September 13, 2020

Answer 1:

In $\mathbb{R}^2$, the origin has the coordinates $(0, 0)$.

We know that, any point (x,y) in $\mathbb{R}^2$ can be represented in $\mathbb{R}^3$ with the homogeneous coordinates, $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, where $x = \frac{x_1}{x_3}$ and $y = \frac{x_3}{x_3}$.

As a result, the origin $(0, 0)$ can be represented in $\mathbb{R}^3$ as $\begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix}$ or $k* \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, where $k \neq 0$ and $k \in \mathbb{R}$

Answer 2:

We know that any point at infinity is represented as $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$ in $\mathbb{R}^3$

x & y can be any real numbers. By changing x & y, we can get a plane of points at infinity. Thus, any point at infinity is along a specific direction that is controlled by x & y

As a result, all points at infinity are not unique.

Answer 3:

We know that a degenrate conic is represented by the equation,

$$C = lm^T + ml^T$$

where l & m are homogeneous representations of the intersecting lines that form the degenerate conic.
lm^T is a vector outer product which results in a matrix with linearly dependent columns. As a result, the rank of the matrix lm^T is 1.
In the same way, we can also say that the rank of the matrix ml^T is also 1.
But we know that,

$$rank(A + B) \leq rank(A) + rank(B)$$

As a result,

$$rank(C) = rank(lm^T + ml^T)$$

$$rank(C) \leq rank(lm^T) + rank(ml^T)$$
$$rank(C) \leq 1 + 1$$
$$rank(C) \leq 2$$

Answer 4:

a)

l_1 passes through the points $(0,0)$ & $(5,-3)$. l_2 passes through the points $(-3,4)$ & $(-7,5)$. Thus, to find the intersection of l_1 & l_2 we need the following three steps:

1) Determining l_1 ,

$$l_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ X \ \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$$

2) Determining l_2,

$$l_2 = \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix} \ X \ \begin{bmatrix} -7 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \\ 13 \end{bmatrix}$$

3) Determining point of intersection P,

$$P = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} \ X \ \begin{bmatrix} -1 \\ -4 \\ 13 \end{bmatrix} = \begin{bmatrix} 39 \\ 65 \\ 23 \end{bmatrix}$$

Thus, in $\mathbb{R}^3$ P is $23 * \begin{bmatrix} 39/23 \\ 65/23 \\ 1 \end{bmatrix}$

In $\mathbb{R}^2$, P is $\begin{bmatrix} 39/23 \\ 65/23 \end{bmatrix}$

b)

In this case, we would take 1 step to determine l_1 again. We would take the second step to determine l_2 as follows:

$$l_2 = \begin{bmatrix} -7 \\ -5 \\ 1 \end{bmatrix} \ X \ \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ 14 \\ 0 \end{bmatrix}$$

We can see that both l_1 & l_2 have their third element as 0. We know that this third component is the intercept of the line because we represent a line as $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$. Without needing to calculate, we can say that thew point of intersection P in this case in $(0,0)$. **Hence, we only need 2 steps in this case.**

Answer 5:

Determining l_1,

$$l_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \ X \ \begin{bmatrix} 5 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix}$$

Determining l_2,

$$l_2 = \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} \ X \ \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 15 \end{bmatrix}$$

Now, the point of intersection P is.

$$P = l_1 \ X \ l_2$$

$$P = \begin{bmatrix} 3 \\ 5 \\ 0 \end{bmatrix} \ X \ \begin{bmatrix} 3 \\ 5 \\ 15 \end{bmatrix} = \begin{bmatrix} 75 \\ -45 \\ 0 \end{bmatrix}$$

We can see that l_1 and l_2 are parallel lines as their first two elements are same and only intercept changes. As a result, these two lines intersect at infinity at an ideal point $\begin{bmatrix} 75 \\ -45 \\ 0 \end{bmatrix}$ or they intersect at a point at infinity along the direction $(75, -45)$

Answer 6:

Given: The Conic C is an ellipse that is centered at $(3, 2)$, has a major axis $a = 2$ and a minor axis $b = \frac{1}{2}$. We know that, the equation of an ellipse centered at a point (a, b) is given as,

$$\frac{(x - c_1)^2}{a^2} + \frac{(y - c_2)^2}{b^2} = 1$$

Thus in our case, we can write this as,

$$\frac{(x - 3)^2}{2^2} + \frac{(y - 2)^2}{(\frac{1}{2})^2} = 1$$

On solving this and bringing it into it's implicit form, we get,

$$x^2 + 4y^2 - 6x - 16y + 24 = 0$$

Comparing this to the equation of a conic in implicit form,

$$ax_1^2 + bx_1x_2 + cx_2^2 + dx_1x_3 + ex_2x_3 + fx_3^2 = 0$$

we see that $a = 1$, $b = 0$, $c = 4$, $d = -6$, $e = -16$, $f = 24$.
Thus, the homogeneous representation of the ellipse can be written as,

$$C = \begin{bmatrix} a & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{bmatrix} = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 4 & -8 \\ -3 & -8 & 24 \end{bmatrix}$$

The point $(0,0)$ in $\mathbb{R}^3$ is $\begin{bmatrix}0\\0\\1\end{bmatrix}$

Thus, the polar line l is,

$$l = Cp$$
$$l = \begin{bmatrix}1 & 0 & -3\\0 & 4 & -8\\-3 & -8 & 24\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix}$$
$$l = \begin{bmatrix}-3\\-8\\24\end{bmatrix}$$

Thus, the equation of the polar line can be written as,

$$-3x - 8y + 24 = 0$$

To find intersection with x axis, put y=0 in the above equation i.e

$$-3x - 8(0) + 24 = 0$$
$$x = 8$$

Hence, the polar line l intersects with the x axis at the point $(8,0)$.
Simialrly, to find intersection with y axis, put x=0 in the equation of polar line i.e

$$-3(0) - 8y + 24 = 0$$
$$y = 3$$

Hence, the polar line l intersects with the y axis at the point $(0,3)$

Answer 7:

Given,

$$l_1 = \begin{bmatrix}1\\0\\-1/2\end{bmatrix}$$

and,

$$l_2 = \begin{bmatrix}0\\1\\1/3\end{bmatrix}$$

Thus, the point of intersection P of the two lines is,

$$P = l_1 \ X \ l_2$$
$$P = \begin{bmatrix}1\\0\\-1/2\end{bmatrix} \ X \ \begin{bmatrix}0\\1\\1/3\end{bmatrix} = \begin{bmatrix}1/2\\-1/3\\1\end{bmatrix}$$

Hence the two lines intersect at the point, $P = (1/2, -1/3)$