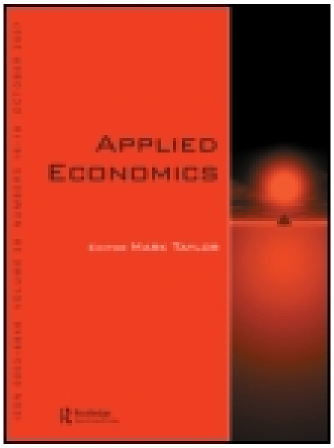

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Does player specialization predict player actions? Evidence from penalty kicks at FIFA World Cup and UEFA Euro Cup

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Penalty kicks are analysed in the literature as 'real life experiments' for assessing the use of rational mixed strategies by professional players. However, each penalty kick cannot be considered a repetition of the same event because of the varying background conditions, in particular the heterogeneous ability of different players. Consequently, aggregate statistics over data sets composed of a large number of penalty kicks mediate the behaviour of the players in *different* games, and the properties of optimal mixed strategies cannot be tested directly because of *aggregation bias*. In this article, we model the heterogeneous ability of players. We then test the hypothesis that differently talented players randomize over different actions. To achieve this aim, we study a data set that collects penalties kicked during shoot-out series in the last editions of FIFA World Cup and UEFA Euro Cup (1994–2012) where kickers are categorized as specialists and non-specialists. The results support our theoretical predictions.

Keywords: mixed strategies; penalty kicks; aggregation bias; players' specialization; action predictability

JEL Classification: C72; C93; L83

I. Introduction

The individual behaviour of professional players during sports events has been often treated in the economic literature as a natural experiment for testing the empirical relevance of game theoretical predictions, particularly those of mixed strategy Nash equilibria in simultaneous two-person, one-shot games. For this aim, sports events present a number of advantages: unlike the participants in many laboratory experiments, sport professionals are expert players, with evident incentives to behave rationally, and their actions are easily observable; moreover, unlike many alternative natural experiments, sports events are characterized by obvious pay-off functions and simple interaction settings.

This strand of the economic literature is essentially based on the empirical analysis of data sets composed of large numbers of independent, similar events (especially penalty kicks) that are selected to obtain a suitable estimate of the behaviour and the pay-off of the players. The outcomes are then compared in many ways with the theoretical predictions to assess whether the behaviour of expert players is consistent with the predictions of Nash equilibria.

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Despite the advantages, the use of strategic interactions in sports events for assessing game theoretical predictions is not without difficulties. The discussion in the literature deals, in particular, with two critical issues: how a game model should represent observed situations (Is the game simultaneous or not? What are the strategic variables and the action sets of the players?) and how to interpret the empirical evidence. In the latter case, while game theoretical predictions hold for individual observations, empirical tests can only be performed on samples of *heterogeneous* observations. Given that different players are likely to be characterized by different pay-offs or even different strategy sets, it is easy to understand that the properties of aggregated estimates do not necessarily coincide with the properties required for individual behaviour.

Restricting our attention to penalty kicks in soccer, some of the questions addressed before find adequate answers in the literature. The evidence that professional players actually behave at the equilibrium as predicted by theory is quite extensive. Moreover, the methodologies proposed so far for testing simultaneity and serial independence of actions are widely accepted and penalty kicks are normally recognized as simultaneous one-shot games. Less attention has been paid in this literature to the analysis of the effects of aggregation of heterogeneous events, in particular considering heterogeneity determined by differences in players' quality. The scarce attention to the issue is justified by the fact that the samples analysed so far in the literature present a population that is not very heterogeneous, as will be discussed below.

In this article, we provide an in-depth analysis of aggregate behavioural outcomes when players are very heterogeneous. For this purpose, we analyse a set of penalty kicks selected to obtain a sample of heterogeneous players acting in a relatively uniform environment to highlight the role of different abilities of the players. Our data set is composed of every penalty kicked during shoot-out series at FIFA World Cup and UEFA Euro Cup over the period 1994 to 2012.[1] Shoot-out series produce valuable data because players other than 'penalty specialists' are called to kick while guaranteeing more uniform incentives and fatigue for every player.[2]

Our main result is that when the ability of the kickers is sufficiently heterogeneous, modelling the actions of the players with only two or three actions associated to the side of the kick – as usually happens – is not sufficient to represent the strategic space. We show that differently skilled players not only randomize with different probability distributions over the same actions, but also randomize over different actions. This evidence cannot be obtained within the framework used in the previous literature.

In Section II, we survey the economic literature on penalty kicks. In Section III, we present our data set. In Section IV, we present a series of theoretical predictions and discuss the results. Section V briefly concludes.

II. The Economic Literature on Penalty Kicks

The empirical significance of theoretical predictions concerning mixed strategy equilibria has been often analysed in the economic literature addressing professional sports events, and in particular penalty kicks in soccer matches. Other sports applications include, among others, first serves in tennis (Walker and Wooders, 2001; Hsu *et al.*, 2007), soccer shots during live action (Moschini, 2004), pitches in baseball and play choice in football (Kovash and Levitt, 2007).

A penalty kick can be simply treated as a generalized matching pennies (GMP) game. A GMP is a constant-sum, one-shot game. The players simultaneously choose an action out of the two available (for both players) and if the choices 'match', one of the players obtains a pay-off higher than the pay-off obtained if the choices do not 'match'. The opposite holds for the rival. Obviously, a GMP presents no Nash equilibria in pure strategies.

In a penalty kick game, the players are the kicker (K) and the goalkeeper (G), and their choices are deciding where to kick and where to jump, respectively. The game varies from a GMP for the following reasons:

- there are actually more than two available actions: K can direct the shot towards any point in the goal area, adjust the power of the kick, etc.; G can direct

[1] A penalty shoot-out is a method used to decide which team progresses to the next stage of a tournament (or wins the tournament) following a tied game. In a nutshell, the shoot-out series is composed of five penalties for every team, but every single penalty must be kicked by a different player, so that even non-penalty-specialists are involved. The team that scores more penalties obviously wins. Details concerning specific rules and the history of shoot-outs can be found at en.wikipedia.org/wiki/Penalty_shoot-out_(association_football)

[2] Shoot-out penalties are kicked at the end of the match and every single kick is obviously crucial. On the contrary, with the relevant exception of the paper by Apesteguia and Palacios-Huerta (2010) discussed in section 'Kicker's specialization', the data sets used in the literature refer to penalties kicked during national league matches. These penalties are usually kicked by the team specialist and can be assigned at any moment of the match. The outcome of these penalties can be more or less crucial, depending on the score and the time remaining to the end of the match.

his jump in many different directions, decide where to stand on the goal line, etc.;

- the action choice of each player does not necessarily result in the outcome observed, because the actual action of the player could, more or less, fail[3];
- every action profile does not determine a certain pay-off but only a scoring probability. For example, when the actions of the players match (i.e. they choose the same region of the goal area), the goalkeeper may not be able to clear the ball; alternatively, when the actions do not match, the ball could miss the goal. Intuition (and the empirical evidence) suggests that the probability of scoring a goal increases, *ceteris paribus*, when the actions of the two players do not match (and decreases when they do match).

As for the first point, the seminal paper of Chiappori *et al.* (2002) first represents the action set of each player as composed of three actions: the kicker and the goalkeeper can kick/jump to their left, central or right side.[4] In most of the works that followed Chiappori *et al.*, the pure strategy set has been further reduced to two elements: left and right.[5] To be more precise, the composition of the action set of the players usually represented in the literature takes into account the fact that the game is not symmetric because the kicker has a preferred or *natural* side. According to conventional wisdom, when kicking to the converse side of his strong foot, a kicker obtains a higher scoring probability because, *ceteris paribus*, shots are placed with more strength and/or precision; this fact is explained by the biomechanical characteristics of the movement. Given this asymmetric performance, the kicker must actually decide wether to kick on his '(N)atural' side (left for a right-footed kicker and right for a left-footed kicker) or on his '(O)pposite' side.

The reduced form of the game (2 × 2 actions) is then represented in Table 1. Since the pay-offs for K and G sum to one, only the pay-off for K is reported. The ranking of

Table 1. Asymmetric 2×2 penalty kick game

	Goalkeeper (G)	
Kicker (K)	Natural	Opposite
Natural	P_N	π_N
Opposite	π_O	P_O

the parameters is $1 > \pi_N > \pi_O > P_N > P_O > 0$ (following empirical evidence, the advantage of wrong-footing by the goalkeeper is assumed to be larger than the natural side advantage). The mixed strategy equilibrium of the game can be easily obtained.

Starting from the paper of Chiappori *et al.* (2002), the typical research strategy in the literature – aimed at assessing the equilibrium behaviour of professional players – selects a large number of penalty kicks, observes the action of the players, measures the frequency of the different actions and the scoring probabilities for every action profile, and then controls whether the estimated figures are coherent with the theoretical predictions. To achieve this aim, the natural test to perform is based on the 'Fundamental Lemma' of mixed strategies. This lemma presumes that, at the equilibrium, the expected pay-offs of every pure strategy that is randomized with positive probability must be equal.[6]

Even in the simplified model represented in Table 1, the actual consistency of the observed behaviour of professional soccer players with the predictions of mixed strategy Nash equilibria is not easy, because of observations' heterogeneity. If the data sets were all composed of homogeneous observations, the obvious strategy for verifying that players actually play Nash equilibrium strategies would be to check whether the expected pay-off for both the kicker and the goalkeeper that choose to kick/jump natural is not significantly different from the expected pay-off they obtain when choosing to kick/jump opposite (applying the Fundamental Lemma). However, every single penalty kick observed is indeed a *different* game

[3] A kick out of the goal is an example of such a failure, even if often reveals at least the broad intended shot direction. The same is less easy to detect for centre kicks, which could be 'failed' left or right kicks.

[4] Chiappori *et al.* and the following literature therefore choose to consider only the 'horizontal direction' of the kick (or jump), deliberately ignoring other decisions, such as the vertical direction of the ball or the power of the kick. The only empirical analysis taking into account the vertical direction of a penalty kick is provided by Bar-Eli and Azar (2009). This choice can be easily understood considering that during data collection (data are usually obtained by direct inspection of the videos of soccer matches), it is difficult to identify vertical directions and almost impossible to measure the power of the kick.

[5] Palacios-Huerta (2003, footnote 11) decides to drop the pure strategy of centre kicks/jumps because of the small proportion observed in his empirical analysis. The same applies in Baumann *et al.* (2011). Leininger and Ockenfels (2008), on the contrary, assign a relevant strategic role to central kicks. They observe an increase of the scoring probability in the German Football League (Bundesliga) in the period 1963–1990, and they attribute this phenomenon to a behavioural innovation that took place when kickers, during the 1970s, recognized the strategic value of occasionally kicking to the centre. On the empirical side, the decision to limit the action set of the players to just two pure strategies eliminates any ambiguity that could be introduced when defining the width of the area of the goal to be considered 'central'.

[6] As said, the literature usually assumes that no one player is able to anticipate the other by perceiving some clue about his choice, so that the game is indeed simultaneous; moreover, it is assumed that the game is one-shot, in that the memory of past repetitions does not induce dynamic strategies. The empirical evidence confirms these assumptions (Miller, 1998; Chiappori *et al.,* 2002; Palacios-Huerta, 2003).

because P and π depend on the ability of the two players and on several other variables (such as the athletic/psychological conditions of the players or field circumstances[7]). If a reasonable number of observations is needed, one has to accept a certain level of heterogeneity. The average scoring probability, for example, is then only a mean of the different values assumed by P and π in every individual observation. As a consequence, because different Ps and πs characterize every observation, the average scoring probabilities do not need to comply with the Fundamental Lemma even at equilibrium. For the same reason, average pay-offs also do not need to comply with the ranking of the pay-offs expected for every single observation.[8] If the observations are heterogeneous, an *aggregation bias* may arise.

Assume, for example, that the population of the kickers is characterized by two types of players, differing only by the value of P_N. Those characterized by higher values of P_N kick more often on their natural side and goalkeepers facing such players jump more often on the natural side. As a consequence, among the shots on the natural side, those kicked by high-P_N players are overrepresented; consequently, the choice to kick on the natural side seems to present a higher (average) scoring probability even at the equilibrium, thus apparently violating the Fundamental Lemma.

To avoid a direct test of the Fundamental Lemma, Chiappori *et al.* (2002, Prop. 3) find a number of properties of the equilibrium that do not depend on the composition of the population, but only require that the pay-offs *for every player* follow the ranking indicated before ($1 > \pi_N > \pi_O > P_N > P_O > 0$).[9] For example, at the equilibrium, kickers must kick opposite less frequently than goalkeepers dive opposite. All of these properties are confirmed by the empirical evidence, which is provided by a data set composed of 459 penalties kicked in the Italian and French first leagues in the period 1997 to 2000.

Coloma (2007) confirms on the same data set the results obtained by Chiappori *et al.*, proposing an original methodology that uses a simultaneous regression approach.

Palacios-Huerta (2003) collects a data set composed of 1417 penalties kicked in different leagues (most Spanish, Italian or English) in the period 1995 to 2000. This database is so large that it is possible to test whether the scoring probabilities are identical across pure strategies for *individual* players: 42 players (both kickers and goalkeepers) were involved at least in 30 penalty games. Palacios-Huerta finds that the null hypothesis of equal scoring probabilities cannot be rejected for 39 players at the 5% significance level.

In summary, the literature confirms that soccer professionals' behaviour is remarkably consistent with equilibrium choices.[10] These results are robust to players' heterogeneity. No evidence that different players behave differently is provided in the literature. The only exception is Baumann *et al.* (2011), who propose a probit analysis on a data set of 999 penalties kicked in the German Bundesliga in the period 1995 to 2007. They observe that high-quality players choose more frequently the natural side, a choice that makes them more predictable; however, high-quality players also present a higher scoring probability. The result points out a bias towards the natural side that increases with the ability of the player.

None of the authors cited challenge the implicit assumption in the literature that different players randomize over the *same* action set.[11] It is important to remember again here that the heterogeneity of the players in the samples analysed in the cited literature is likely to be fairly limited. In our article, we will discuss this assumption and provide empirical evidence derived from a data set where heterogeneity across players is much more pronounced.

Similarly, the literature does not discuss the fact that the direction of the kick that is observed could be an imperfect realization of the intentions of the kicker. In other words, every data set always contains a number of 'observational mistakes'. A corollary to the previous observation is that, not surprisingly, in most of the literature on penalty kicks, a blocked kick is treated the same way as an outside-the-goal kick. The following empirical analysis will also distinguish the information associated to these two types of events.

III. Data and Variables

Our data set comprises all the penalties kicked during all shoot-out series at the FIFA World Cup (over the period 1994 to 2010) and the UEFA Euro Cup (over the period

[7] Apesteguia and Palacios-Huerta (2010) analyse a large set of penalty shoot-outs in order to understand the effect of psychological pressure on the performance of the players. Their main findings are discussed in section 'Kicker's specialization'. Dohmen (2008) documents that penalty kickers are more likely to choke on a penalty kick when the match takes place in their home stadium.
[8] To be more precise, it is easy to prove that the ranking of average pay-offs respect the expected ranking for every single observation if Ps and πs differ only by a constant among players. The same applies for the Fundamental Lemma.
[9] Actually, another assumption is needed: $\pi_N - P_N$ must be lower than $\pi_O - P_O$ for every player. More properties characterizing the mixed strategy Nash equilibrium can be derived under the hypothesis that heterogeneity regards only one of the two players.
[10] Bar-Eli *et al.* (2007), however, discuss possible nonoptimal behaviour of goalkeepers driven by 'inaction aversion'.
[11] Chiappori *et al.* (Prop. 1) find that a 'restricted randomization' is possible when the scoring probability of kicking central is sufficiently low. In this case, the kicker never kicks to the centre and the goalkeeper never remains at the centre. The authors do not provide any empirical test about this prediction because of the small number of central kicks in their data set.

Table 2. Descriptive statistics of the data set

Tournament	Play-off matches	Matches ended with shoot-out	Penalties (a + b + c)	Goals (a)	%	Kicks saved by the goalkeeper (b)	%	Kicks out of the goal (c)	%
FIFA 1994	16	3	29	18	62.1	7	24.1	4	13.8
UEFA 1996	7	4	42	37	88.1	4	9.5	1	2.4
FIFA 1998	16	3	28	20	71.4	7	25.0	1	3.6
UEFA 2000	7	1	8	4	50.0	3	37.5	1	12.5
FIFA 2002	16	2	19	13	68.4	3	15.8	3	15.8
UEFA 2004	7	2	26	20	76.9	2	7.7	4	15.4
FIFA 2006	16	4	33	21	63.6	8	24.2	4	12.1
UEFA 2008	7	2	16	10	62.5	4	25.0	2	12.5
FIFA 2010	16	2	18	14	77.8	2	11.1	2	11.1
UEFA 2012	7	2	18	12	66.7	3	16.7	3	16.7
Total	115	25	237	169	71.3	43	18.1	25	10.6

1996 to 2012). Descriptive statistics of the data set are provided in Table 2.[12]

Each penalty is an observation. For each observation, we have collected data on the identity of the kicker and the goalkeeper, as well as the foot used by the kicker for the shot and the action chosen by both players. For the action sets, we have made the following choices:

- we have vertically divided the goal area into two equal parts. Kicks have been classified depending on the half of the goal area where they have been directed.[13] Combining this information with the foot used for the kick allows us to classify actions as natural side shots (N) or opposite side shots (O);
- for the goalkeeper, three possible alternatives are evaluated: the goalkeeper jumps to his left, the goalkeeper jumps to his right or the goalkeeper stands still. Similarly, jumps have been classified into natural side jumps (N) or opposite side jumps (O).

The data set was assembled by consulting the FIFA and UEFA official reports, as well as videos of every match. However, missing or unclear videos allowed us to obtain full information for only 220 out of 237 penalties of our data set.[14] The 237 observations involve 198 kickers and 35 goalkeepers. The number of observations on individual players is too small to allow us to analyse the behaviour of single kickers (specifically, no single player in the sample kicks more than three penalties). On the other hand, the composition of this sample guarantees a relevant heterogeneity in terms of players' ability, because nonspecialists as well as specialists are called to kick in shoot-out series.[15]

Kicker's specialization

The presence of a relevant number of nonspecialists among the kickers in our sample is first suggested by the comparison of the scoring probability of our shoot-out penalties (71.3%, see Table 2) with the success rates in the samples of penalties during regular time proposed by Chiappori *et al.* (2002), Palacios-Huerta (2003) and Dohmen (2008) (75.0%, 80.1% and 74.3%, respectively).[16] Our figures are more similar to the scoring probability in Apesteguia and Palacios-Huerta (2010) (73.1%), also obtained from a data set composed of shoot-out penalties. An alternative interpretation of the lower performance of the kickers during shoot-outs could be associated to the higher psychological pressure of the players in this context.

In this section, we try to understand whether the lower success rate of the kickers during shoot-outs has to be attributed to the presence of nonspecialists or to

[12] The shoot-out series take place only at the end of play-off matches that end in a tie. It must be remarked that there need not to be 10 penalties in each series. Teams take turns to kick until each has taken five kicks. However, if one side scores more goals than the other could possibly reach with all of their remaining kicks, the shoot-out ends regardless of the number of kicks remaining. On the other hand, if the five penalty series end in a tie, further penalties are kicked until one team scores and the other team does not.

[13] Consequently, we have assigned a left or right side even to kicks that in other papers would have been classified as 'central'. Thus, we avoid a discretionary width of the central area of the goal to classify kicks as central. In some cases, it is difficult to determine the precise position of the penalty, but this difficulty would have been the same (or worse) with other conventions.

[14] We were not able to find the video for 8 out of 12 penalties of the match Czech Rep. vs. France at the UEFA Euro Cup 1996. We know that all of them were scored and the identity of the players, but we cannot classify the underlying actions. Additionally, the videos of nine other penalties were taken from an unfortunate perspective which made it impossible to identify the half of the goal area where they were directed. These penalties were obviously rather 'central'.

[15] In particular, we are thinking of the heterogeneity of kickers, because in a shoot-out series the goalkeeper remains the same (i.e. they are all specialists), while there must be at least five kickers for every team, so that even nonspecialists must kick.

[16] The mean difference is significant ($p = 0.001$) only for the sample of Palacios-Huerta (2003).

Table 3. The distribution of shot failures between specialists and nonspecialists

	Penalties kicked (a + b + c)	Goals (%) (a)	Penalties saved (% of failed kicks) (b)	Penalties out (% of failed kicks) (c)
Specialists	119	91(76.5%)	15 (53.6%)	13 (46.4%)
Nonspecialists	118	78 (66.1%)	28 (70.0%)	12 (30.0%)
Total	237	169 (71.3%)	43 (63.2%)	25 (36.8%)

performance decrements induced by psychological pressure. With this aim, we first introduce a specialization variable. We define as 'specialist' a kicker who scored at least one penalty kick in his club (in league matches) or in his national team (excluding shoot-out penalties) during a four-year interval centred around the observation date.[17]

The explanatory power of our variable is confirmed by performance measures. As illustrated in Table 3, we have identified 119 penalties kicked by specialists and 118 penalties kicked by nonspecialists; the scoring probability in the first group is 76.5%, whereas the scoring probability in the second group is 66.1%. This difference is statistically significant, with $p = 0.039$. In the following discussion, we will use the term 'specialist' and 'nonspecialist' to identify players from these two subsamples.

Apesteguia and Palacios-Huerta (2010) provide strong empirical evidence of the detrimental effects on performance of the 'importance' of the kick, as measured in terms of his weight on the *ex ante* probability of winning the shoot-out. This evidence determines that:

- the kickers of the teams that are lagging in the score present lower scoring performance, and vice versa (a 'behind-ahead asymmetry');
- the detrimental effects on performance become more pronounced as the final rounds are approached.

Linked to these findings is the common wisdom that specialist players are usually allocated 'early' kicks in the shoot-out series, even if this choice is not always optimal.[18] Our classification of players' specialization supports such common wisdom: as reported in Table 4, more than three quarters of the kickers in the first round are specialists, while their incidence falls to about one half and then one quarter in the following rounds.

Table 4. Specialists distribution and scoring performance in the different rounds of the shoot-outs

Round	Kickers	Of which specialists (%)	Goals (%)
1	50	38 (76.0%)	36 (72.0%)
2	50	26 (52.0%)	37 (74.0%)
3	50	25 (50.0%)	37 (74.0%)
4	48	13 (27.1%)	28 (58.3%)
5	27	14 (51.9%)	24 (88.9%)
+5	12	3 (25.0%)	7 (58.3%)
Total	237	119 (50.2%)	169 (71.3%)

[17] Penalty statistics have been assembled from various public databases, in particular www.transfermarkt.de (for national team penalties) and www.worldfootball.net (for club penalties). We were able to obtain complete information concerning all penalty takers in the national team, while in 34 (out of 237) cases penalty scorers in the club were not available in the whole four-year period. Notice that available statistics include only penalties scored (not kicked) during league matches, thus introducing a (negligible) endogenity issue. In all these cases, we implicitly took a conservative approach, by assuming that no penalties have been kicked in absence of the information. In this sense, the category of nonspecialists could include some specialist (and not vice versa). We also tried different alternative specifications for this variable, taking into consideration the general goal-scoring attitude of the players or collecting the penalties kicked during other tournaments (in particular, the continental club cups). Results are not significantly different; for a better illustration of the methodology used for building the specialization variable, an example can be useful. Germany vs. Argentina on 30 June 2006, the quarter-final of the FIFA World Cup, ended after a shoot-out. Eight penalties were kicked in the series, all scored for Germany, only two scored for Argentina. Germany consequently went through to the semi-finals. The kickers for Germany were Ballack, Neuville, Podolski and Borowski. The kickers for Argentina were Cruz, Rodriguez, Cambiasso and Ayala. The observation period for assessing the specialization of the kickers is, as said, July 2004–June 2008. During this period, only Ballack – out of the eight kickers – scored at least one penalty (three, in 2005, excluding shoot-outs) for his national team. During the same interval, he also scored one penalty kick for his club team; Bayern München, in the 2004/05 season. Cruz during the same period never scored a penalty for Argentina, but he scored five penalties for his club team, Inter Milan (one, three and one, respectively, during the 2004/05, 2005/06 and 2007/08 seasons). Podolski, Neuville and Rodriguez never scored a penalty for their national team, but they scored, respectively, eight, eight and two penalty kicks for their club teams. All of these players were consequently classified as 'specialists', whereas Borowski, Cambiasso and Ayala never scored a penalty kick for their national or club team during those four years and were then considered as 'nonspecialists'.

[18] Specialized players could be used early in order to obtain a greater chance to put pressure on the rivals, but it could also be rational to exploit their talent in the (possible) more 'important' final rounds.

Table 5. Success rate for specialists and nonspecialists in different score situations

Team of the kicker	Penalties kicked	Goals (%)	Specialists	Goals (%)	Nonspecialists	Goals (%)
Ahead	31	25 (80.6%)	15	15 (100.0%)	16	10 (62.5%)
Even	111	80 (72.1%)	57	43 (75.4%)	54	37 (68.5%)
Behind	95	64 (67.4%)	47	33 (70.2%)	48	31 (64.6%)
Total	237	169 (71.3%)	119	91 (76.5%)	118	78 (66.1%)

One can then wonder whether specialists perform better because they are actually more talented, or because they simply kick 'less important' penalties.[19]

We have consequently tried to examine more in detail the effect of the 'importance' of every single penalty, and to verify whether specialists actually perform better than nonspecialists also when psychological pressure is similar. Following the insight of Apesteguia and Palacios-Huerta (2010) concerning the 'behind-ahead asymmetry', we have then compared the scoring performance of specialists and nonspecialists in the three cases when the player is called to kick when his team is ahead, even or behind in the score. Results are given in Table 5. As expected, the scoring performance is the highest when the team of the kicker is ahead in the score and the lowest when the kicker is lagging in the score. Interestingly, in every different situation, specialists always perform better than nonspecialists.[20] We interpret this result as a confirmation that specialists – as defined by our classification – are actually more talented than nonspecialists.

IV. A model of Heterogeneous Behaviour of the Kickers

Two actions are not enough

The analysis of the data presented in the previous section is driven by the purpose of highlighting the role of the heterogeneous quality of the players. We are interested to understand how kicking ability affects the strategies chosen by professional players, and in particular whether two (or three) actions concerning the direction of the shot are sufficient to illustrate their different behaviour.

A first puzzling evidence is obtained by analysing 'how' shots failed. As shown in Table 2, we see that 68 out of 237 penalties failed. Of these penalties, approximately two-thirds of them (43) were cleared by the goalkeeper. The remaining (25) missed the goal area. In Table 3, we illustrate how this proportion varies among specialists and nonspecialists. Table 3 shows that specialists miss the goal area almost half of the times when they fail to score, whereas nonspecialists miss the area only in 30% of the cases (difference is statistically significant with $p = 0.083$). Because specialists demonstrate superior skill (their overall performance is significantly higher), this empirical evidence is hard to understand within the framework proposed in the literature.

A different explanation for this puzzle might be the possibility that the side of the shot is not sufficient for adequately describing the strategic options of the kickers. In particular, each kicker must also select the difficulty level of his shot – for example, a kick that is more or less powerful, or closer to the post or to the centre.

Why would a kicker make his task more complicated? On the one hand, a kick close to the post (or very powerful) performs better – *ceteris paribus* – if the goalkeeper correctly guesses the side; on the other hand, a 'difficult' kick is more likely to miss the goal area. The ability to direct difficult kicks *within* the goal area is a trait of high-quality kickers. If this is the strategic framework, only two actions in the set of the kicker are not sufficient for illustrating the behaviour of the players.

When more than two actions are available to the players, the analysis becomes more complicated as it becomes possible that different types of players actually randomize over different actions. Furthermore, if there are more than two actions of the kicker to be analysed, representing the outcome of the game within a 2×2 matrix obtains wrong, or at least ambiguous, results.

In the next section, we develop an extended version of the penalty kick game. In the new representation, the set of actions available to the kicker is composed of more strategic options than those usually analysed in the literature.

More options for kicking a penalty

In our version of the penalty game, the goalkeeper still decides only the side of his jump ((N)atural or (O)pposite), while the kicker decides the side *and* the difficulty level of the kick. It is not important here what makes a kick difficult; for our purpose, a (S)afe kick simply obtains a better pay-off for the kicker than a (D)ifficult kick when players choose different sides and a worse pay-off when

[19] We thank a referee for attracting our attention to this fact.
[20] The mean difference is actually significant at the 1% level only when the team of the kicker is ahead in the score.

Table 6. Our extended version of the penalty kick game

	Goalkeeper (G)	
Kicker (K)	N	O
ND	P_{ND}	π_{ND}
NS	P_{NS}	π_{NS}
OS	π_{OS}	P_{OS}
OD	π_{OD}	P_{OD}

they choose the same side.[21] The kicker, then, has four options (difficult/safe and natural/opposite). The normal form of the game is represented in Table 6.

In line with our definition of a kick's difficulty, the action of the goalkeeper will be more crucial for S-type kicks. Formally, this means that $\pi_{iD} - P_{iD} \ll \pi_{iS} - P_{iS}$ $(i = N, O)$). π_{iD} is not much lower than the complement to 1 of the probability of missing the goal. Moreover, it is assumed that:

- the scoring probability is lower if the goalkeeper, *ceteris paribus*, correctly guesses the side (that is $\pi_{ij} > P_{ij}$ $(i = N, O; j = D, S)$;
- the scoring probability is higher, *ceteris paribus*, on the natural side of the kicker (that is $\pi_{Nj} > \pi_{Oj}$ and $P_{Nj} > P_{Oj}$).

Consequently, the ranking of the values assumed by the scoring is as follows:

$$\pi_{NS} > \pi_{OS} > \pi_{ND} > P_{ND}; \pi_{OD} > P_{OD} > P_{NS} > P_{OS} \tag{1}$$

The relative ranking between π_{OD} and P_{ND} is undetermined: it depends on the relative size of the advantage of a difficult kick when the goalkeeper correctly guesses the side with respect to the advantage obtained by kicking on the natural side. Notice that if $\pi_{OD} < P_{ND}$, the ND strategy dominates the OD strategy.

Being strictly competitive, this game obviously presents no pure strategy Nash equilibria. The characterization of mixed strategy equilibria asks for the analysis of every case where both K and G randomize over two actions. Moreover, at the equilibrium, both players randomize over one kick/jump to the natural side and one to the opposite side. Hence, there are four strategy profiles to be analysed, as listed in Table 7.

The profile that corresponds to the equilibrium of the game obviously depends on the actual values of the scoring probabilities, which in turn depend on the skills of the players. To make results more intuitive, we now introduce a simplifying restriction on the parameters. In particular, we fix $P_{iS} = 0$ and $\pi_{iS} = 1$. Such restrictions do not alter the nature of the results[22] and, moreover, are rather close to the empirical evidence. In other words, this assumption states that if the difficulty level of the kick is low (i) the side advantage disappears, (ii) the quality of the players does not matter, (iii) the probability of missing the goal is null and (iv) should the goalkeeper guess correctly, he is almost sure to block the ball.[23]

The conditions for every single profile reported to be an equilibrium are detailed in Appendix. The summary of the results is reported in Table 8.

These results reveal that only three types of equilibria are possible.

If the kicker is characterized by limited kicking skills (profile II; $\pi_{ND} + P_{ND} < 1$), the rational choice is to rely on safe kicks that will never miss the goal area, hoping to wrong-foot the goalkeeper. Because in this case we do not model a natural side advantage, randomization equals the probability of kicking left or right.

Table 7. Possible equilibrium profiles

Kicker				Goalkeeper		
$prob(ND)$	$prob(NS)$	$prob(OS)$	$prob(OD)$	$prob(N)$	$prob(O)$	Profile
p_1	0	$1-p_1$	0	q_1	$1-q_1$	I
0	p_2	$1-p_2$	0	q_2	$1-q_2$	II
p_3	0	0	$1-p_3$	q_3	$1-q_3$	III
0	p_4	0	$1-p_4$	q_4	$1-q_4$	IV

[21] A safe kick, for example, has limited power and angle, but it is substantially always directed within the goal: if the goalkeeper correctly guesses the side, the probability of saving the goal is very high. On the contrary, a difficult kick can more easily miss the goal or hit the post/bar; if not, it is quite difficult to be blocked even if the goalkeeper jumps on the right side.

[22] The characterizations of mixed strategy equilibria in the general case are available upon request.

[23] Because we do not claim to be able to empirically distinguish between difficult and safe kicks, a direct confirmation to our reasonable assumption is limited to π_s. In this case, consider that in our entire data set, when the goalkeeper was wrong-footed, he never saved the ball.

Table 8. Equilibrium profiles

Profiles	I	II	III
p_{ND}	$\frac{1}{1+\pi_{ND}-P_{ND}}$	0	$\frac{\pi_{OD}-P_{OD}}{\pi_{ND}+\pi_{OD}-P_{ND}-P_{OD}}$
p_{NS}	0	$\frac{1}{2}$	0
p_{OS}	$\frac{\pi_{ND}-P_{ND}}{1+\pi_{ND}-P_{ND}}$	$\frac{1}{2}$	0
p_{OD}	0	0	$\frac{\pi_{ND}-P_{ND}}{\pi_{OD}+\pi_{ND}-P_{OD}-P_{ND}}$
q_N	$\frac{\pi_{ND}}{1+\pi_{ND}-P_{ND}}$	$\frac{1}{2}$	$\frac{\pi_{ND}-P_{OD}}{\pi_{ND}+\pi_{OD}-P_{ND}-P_{OD}}$
q_O	$\frac{1-P_{ND}}{1+\pi_{ND}-P_{ND}}$	$\frac{1}{2}$	$\frac{\pi_{OD}-P_{ND}}{\pi_{ND}+\pi_{OD}-P_{ND}-P_{OD}}$
Equilibrium conditions	$\pi_{ND}+P_{ND}>1$ and $\pi_{ND}(1-\pi_{OD})\geq P_{OD}(1-P_{ND})$	$\pi_{ND}+P_{ND}\leq 1$	$\pi_{ND}+P_{ND}>1$ and $\pi_{ND}(1-\pi_{OD})<P_{OD}(1-P_{ND})$

In the case of talented kickers (profiles I and III; $\pi_{ND}+P_{ND}>1$), we have two sub cases: the equilibrium conditions distinguish when $\pi_{ND}(1-\pi_{OD})$ is greater or lower than $P_{OD}(1-P_{ND})$.

A lower value for $\pi_{ND}(1-\pi_{OD})$ means that the kicker's ability to perform 'difficult' kicks regardless of the side-choice of the goalkeeper is prevalent over natural side advantages (profile III).[24] If this is the case, kickers will prefer to make difficult kicks on both sides.

On the contrary, when $\pi_{ND}(1-\pi_{OD})$ is higher, the natural side advantage prevails (profile I). If this is the case, kickers will prefer to attempt difficult shots on the natural side and sometimes shoot safe on the opposite side to make their actions less predictable.

In conclusion, if we imagine a generic distribution of the three types of players in our sample, we expect that only talented players (that is, players with a sufficient ability to kick difficult) will try difficult kicks and that difficult kicks will be much more frequent on the natural side.

Empirical evidence about different behaviours of differently talented kickers

In this section, we illustrate the empirical evidence drawn from our sample in support of the theoretical predictions proposed in the previous section. First, in Table 9, we report the strategies chosen by the players in every penalty and the related outcome in terms of scoring frequencies. Because we are not able to distinguish between safe and difficult kicks, each cell aggregates the shots of both types. Thus, a significant phenomenon of aggregation bias may affect results. An evidence of aggregation bias is offered, for example, by considering that in Table 9. This contradicts the widely accepted assumption of a natural side advantage.[25] If we assume that players behave as predicted by equilibrium mixed strategies, further evidence of aggregation bias is offered by violations of the properties of the Fundamental Lemma. In particular, the pay-off of the goalkeeper associated with the pure strategy N $(1-73.8\%)$ is lower than his pay-off associated with the

Table 9. Players' behaviour and outcomes in the whole data set: the number of observations and scoring probability (in parentheses)

	Goalkeeper (G)				
Kicker (K)	Natural	Opposite	No move	No info	Total
Natural	72 (61.1%)	45 (84.4%)	2 (100.0%)	– (–)	119 (70.6%)
Opposite	49 (91.8%)	47 (40.4%)	5 (100.0%)	– (–)	101 (68.3%)
No info	5 (80.0%)	4 (100.0%)	– (–)	8 (100.0%)	17 (94.1%)
Total	126 (73.8%)	96 (63.5%)	7 (100.0%)	8 (100.0%)	237 (71.3%)

[24] If no side advantage is in place, then $\pi_{ND}=\pi_{OD}>P_{ND}=P_{OD}$ and $\pi_{ND}(1-\pi_{OD})<P_{OD}(1-P_{ND})$ because $\pi_{ND}+P_{ND}>1$.
[25] See again footnote 8. This difference is actually not significant at the 10% level.

pure strategy O ($1 - 63.5\%$) with $p = 0.092$.[26] A more clear evidence will be proposed below, when we will separately analyse specialists and nonspecialists.

As for the elements in favour of the hypothesis of rational behaviour of the players, as discussed in Section II, Chiappori *et al.* (2002) obtain several properties of the equilibrium that need to be verified, even when aggregating the observations in a sample of heterogeneous players. The only condition for these properties to be true is that the ranking of the parameters must hold for every single player. The properties that survive in our setting predict that (i) the number of jumps to the natural side will be larger than the number of jumps to the opposite side, (ii) the number of kicks to the natural side will be larger than the number of jumps to the opposite side and (iii) the profile (N,N) (i.e. the kicker chooses N and the goalkeeper chooses N) will be more frequent than the profile (O,O).[27] As observed in Table 9, every property is confirmed in our sample. Testing the null hypothesis of equal propensities for each property leads to rejection at the 5% significance level. Notice that the empirical evidence also confirms the natural side advantage when the goalkeeper correctly guesses the side of the kick (the scoring probability of (N,N) is significantly higher than the scoring probability of (O,O), $p = 0.014$).

These results are confirmed within each of the subsamples of the specialists (see Table 10) and nonspecialists (see Table 11). We expect lower heterogeneity (more limited effects of aggregation bias) in the first one because we cannot exclude the presence of some talented players in the subsample of nonspecialists.

The higher talent of specialists is confirmed for every cell of the game.[28] The natural side advantage is particularly relevant for low-quality players, which is in line with the theoretical predictions. Interestingly, in Table 10 (where heterogeneity among players is fairly limited), the expected pay-offs between pure strategies (Fundamental Lemma) are very similar.[29]

Another interesting result concerns the predictability of the players' behaviour. Baumann *et al.* (2011), as discussed in Section II, offer empirical evidence to suggest that high-quality kickers (as defined by a ranking provided by a specialized magazine) are more predictable, in that they kick more often on the natural side; nonetheless, their scoring performance proves to be higher. Our empirical evidence confirms this result (specialist kickers kick approximately 56% of their shots on the natural side, whereas nonspecialists kick approximately 52% of their shots on the natural side; the difference is actually not statistically significant).

Our conceptual model supports these findings. If we define a kicker's talent as the ability to score independently from the action of the goalkeeper, given a side advantage, player quality is associated with high values

Table 10. Specialists behaviour and outcomes: the number of observations and scoring probability (in parentheses)

	Goalkeeper (G)				
Kicker (K)	Natural	Opposite	No move	No info	Total
Natural	33 (63.6%)	27 (85.2%)	1 (100.0%)	− (−)	61 (73.8%)
Opposite	23 (95.7%)	23 (56.5%)	2 (100.0%)	− (−)	48 (77.1%)
No info	3 (66.7%)	4 (100.0%)	− (−)	3 (100.0%)	10 (90.0%)
Total	59 (76.3%)	54 (74.1%)	3 (100.0%)	3 (100.0%)	119 (76.5%)

Table 11. Nonspecialists behaviour and outcomes: the number of observations and scoring probability (in parentheses)

	Goalkeeper (G)				
Kicker (K)	Natural	Opposite	No move	No info	Total
Natural	39 (59.0%)	18 (83.3%)	1 (100.0%)	–(–)	58 (67.2%)
Opposite	26 (88.5%)	24 (25.0%)	3 (100.0%)	–(–)	53 (60.4%)
No info	2 (100.0%)	– (–)	–(–)	5 (100.0%)	7 (100.0%)
Total	67 (71.6%)	42 (50.0%)	4 (100.0%)	5 (100.0%)	118 (66.1%)

[26] Based on Pearson's χ^2 goodness-of-fit test of equality of two distributions (see Palacios-Huerta (2003, p. 403) for details).
[27] Actually, those properties are verified for profiles I and III, while profile II predicts equal number of shots and jumps on natural and opposite side (see Table 8).
[28] However, the difference is significant ($p = 0.014$) only for the action profile (O, O).
[29] See again footnote 26. The hypothesis that the scoring probability is identical across strategies for the kickers and the goalkeeper cannot be rejected at the 10% significance level.

of P_{ND} and with low values of $\pi_{ND} - P_{ND}$. Given this definition of quality, the probability of kicking on the natural side (see again Table 8, profiles I and III) actually increases when the kicker's quality increases.[30] This result is also confirmed by the empirical evidence provided by Baumann *et al.* (2011). Notice that it strictly depends on the asymmetry of the game.

Of course, the term 'predictability', as in the paper by Baumann *et al.* (2011), has to be understood within a context of simultaneous moves. In this sense, we tried to verify whether this simultaneity assumption holds by analysing the contingency tables associated with strategies N and O for both specialists and nonspecialists. As for the specialists, the statistical dependence can be rejected with $p = 0.61$, whereas in the subsample of nonspecialists, statistical dependence can be rejected with $p = 0.08$, implying some ability of the goalkeeper to anticipate the move of the kicker. This evidence suggests another way to define ability in kicking penalties and produces results, as shown in Table 11, that are less robust than the ones observed in Table 10.

The results presented so far in this section confirm some properties of professional soccer player behaviour already obtained in similar previous applications and demonstrate that in more heterogeneous samples the effects of aggregation bias are increased. We want to conclude this section adding further direct support to the conceptual model presented in section 'More option for kicking a penalty'. To achieve this aim, we have analysed the distribution of shots outside of the goal, i.e., shots that miss the goal or hit the post/bar. The predictions of our model in this direction are very clear: out-of-the-goal shots must be more frequent among specialists and on the natural side. Both predictions go against intuition, which might expect such major mistakes on the unfavoured side and from less talented players.

As for the distribution of out-of-the-goal shots between specialists and not specialists, see again Table 3. The distribution of out-of-the-goal shots between natural and opposite side is presented in Table 12. In both cases, the evidence confirms the predictions of our conceptual model.

Table 12. The distribution of out-of-the-goal shots among action profiles (frequency over the whole number of penalties in parentheses)

	Goalkeeper (G)		
Kicker (K)	Natural	Opposite	Total
Natural	8 (11.1%)	7 (15.6%)	15 (12.6%)
Opposite	4 (8.2%)	5 (10.6%)	9 (8.9%)
Total	12 (9.5%)	12 (12.5%)	24 (10.1%)

V. Concluding Remarks

In this article, we have proposed an analysis of the behaviour of professional soccer players when kicking a penalty. With respect to the extant literature, to focus on the effects of heterogeneity that determines aggregation bias, we have proposed a more general game setting that expands the range of actions available to the kickers. The empirical implications of our model have been tested within an original data set composed of every penalty kicked during shoot-outs at World Cup and Euro Cup since 1994.

First, in line with the literature, we find that the players in our sample – especially the high-quality ones – behave rationally, meaning that they move as suggested by game theoretical predictions. Moreover, the observed outcomes support the framework proposed in our model.

Second, we claim that players' behaviours cannot be completely understood unless more than one strategic variable (a shot's difficulty level in addition to its direction) is considered. The analysis of the strategic choices concerning shot's difficulty is essential to deal with aggregation bias problems because it is along this variable that players differentiate their strategies conditional upon their own quality.

Finally, as far as predictability of the strategies of players, our model confirms that more talented players are more predictable, as suggested by the empirical evidence in the literature. The intuition for such a result is quite straightforward: a quality player will favour his preferred action (the natural side, in our case) because the action of the rival is less crucial for him.

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[30] Imagine a very talented kicker who never fails the goal area even when shooting with great power and angle, especially on the natural side. This kicker is characterized by $\pi_{ND} = 1$ and $P_{ND} \to 1$ and will always kick difficult on the natural side; thus, his action will be easily predicted by the goalkeeper. This correct prediction will be of no use for the goalkeeper because the scoring frequency will be close to one.

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Appendix

For every strategy profile reported in the table below, the equilibrium conditions are obtained.

Kicker				Goalkeeper		
$prob(ND)$	$prob(NS)$	$prob(OS)$	$prob(OD)$	$prob(N)$	$prob(O)$	Profile
p_1	0	$1-p_1$	0	q_1	$1-q_1$	I
0	p_2	$1-p_2$	0	q_2	$1-q_2$	II
p_3	0	0	$1-p_3$	q_3	$1-q_3$	III
0	p_4	0	$1-p_4$	q_4	$1-q_4$	IV

Profile I

This strategy combination is an equilibrium if the following conditions are met:

- The actions over which player K randomizes (ND and OS) determine equal expected pay-offs. Thus,

$$\hat{E}_K = \hat{E}(\Pi_K(ND)) = \hat{E}(\Pi_K(OS)) \rightarrow q_1 P_{ND} + (1-q_1)\pi_{ND} = q_1$$

Consequently,

$$q_1 = \hat{E}_K = \frac{\pi_{ND}}{1+\pi_{ND}-P_{ND}} \quad (2)$$

Moreover, we need that

$$\hat{E}_K \geq \hat{E}(\Pi_K(NS)) = 1-q_1 \rightarrow \pi_{ND}+P_{ND} \geq 1$$

and

$$\hat{E}_K \geq \hat{E}(\Pi_K(OD)) = q_1\pi_{OD} + (1-q_1)P_{OD} \rightarrow \pi_{ND}(1-\pi_{OD}) \geq P_{OD}(1-P_{ND})$$

- The actions over which player G randomizes (N and O) determine equal expected pay-offs; thus,

$$\hat{E}_G = \hat{E}(\Pi_G(N)) = \hat{E}(\Pi_G(O)) \rightarrow p_1(1-P_{ND}) = p_1(1-\pi_{ND}) + 1 - p_1$$

Consequently,

$$p_1 = \frac{1}{1+\pi_{ND}-P_{ND}} \quad (3)$$

and

$$\hat{E}_G = \frac{1-P_{ND}}{1+\pi_{ND}-P_{ND}} = 1-\hat{E}_K \quad (4)$$

Profile II

This strategy combination is an equilibrium if the following conditions are met:

- The actions over which player K randomizes (NS and OS) determine equal expected pay-offs. Thus,

$$\hat{E}_K = \hat{E}(\Pi_K(NS)) = \hat{E}(\Pi_K(OS)) \quad \rightarrow \quad 1 - q_2 = q_2$$

Consequently,

$$q_2 = \hat{E}_K = \frac{1}{2} \tag{5}$$

Moreover, we need that

$$\hat{E}_K \geq \hat{E}(\Pi_K(ND)) = (P_{ND} + \pi_{ND})/2 \rightarrow \pi_{ND} + P_{ND} \leq 1$$

and

$$\hat{E}_K \geq \hat{E}(\Pi_K(OD)) = (P_{OD} + \pi_{OD})/2 \rightarrow \pi_{OD} + P_{OD} \leq 1$$

If the first of these two conditions is true, the second one also needs to be true, given the rank of the values of the parameters.

- The actions over which player G randomizes (N and O) determine equal expected pay-offs; thus

$$\hat{E}_G = \hat{E}(\Pi_G(N)) = \hat{E}(\Pi_G(O)) \quad \rightarrow \quad p_2 = 1 - p_2$$

Consequently,

$$p_2 = \hat{E}_G = \frac{1}{2} = 1 - \hat{E}_K \tag{6}$$

Profile III

First of all, notice that a necessary condition for the Profile III to be an equilibrium is that $\pi_{OD} > P_{ND}$; otherwise, OD is dominated by ND. The profile III is an equilibrium if:

- The actions over which player K randomizes (ND and OD) determine equal expected pay-offs; thus,

$$\begin{aligned}\hat{E}_K &= \hat{E}(\Pi_K(ND)) = \hat{E}(\Pi_K(OD)) \\ &\rightarrow q_3 P_{ND} + (1 - q_3)\pi_{ND} \\ &= q_3 \pi_{OD} + (1 - q_3) P_{OD}\end{aligned}$$

Consequently, we obtain

$$q_3 = \frac{\pi_{ND} - P_{OD}}{\pi_{ND} + \pi_{OD} - P_{ND} - P_{OD}} \tag{7}$$

and

$$\hat{E}_K = \frac{\pi_{ND}\pi_{OD} - P_{ND}P_{OD}}{\pi_{ND} + \pi_{OD} - P_{ND} - P_{OD}} \tag{8}$$

Notice that $\pi_{OD} > P_{ND}$ implies that $q_3 < 1$. Moreover, we need that

$$\hat{E}_K \geq \hat{E}(\Pi_K(OS)) = q_3 \rightarrow \pi_{ND}(1 - \pi_{OD}) \leq P_{OD}(1 - P_{ND})$$

and

$$\begin{aligned}\hat{E}_K \geq \hat{E}(\Pi_K(NS)) = 1 - q_3 &\rightarrow \pi_{OD}(1 - \pi_{ND}) \\ &\leq P_{ND}(1 - P_{OD})\end{aligned}$$

Given the rank of the values of the parameters, $\pi_{OD} < \pi_{ND}$ implies that $\pi_{ND}(1 - \pi_{OD}) > \pi_{OD}(1 - \pi_{ND})$ and $P_{OD} < P_{ND}$ implies that $P_{ND}(1 - P_{OD}) > P_{OD}(1 - P_{ND})$. Thus, if $\hat{E}_K \geq \hat{E}(\Pi_K(OS))$, also $\hat{E}_K \geq \hat{E}(\Pi_K(NS))$ must be true. Note that $\pi_{OD} > P_{ND}$ is a necessary condition for $\hat{E}_K \geq \hat{E}(\Pi_K(OS))$. In addition consider that the condition $\pi_{DN} + P_{DN} < 1$ implies that $\pi_{ND}(1 - \pi_{OD}) > P_{OD}(1 - P_{ND})$ so that $\pi_{DN} + P_{DN} \geq 1$ is a necessary condition for making the Profile III an equilibrium.

- The actions over which player G randomizes (N and O) determine equal expected pay-offs; thus,

$$\begin{aligned}\hat{E}_G = \hat{E}(\Pi_G(N)) = \hat{E}(\Pi_G(O)) &\rightarrow p_3(1 - P_{ND}) \\ &+ (1 - p_3)(1 - \pi_{OD}) = p_3(1 - \pi_{ND}) \\ &+ (1 - p_3)(1 - P_{OD})\end{aligned}$$

Consequently, we obtain

$$p_3 = \frac{\pi_{OD} - P_{OD}}{\pi_{OD} + \pi_{ND} - P_{OD} - P_{ND}} \tag{9}$$

and

$$\begin{aligned}\hat{E}_G &= \frac{\pi_{OD} + \pi_{ND} - P_{OD} - P_{ND} - \pi_{ND}\pi_{OD} + P_{ND}P_{OD}}{\pi_{OD} + \pi_{ND} - P_{OD} - P_{ND}} \\ &= 1 - \hat{E}_K\end{aligned} \tag{10}$$

Profile IV

This strategy combination is an equilibrium if the actions over which player K randomizes (NS and OD) determine equal expected pay-offs; thus,

$$\begin{aligned}\hat{E}_K = \hat{E}(\Pi_K(NS)) = \hat{E}(\Pi_K(OD)) \quad &\rightarrow \quad 1 - q_4 \\ &= q_4\pi_{OD} + (1 - q_4)P_{OD}\end{aligned}$$

Moreover, we need that

$$\hat{E}_K \geq \hat{E}(\Pi_K(ND)) = q_4\pi_{ND} + (1 - q_4)P_{ND}$$
$$\rightarrow \pi_{OD}(1 - P_{ND}) \geq \pi_{ND}(1 - P_{OD})$$

and

$$\hat{E}_K \geq \hat{E}(\Pi_K(OS)) = q_4 \quad \rightarrow \quad 1 - P_{OD} \leq \pi_{OD}$$

The first condition, given the assumptions on the values of the parameters, is always false because $\pi_{OD} < \pi_{ND}$ and $1 - P_{ND} < 1 - P_{OD}$, so that the Profile IV can never be an equilibrium.