27/1/22

RANDOM VARIABLE:

A random variable is a function that assigns a real number $X(s)$ to every element in the sample space $s \in S$ corresponding to a random experiment E

Types of random variables:

* discrete random variables
* continuous random variables.

Discrete random variables:

A random variable x is discrete if it assumes only discrete values.

Example: No. of students inside a class.

Continuous random variables:

If X is a random variable which can take all possible values between certain limits/ in an interval, which may be a finite (or) infinite then X is a continuous random variable.

Eg: $f(x) = \{ x^2 - 1 \leq x \leq 5 \}$

PROBABILITY MASS FUNCTION (PMF):

If X is a discrete random variable which can take the values $x_1, x_2, x_3, \dots$ such that, $P(X = x_i) = P_i$

then P_i is called probability mass function (or) probability function and it satisfies the condition.

(i) $P_i \geq 0$

(ii) $\sum P_i = 1$

PROBABILITY DENSITY FUNCTION (PDF):

If X is a continuous random variable such that

$$P(X - \tfrac{1}{2}dx \leq X \leq X + \tfrac{1}{2}dx) = f(x)\, dx.$$

then $f(x)$ is said to be probability density function and it also obeys

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

COMMULATIVE DISTRIBUTION FUNCTION [CDF]:

If X is a random variable, discrete or continuous then $P(X \leq x)$ is called cummulative distribution function. It is represented by $F(x)$.

if $F(x) = \sum P_i$, $x_i \in x$ Then x is discrete

if $F(x) = \int_{-\infty}^{\infty} f(x)\, dx$, then x is continuous.

30/1/23

1. A discrete random variable X has the following probability distribution.

x	0	1	2	3	4	5	6	7	8
$P(X=x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i) Find the value of a.

ii) Find $P(X<3)$, $P(0<X<3)$, $P(X\geq 3)$ $= 1 - P(X\geq 3)$

iii) Find the distribution function dx.

i) We know that,

$$\sum P(x_i) = 1$$

$$a+3a+5a+7a+9a+11a+13a+15a+17a = 1$$

$$81a = 1$$

$$\boxed{a = 1/81}$$

ii) $P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$$= a+3a+5a$$

$$= 9a$$

$$= \frac{9}{81}$$

$$= \frac{1}{9}$$

$P(0<X<3) = P(X=1) + P(X=2)$

$$= 3a+5a$$

$$= 8a$$

$$= \frac{8}{81}.$$

$P(X \geq 3) = 1 - P(X < 3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \frac{1}{9}$

$= \frac{8}{9}$.

iii) distribution fn $F(x)$

$F(0) = P(X=0) = a = 1/81$

$F(1) = P(X=0) + P(X=1) = a + 3a = 4/81$

$F(2) = P(X=0) + P(X=1) + P(X=2) = \frac{9}{81}$.

$F(3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = \frac{16}{81}$.

$F(4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = \frac{25}{81}$

$F(5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) = \frac{36}{81}$

$F(6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$
$+ P(X=6) = \frac{49}{81}$.

$F(7) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$
$+ P(X=6) + P(X=7) = \frac{64}{81}$.

$F(8) = 81/81 = 1$

x	0	1	2	3	4	5	6	7	8
$F(x)$	$\frac{1}{81}$	$\frac{4}{81}$	$\frac{9}{81}$	$\frac{16}{81}$	$\frac{25}{81}$	$\frac{36}{81}$	$\frac{49}{81}$	$\frac{64}{81}$	1

If X is a discrete random variable

then mean $E(X) = \sum x_i \, P(x_i)$ mean $\sum p(x_i) = 1$ discrete

$$E(X^2) = \sum x_i^2 \, P(x_i)$$

$$Var(X) = E[x^2] - [E[x]]^2$$

2. Determine the constant k given the following probability distribution of discrete random variable X. Also find mean and variance of X.

$X = x$	1	2	3	4	5
$P(X = x_i)$	0.1	0.2	k	$2k$	0.1

to find k:

$$\sum P(x_i) = 1$$

$$0.1 + 0.2 + k + 2k + 0.1 = 1$$

$$3k + 0.4 = 1$$

$$3k = 0.6$$

$$k = 0.2$$

$$\text{Mean} = \sum x_i \, P(x_i)$$

$$= 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.2 + 4 \times 0.4 + 5 \times 0.1$$

$$= 3.2$$

$$E(x^2) = \sum x_i^2 \, P(x_i)$$

$$= 1^2 \times 0.1 + 2^2 \times 0.2 + 3^2 \times 0.2 + 4^2 \times 0.4 + 5^2 \times 0.1$$

$$= 11.6$$

$$Var(X) = E(X^2) - [E(x)]^2$$

$$= 11.6 - (3.2)^2$$

$$= 1.36.$$

CONDITIONAL PROBABILITY

$$P(A/B) = \frac{P(A \cap B)}{P(B)}.$$

2) A random variable X has the following Probability fn

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

i) Find K

ii) $P[X < 6]$, $P[X \geq 6]$

iii) If $P[X \leq C] > \frac{1}{2}$ Find minimum value of C.

iv) Evaluate $P[X < 2]$, $P[X > 3]$, $P[1 < X < 5]$

v) $P[1.5 < X < 4.5 / X > 2]$

i) $0 + K + 2K + 2K + 3K + K^2 + 2K^2 + 7K^2 + K = 1$

$$9K + 10K^2 = 1$$

$$10K^2 + 9K - 1 = 0$$

$$\boxed{K = 1/10} \quad K = -1$$

ii) $P[x<6] = P[x=0] + P[x=1] + P[x=2] + P[x=3]$

$+ P[x=4] + P[x=5]$

$= 0 + k + 2k + 2k + 3k + k^2$

$= 8k + k^2$

$= \frac{8}{10} + \frac{1}{100}$

$= \frac{80+1}{100}$

$= \frac{81}{100}$

$P[x \geq 6] = 1 - P[x<6]$

$= 1 - \frac{81}{100}$

$= \frac{100-81}{100}$

$= \frac{19}{100}$

iv) $P[x<2] = P[x=0] + P[x=1]$

$= 0 + k$

$= \frac{1}{10}$

$P[x>3] = P[x=4] + P[x=5] + P[x=6] + P[x=7]$

$= 3k + k^2 + 2k^2 + 7k^2 + k$

$= 10k^2 + 4k$

$= 10\left(\frac{1}{10}\right)^2 + \frac{4}{10}$

$= \frac{10}{100} + \frac{4}{10}$

$= \frac{10+40}{100} = \frac{50}{100} = \frac{1}{2}$

$P[1 < X < 5] = P[X=2] + P[X=3] + P[X=4]$

$= 2K + 2K + 3K$

$= 7K$

$= 7/10$

iii) $P[X \leq C] > 1/2$

$P(X=0) = 0$

$P(X \leq 1) = 0 + K = 1/10$

$P(X \leq 2) = 3K = 3/10$

$P(X \leq 3) = 5K = 1/2$

$P(X \leq 4) = 8K > 1/2$

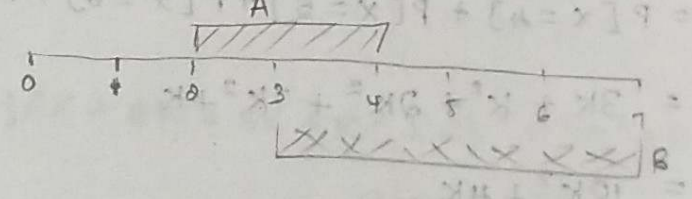
$\therefore \boxed{C = 4}$

v) $P[1.5 < X < 4.5 / X > 2]$

we know that,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Here.



$$P[1.5 < X < 4.5 / X > 2] = \frac{P[1.5 < X < 4.5] \cap P[X > 2]}{P[X > 2]}$$

$$= \frac{P(X=3) + P(X=4)}{1 - P(X \leq 2)}$$

$$= \frac{5K}{1 - [P(X=0) + P(X=1) + P(X=2)]}$$

$$= \frac{5K}{1-3K} = \frac{5/10}{1-3/10} = \frac{5/10}{\frac{10-3}{10}} = \frac{5/10}{7/10} = \frac{5}{10} \times \frac{10}{7}$$

$$= 5/7$$

4) The probability function of an infinite discrete distribution is given by, $P[X = x_j] = 1/2^j$, $j = 1, 2, \ldots \infty$

Find mean and $P(X \text{ is even})$, $P(X \geq 5)$

$P[X \text{ divisible by } 3]$.

Soln:

$$\text{Mean} = \sum_{i=1}^{\infty} x_i p_i$$

$$= 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2^2} + 3 \frac{1}{2^3} + \ldots$$

$$= \frac{1}{2}\left[1 + 2(1/2) + 3(1/2)^2 + \ldots\right].$$

$$\because (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots$$

$$\text{Mean} = 1/2\,[1 - (1/2)]^{-2}. \Rightarrow \frac{1}{2}\left[\frac{2-1}{2}\right]^{-2} \Rightarrow \frac{1}{2}\left(\frac{1}{2}\right)^{-2}$$

$$= 1/2\,(1/2)^{-2}$$

$$\frac{1}{2}\,\frac{1}{(1/2)^2}$$

$$= \frac{2^2}{2}$$

$$= 4/2$$

$$\boxed{\text{mean} = 2.}$$

$$P[X=\text{even}] = P(X=2)+P(X=4)+P(X=6)+\cdots$$

$$= \frac{1}{2^2}+\frac{1}{2^4}+\frac{1}{2^6}+\cdots$$

$2^6 = (2^2)^3$

$= 2^2 \cdot 2^2 \cdot 2^2$

$= 2^2(2^4)$

$$= \frac{1}{2^2}\left[1+\frac{1}{2^2}+\frac{1}{2^4}+\cdots\right]$$

$$= \frac{1}{4}\left[1-\left(\frac{1}{2^2}\right)\right]^{-1} \Rightarrow \frac{1}{4}\left[1-\frac{1}{4}\right]^{-1}$$

$$\cancel{= \frac{1}{4}\left(\frac{1}{2^2}\right)^{-1}} \qquad = \frac{1}{4}\left[\frac{4-1}{4}\right]^{-1}$$

$2^2 \cdot 2^2$

$$\cancel{= \left(\frac{1}{4}\right)(2)} \qquad = \frac{1}{4}\left(\frac{3}{4}\right)^{-1}$$

$= 2^2$

$$\cancel{= \frac{1}{2}.} \qquad = \frac{1}{4}\left(\frac{4}{3}\right)$$

$$\boxed{P[X=\text{even}] = \frac{1}{3}}$$

$$P(X\geq 5) = P(X=5)+P(X=6)+P(X=7)+\cdots$$

$$= \frac{1}{2^5}+\frac{1}{2^6}+\frac{1}{2^7}+\cdots$$

$$= \frac{1}{2^5}+\frac{1}{2^5\cdot 2^1}+\frac{1}{2^5\cdot 2^2}+\cdots$$

$$= \frac{1}{2^5}\left[1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\cdots\right]$$

$$= \frac{1}{2^5}\left[1-\frac{1}{2}\right]^{-1} \Rightarrow \left(\frac{2-1}{2}\right)^{-1}\times\frac{1}{2^5}$$

$$= \frac{1}{2^5}\left(\frac{1}{2}\right)^{-1} \qquad = \left(\frac{1}{2}\right)^{-1}\times\frac{1}{2^5}$$

$$= \frac{1}{2^4} \qquad = \frac{1}{(1/2)}\times\frac{1}{2^5} = 2\times\frac{1}{2^{5-4}}$$

$$= \frac{1}{2^4}$$

$P[X \text{ divisible by } 3] = P[X=3] + P[X=6] + \dots$

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^9 + \dots$$

$$= \frac{1}{2^3} + \left(\frac{1}{2^6}\right) + \frac{1}{2^9} + \dots$$

$$= \frac{1}{2^3}\left[1 + \frac{1}{2^3} + \frac{1}{2^6} + \dots\right]$$

$$= \frac{1}{2^3}\left[1 - \frac{1}{2^3}\right]^{-1}$$

$$= \frac{1}{2^3}\left(\frac{7}{8}\right)^{-1}$$

$$= \frac{1}{8} \times \frac{8}{7}$$

$$= \frac{1}{7}.$$

Properties:

* $V(ax+b) = a^2\, var(x)$

* $E[ax+b] = a\, E(x) + b$. [Mean]

1. If variance of x is 4. Find $var[3x+8]$.

Soln:

Given, $var(x) = 4$

$$var(3x+8) = 3^2\, Var(x)$$
$$= 9 \times 4$$
$$= 36.$$

2. If X is a random variable with $E(X) = 1$ & $E(X(X-1)) = 4$. Find $var[x]$, $var[X/2]$ and $var[2-3x]$.

$E(X^2) - [E(X)]^2$

Given,

$E(x) = 1$

$E[x(x-1)] = 4$

$E[x^2] - E[x] = 4$

$E[x^2] = 5$

$V[x] = E[x^2] - [E[x]]^2$

$= 5 - 1^2$

$V[x] = 4$

$Var\left[\frac{x}{2}\right] = \left[\frac{1}{2}\right]^2 \cdot Var[x]$

$= \frac{1}{4} \cdot 4$

$= 1$

$Var[2-3x] = [-3]^2 \; Var[x]$

$= 9 \times 4$

$= 36.$

3/2/23

1. A continuous random variable X has probability density function $f(x) = k$, $0 \le x \le 1$. Determine the value of $\underline{k}$ and also $P\left[x \le \frac{1}{4}\right]$.

$f(x) = k.$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\int_0^1 k\,dx = 1.$

$$K[x]_0^1 = 1$$

$$K[1-0] = 1$$

$$\boxed{K=1}$$

$$P(X \le 1/4) = \int_0^{1/4} f(x)\, dx$$

$$= \int_0^{1/4} 1.\, dx$$

$$= [x]_0^{1/4}$$

$$= \frac{1}{4} - 0$$

$$= 1/4$$

2. If X is a continuous Random variable with Pdf

$$f(x) = \begin{cases} c(4x - 2x^2), & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

i) Find c

ii) Find $P[x > 1]$

Soln:

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_0^2 c\,[4x - 2x^2]\, dx = 1$$

$$c\left[\frac{4x^2}{2} - \frac{2x^3}{3}\right]_0^2 = 1$$

$$c\left[\frac{4\times 4}{2}-\frac{2\times 8}{3}\right]=1$$

$$c\left[8-\frac{16}{3}\right]=1$$

$$c\left[\frac{24-16}{3}\right]=1$$

$$\boxed{c=\frac{3}{8}}$$

$$P(X>1)=\int_{1}^{\infty} f(x)\,dx$$

$$=\int_{1}^{2} 3/8\,(4x-2x^2)\,dx$$

$$=\frac{3}{8}\left[\frac{4x^2}{2}-\frac{2x^3}{3}\right]_1^2$$

$$=\frac{3}{8}\left[2\times 4-\frac{2\times 8}{3}-\frac{4}{2}+\frac{2}{3}\right]$$

$$=\frac{3}{8}\left[8-\frac{16}{3}-\frac{4}{2}+\frac{2}{3}\right]$$

$$6-\frac{14}{3}$$

$$\frac{18-14}{3}$$

$$\frac{4}{3}$$

$$=\frac{3}{8}\left[\frac{48-32-12+4}{6}\right]$$

$$=\frac{3}{8}\left[\frac{52-44}{6}\right]$$

$$=\cancel{\frac{3}{8}\left[\frac{80}{6}\right]}=\frac{3}{8}\left[\frac{8}{6}\right]=3/8\,(4/3)$$

$= 1/2.$

3. If $f(x) = \begin{cases} Kxe^{-x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$ is the pdf of a continuous random variable X. Find K.

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{0}^{\infty} \underset{(u\,v)}{Kxe^{-x}}\, dx = 1$$

$$K\left[\frac{xe^{-x}}{-1} - e^{-x}\right]_0^{\infty} = 1$$

$u = x \quad v = e^{-x}$

$u' = 1 \quad v_1 = \frac{e^{-x}}{-1}$

$u'' = 0 \quad v_2 = e^{-x}$

$$K[0 - 0 + 0 + e^0] = 1$$

$$\boxed{K = 1}$$

4. Let X be a continuous random variable with probability density function

$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ Find. (i) $P[X \leq 0.4]$

(ii) $P[X > 3/4]$ (iii) $P[X \geq 1/2]$ (iv) $P[1/2 < X < 3/4]$

v) $P\left[X > 3/4 \,/\, X > 1/2\right]$.

i) $P[X \le 0.4] = \int_0^{0.4} 2x \, dx$

$= 2\left[\frac{x^2}{2}\right]_0^{0.4}$

$= 0.4 \times 0.4$

$= 0.16.$

ii) $P[X > \frac{3}{4}] = \int_{3/4}^{1} 2x \, dx$

$= 2\left[\frac{x^2}{2}\right]_{3/4}^{1}$

$= 2 \cdot \frac{1}{2} [x^2]_{3/4}^{1}$

$= 1 - \left(\frac{3}{4}\right)^2$

$= 1 - \frac{9}{16}$

$= \frac{16-9}{16}$

$= 7/16$

iii) $P[X > 1/2] = \int_{1/2}^{1} 2x \, dx$

$= 2\left[\frac{x^2}{2}\right]_{1/2}^{1}$

$= 2 \cdot \frac{1}{2} [x^2]_{1/2}^{1}$

$$= 1 - \left(\frac{1}{2}\right)^2$$

$$= 1 - \frac{1}{4}$$

$$= \frac{4-1}{4}$$

$$= 3/4$$

iv) $P\left[\frac{1}{2} < x < \frac{3}{4}\right]$

$$= \int_{1/2}^{3/4} 2x \, dx$$

$$= 2\left[\frac{x^2}{2}\right]_{1/2}^{3/4}$$

$$= \cancel{2} \cdot \frac{1}{\cancel{2}}\left[x^2\right]_{1/2}^{3/4}$$

$$= \left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \frac{9}{16} - \frac{1}{4}$$

$$= \frac{9-4}{16}$$

$$= \frac{5}{16}.$$

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

v) $P\left[X > \frac{3}{4} / X > 1/2 \right]$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(X > 3/4)}{P(X > 1/2)}$$

$$= \frac{7/16}{3/4}$$

$$= \frac{7}{16} \times \frac{4}{3}$$

$$= 7/12.$$

In a continuous random variable X having probability density function $f(x) = \begin{cases} \frac{x^2}{3} ; & -1 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

i) verify $\int_{-\infty}^{\infty} f(x)\, dx = 1$

(ii) $P(0 < X \leq 1)$

(iii) Find $F(x)$

iv) $\int_{-1}^{2} \frac{x^3}{3}\, dx \cdot \frac{1}{3}\left[\frac{x^4}{4}\right]_{-1}^{2}$

$= \frac{1}{3} \times \frac{1}{4} \left[2^4 - 1^4\right] =$

$\frac{1}{12}\left[16 - 1\right]$

$\frac{15}{12}$

i) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$= \int_{-1}^{2} \frac{x^2}{3}\, dx$$

$$= \frac{1}{3} \int_{-1}^{2} x^2\, dx$$

$$= \frac{1}{3} \left[\frac{x^3}{3}\right]_{-1}^{2}$$

$$= \frac{1}{9} \left[2^3 - (-1)^3\right]$$

$$= \frac{1}{9} (8+1)$$

$$= \frac{9}{9}$$

$$= 1$$

ii) $P(0 < X \leq 1) = \int_{0}^{1} \frac{x^2}{3}\, dx$

$$= \frac{1}{3} \int_{0}^{1} x^2\, dx$$

$$= \frac{1}{3} \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$= \frac{1}{9} [1-0]$$

$$= 1/9$$

iii) Find $F(x)$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-1}^{x} \frac{x^2}{3}\,dx$$

$$= \left[\frac{x^3}{9}\right]_{-1}^{x}$$

$$= \left[\frac{x^3}{9} - \frac{(-1)^3}{9}\right]$$

$$= \frac{x^3+1}{9}$$

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{x^3+1}{9} & -1 < x < 2 \\ 1 & x \geq 2. \end{cases}$$

2. If $f(x) = \begin{cases} K(1-x^2) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ is a probability distributive function of random variable X Find K and distribution function of X.

$$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$$

$$\int_{0}^{1} K(1-x^2)\,dx = 1$$

$$K\int_0^1 (1-x^2)\,dx = 1$$

$(1-x)^{-2} = 1+2x+3x^2+\ldots$

$(1-x)^{-1} = 1+x+x^2+x^3+\ldots$

$$K\left[x-\frac{x^3}{3}\right]_0^1 = 1$$

$$K\left[\frac{3x-x^3}{3}\right]_0^1 = 1$$

$$K\left[\frac{3-1}{3}-0\right] = 1$$

$$K\left[2/3\right] = 1$$

$$\boxed{K = 3/2}$$

$$\boxed{F(x) = \int_{\infty}^{x} f(x)\,dx}$$

$$= \int_0^x \frac{3}{2}\left[1-x^2\right]dx$$

$$= \frac{3}{2}\int_0^x \left[1-x^2\right]dx$$

$$= \frac{3}{2}\left[x-\frac{x^3}{3}\right]_0^x$$

$$= \frac{3}{2}\left[x-\frac{x^3}{3}\right]$$

$$= \frac{\cancel{3}}{2}\left[\frac{3x-x^3}{\cancel{3}}\right]$$

$$F(x) = \frac{3x - x^3}{2}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3x - x^3}{2} & 0 < x < 1 \\ 1 & x > 1 \end{cases}$$

3. A continuous random variable x has the probability density function

$$f(x) = \frac{k}{1+x^2} \quad -\infty < x < \infty$$. Find k and distribution function of x.

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\boxed{\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}.}$$

$\tan^{-1}\infty = \frac{\pi}{2}$

$\tan^{-1} 0 = 0$

$$\int_{-\infty}^{\infty} \frac{k}{1+x^2}\, dx = 1$$

$\because$ The given function is even, then

$$2 \int_{0}^{\infty} \frac{k}{1+x^2}\, dx = 1$$

$$2k \left[\frac{1}{1} \tan^{-1} \frac{x}{1} \right]_0^{\infty} = 1$$

$$2K\left[\tan^{-1}x\right]_0^{\infty} = 1$$

$$2K\left[\tan^{-1}\infty - \tan^{-1}0\right] = 1$$

$$2K\left[\frac{\pi}{2} - 0\right] = 1$$

$$\cancel{2}K\frac{\pi}{\cancel{2}} = 1$$

$$\boxed{K = 1/\pi}$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\pi(1+x^2)}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1}x\right]_{-\infty}^{x}$$

$$= \frac{1}{\pi}\left[\tan^{-1}x - \tan^{-1}(-\infty)\right]$$

$$= \frac{1}{\pi}\left[\tan^{-1}x - \left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{\pi}\left[\tan^{-1}x + \frac{\pi}{2}\right]$$

$$F(x) = \frac{1}{\pi}\left[\tan^{-1}x + \frac{\pi}{2}\right]$$

4. If a continuous random variable x having probability density function $f(x) = k e^{-|x|}$ find k and distribution function $F(x)$

$e^{-\infty} = 0$

$e^{0} = 1$

$e^{\infty} = \infty$

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} k e^{-|x|}\, dx = 1$$

$$2k \int_{0}^{\infty} e^{-|x|}\, dx = 1$$

$$2k \int_{0}^{\infty} e^{-x}\, dx = 1$$

$$2k \left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$$

$$2k \left[\frac{e^{-\infty}}{-1} - \frac{e^{0}}{-1}\right] = 1$$

$$2k \left[e^{-\infty} + e^{0}\right] = 1$$

$f(x) = \frac{1}{2} e^{+x} \quad -\infty \leq x < 0$

$= \frac{1}{2} e^{-x} \quad 0 < x < \infty$

$$2k [0+1] = 1$$

$$2k = 1$$

$$\boxed{k = 1/2}$$

$$F[x] = \int_{-\infty}^{\infty} \boxed{\frac{e^{-|x|}}{2}} \, dx$$

e^{∞} $e^{-x-\infty}$

$-\infty$ 0 ∞

When, $-\infty < x < 0$

$x<0$

$$F(x) = \int_{-\infty}^{x} \frac{e^{x}}{2} \, dx$$

$$= \frac{1}{2} \int_{-\infty}^{x} e^{x} \, dx$$

$$= \frac{1}{2} \left[e^{x} \right]_{-\infty}^{x}$$

$$= \frac{1}{2} \left[e^{x} - e^{-\infty} \right]$$

$$= \frac{1}{2} \left[e^{x} - 0 \right]$$

$$= \frac{e^{x}}{2}$$

$x>0$

$$F(x) = \int_{-\infty}^{0} f(x) \, dx + \int_{0}^{x} f(x) \, dx$$

$$= \int_{-\infty}^{0} \frac{1}{2} e^{x} \, dx + \int_{0}^{x} \frac{1}{2} e^{-x} \, dx$$

$$= \frac{1}{2} \left[e^{x} \right]_{-\infty}^{0} + \frac{1}{2} \left[\frac{e^{-x}}{-1} \right]_{0}^{x}$$

$$= \frac{1}{2} \left[e^{0} - e^{-\infty} \right] + \frac{1}{2} \left[-e^{-x} + 1 \right]$$

$$= \frac{1}{2} - \frac{e^{-x}}{2} + \frac{1}{2}$$

$$= 1 - \frac{e^{-x}}{2}.$$

5. Find the cumulative distributive function of

$$f(x) = \begin{cases} \frac{x}{2} & 0 \le x \le 1 \\ 1/2 & 1 \le x \le 2 \\ 1/2(3-x) & 2 \le x \le 3. \\ 0 & \text{otherwise.} \end{cases}$$

F(x) when $x < 0$

$F(x) = 0$

when $x \le 1$,

$$F(x) = \int_0^x \frac{x}{2}\, dx$$

$$= \left[\frac{x^2}{4}\right]_0^x$$

$$= \frac{x^2}{4}$$

when $1 \le x \le 2$

$$F(x) = \int_0^1 \frac{x}{2}\, dx + \int_1^x \frac{1}{2}\, dx$$

$$= \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^x$$

$$= \frac{1}{4} + \frac{x}{2} - \frac{1}{2}$$

$$= \frac{x}{2} - \frac{1}{4}$$

when $2 \leq x \leq 3$.

$$F(x) = \int_0^1 \frac{x}{2}\,dx + \int_1^2 \frac{1}{2}\,dx + \int_2^x \frac{3-x}{2}\,dx.$$

$$= \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^2 + \frac{1}{2}\int_2^x 3-x\,dx$$

$$= \frac{1}{4} + \frac{2}{2} - \frac{1}{2} + \frac{1}{2}\left[3x - \frac{x^2}{2}\right]_2^x$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[\frac{6x-x^2}{2}\right]_2^x$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[\frac{6x-x^2}{2}\right] - \left[\frac{12-4}{2}\right]$$

$$= \frac{1}{4} + \frac{2}{4} + \frac{1}{2}\left[\frac{6x-x^2}{2}\right] - \left[\frac{8}{2}\right]$$

$$= \frac{1}{4} + \frac{2}{4} + \frac{1}{2}\left[\frac{6x-x^2-8}{2}\right]$$

$$= \frac{1}{4} + \frac{2}{4} + \frac{6x-x^2-8}{4}$$

$$= \frac{3}{4} + \frac{6x-x^2-8}{4}$$

$$= \frac{6x-x^2-5}{4}$$

when $F(x) > 3$

$F(x) = 1$, because sum of the probability is 1.

6. The cumalative distributive function

$\boxed{F(x) = 1-(1+x)e^{-x}}$, $x>0$. Find $f(x)$, mean & variance.

$$f(x) = \frac{d}{dx} F(x)$$

$$= \frac{d}{dx}\left[1-e^{-x} - \underbrace{x e^{-x}}_{u\,v}\right]$$

$(uv)' = uv_1 - u'v_2 +$

$$= e^{-x} - e^{-x} - x(-e^{-x})$$

$$= x e^{-x}, \quad x>0$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_{0}^{\infty} x\, x\, e^{-x}\, dx$$

$u = x^2 \quad v = e^{-x}$

$u' = 2x \quad v_1 = -e^{-x}$

$u'' = 2 \quad v_2 = e^{-x}$

$u''' = 0 \quad v_3 = e^{-x}$

$$= \int_{0}^{\infty} x^2 e^{-x}\, dx$$

$x^2 \; -e^{-x}$

$-2x e^{-x}$

$+2(-e^{-x})$

$$uv_1 - u'v_2 + u''v_3$$

$$= \left[x^2(-e^{-x}) - 2xe^{-x} + 2(-e^{-x})\right]_0^{\infty}$$

$$= 2e^{0}$$

$$= 2.$$

$$E[x^2] = \int_{0}^{\infty} x^3 e^{-x}\, dx.$$

$u = x^3 \qquad v = e^{-x}$

$u' = 3x^2 \qquad v_1 = -e^{-x}$

$u'' = 6x \qquad v_2 = e^{-x}$

$u''' = 6$ $\quad v_3 = -e^{-x}$

$u'''' = 0$ $\quad v_4 = e^{-x}.$

$$E[x^2] = \Big[x^3(-e^{-x}) - 3x^2 e^{-x} + 6x(-e^{-x}) - 6(e^{-x})\Big]_0^{\infty}$$

$$= [-[-6e^0]]$$

$$= 6.$$

$$Var[x] = E[x^2] - [E[x]]^2$$

$$= 6 - (2)^2$$

$$= 2.$$

MOMENT AND MOMENT GENERATING FUNCTIONS

Moment about origin :

Let X be a random variable, then r^{th} moment about origin is

$$E[X^r] = \sum x_i^r P(x_i) \text{ if } X \text{ is discrete}$$

$$E[X^r] = \int_{-\infty}^{\infty} x^r f(x)\,dx \text{ if } X \text{ is continuous.}$$

Probability mass function

Probability density function

$E[X^r]$ is represented as μ_r'

i.e. $\mu_1' = \text{mean}$, $\mu_2' = E[X^2]$

$$\text{var}[X] = \mu_2' - (\mu_1')^2$$

r^{th} moment about ~~origin~~ mean / central moment :

$$E[X - \bar{x}]^r$$

$$= \sum (x - \bar{x})^r P(x_i) \text{ if } X \text{ is discrete}$$

$$\int_{-\infty}^{\infty} (x - \bar{x})^r f(x)\,dx \text{ if } X \text{ is continuous.}$$

It is represented as μ_r

Moment Generating function :

moment generating function of a random variable about the origin is

$$M_X(t) = E[e^{tx}].$$

$$= \sum e^{tx} p(x_i), \quad x \text{ is discrete.}$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\, dx, \quad x \text{ is continuous.}$$

properties of moment generating function. (2m)

Let $Y = ax + b$, where X is a random variable with moment generating function $M_X(t)$

$$M_Y(t) = e^{bt} M_X(at)$$

If X and Y are two independent random variable then,

$$M_{X+Y}(t) = M_X(t)\, M_Y(t)$$

$$e^{(X+Y)t} \quad e^{Xt}\, e^{Yt}.$$

Relation between moment generating function & moment about origin.

$$\mu_r' = \frac{d^r}{dt^r}\left[M_X(t)\right]_{t=0}$$

at $t=0$, $\mu_1' =$ mean

$\mu_1' = E(X)$

$$\boxed{M_X(t) = E[e^{tx}]}$$

$$\boxed{\frac{t^r}{r!} \text{ coeff } \mu_r'}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

1. Find the moment generating function of an random variable X whose probability function is $P(X = x) = \frac{1}{2^x}$, $x = 1, 2, \dots$ Hence find its mean.

Soln:

The given probability function is a discrete random variable,

$$M_X(t) = \sum e^{tx} P(x_i).$$

$$M_X(t) = \sum_{x=1}^{\infty} e^{tx} \frac{1}{2^x}$$

$$= e^t \frac{1}{2} + e^{2t} \frac{1}{2^2} + e^{3t} \cdot \frac{1}{2^3} + \dots$$

$$= \frac{e^t}{2} \left[1 + \frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \dots \right]$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$$

$$= \frac{e^t}{2} \left[1 - \frac{e^t}{2} \right]^{-1}$$

$$= \frac{e^t}{2} \left[\frac{2 - e^t}{2} \right]^{-1}$$

$$= \frac{e^t}{\cancel{2}} \left[\frac{\cancel{2}}{2 - e^t} \right]$$

$$= \frac{e^t}{2-e^t}$$

$M_x'(t) = \mu_1'$ mean $= \frac{d}{dt}\left[M_x(t)\right]_{t=0}$

$\frac{u}{v} = \frac{vu' - uv'}{v^2}$

$$= \frac{d}{dt}\left[\frac{e^t}{2-e^t}\right]_{t=0}$$

$$= \left[\frac{(2-e^t)\,e^t - e^t\,(-e^t)}{(2-e^t)^2}\right]_{t=0}$$

$$= \frac{(2-1)\,(1)+1}{(2-1)^2}$$

$$= 2.$$

2. If X represent the outcome when a fair die is tossed. Find the moment generating function of X and hence find mean and variance of X.

x	1	2	3	4	5	6
P(X=x)	1/6	1/6	1/6	1/6	1/6	1/6

$$M_x(t) = \sum_{x=1}^{6} e^{tx}\,p(x_i)$$

$$= \frac{1}{6}e^t + \frac{1}{6}e^{2t} + \frac{1}{6}e^{3t} + \frac{1}{6}e^{4t} + \frac{1}{6}e^{5t} + \frac{1}{6}e^{6t}$$

$$M_X(t) = \frac{1}{6}\left[e^{t} + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}\right]$$

$$M_X'(t) = \mu_1' \text{ (mean)} = \frac{d}{dt}\left[M_X(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[\frac{1}{6}\left[e^{t} + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}\right]\right]_{t=0}$$

$$= \frac{1}{6}\left[e^{t} + 2e^{2t} + 3e^{3t} + 4e^{4t} + 5e^{5t} + 6e^{6t}\right]_{t=0}$$

$$= \frac{1}{6}\left[e^{0} + 2e^{0} + 3e^{0} + 4e^{0} + 5e^{0} + 6e^{0}\right]$$

$$= \frac{1}{6}\left[1+2+3+4+5+6\right]$$

$$= 21/6$$

$$\boxed{M_X''(t) = \mu_2' \; \cancel{(\text{Variance})} = E[X^2]}$$

$$M_X''(t) = \frac{1}{6}\left[e^{t} + 4e^{2t} + 9e^{3t} + 16e^{4t} + 25e^{5t} + 36e^{6t}\right]_{t=0}$$

$$= \frac{1}{6}\left[1+4+9+16+25+36\right]$$

$$= \frac{1}{6}[91]$$

$$= 91/6$$

$$\boxed{\text{Var}(X) = \mu_2' - (\mu_1')^2}$$

$$= \frac{91}{6} - \left(\frac{21}{6}\right)^2$$

$$= \frac{546}{36} - \frac{441}{36}$$

$$= \frac{105}{36}$$

3 If X is a random variable represents the outcome of no. of. ~~ta~~ heads obtaining four toss of a coin. Find moment generating function, mean, variance

$S = \{HHHH, HHHT, HHTH, HTHH, THHH,$
$TTTT, TTTH, TTHT, THTT, HTTT,$
$HTTH, THHT, HTHT, THTH, TTHH, HHTT\}$

$P(S) = \frac{1}{16}$

x	0	1	2	3	4
$P(x)$	1/16	4/16	6/16	4/16	1/16

$$M_X(t) = \sum_{x=0}^{4} e^{tx} P(x_i)$$

$$= e^0 \frac{1}{16} + e^t \frac{4}{16} + e^{2t} \frac{6}{16} + e^{3t} \frac{4}{16} + e^{4t} \frac{1}{16}$$

$$= \frac{1}{16}\left[e^0 + 4e^t + 6e^{2t} + 4e^{3t} + e^{4t}\right]$$

Mean $= E[X] = M_X'(t)/_{t=0}$

$$M_X'(t) = \frac{1}{16}\left[0 + 4e^t + 12e^{2t} + 12e^{3t} + 4e^{4t}\right]$$

$$M_X'(t) = \frac{1}{16}\left[4 + 12 + 12 + 4\right]$$

$$= \frac{32}{16}$$

$E[X^2] = M_X''(t)/_{t=0}$

$M_x''(t) = \frac{1}{16}\left[4e^t + 24e^{2t} + 36e^{3t} + 16e^{4t}\right]$

$M_X''(t) = \frac{1}{16}\left[4 + 24 + 36 + 16\right]$

$= 5$

$Var(X) = \mu_2' - (\mu_1')^2$

$= 5 - 4$

$= 1$

4. Let X be a random variable of a probability density function, $f(x) = \begin{cases} \frac{1}{3}e^{-x/3} & , x > 0 \\ 0 & \text{otherwise.} \end{cases}$

Find i) $P(X > 3)$

ii) m.g.f

iii) $E[X]$, var$[X]$.

i) $P(X>3) = \int_3^{\infty} \frac{1}{3} e^{-x/3} \, dx$

$= \frac{1}{3}\left[\frac{e^{-x/3}}{-1/3}\right]_3^{\infty}$

$= -e^{-\infty} + e^{-1}$

$= 1/e$

$$M_X(t) = \int_0^\infty \frac{e^{tx} e^{-x/3}}{3} dx \qquad e^{-x/3} e^{tx}$$

$$e^{x[-\frac{1}{3}+t]}$$

$$= \frac{1}{3} \int_0^\infty e^{-x[1/3 - t]} dx \qquad e^{-x[\frac{1}{3}-t]}$$

$$= \frac{1}{3} \left[\frac{e^{x[1/3-t]}}{-(1/3 - t)} \right]_0^\infty$$

$$= \frac{1}{3} \left[\frac{e^{-\infty}}{-[1/3 - t]} + \frac{e^0}{1/3 - t} \right].$$

$$= \cancel{\frac{1}{3}} \frac{1}{\frac{1-3t}{\cancel{3}}}$$

multiplied by -

$$= \frac{1}{1-3t}$$

$$\cancel{Var[X] = \mu_2' - (\mu_1')^2}$$

$$\cancel{= 5 - 4}$$

$$\cancel{= 1}$$

$$M_X'(t) = \to \frac{1}{(1-3t)^2} (-3)$$

$$= \frac{3}{(1-3t)^2}$$

$$= 3$$

$$M_x''t = 3\frac{(-2)}{(1-3t)^3}(-3) \qquad \frac{u}{v} = \frac{vu'-uv'}{v^2}$$

$$= \frac{18}{(1-3t)^3}$$

$$= 18$$

Probability density function of a random variable x is $f(x) = \begin{cases} \frac{x}{2}, & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$ find the moment generating function & 1st two moments about the origin.

$$M_x(t) = \frac{1}{2}\int_0^2 e^{tx}\, x \, dx.$$

$u = x \qquad v_1 = e^{tx}$

$u' = 1 \qquad v_2 = \frac{e^{tx}}{t}$

$u'' = 0 \qquad v_3 = \frac{e^{tx}}{t^2}$

$$M_x(t) = \left[\frac{xe^{tx}}{t} - \frac{e^{tx}}{t^2}\right]_0^2$$

$$= \frac{2e^{2t}}{t} - \frac{e^{2t}}{t^2} - 0 + \frac{1}{t^2} \qquad \frac{2te^{2t}}{t^2} - \frac{e^{2t}}{t^2} + \frac{1}{t^2} \qquad \frac{1}{t^2}\left(1 - e^{2t} + 2te^{2t}\right)$$

$$= \frac{1}{2t^2}\left[1 + 2te^{2t} - e^{2t}\right].$$

$$E[X] = \int_0^2 x \; f(x) \, dx$$

$$= \int_0^2 x \cdot \frac{x}{2} \, dx$$

$$= \frac{1}{2} \int_0^2 x^2 \, dx$$

$$= \frac{1}{2} \left[\frac{x^3}{3} \right]_0^2$$

$$= \frac{1}{\cancel{2}} \left[\frac{\cancel{8}^{\,4}}{3} \right]$$

$$= \frac{4}{3}.$$

$$E[X^2] = \int_0^2 x^2 \; f(x) \, dx$$

$$= \int_0^2 x^2 \cdot \frac{x}{2} \, dx$$

$$= \frac{1}{2} \int_0^2 x^3 \, dx$$

$$= \frac{1}{2} \left[\frac{x^4}{4} \right]_0^2$$

$$= \frac{1}{\cancel{2}} \left[\frac{\cancel{16}^{\,\cancel{4}\,2}}{\cancel{4}} \right]$$

$$= 2.$$

The density function of a random variable x is given by $f(x) = kx(2-x),\ 0 \le x \le 2$. Find k, mean, variance & r^{th} moment.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{2} k(2x - x^2)\,dx = 1$$

$$k\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$k\left[4 - \frac{8}{3}\right] = 1$$

$$\frac{4}{3}k = 1$$

$$k = \frac{3}{4}$$

r^{th} $$\mu_r' = \int_{-\infty}^{\infty} x^r f(x)\,dx$$

$$\mu_r' = \int_0^2 x^r \frac{3}{4}(2x - x^2)\,dx$$

$$= \frac{3}{4}\int_0^2 2x^{r+1} - x^{r+2}\,dx$$

$$= \frac{3}{4}\left[2\frac{x^{r+2}}{r+2} - \frac{x^{r+3}}{r+3}\right]_0^2$$

$$= \frac{3}{4}\left[\frac{2 \cdot 2^{r+2}}{r+2} - \frac{2^{r+3}}{r+3}\right]$$

$$= \frac{3}{4}\left[\frac{2^{r+3}}{r+2} - \frac{2^{r+3}}{r+3}\right]$$

$$= \frac{3}{4}\left[2^{r+3}\right]\left[\frac{1}{r+2} - \frac{1}{r+3}\right].$$

$$= \frac{3}{4}\, 2^{r+3}\left[\frac{r+3-r-2}{(r+2)(r+3)}\right]$$

$$= \frac{3}{4}\, 2^{r+3}\left[\frac{1}{(r+2)(r+3)}\right]$$

$$\text{Mean} = \mu_1' = \frac{3}{\cancel{4}}\cdot \cancel{2^4}^{\,2}\cdot \frac{1}{3\times 4}$$

$$= \frac{3\times 4}{3\times 4}$$

$$= 1$$

$$E[x^2] = \mu_2' = \frac{3}{\cancel{4}}\cdot \cancel{2^5}^{\,3}\cdot \frac{1}{4\times 5}$$

$$= \frac{3\times \cancel{8}^{\,2}}{\cancel{4}\times 5}$$

$$= \frac{6}{5}$$

2M

Functions of RV.

If X is a random variable with pdf $f(x)$ and Y is a random variable such that $y = G(x)$ then the pdf of Y is

$$f(y) = f(x) * \left|\frac{dx}{dy}\right|$$

If X is a continuous random variable with the interval $(0, 2)$ and $y = 4x+3$ and $f(x) = \frac{1}{2}$. Find $f(y)$.

$$f(y) = f(x) * \left|\frac{dx}{dy}\right|$$

$$\frac{dy}{dx} = 4$$

$$\frac{dx}{dy} = \frac{1}{4}$$

$$f(y) = \frac{1}{2} * \frac{1}{4}$$

$$= \frac{1}{8}$$

when $x = 0$,

$$y = 4x+3$$

$$y = 3$$

when $x = 2$

$$y = 4(2)+3$$

$$y = 11$$

$$f(y) = \frac{1}{8}, \quad (3, 11)$$

Consider a random variable X, with the pdf $f(x) = e^{-x}; x>0$ with the transformation $y = e^{-x}$. Find $f(y)$

$$f(y) = f(x) \times \left|\frac{dx}{dy}\right|$$

$$\frac{dy}{dx} = -e^{-x}$$

$$\frac{dx}{dy} = -\frac{1}{e^{-x}}$$

$$\left|\frac{dx}{dy}\right| = \frac{1}{e^{-x}}$$

$$f(y) = e^{-x} \times \frac{1}{e^{-x}}$$

$$= 1$$

$$0 \le x \le \infty$$

when $x = 0$

$y = 1$

$x = \infty$

$y = 0$

$f(y) = 1 \quad 0 \le x \le 1.$

A RV X has the pdf $x = \frac{1}{\pi}$ with the transformation

$y = \tan x$, x lies within the interval $(-\pi/2, \frac{\pi}{2})$

$$f(y) = f(x) * \left| \frac{dx}{dy} \right|$$

$$\frac{dy}{dx} = \sec^2 x.$$

$$\frac{dx}{dy} = \frac{1}{\sec^2 x}$$

$$= \frac{1}{1+\tan^2 x}$$

$$= \frac{1}{1+y^2}$$

$$f(y) = \frac{1}{\pi} * \frac{1}{1+y^2}$$

when $x = -\frac{\pi}{2}$

$$y = \tan\left(-\frac{\pi}{2}\right) = -\infty$$

$$y = \tan\left(\frac{\pi}{2}\right) = \infty$$

$$f(y) = \frac{1}{\pi(1+y^2)} \quad -\infty \le x \le \infty$$

$p+q=1$

BINOMIAL DISTRIBUTION.

A random variable X is said to follow binomial distribution if it assumes only non-negative values and its probability mass function

$$P(X=x)=f(x)=\begin{cases} n_{C_x}\, p^x q^{n-x} & x=0,1,2,\dots \\ 0 & \text{otherwise} \end{cases}$$

Here n and p are called parameters of the distribution.

standard deviation $\sqrt{var}$ $\quad n_{c_1}=n \quad {}^nC_0=1,\ {}^nC_n=1$

Moment generating function of a Binomial distribution

$$M_X(t)=\sum_{x=0}^{\infty} e^{tx} f(x)$$

$$=\sum_{x=0}^{\infty} e^{tx}\, n_{C_x}\, p^x q^{n-x}$$

$$=e^{0}\, n_{C_0}\, p^0 q^n + e^t\, n_{C_1}\, p^1 q^{n-1} + e^{2t}\, n_{C_2}\, p^2 q^{n-2} + \dots + e^{nt}\, n_{C_n}\, p^n q^{n-n}$$

$$=q^n + n_{C_1}(e^t p)^1 q^{n-1} + n_{C_2}(e^t p)^2 q^{n-2} + \dots + (pe^t)^n$$

$$=(e^t p+q)^n$$

$$mean = E[X] = \frac{d}{dt}\left[M_X(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[e^t p + q\right]^n_{t=0} \qquad \frac{d}{dx} x^n = nx^{n-1}$$

$$= n(e^t p + q)^{n-1} e^t p / _{t=0}$$

$$= n[p+q]^{n-1} p$$

$$= n(1)^{n-1} p$$

$$= np.$$

$$E[X^2] = \frac{d^2}{dt^2}\left[M_X(t)\right]_{t=0}$$

$$uv = uv' + vu'$$

$$= \frac{d}{dt}\left[\underbrace{npe^t}_{u} \underbrace{(e^t p + q)^{n-1}}_{V}\right]_{t=0}$$

$$= np\left[\underbrace{e^t}_{u}\underbrace{(n-1)(e^t p + q)^{n-2} e^t p}_{V'} + \underbrace{(e^t p + q)^{n-1}}_{V} e^t\right]_{t=0}$$

$$= np\left[(n-1)(p+q)^{n-2} p + (p+q)^{n-1}(1)\right]$$

$$= np\left[(n-1)p + 1\right]$$

$$= np\left[np - p + 1\right]$$

$$E[X^2] = n^2p^2 - np^2 + np$$

$$Var[X] = E[X^2] - [E[X]]^2$$

$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$

$= -np^2 + np$

$= np[1-p]$

$= npq$

2m

1. The mean of a Binomial distribution is 20 and standard deviation is 4. Determine the parameters of the distribution.

$np = mean = 20 \rightarrow (1)$

$sd = 4$

$\sqrt{var} = sd$

$var = (sd)^2$

$var(x) = 16$

$npq = 16$

$20q = 16$

$q = \frac{16}{20} = \frac{4}{5}$

$p = 1 - q$

$= 1 - \frac{4}{5}$

$\boxed{p = 1/5}$

$np = 20$

$n = 20 \times 5$

$\boxed{n = 100}$

1. Is it possible for a binomial distribution to have mean 2 and variance 4.

mean $\Rightarrow np = 2$

variance $\Rightarrow npq = 4$

$2q = 4$

$q = 4/3$

2. The mean & the variance of a binomial distribution is 8 and 6 respectively. Find $P(X) \geq 2$

mean $\Rightarrow np = 8$

variance $\Rightarrow npq = 6$

$8q = 6$

$q = \cancel{6}^{3}/\cancel{8}_{4}$

$\boxed{q = 3/4}$

$p + q = 1$

$p + \frac{3}{4} = 1$

$\frac{4p+3}{4} = 1$

$4p + 3 = 4$

$4p = 4 - 3$

$4p = 1$

$\boxed{p = 1/4}$

$\Rightarrow np = 8$

$n\frac{1}{4} = 8$

$$\frac{n}{4} = 8$$

$$n = 32$$

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[{}^{32}C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{32-0} + {}^{32}C_1 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^{31} \right]$$

$$= 1 - \left[{}^{32}C_0 (3/4)^{32} + {}^{32}C_0 \left(\frac{1}{4}\right) (3/4)^{31} \right]$$

$$= 1 - \left(\frac{3}{4}\right)^{31} \left[\frac{3}{4} + \frac{32}{4} \right]$$

$$= 1 - \left(\frac{3}{4}\right)^{31} \left[\frac{35}{4} \right]$$

BISON DISTRIBUTION.

Four coins are tossed simultaneously what is the probability of getting (i) 2 heads (ii) atleast two heads (iii) atmost two heads.

i) Probability of getting head. $p = 1/2$

$q = 1 - 1/2 = 1/2$

$n = 4$

$P(X = x) = {}^nC_n \, p^x q^{n-x}$

$P(X=2) = {}^4C_2 \cdot (1/2)^2 (1/2)^{4-2}$

$= {}^4C_2 \, (1/2)^2 (1/2)^2$

$= {}^4C_2 \, (1/2)^4$

$= \frac{\cancel{4}^2}{1} \cdot \frac{3}{\cancel{2}} \cdot \frac{1}{2^4}$

$= \frac{\cancel{6}^3}{\cancel{2^4}\,{}_3}$

$= \frac{3}{2^3}$

$= \frac{3}{8}$

ii) Probability of getting atleast twoheads

$P(x \geq 2) = P(X = 2) + P(X = 3) + P(X = 4)$

$= {}^4C_2 \, (1/2)^2 (1/2)^{4-2} + {}^4C_3 \, (1/2)^3 (1/2)^{4-3}$

$+ {}^4C_4 \, (1/2)^4 (1/2)^0$

2.

$$= \frac{3}{8} + \frac{4 \times \cancel{3} \times \cancel{2}}{1 \times \cancel{2} \times \cancel{3}} \left(\frac{1}{2}\right)^4 + 1\left(\frac{1}{2}\right)^4$$

$$= \frac{3}{8} + \frac{1}{4} + \frac{1}{16}.$$

$$= \frac{6+4+1}{16}$$

$$= \frac{11}{16}.$$

probability of getting atmost two heads $P(X \leq 2)$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= {}^4C_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^{4-0} + 4C_1 \left(\tfrac{1}{2}\right)^1 \left(\tfrac{1}{2}\right)^{4-1}$$

$$+ 3/8$$

$$= \left(\tfrac{1}{2}\right)^4 + 4\left(\tfrac{1}{2}\right)^4 + \frac{3}{8}.$$

$$= \frac{1}{2^4} + \frac{4}{2^4} + \frac{3}{8}$$

$$= \frac{1}{2^4} + \frac{4}{2^4} + \frac{3 \times 2}{2^4}$$

$$= \frac{1+4+6}{2^4}$$

$$= \frac{11}{16}.$$

A pair of dice is thrown 4 times if getting a doublet is consider a success find the probability of two success.

probability of getting double $= \frac{6}{36} = \frac{1}{6}$

$p = \frac{1}{6}$; $q = 1 - \frac{1}{6}$

$= \frac{5}{6}$

$n = 4$

probability of getting two success $= P(X=2)$

$= {}^4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{4-2}$

$= \frac{4 \times 3}{1 \times 2} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{25}{216}$.

A large consignment of electric bulb 10% are defective

A random sample of 20 is taken for inspection

Find i) the probability that all are good bulbs.

ii) almost three is defective $P(X \le 3)$

iii) Exactly three is defective.

p is probability of getting defective, $p = \frac{10}{100} = \frac{1}{10}$

$n = 20$

$p = \frac{1}{10}$, $q = 1 - \frac{1}{10} = \frac{9}{10}$.

i) $P(X=0) = {}^nC_x\, p^x\, q^{n-x}$

$= {}^{20}C_0\, p^0\, q^{20-0}$

$= {}^{20}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20}$

$^nC_n = 1$

$= \left(\frac{9}{10}\right)^{20}$

$^nC_0 = 1$

$^nC_1 = n$

$= 0.1216.$

ii) atmost three is defective:

$P(X \leq 3)$

$P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= {}^{20}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20-0} + {}^{20}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{20-1}$

$+ {}^{20}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{20-2} + {}^{20}C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{20-3}$

$\frac{20 \times 19}{1 \times 2} \times \frac{1}{10} \times \frac{1}{10} \times \left(\frac{9}{10}\right)^{18}$

$= \left(\frac{9}{10}\right)^{20} + 20 \left(\frac{1}{10}\right)\left(\frac{9}{10}\right)^{19} + \frac{19}{10}\left(\frac{9}{10}\right)^{18} + \frac{57}{50}\left(\frac{9}{10}\right)^{17}$

$= \left(\frac{9}{10}\right)^{20} + 2 \cdot \left(\frac{9}{10}\right)^{19} + \frac{19}{10}\left(\frac{9}{10}\right)^{18} + \frac{57}{50}\left(\frac{9}{10}\right)^{17}$

$= \left(\frac{9}{10}\right)^{17} \left[\left(\frac{9}{10}\right)^3 + 2\left(\frac{9}{10}\right)^2 + \frac{19}{10} \cdot \frac{9}{10} + \frac{114}{100}\right]$

$= 0.866$

iii) Exactly 3,

$P(X=3) = {}^nC_x\, p^x\, q^{n-x}$

$= {}^{20}C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{20-3}$

$$= \frac{20 \times 19 \times 18}{3 \times 2 \times 1} \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{17}$$

$$= 1140 \times \frac{1}{1000} \times \left(\frac{9}{10}\right)^{17}$$

$$= 1140 \times \frac{1}{1000} \times 0.166$$

$$= \frac{189.24}{1000}$$

$$= 0.189$$

$$= 0.19$$

It is known that screws produced by a certain company will be defective with a probability 0.01 the companies sells the screws in pack of 10. & offers a money back guarantee that atmost one of these screw is defective what proportion must the Company replace.

$P(x=0) + P(x=1)$

$$\begin{array}{r} 1.00 \\ 0.01 \\ \hline 0.99 \end{array}$$

$n = 10$, $P = 0.01$, $q = 1 - 0.01$
$= 0.99$

$$P(x>1) = 1 - P(x \leq 1)$$

$$= 1 - [P(x=0) + P(x=1)]$$

$$= 1 - [10C_0 (0.01)^0 (0.99)^{10} + 10C_1 (0.01)(0.99)^9]$$

$$= 1 - [(0.99)^{10} + 10 (0.01)(0.99)^9]$$

$$= 1 - [0.904 + 0.0913]$$

$$= 1 - 0.9953$$

$$P(x>1) = 0.0047$$

In a certain town 20% samples of the population is literate and assume that 200 investigators take samples of 10 individuals to see whether they are literate. How many investigators would you expect to report that three people or less literate in the sample.

$N = 200$

$n = 10$

N = no. of investigators.

$n = 10$ [no. of samples]

$^nC_x\, p^x\, q^{n-x}$

$p = \frac{20}{100} = 0.2$

$q = 1 - p = 1 - 0.2 = 0.8$

0.04×0.2
0.008

for 1 investigator $P(X \leq 3)$

for 200 investigator $200\, P(X \leq 3)$

$P(X \leq 3) = P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$

$200\, P(X \leq 3) = 200\left[P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)\right]$

$= 200\left[{}^{10}C_0\,(0.2)^0\,(0.8)^{10} + {}^{10}C_1\,(0.2)^1\,(0.8)^9 + {}^{10}C_2\,(0.2)^2\,(0.8)^8 + {}^{10}C_3\,(0.2)^3\,(0.8)^7\right]$

$= 200\left[(1)(1)(0.1073) + 10(0.2)(0.1342) + \frac{10 \times 9}{1 \times 2}(0.04)(0.1677) + \frac{10 \times 9 \times 8}{1 \times 2 \times 3}(0.008)[0.2097]\right]$

$= 200\left[0.1073 + 0.2684 + 0.30186 + 0.2013\right]$

$= 200\,[0.87886]$

$= 175.772$

$= 176$

Out of 800 families with four children each. How many families do you expect to have.

i) Two boys and two girls

ii) atleast one boy

iii) atmost two girls

iv) children of both genders

MMMF
MMFF
MFFF
FFMM

Assume equal probability for both boys & girls.

Probability of getting boy $p = 1/2$

$q = 1 - 1/2 = 1/2$

//no. of samples $n = 4$

$N = 800$

i) probability of getting two boys & two girls =

$$800 \left[{}^4C_2 \; p^2 \; q^2 \right]$$

$$= 800 \left[\frac{\cancel{4}^{2} \times 3}{1 \times \cancel{2}} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2 \right]$$

$$= \cancel{800}^{200} \left[\cancel{6}\,3 \left(\frac{1}{\cancel{4}_2}\right) \left(\frac{1}{\cancel{4}}\right) \right]$$

$$= \cancel{200}^{100} \times 3 \times \frac{1}{2}$$

$= 300.$

ii) probability of getting atleast one boy = $P(X \geq 1)$

$= 800\,[P(X=1) + P(X=2) + P(X=3) + P(X=4)]$

$$= 800\left[{}^4C_1\, p^1 q^3 + {}^4C_2\, p^2 q^2 + {}^4C_3\, p^3 q^1 + {}^4C_4\, p^4 q^0\right]$$

${}^nC_0 = 1$

${}^nC_1 = n$

${}^nC_n = 1$

$$= 800\left[\cancel{4}^{2}\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)^3 + \frac{\cancel{4}^{2}\times 3}{1\times \cancel{2}}\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^2 + \frac{4\times\cancel{3}\times\cancel{2}}{1\times\cancel{2}\times\cancel{3}}\left(\tfrac{1}{2}\right)^3\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{2}\right)^4\left(\tfrac{1}{2}\right)^0\right]$$

$$= 800\left[\frac{\cancel{2}^{1}}{\cancel{8}_{4}} + \cancel{6}^{3}\left(\frac{1}{\cancel{4}_{2}}\right)\left(\frac{1}{4}\right) + 4\left(\frac{1}{8}\right) + \frac{1}{16}\right]$$

$$= 800\left[\frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}\right]$$

$$= 800\left[\frac{4+6+4+1}{16}\right]$$

$$= 800\left[\frac{15}{16}\right] = \frac{12000}{16} = 750$$

iii) probability of getting atmost two girls = $800\,[P(X \leq 2)]$

$$= 800\left[{}^4C_0\, p^0 q^4 + {}^4C_1\, p^1 q^3 + {}^4C_2\, p^2 q^2\right]$$

$$= 800\left[1\cdot\left(\tfrac{1}{2}\right)^0\left(\tfrac{1}{2}\right)^4 + 4\cdot\left(\tfrac{1}{2}\right)\left(\tfrac{1}{2}\right)^3 + 6\cdot\left(\tfrac{1}{2}\right)^2\left(\tfrac{1}{2}\right)^2\right]$$

$$= 800\left[\frac{1}{16} + 4\cdot\frac{1}{16} + 6\cdot\frac{1}{16}\right]$$

$= 550.$

iv) Probability of getting children of both genders

$$= 800\,\{1 - [P(X=4) + P(X=0)]\}$$

$$= 800\left[1 - \left(\frac{1}{16} + \frac{1}{16}\right)\right]$$

$$= 800\left[1 - \frac{\cancel{2}^{1}}{\cancel{16}_{8}}\right]$$

$$= 800\left[\frac{8-1}{8}\right] = \cancel{800}^{100}\,\frac{7}{\cancel{8}} = 700.$$

Poission Distribution

A R.V X is said to follow poission distribution if it assumes only non-negative values and its probability mass function is given by

$$P(X=x) = \begin{cases} \frac{e^{-\lambda}\lambda^{x}}{x!} & x=0,1,2,\ldots \quad \lambda>0 \\ 0 & \text{otherwise} \end{cases}$$

where λ is the parameter of the poission distribution.

Moment generating function:

$$M_X(t) = E[e^{tx}]$$

$$= \sum_{x=0}^{\infty} e^{tx}\, P(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \sum_{x=0}^{\infty} \frac{e^{-\lambda}(\lambda e^{t})^{x}}{x!}$$

$$= e^{-\lambda} \sum_{x=0}^{\infty} \frac{(\lambda e^{t})^{x}}{x!}$$

$$= e^{-\lambda}\left[1+\frac{\lambda e^{t}}{1!}+\frac{(\lambda e^{t})^{2}}{2!}+\ldots\right]$$

$$= e^{-\lambda}\, e^{\lambda e^{t}} \qquad 1+\frac{x}{1!}+\frac{x^{2}}{2!}+\ldots = e^{x}$$

$$= e^{\lambda[e^{t}-1]}$$

$$\text{Mean} = M_x'(t)/_{t=0}$$

$$M_x'(t) = \frac{d}{dt}[e^{-\lambda} e^{\lambda e^t}]$$

$$\frac{d}{dx}e^{2x} = 2e^{2x}$$

$$= e^{-\lambda} \underline{\lambda e^t e^{\lambda e^t}}$$

$$\frac{d}{dt} e^{\lambda e^t} = \underline{\lambda e^t e^{\lambda e^t}}$$

$$M_x'(t)/_{t=0} = e^{-\lambda} \lambda (1) e^{\lambda}$$

$$= \lambda$$

$$M_x''(t) = \frac{d}{dt} M_x'(t)$$

$$= \frac{d}{dt}\left[e^{-\lambda} \lambda e^t e^{\lambda e^t}\right]$$

$$= e^{-\lambda} \lambda \frac{d}{dt}\left[\underset{u}{e^t} \underset{v}{e^{\lambda e^t}}\right] \qquad uv = uv' + vu'$$

$$= e^{-\lambda} \lambda \left[e^t \lambda e^t e^{\lambda e^t} + e^{\lambda e^t} e^t\right]$$

$$M_x''(t)/_{t=0} = e^{-\lambda} \lambda \left[(1) \lambda (1) e^{\lambda} + e^{\lambda} (1)\right]$$

$$= \lambda^2 + \lambda$$

$$\text{var}[x] = E[x^2] - [E[x]]^2$$

$$= \lambda^2 + \lambda - \lambda^2$$

$$= \lambda$$

NOTE : $\lambda = np$ for a Poisson distribution

Binomial distribution	Poission distribution
* $P(x) = {}^nC_x \, p^x \, q^{n-x}$	* $P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$
* parameters n, p	* parameters $= \lambda$ $= np$
* mean $= np$ var $= npq$	* mean $=$ var $= \lambda$

If x is a poission variate such that $E[x^2] = 6$ find $E[x]$

Given: $E[x^2] = 6$

Find $E[x]$

Sol: $var(x) = E[x^2] - [E[x]]^2$

$\lambda = 6 - \lambda^2$

$\lambda^2 + \lambda - 6 = 0$

$(\lambda + 3)(\lambda - 2) = 0$

$\lambda = -3, 2$

$\boxed{\lambda = 2}$ $\because \lambda > 0$.

If x is a poission distribution function such that $P(x=0) = 0.5$ Find var(x)

$P(x=0) = \frac{e^{-\lambda} \lambda^x}{x!}$

$= \frac{e^{-\lambda} \lambda^0}{0!}$

$$0.5 = e^{-\lambda}$$

$$-\lambda = \log 0.5$$

$$\lambda = -\log(1/2)$$

$$= \log(2)$$

If three % ~~person~~ of the electric bulbs manufactured by a company are defective. Find the probability that in a sample of 100 bulbs exactly five bulbs are defective.

$$P(X=5) = \frac{e^{-\lambda}\lambda^5}{5!} \qquad \because \frac{e^{-\lambda}\lambda^x}{x!}$$

$$P = \frac{3}{100} \; ; \; n = 100$$

$$\boxed{\lambda = np}$$

$$= 100 \times \frac{3}{100}$$

$$\boxed{\lambda = 3}$$

$$P(X=5) = \frac{e^{-\lambda}\lambda^5}{5!}$$

$$= \frac{e^{-3}3^5}{5!}$$

$$= \frac{0.0497 \times 243}{120}$$

$$= \frac{12.098}{120}$$

$= 0.10081$

A radio active elements are randomly distributed

The atoms of radioactive elements are randomly disintegrated if every gram of this element on average emits 3.9 alpha particles per second. what is the probability that during the next second. The no of alpha particle emitted from 1 gram is

i) atmost 6 ii) atleast 2 iii) atleast 3 & atmost 6.

Mean $= \lambda = 3.9$ $\quad 1-(x<2) \quad \dfrac{e^{-\lambda}\lambda^{x}}{x!}$

atmost 6

$P(X \leq 6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$
$+ P(X=5) + P(X=6)$

$= \dfrac{e^{-3.9}\lambda^{0}}{0!} + \dfrac{e^{-3.9}\lambda^{1}}{1!} + \dfrac{e^{-3.9}\lambda^{2}}{2!} + \dfrac{e^{-3.9}\lambda^{3}}{3!} + \dfrac{e^{-3.9}\lambda^{4}}{4!}$

$+ \dfrac{e^{-3.9}\lambda^{5}}{5!} + \dfrac{e^{-3.9}\lambda^{6}}{6!}$

$\begin{matrix} 2\times 3 \\ 120\times 6 \\ 720 \end{matrix}$

$= 0.020 + 0.0789 + \dfrac{0.3078}{1\times 2} + \dfrac{1.2007}{1\times 2\times 3} + \dfrac{4.682}{1\times 2\times 3\times 4} + \dfrac{18.263}{120}$
$+ \dfrac{71.226}{}$

$= 0.020 + 0.0789 + 0.1539 + 0.2001 + 0.1951 + 0.1521 +$
0.0989

$= 0.899$

atleast 2 $= P(X \geq 2)$

$= 1 - P(X<2)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - [0.020 + 0.0789]$

$= 1 - 0.0989 = 0.9011$

iii) $P(3 \leq X \leq 6)$

$P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) + P(X=6)$

$= \dfrac{e^{-3.9}(3.9)^3}{3!} + \dfrac{e^{-3.9}(3.9)^4}{4!} + \dfrac{e^{-3.9}(3.9)^5}{5!}$

$+ \dfrac{e^{-3.9}(3.9)^6}{6!}$

$= \dfrac{1.2007}{6} + \dfrac{4.6828}{24} + \dfrac{18.2631}{120} + \dfrac{71.2260}{720}$

$= 0.2001 + 0.1951 + 0.1521 + 0.0989$

$= 0.6462$

The number of monthly breakdown of a computer is a random variable having poission distribution with mean = 1.8. Find the probability that this computer will function for a month without a breakdown. $P(X=0)$

ii) With only one breakdown $P(X=1)$

iii) With atleast one breakdown $P(X \geq 1)$ (or $1 - P(X<1)$)

$\lambda = 1.8$

$np = 1.8$

mean $= \lambda = 1.8 = np$

i) $P(X=0) = \dfrac{e^{-\lambda} \lambda^{x}}{x!}$

$= \dfrac{e^{-1.8} (1.8)^0}{0!}$ $\qquad 0! = 1$

$= 0.16529$

ii) $P(X=1) = \dfrac{e^{-\lambda} \lambda^{x}}{x!}$

$= \dfrac{e^{-1.8} (1.8)^1}{1!}$

$= \dfrac{0.16529 \times 1.8}{1}$

$= 0.297522$

iii) $P(X \geq 1) = 1 - P(X<1)$

$= 1 - P(X=0)$

$= 1 - 0.16529$

$=$

A book of 500 pages contains 500 mistakes. Find the probability that there are atleast 4 mistakes per page.

$P(X \geq 4)$

for 1 page the average mistake $\lambda = \frac{500}{500} = 1$

$\frac{e^{-\lambda} \lambda^{x}}{x!}$

$$P(X \geq 4) = 1 - P(X < 4)$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2) + P(X=3)]$$

$$= 1 - \left[\frac{e^{-1}(+1)^0}{0!} + \frac{e^{-1}(+1)^1}{1!} + \frac{e^{-1}(+1)^2}{2!} + \frac{e^{-1}(+1)^3}{3!}\right]$$

$$= 1 - \left[\frac{0.368 \times 1}{1} + \frac{0.368 \times 1}{1!} + \frac{0.368 \times 1}{1 \times 2} + \frac{0.368 \times 1}{1 \times 2 \times 3}\right]$$

$$= 1 - \left[0.368 + 0.368 + \frac{0.368}{2} + \frac{0.368}{6}\right]$$

$$= 1 - \left[\frac{2.208 + 2.208 + 1.104 + 0.368}{6}\right]$$

$$= 1 - \left[\frac{5.888}{6}\right]$$

$$= 1 - 0.981$$

$$= 0.01867$$

characteristics of poission distribution:

It is the limiting form of binomial distribution when n is large and p is small.

It consists of single parameter lambda λ and hence the entire distribution can be obtained by using the mean value.

properties of poission distribution

If X_1 and X_2 are two independent poission random variable with parameters λ_1 & λ_2 then $X_1 + X_2$ is a poission random variable with parameter $\lambda_1 + \lambda_2$

Suppose that the number of telephone calls comming into a telephone exchange between 9am & 10am is a poission random variable with parameter 2 & the no. of telephone calls comming between 10am & 11am is a poission random variable with parameter 6. If these two random variables are independent what is the probability that more than five calls come inbetween 9am & 11am?

X_1 is poission random variable [call between 9am to 10am and 10 am to 11am]

$P[x>5] = ?$ from 9 to 11

X_1 and X_2 are independent, $\lambda = \lambda_1 + \lambda_2$
$= 2 + 6$

$$P(X>5) = 1 - P(X \le 5)$$

$$= 1 - \left[P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)\right]$$

$$= 1 - \left[\frac{e^{-8}\,8^0}{0!} + \frac{e^{-8}\,8^1}{1!} + \frac{e^{-8}\,8^2}{2!} + \frac{e^{-8}\,8^3}{3!} + \frac{e^{-8}\,8^4}{4!} + \frac{e^{-8}\,8^5}{5!}\right]$$

$$= 1 - e^{-8}\left[1 + 8 + \frac{64}{2!} + \frac{512}{3!} + \frac{4096}{4!} + \frac{32768}{5!}\right]$$

$$= 1 - e^{-8}\left[1 + 8 + \frac{64}{1\times\cancel{2}} + \frac{\overset{256}{\cancel{512}}}{1\times\cancel{2}\times 3} + \frac{4096}{4\times3\times2\times1} + \frac{32768}{5\times4\times3\times2\times1}\right]$$

Atom

atmost 6

0.899

$$= 1 - e^{-8}\left[1 + 8 + 32 + 85.33 + 170.67 + 273.067\right]$$

atleast 2

0.901

$$= 1 - e^{-8}(570.067)$$

Un 5.6

0.646

$$= 1 - (0.000335)(570.067)$$

$$= 1 - 0.191236$$

$$= 0.808763$$