Unit - 3.

Two Dimensional Random Variables :-

⇒ 2D Random Variables :-

Let S be the sample space

Let $X = X(s)$ & $Y = Y(s)$ be two functions each assigning a real number to each outcome $s \in S$ then (x, y) are called 2D Random variables.

⇒ 2D Discrete Random variable :-

If the possible values of (x, y) are finite (countable) then x, y is called a 2D Discrete Random variable and it is denoted by

$$P(x_i, y_i), \quad i = 1, 2 \ldots n$$
$$j = 1, 2, \ldots m$$

⇒ Join Probability mass function :-

The function $P(X = x_i, Y = y_j) = P(x_i, y_i)$ is called JPmf, for discrete random variable x and y, if its satisfied the following conditions

i) $P(x_i, y_j) \geq 0 \quad \forall i, j$

ii) $\sum_j \sum_i P(x_i, y_i) = 1.$

Marginal Distribution :

If the JPMF of 2 Random variables x and y is given then the marginal distribution of x is given by $P(X = x_i) = \sum_{j=1}^{m} P_{ij} = \sum_{j=1}^{m} P(x_i, y_j)$

The marginal distribution function of y is given by

$$P(Y = y_j) = \sum_{i=1}^{n} P_{ij} = \sum_{i=1}^{n} P(x_i, y_j)$$

Conditional Distribution :-

The conditional distribution function of x given y is defined by $P(x = x_i | y = y_j) = \dfrac{P(X = x_i, Y = y_j)}{P(Y = y_j)}$

The conditional distribution function of y given x is defined by $P(Y = y_j | X = x_i) = \dfrac{P(X = x_i, Y = y_j)}{P(X = x_i)}$

Independent :

2 Random variables (x and y) are set to be independent if $P(x, y) = P(X).P(Y)$

2D Continuous Random Variable :

If x, y can take all the values in a region R in the x, y plane, then x, y is called as 2D continuous random variable and is denoted by

Join Probability Density function :

If x, y are continuous random variables then $f(x, y)$ is called JPDF if it satisfies the following condition.

i) $f(x, y) \geq 0 \quad -\infty < x < \infty \quad -\infty < y < \infty$.

ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)\, dy\, dx = 1$.

Marginal Distribution :

If the ~~JPP~~ Joint probability dis of x, y is given then the marginal distribution fn of x is defined by

$$f(x) = \int_{-\infty}^{\infty} f(x, y)\, dy \quad (y \text{ limit})$$

The marginal distribution fn of y is defined by

$$f(x) = \int_{-\infty}^{\infty} f(x, y)\, dx \quad (x \text{ limit})$$

Conditional distribution :

If the JPDF (x, y) are given then the conditional distribution of x given by

$$f(x/y) = \frac{f(x,y)}{f(y)} \qquad f(y/x) = \frac{f(x,y)}{f(x)}$$

Independent:-

2 Random variables x and y are set to be independent $f(x,y) = f(x).f(y)$

Join Cumulative Probability Distribution function:-

The join cumulative distribution function of the random variables x and y is defined as

For discrete case:-

$$F(x,y) = P(X \leq x, Y \leq y) = \sum_{j} \sum_{i} P(x_i, y_j)$$

$$Y \leq y_j \quad X = x_i$$

For continuous case:-

$$F(x,y) = \int_{-\infty}^{y} \int_{-\infty}^{x} f(x,y)\, dx\, dy$$

Note:-

$$f(x,y) = \frac{\partial^2}{\partial x \partial y} \cdot F(x,y)$$

Discrete:

1D - $X \rightarrow$ finite 0, 1, 2 ...

2D $X \rightarrow 0, 1, 2$

Problems for Discrete case:

1.) Gn the following distribution x and y

$X \backslash Y$	-1	0	1
0	$1/15$	$2/15$	$1/15$
1	$3/15$	$2/15$	$1/15$
2	$2/15$	$1/15$	$2/15$

i) Find the MD of x & y

ii) Find the CD of x gn $y=1$

i) Marginal Distribution of x:-

$P(X=0) = 1/15 + 2/15 + 1/15 = 4/15$

$P(X=1) = 3/15 + 2/15 + 1/15 = 6/15$

$P(X=2) = 2/15 + 1/15 + 2/15 = 5/15.$

X	0	1	2
P(X)	$4/15$	$6/15$	$5/15$

ii) Marginal Distribution of y:

$P(Y=-1) = 1/15 + 3/15 + 2/15 = 6/15$

$P(Y=0) = 2/15 + 2/15 + 1/15 = 5/15$

$P(Y=1) = 1/15 + 1/15 + 2/15 = 4/15$

Y	-1	0	1
P(Y)	$6/15$	$5/15$	$4/15$

ii) Conditional distribution X gn Y = 2.

$$P(X/Y=2) = \frac{P(X=0, Y=2)}{Y=2}$$

$$P(X=0/Y=2) = \frac{P(X=0, Y=2)}{P(Y=2)} = \frac{1/15}{4/15} = 1/4.$$

$$P(X=1/Y=2) = \frac{P(X=1, Y=2)}{P(Y=2)} = \frac{1/15}{4/15} = 1/4.$$

$$P(X=2/Y=2) = \frac{P(X=2, Y=2)}{P(Y=2)} = \frac{2/15}{4/15} = 1/2$$

2) If the join probability mass function of x, y is given by $P(x,y) = K(2x+3y)$, $x = 0, 1, 2$ $y = 1, 2, 3$.

i) Find the Marginal Distribution of x and y

ii) Find the conditional Distribution of y gn x

iii) Find the probability Distribution of $x+y$.

y \ x	0	1	2
1	3K	5K	7K
2	6K	8K	10K
3	9K	11K	13K
	18K	24K	30K

Since its a JPmf:

$$\sum_j \sum_i P(x_i, y_j) = 1$$

$$72K = 1$$

$$\boxed{K = 1/72}$$

18
24
30
72

i) MD of x.

$$P(x=0) = 3/72 + 6/72 + 9/72 = 18/72$$

$$P(x=1) = 5/72 + 8/72 + 11/72 = 24/72$$

$$P(X=2) = 7K + 10K + 13K = 30/72.$$

Marginal Distribution of Y.

$P(Y=1) = 15/72 \quad P(Y=2) = 24/72 \quad P(Y=3) = 33/72$

X	0	1	2
P(x)	18/72	24/72	30/72

Y	1	2	3
P(Y)	15/72	24/72	33/72

ii)

Conditional distribution :-

$P(Y=1/X=0) = \frac{3/72}{18/72} = 3/18 = 1/6 \qquad \frac{P(Y=1, X=0)}{P(X=0)}$

$P(Y=2/X=0) = \frac{6/72}{18/72} = 1/3$

$P(Y=3/X=0) = \frac{9/72}{18/72} = 1/2$

$P(Y=1/X=1) = \frac{5/72}{24/72} = 5/24$

$P(Y=2/X=1) = \frac{8/72}{24/72} = 1/3$

$P(Y=3/X=1) = \frac{11/72}{24/72} = 11/24$

$P(Y=1/X=2) = \frac{7/72}{30/72} = 7/30$

$P(Y=2/X=2) = 10/30 = 1/3$

$P(Y=3/X=2) = 13/30$

iii)

X+Y	1	2	3	4	5
P(X+Y)	$3/72$	$11/72$	$24/72$	$21/72$	$13/72$

③ The JPMF of x & Y is given by

Y \ X	0	1	2
0	0.10	0.04	0.02
1	0.08	0.2	0.06
2	0.06	0.14	0.3

i) Find $P(X \le 1)$
ii) $P(Y \le 1)$ iii) $P(X \le 1, Y \le 1)$
iv) $P(X \le 1 / Y \le 1)$
v) $P(Y \le 1 / X \le 1)$ vi) $P(X+Y \le 3)$

Marginal Distribution of X :-

X	0	1	2
P(X)	0.24	0.38	0.38

Marginal Distribution of Y :-

Y	0	1	2
P(Y)	0.16	0.34	0.5

i) $P(X \le 1) = P(X=0) + P(X=1) = 0.62$

ii) $P(Y \le 1) = P(Y=0) + P(Y=1) = 0.5$

iii) $P(X \le 1, Y \le 1) = P(X=0, Y=0) + P(X=1, Y=1) + P(X=0, Y=1) + P(X=1, Y=0)$

$= 0.42$

iv) $P(X \le 1 / Y \le 1) = \dfrac{P(X \le 1, Y \le 1)}{P(Y \le 1)} = \dfrac{0.42}{0.5}$

v) $P(Y \le 1 / X \le 1) = \dfrac{P(Y \le 1, X \le 1)}{P(X \le 1)} = \dfrac{0.42}{0.62}$

vi) $P(X+Y \le 3) = 0.7$

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4.) The JPDF of a random variable x, y is gn

$$f(x,y) = \begin{cases} \frac{x^3 y^3}{16}, & 0 < x < 2 \\ & 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

i) Find MD of x & y

ii) $f(Y/X)$ and $f(X/Y)$

iii) Are X & Y independent

i) Marginal Dist of x :-

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

$$= \int_0^2 \frac{x^3 y^3}{16}\, dy$$

$$= \frac{x^3}{16}\left[\frac{y^4}{4}\right]_0^2$$

$$= \frac{x^3}{4} \quad 0 < x < 2$$

Marginal Dist of y :-

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$$

$$= \int_0^2 \frac{x^3 y^3}{16}\, dx$$

$$= \frac{y^3}{16}\left[\frac{x^4}{4}\right]_0^2$$

$$= \frac{y^3}{4}, \quad 0 < y < 2$$

ii) $$f(Y/X) = \frac{f(x,y)}{f(x)} = \frac{x^3y^3/16}{x^3/4} = y^3/4$$

$$f(X/Y) = \frac{f(x,y)}{f(y)} = \frac{x^3y^3/16}{y^3/4} = x^3/4.$$

iii) $$f(x).f(y) = \frac{x^3}{4} \cdot \frac{y^3}{4} = \frac{x^3y^3}{16} = f(x,y)$$

$\therefore x$ & y are independent

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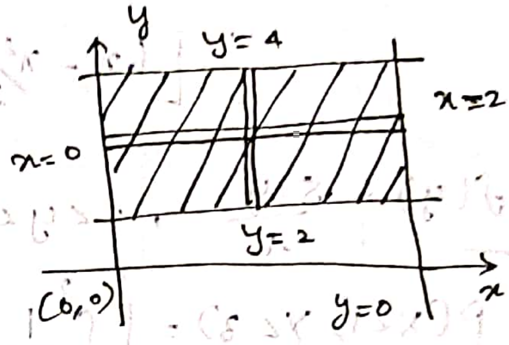
5.) The JPDF of x, Y is gn by, $f(x,y) = K(6-x-y)$, $0<x<2$, $2<y<4$. Find

i) K ii) MD of x, y iii) $P(X<1, Y<3)$ iv) $P(X<1 / Y<3)$

i) Since its JPDF



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

$$\int_{2}^{4}\int_{0}^{2} K(6-x-y)\,dx\,dy = 1$$

$$K\int_{2}^{4}\left[6x-\frac{x^2}{2}-yx\right]_0^2 dy = 1$$

$$K\int_{2}^{4}\left[12-\frac{4}{2}-2y\right]dy = 1 \Rightarrow K\int_{2}^{4}\left[10-2y\right]dy = 1$$

$$K\left[10y-\frac{2y^2}{2}\right]_2^4 = 1 \Rightarrow K\left[40-\frac{2\times\cancel{16}^{8}}{\cancel{2}}-20+\frac{2\times\cancel{4}^{2}}{\cancel{2}}\right] = 1$$

$$K[40-16-20+4] = 1 \Rightarrow \boxed{K = 1/8}$$

Marginal Distribution of x:

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{2}^{4}\frac{1}{8}(6-x-y)\,dy = \frac{1}{8}\left[6x-xy-y^2/_2\right]_2^4$$

$$= \frac{1}{8}\left[(24-4x-8)-(12-2x-2)\right] = \frac{1}{8}\left[6-2x\right] = \frac{3-x}{4}$$

$$f(x) = \frac{3-x}{4}, \quad 0<x<2$$

Marginal Distribution of Y.

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^2 \frac{1}{8}(6-x-y)\,dx$$

$$= \frac{1}{8}\left[6x - \frac{x^2}{2} - xy\right]_0^2 = \frac{10-2y}{8} = \frac{5-y}{4}$$

$$f(y) = \frac{5-y}{4}, \quad 2<y<4.$$

$$P(X<1, Y<3) = \int_0^1\int_2^3 \frac{1}{8}(6-x-y)\,dy\,dx$$

$$= \frac{1}{8}\int_0^1 \left(6y - xy - \frac{y^2}{2}\right)_2^3 dx = \frac{1}{8}\int_0^1 \left\{\left[18-3x-\frac{9}{2}\right]-\left[12-2x-2\right]\right\}$$

$$= \frac{1}{8}\int_0^1 \left(8-\frac{9}{2}-x\right)dx = \frac{1}{16}\int_0^1 (7-2x)\,dx = \frac{1}{16}\left[7x - \frac{2x^2}{2}\right]_0^1$$

$$= \frac{6}{16} = \frac{3}{8}$$

$$P(X<1 / Y<3) = \frac{P(X<1, Y<3)}{P(Y<3)} = \frac{3/8}{5/8} = \frac{3}{5}$$

If MP knows $\Downarrow$ If doesn't know MP $\Downarrow$

$$P(Y<3) = \int_2^3 \frac{5-y}{4}\,dy \quad (\text{or}) \quad P(Y<3) = \int_0^2\int_2^3 \frac{1}{8}(6-x-y)\,dy\,dx$$

$$= \frac{1}{4}\left[5y - \frac{y^2}{2}\right]_2^3 = \frac{1}{4}\left[15 - \frac{9}{2} - 10 + \frac{4}{2}\right]$$

$$= \frac{1}{4}\left[5 - \frac{5}{2}\right] = \frac{1}{4}\left[\frac{10-5}{2}\right] = \frac{5}{8}$$

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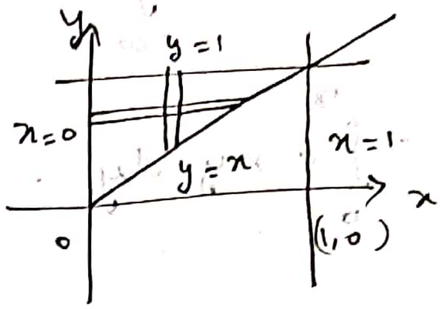
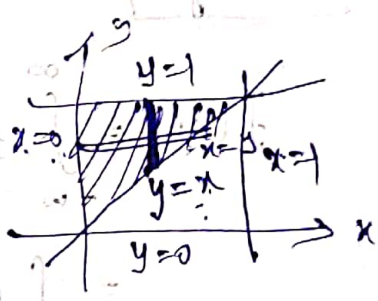
6) Let JPDF of x and y is given by $f(x,y) = 2$, $0 < x < y < 1$. Find the marginal & conditional distribution.



MD of x:

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_{x}^{1} 2\,dy$$



$$f(x) = [2y]_x^1 = 2 - 2x, \quad 0 < x < 1$$

$$f(x) = 2(1-x), \quad 0 < x < 1$$

MD of Y:

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

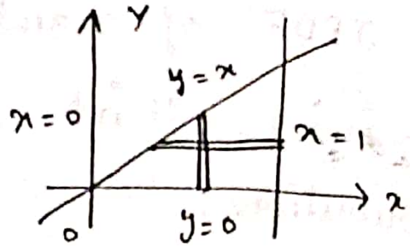
$$= \int_{0}^{y} 2\,dx = 2[x]_0^y = 2y, \quad 0 < y < 1.$$

$$f(Y/x) = \frac{f(x,y)}{f(x)} = \frac{2}{2(1-x)} = \frac{1}{1-x}$$

$$f(X/y) = \frac{f(x,y)}{f(y)} = \frac{2}{2y} = 1/y.$$

7) The JPDF of the random variables x, y is given by $f(x,y) = 8xy$, $0 < x < 1$, $0 < y < x$. Find $f_X(x)$, $f_Y(y)$, $f(Y/x)$, $f(X/y)$.

MD of x :



$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_0^x 8xy\,dy = 8x\left[\frac{y^2}{2}\right]_0^x = 4x^3, \quad 0<x<1$$

MD of y :

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$= \int_y^1 8xy\,dx$$

$$= 8y\left(\frac{x^2}{2}\right)_y^1$$

$$= 4y(1-y^2), \quad 0<y<1.$$

$$f(Y/x) = \frac{f(x,y)}{f(x)} = \frac{8xy}{4x^3} = \frac{2y}{x^2}$$

$$f(X/Y) = \frac{f(x,y)}{f(Y)} = \frac{8x\cancel{y}}{4\cancel{y}(1-y^2)} = \frac{2x}{1-y^2}$$

7.) The join PDF of x & y is $f(x,y) = Kxye^{-(x^2+y^2)}$ $x>0,\ y>0$. Prove that x and y are independent

$f(x,y) = kxye^{-(x^2+y^2)}$, $x>0, y>0$

Since $f(x,y)$ is a JPDF.

$$\Rightarrow \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = 1$$

$$\int_0^{\infty}\int_0^{\infty} kxye^{-x^2}e^{-y^2}\,dy\,dx = 1$$

$$k\int_0^{\infty} xe^{-x^2}dx \int_0^{\infty} ye^{-y^2}dy = 1$$

$u = x^2$ $\quad$ $v = y^2$

$du = 2x\,dx$ $\quad$ $dv = 2y\,dy$

$$k\int_0^{\infty}\frac{du}{2}e^{-u}\int_0^{\infty}\frac{dv}{2}e^{-v} = 1$$

$$\frac{k}{4}\left\{\left[\frac{e^{-u}}{-1}\right]_0^{\infty}\left[\frac{e^{-v}}{-1}\right]_0^{\infty}\right\} = 1$$

$$\boxed{k = 4}$$

MD of x :-

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0 4xye^{-x^2}e^{-y^2}dy$$

$$= 4xe^{-x^2}\int^{\infty} ye^{-y^2}dy$$

$$= 4xe^{-x^2}\left[\frac{e^{-u}}{-2}\right]_0^{\infty} = 2xe^{-x^2}$$

$f(x) = 2xe^{-x^2}$, $0<x<\infty$.

MD of Y:-

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{0}^{\infty} 4xy\,e^{-x^2} e^{-y^2}\,dx$$

$$= 4y\,e^{-y^2}\int_{0}^{\infty} x\,e^{-x^2}\,dx$$

$$= 4y\,e^{-y^2}\left[\tfrac{1}{2}\right] = 2y\,e^{-y^2}.$$

$$f(x)\cdot f(y) = 2y\,e^{-y^2}\cdot 2x\,e^{-x^2} = 4xy\,e^{-(x^2+y^2)}$$

Covariance:-

~~Let~~

If x and y are two random variables, then the covariance of x and y is defined by

$$\operatorname{cov}(X,Y) = E(XY) - E(X)E(Y).$$

If x and y are independent $E(XY) = E(X)E(Y)$ then $\operatorname{cov}(X,Y) = 0$.

Properties:-

$$\operatorname{cov}(aX, bY) = ab\operatorname{cov}(X,Y)$$

$$\operatorname{cov}(X+a, Y+b) = \operatorname{cov}(X,Y)$$

$$\operatorname{cov}(aX+b, cY+d) = ac\operatorname{cov}(X,Y).$$

$$\operatorname{Var}(X_1+X_2) = \operatorname{Var}(X_1) + \operatorname{Var}(X_2) + 2\operatorname{cov}(X_1,X_2)$$

Correlation:-

If change in one variable affects the change in other variable then the variables are set to be correlated.

+ve correlation :-

If the 2 variables deviated in the same direction then it is called as +ve correlation.

Example : Income & Expenditure

-ve correlation :-

If the 2 variables deviated in the opposite direction . then it is -ve correlation .

Example : Price & Demand of commodity

Correlation coefficient between x & y is given by

$$r = \rho = \frac{Cov(x, y)}{\sigma_x \sigma_y} = \frac{Cov(x, y)}{\sqrt{var(x)}\sqrt{var(y)}}$$

data.

Note : $-1 \leq r \leq 1$

$$r = \frac{\frac{1}{n}\Sigma xy - \bar{x}\bar{y}}{\sqrt{\frac{1}{n}\Sigma x^2 - (\bar{x})^2}\sqrt{\frac{1}{n}\Sigma y^2 - (\bar{y})^2}}$$

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Regression:

Regression is a mathematical measure of the average relationship between two or more variables in terms of the original limits of the data.

There are 2 lines of regression

1) Regression line y on x:

$$(y-\bar{y}) = b_{yx}(x-\bar{x})$$

where $b_{yx} = r\frac{\sigma y}{\sigma x}$ is called the regression coefficient of y on x.

$$b_{yx} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2}$$

2) Regression line x on y:

$(x-\bar{x}) = b_{xy}(y-\bar{y})$ where $b_{xy} = r\frac{\sigma x}{\sigma y}$ is called the regression coefficient of x on y.

$$b_{xy} = \frac{\Sigma(y-\bar{y})(x-\bar{x})}{\Sigma(y-\bar{y})^2}.$$

Note: 2 Regression lines always passes through $(\bar{x},\bar{y})$

Problem:

1) From the following data find

i) 2 regression lines (or) eqn

ii) Coefficient of correlation b/w x & y.

iii) The most likely marks in statistics when the marking economics are 30.

Marks in economics (X)	25	28	35	32	31	36	29	38	34	32
Marks in statistics (Y)	43	46	49	41	36	32	31	30	33	39

X	Y	$X-\bar{X}$	$Y-\bar{Y}$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y})$
25	43	-7	5	49	25	-35
28	46	-4	8	16	64	-32
35	49	3	11	9	121	33
32	41	0	3	0	9	0
31	36	-1	-2	1	4	2
36	32	4	-6	16	36	-24
29	31	-3	-7	9	49	-21
38	30	6	-8	36	64	-48
34	33	2	-5	4	25	-10
32	39	0	1	0	1	0
320	380			$\Sigma = 140$	$\Sigma = 398$	$\Sigma(X-\bar{X})(Y-\bar{Y}) = -93$

$\bar{X} = \frac{\Sigma X}{n} = \frac{320}{10} = 32$ $\quad \bar{Y} = \frac{\Sigma Y}{n} = \frac{380}{10} = 38$

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Regression line y on x.:

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$b_{yx} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(x-\bar{x})^2} = \frac{-93}{140} = -0.664$$

$$y - 38 = -0.664(x - 32)$$

$$y - 38 = -0.664x + 21.24.$$

$$\boxed{y = -0.664x + 59.24.}$$

Regression line x on y.:

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$b_{xy} = \frac{\Sigma(x-\bar{x})(y-\bar{y})}{\Sigma(y-\bar{y})^2} = \frac{-93}{398} = -0.234.$$

$$x - 32 = -0.234(y - 38)$$

$$\boxed{x = -0.234y + 40.892.}$$

ii) $r = \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{(-0.664)(-0.234)}$

$$\boxed{r = 0.3941}$$

iii) $x = 30$, $y = ?$

$y = -0.664 \times 30 + 59.21$

$y = 39.32$

$\boxed{y \simeq 39}$

2) The 2 regression lines are given by $4x - 5y + 33 = 0$ and $20x - 9y = 107$. Find the means of x & y. Find the correlation coefficient b/w x & y.

$4x - 5y + 33 = 0 \rightarrow (1)$
$20x - 9y = 107 \rightarrow (2)$

$(1) \times 5 \Rightarrow 20x - 25y = -165$
$(2) \Rightarrow 20x - 9y = 107$
$\quad (-) \quad (+) \quad (-)$

$-16y = -272$

$\boxed{y = 17}$

$4x - 85 = -33$

$4x = +52 \Rightarrow \boxed{x = 13}$

$4x - 5y = -33$

$5y = 4x + 33$

$y = \frac{4}{5}x + \frac{33}{5}$

$b_{yx} = \frac{4}{5}$

$\cancel{4x = -33 - 5y}$
$x = -\frac{5}{4}y$

$20x = 9y + 107$

$x = \frac{9}{20}y + \frac{107}{20}$

$b_{xy} = \frac{9}{20}$

$r = \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{\frac{4}{5} \times \frac{9}{20}} = \sqrt{9/25} = 3/5$

3) Find the regression equations b/w industrial production and export using the following data

Production :	55	56	58	59	60	60	62
Export :	35	38	37	39	44	43	44

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Find the correlation coefficient, also find the production export is 40.

X	Y	$X-\bar{X}$	$Y-\bar{Y}$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y})$
55	35	-3.57	-5	12.74	25	17.85
56	38	-2.57	-2	6.60	4	5.14
58	37	-0.57	-3	6.32	9	1.71
59	39	0.43	-1	0.18	1	-0.43
60	44	1.43	4	2.04	16	5.72
60	43	1.43	3	2.04	9	4.29
62	44	3.43	4	11.76	16	13.72
410	280	3.01		35.68	80	48.0

$$\bar{X} = \frac{410}{7} = 58.57$$

$$\bar{Y} = \frac{280}{7} = 40$$

Regression line y on x.

$$Y-\bar{Y} = b_{yx}(X-\bar{X})$$

$$b_{yx} = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\Sigma(X-\bar{X})^2} = \frac{48}{35.68} = 1.345$$

$$Y-40 = 1.345(X-58.57)$$

$$Y-40 = 1.345X - 78.77 \Rightarrow \boxed{Y = 1.345X - 38.77}$$

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Regression line X on Y :-

$$X - \bar{X} = b_{XY}(Y - \bar{Y})$$

$$b_{XY} = \frac{\Sigma(X-\bar{X})(Y-\bar{Y})}{\Sigma(Y-\bar{Y})^2} = \frac{48}{80} = 0.6$$

$$X - 58.57 = 0.6(Y - 40)$$

$$X - 58.57 = 0.6Y - 24$$

$$X = 0.6Y + 34.57$$

ii) $r = \sqrt{b_{xy} \cdot b_{yx}} = \sqrt{0.6 \times 1.345} = 0.8980.$

iii) when $Y = 40$, $X = ?$

$$X = 0.6(40) + 34.57.$$

$$= 24 + 34.57$$

$$= 58.57 \simeq 59.$$

Transformation of a 2D random variable :-

Consider the problem of transforming a 2D random variable X, Y into another random variable U, V

Transformation is defined by $f(u,v) = f(x,y)|J|$ where J denotes the Jacobian of the random variables

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

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1.) If the JPDF of x, y is given by $f(x,y) = x+y$

$0 \le x \le 1,\ 0 \le y \le 1$. Find the PDF of $u = x+y$.

Let $u = x+y$, $v = y$

$f(u,v) = f(x,y)\,|J|$

$$J = \frac{\partial(x,y)}{\partial(u,v)} \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$u = x+v$, $x = u-v$, $y = v$

$$J = \begin{vmatrix} 1 & -1 \\ 0 & 1 \end{vmatrix} = 1$$

$f(u,v) = (x+y)1 = u\cancel{-v}\cancel{+v} = u$. $\quad 0 \le x \le 1$

$0 \le u-v \le 1$

$0 \le y \le 1$

$v \le u \le v+1$

$0 \le v \le 1$

$$\boxed{f(u,v) = u,\quad v \le u \le v+1,\quad 0 \le v \le 1}$$

2.) If the JPDF of x, y is gn by $f(x,y) = e^{-(x+y)}$,

$0 \le x \le 1,\ 0 \le y \le 1$, Find the PDF of $u = x/y$.

Let $u = x/y$, $v = y$. $\quad u = x/v \Rightarrow x = uv,\ y = v$

$$J = \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} v & u \\ 0 & 1 \end{vmatrix} = v - 0 = v$$

$f(u,v) = e^{-(uv+v)}\, v$

$f(u,v) = v\, e^{-(1+u)}$ $\rightarrow 0 \le w \le 1$

$0 \le u \le 1/v$

$0 \le v \le 1$

Unit - 4.

Testing of Hypothesis :

⇒ Introduction

→ Population and Samples.

⇒ Large Samples ⇒ Z - Test for single proportion

⇒ Z - test for difference of proportion

⇒ Z - test for single mean

⇒ Z - test for difference of means.

⇒ Small Samples ⇒ t - test

⇒ F - test

⇒ chi - square test

Population:

The totality of any finite ~~edge~~ or infinite collection of individuals with which we are consirned is called a population.

Sample:

A sample is a finite subset of a population number of elements in a sample is called size of the sample. In general, the samples classified ... two types. 1.) Large samples $(n \ge 30)$