

RANDOM VARIABLE:

A random variable is a function that assigns a real number $X(s)$ to every element in the sample space $s \in S$ corresponding to a random experiment E .

Types of random variables:

- * discrete random variables

- * continuous random variables.

Discrete random variables:

A random variable X is discrete if it assumes only discrete values.

Example: No. of students inside a class.

Continuous random variables:

If X is a random variable which can take all possible values between certain limits/in an interval, which may be finite or infinite then X is a continuous random variable.

$$\text{Eg: } f(x) = \{x^2 - 1 \leq x \leq 5\}$$

PROBABILITY MASS FUNCTION (PMF):

If X is a discrete random variable which can take the values x_1, x_2, x_3, \dots such that,

$$P(X = x_i) = p_i$$

then P_i is called probability mass function
(or) probability function and it satisfies the
condition.

(i) $P_i \geq 0$ for all i

(ii) $\sum P_i = 1$

PROBABILITY DENSITY FUNCTION (PDF):

If X is a continuous random variable such that

$$P\left(X - \frac{1}{2}dx \leq X \leq X + \frac{1}{2}dx\right) = f(x) dx.$$

then $f(x)$ is said to be probability density function and it also obeys

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x) dx = 1$

CUMMULATIVE DISTRIBUTION FUNCTION (CDF):

If X is a random variable discrete or continuous then $P(X \leq x)$ is called

cumulative distribution function. It is represented by $F(x)$.

If $F(x) = \sum P_i$, $x_i \in x$ Then x is discrete.

Id est if $F(x) = \int_{-\infty}^x f(x) dx$, then x is continuous.

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$$(8x^0)^9 + \dots + (8x^8)^9$$

1. A discrete random variable x has the following probability distribution.

x	0	1	2	3	4	5	6	7	8
$P(X=x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

(x)7 of question 2b (iii)

i) Find the value of a .

ii) Find $P(X \leq 3)$, $P(0 \leq X \leq 3)$, $P(X \geq 3)$

iii) Find the distribution function $F(x)$.

$$P = (6=x)^9 + (7=x)^9 + (8=x)^9 = (6)^9$$

if we know that,

$$\sum_{i=0}^{18} P(X_i) = 1 \Rightarrow (6=x)^9 + (7=x)^9 + (8=x)^9 = (8)^9$$

$$a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$$

$$81a = 1$$

$$(6=x)^9 + (7=x)^9 + (8=x)^9 = (8)^9$$

$$i) P(X \leq 3) = P(X=0) + P(X=1) + P(X=2)$$

$$= a + 3a + 5a$$

$$= 9a$$

$$\frac{P}{18} = (d=x)^9 +$$

$$(6=x)^9 + (7=x)^9 + (8=x)^9 + (9=x)^9 + (10=x)^9 = (9)^9$$

$$= \frac{1}{9}$$

$$\frac{P}{18} = (d=x)^9 + (e=x)^9 +$$

$$18 = (8)^9$$

$$P(0 \leq X \leq 3) = P(X=1) + P(X=2)$$

$$= 3a + 5a$$

$$1 \quad \frac{1}{18} \quad \frac{3}{18} \quad \frac{5}{18} \quad \frac{7}{18} \quad \frac{9}{18} \quad \frac{11}{18} \quad \frac{13}{18} \quad \frac{15}{18} \quad \frac{17}{18}$$

$$= \frac{8}{18}$$

$$P(X \geq 3) = 1 - P(X \leq 2)$$

and $X = 1 - [P(X=0) + P(X=1) + P(X=2)]$

$$= 1 - \frac{1}{9}$$

$x = 0, 1, 2, 3, 4, 5, 6, 7, 8$
 $P(X=x) = \frac{1}{81}, \frac{4}{81}, \frac{9}{81}, \frac{18}{81}, \frac{25}{81}, \frac{36}{81}, \frac{49}{81}, \frac{64}{81}, 1$

iii) distribution fn $F(x)$

$$F(0) = P(X=0) = \frac{1}{81}$$

$$F(1) = P(X=0) + P(X=1) = \frac{5}{81}$$

$$F(2) = P(X=0) + P(X=1) + P(X=2) = \frac{9}{81}$$

$$F(3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = \frac{16}{81}$$

$$F(4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = \frac{25}{81}$$

$$F(5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) = \frac{36}{81}$$

$$F(6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) = \frac{49}{81}$$

$$F(7) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) = \frac{64}{81}$$

$$F(8) = \frac{81}{81} = 1$$

x	0	1	2	3	4	5	6	7	8
$F(x)$	$\frac{1}{81}$	$\frac{4}{81}$	$\frac{9}{81}$	$\frac{18}{81}$	$\frac{25}{81}$	$\frac{36}{81}$	$\frac{49}{81}$	$\frac{64}{81}$	1

$$\frac{8}{18}$$

If x is a discrete random variable
then mean $E(x) = \sum x_i P(x_i)$

$$E(x^2) = \sum x_i^2 P(x_i)$$

$$\text{Var}(x) = E[x^2] - [E[x]]^2$$

- Q. Determine the constant K given the following probability distribution of discrete random variable x
also find mean and variance of x .

$x = x_i$	1	2	3	4	5
$P(x=x_i)$	0.1	0.2	K	$2K$	0.1

to find K : $0.1 + 0.2 + K + 2K + 0.1 = 1$

$$0.1 + 0.2 + K + 2K + 0.1 = 1$$

$$0.4 + 3K = 1$$

$$3K = 0.6$$

$$K = 0.2$$

$$\text{Mean} = \sum x_i P(x_i)$$

$$= 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.2 + 4 \times 0.4 + 5 \times 0.1$$

$$= 3.2$$

$$E(x^2) = \sum x_i^2 P(x_i)$$

$$= 1^2 \times 0.1 + 2^2 \times 0.2 + 3^2 \times 0.2 + 4^2 \times 0.4 + 5^2 \times 0.1$$

$$= 11.6$$

$$\text{standard deviation } (\sigma) = \sqrt{E(X^2)} - [E(X)]^2$$

$$= (11.6) - (3.2)^2$$

$$(11.6) - 10.24 = 1.36$$

$$\sigma = \sqrt{1.36} = 1.16$$

CONDITIONAL PROBABILITY

What does $P(A|B) = \frac{P(AB)}{P(B)}$ mean?

- 2) A random variable X has the following

Probability fn $\begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ P(X) & 0 & k & 2k & 2k & 3k & k^2 & 2k^2 & 7k^2 + k \end{matrix}$

x	0	1	2	3	4	5	6	7	$(x^2 - \bar{x})^2$
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2 + k$	

- Find k
- $P[X \leq 6], P[X \geq 6]$
- If $P[X \leq c] \geq \frac{1}{2}$ Find minimum value of c .
- Evaluate $P[X \leq 2], P[X \geq 3], P[1 < X \leq 5]$
- $P[1.5 < X \leq 4.5 / X \geq 2]$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$9k + 10k^2 = 1$$

$$10k^2 + 9k - 1 = 0$$

$$\boxed{k = \frac{1}{10}, k = -1}$$

$$1.0 \times 0.0 +$$

$$0.11 =$$

$$\text{iii) } P[X \geq 6] = P[X=0] + P[X=1] + P[X=2] + P[X=3]$$

$$+ P[X=4] + P[X=5]$$

$$= 0 + K + 2K + 2K + 3K + K^2$$

$$= 8K + K^2$$

$$= \frac{8}{10} + \frac{1}{100}$$

$$= \frac{80+1}{100}$$

$$= \frac{81}{100}$$

$$P[X \geq 6] = 1 - P[X < 6]$$

$$= 1 - \frac{81}{100}$$

$$= \frac{100 - 81}{100}$$

$$= \frac{19}{100}$$

$$\text{iv) } P[X < 2] = P[X=0] + P[X=1]$$

$$= 0 + K$$

$$= \frac{1}{10}$$

$$P[X \geq 3] = P[X=4] + P[X=5] + P[X=6] + P[X=7]$$

$$= 3K + K^2 + 2K^2 + 7K^2 + K$$

$$= 10K^2 + 4K$$

$$= 10\left(\frac{1}{10}\right)^2 + \frac{4}{10}$$

$$= \frac{10}{100} + \frac{4}{10}$$

$$= \frac{10+40}{100} = \frac{50}{100} = \frac{1}{2}$$

$$\begin{aligned}
 P[1 < X \leq 5] &= P[X=2] + P[X=3] + P[X=4] \\
 &= 2K + 2K + 3K \\
 &= 7K \\
 &= \frac{7}{10} \\
 &= \frac{7}{10} + \frac{8}{10} = \frac{15}{10} = \frac{3}{2}
 \end{aligned}$$

iii) $P[X \leq c] \geq \frac{1}{2}$

$$P(X=0) = 0$$

$$P(X \leq 1) = 0 + K = \frac{1}{10}$$

$$P(X \leq 2) = 3K = \frac{3}{10} \Rightarrow 1 = [0 \leq X] 9$$

$$P(X \leq 3) = 5K = \frac{5}{10} = \frac{1}{2}$$

$$P(X \leq 4) = 8K \geq \frac{1}{2}$$

$$\therefore \boxed{c = 4}$$

iv) $P[1.5 < X < 4.5 / X > 2]$

We know that,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Here,

$$\begin{aligned}
 P[1.5 < X < 4.5] + P[2 = X] 9 + P[3 = X] 9 + P[4 = X] 9 &= [2 < X] 9 \\
 \text{Shaded Area} &= 3K + 4K + 5K = 12K = \frac{12}{10} = \frac{6}{5}
 \end{aligned}$$

$$P[1.5 < X < 4.5 / X > 2] = \frac{P[1.5 < X < 4.5] \cap P[X > 2]}{P[X > 2]}$$

$$\begin{aligned}
 &= \frac{P(X=3) + P(X=4)}{1 - P(X \leq 2)} \\
 &= \frac{\frac{5}{10} + \frac{8}{10}}{1 - \frac{15}{10}} = \frac{\frac{13}{10}}{\frac{-5}{10}} = \frac{13}{5}
 \end{aligned}$$

$$= 1 - [P(X=0) + P(X=1) + P(X=2)]$$

$$= \frac{5K}{1-3K} = \frac{\frac{5}{10}}{1-\frac{3}{10}} = \frac{\frac{5}{10}}{\frac{10-3}{10}} = \frac{\frac{5}{10}}{\frac{7}{10}} = \frac{5}{7} = \frac{5}{10} \times \frac{10}{7}$$

$$= \frac{5}{7}$$

a) The probability function of an infinite discrete distribution is given by; $P[X=x_j] = \frac{1}{2^j}, j=1, 2, \dots$

Find mean and $P(X \text{ is even})$, $P(X \geq 5)$

$P[X \text{ divisible by 3}]$.

Soln:

$$\text{Mean} = \sum_{j=1}^{\infty} x_j p_j$$

$$= 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{2^2} + 3 \cdot \frac{1}{2^3} + \dots$$

$$= \frac{1}{2} [1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + \dots]$$

$$\therefore (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$\text{Mean} = \frac{1}{2} [1 - (1/2)]^{-2} \Rightarrow \frac{1}{2} \left[\frac{2-1}{2}\right]^{-2} \Rightarrow \frac{1}{2} \left(\frac{1}{2}\right)^{-2}$$

$$= \frac{1}{2} \left(\frac{1}{2}\right)^{-2} = \frac{1}{2} \cdot \frac{1}{(1/2)^2}$$

$$= \frac{1}{2} \cdot \frac{1}{(1/4)} = \frac{1}{2} \cdot 4 = 2$$

$$= \frac{1}{2} \cdot 4$$

$$\text{mean} = 2.$$

$$P[X = \text{even}] = P(X=2) + P(X=4) + P(X=6) + \dots$$

$$= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \dots$$

$$= 2^6 = (2^2)^3$$

$$= \frac{1}{2^2} \left[1 + \frac{1}{2^2} + \frac{1}{2^4} + \dots \right]$$

$$= 2^2 \left[\frac{1}{2^2} \right]$$

$$= \frac{1}{4} \left[1 - \left(\frac{1}{2}\right)^{-1} \right]^{-1} \Rightarrow \frac{1}{4} \left[1 - \frac{1}{4} \right]^{-1}$$

$$= \frac{1}{4} \left[\frac{3}{4} \right]^{-1}$$

$$= \left(\frac{1}{4}\right)^{-1} = \frac{1}{4} \left(\frac{3}{4}\right)^{-1}$$

$$= \frac{1}{2},$$

$$\boxed{P[X = \text{even}] = \frac{1}{3}}$$

$$P(X \geq 5) = P(X=5) + P(X=6) + P(X=7) + \dots$$

$$= \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \dots$$

$$= \frac{1}{2^5} + \frac{1}{2^5 \cdot 2^1} + \frac{1}{2^5 \cdot 2^2} + \dots$$

$$= \frac{1}{2^5} \left[1 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \dots \right]$$

$$= \frac{1}{2^5} \left[1 - \left(\frac{1}{2}\right)^{(n-1)} \right]^{-1} \Rightarrow \left(\frac{1}{2}\right)^{-1} \times \frac{1}{2^5}$$

$$= \frac{1}{2^5} \left(\frac{1}{2}\right)^{-1} \times \left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{-1} \times \frac{1}{2^5}$$

$$= \frac{1}{2^4} \times \frac{1}{2^5} = 2^2 \times \frac{1}{2^{14}}$$

$$= \frac{1}{2^9}$$

$$\begin{aligned}
 P[X \text{ divisible by } 3] &= P[X=3] + P[X=6] + \dots \\
 &= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^9 + \dots \\
 &= \frac{1}{2^3} + \left(\frac{1}{2^6}\right) + \frac{1}{2^9} + \dots \\
 &= \frac{1}{2^3} \left[1 + \frac{1}{2^3} + \frac{1}{2^6} + \dots \right] \\
 &= \frac{1}{2^3} \left[1 + \frac{1}{8} \right] = \frac{1}{2^3} \cdot \frac{9}{8} \\
 &= \frac{1}{2^3} \left[\frac{9}{8} \right] = \frac{9}{8} \cdot \frac{1}{2^3} \\
 &= \frac{9}{8} \cdot \frac{1}{8} = \frac{9}{64} \\
 &= \frac{9}{64} \cdot \frac{1}{2^3} = \frac{9}{128} \\
 &= \frac{9}{128} \cdot \frac{1}{2^3} = \frac{9}{256} \\
 &= \frac{9}{256} \cdot \frac{1}{2^3} = \frac{9}{512} \\
 &= \frac{9}{512} \cdot \frac{1}{2^3} = \frac{9}{1024} \\
 &= \frac{9}{1024} \cdot \frac{1}{2^3} = \frac{9}{2048} \\
 &= \frac{9}{2048} \cdot \frac{1}{2^3} = \frac{9}{4096} \\
 &= \frac{9}{4096} \cdot \frac{1}{2^3} = \frac{9}{8192} \\
 &= \frac{9}{8192} \cdot \frac{1}{2^3} = \frac{9}{16384} \\
 &= \frac{9}{16384} \cdot \frac{1}{2^3} = \frac{9}{32768} \\
 &= \frac{9}{32768} \cdot \frac{1}{2^3} = \frac{9}{65536} \\
 &= \frac{9}{65536} \cdot \frac{1}{2^3} = \frac{9}{131072} \\
 &= \frac{9}{131072} \cdot \frac{1}{2^3} = \frac{9}{262144} \\
 &= \frac{9}{262144} \cdot \frac{1}{2^3} = \frac{9}{524288} \\
 &= \frac{9}{524288} \cdot \frac{1}{2^3} = \frac{9}{1048576} \\
 &= \frac{9}{1048576} \cdot \frac{1}{2^3} = \frac{9}{2097152} \\
 &= \frac{9}{2097152} \cdot \frac{1}{2^3} = \frac{9}{4194304} \\
 &= \frac{9}{4194304} \cdot \frac{1}{2^3} = \frac{9}{8388608} \\
 &= \frac{9}{8388608} \cdot \frac{1}{2^3} = \frac{9}{16777216} \\
 &= \frac{9}{16777216} \cdot \frac{1}{2^3} = \frac{9}{33554432} \\
 &= \frac{9}{33554432} \cdot \frac{1}{2^3} = \frac{9}{67108864} \\
 &= \frac{9}{67108864} \cdot \frac{1}{2^3} = \frac{9}{134217728} \\
 &= \frac{9}{134217728} \cdot \frac{1}{2^3} = \frac{9}{268435456} \\
 &= \frac{9}{268435456} \cdot \frac{1}{2^3} = \frac{9}{536870912} \\
 &= \frac{9}{536870912} \cdot \frac{1}{2^3} = \frac{9}{1073741824} \\
 &= \frac{9}{1073741824} \cdot \frac{1}{2^3} = \frac{9}{2147483648} \\
 &= \frac{9}{2147483648} \cdot \frac{1}{2^3} = \frac{9}{4294967296} \\
 &= \frac{9}{4294967296} \cdot \frac{1}{2^3} = \frac{9}{8589934592} \\
 &= \frac{9}{8589934592} \cdot \frac{1}{2^3} = \frac{9}{17179869184} \\
 &= \frac{9}{17179869184} \cdot \frac{1}{2^3} = \frac{9}{34359738368} \\
 &= \frac{9}{34359738368} \cdot \frac{1}{2^3} = \frac{9}{68719476736} \\
 &= \frac{9}{68719476736} \cdot \frac{1}{2^3} = \frac{9}{137438953472} \\
 &= \frac{9}{137438953472} \cdot \frac{1}{2^3} = \frac{9}{274877906944} \\
 &= \frac{9}{274877906944} \cdot \frac{1}{2^3} = \frac{9}{549755813888} \\
 &= \frac{9}{549755813888} \cdot \frac{1}{2^3} = \frac{9}{1099511627776} \\
 &= \frac{9}{1099511627776} \cdot \frac{1}{2^3} = \frac{9}{2199023255552} \\
 &= \frac{9}{2199023255552} \cdot \frac{1}{2^3} = \frac{9}{4398046511104} \\
 &= \frac{9}{4398046511104} \cdot \frac{1}{2^3} = \frac{9}{8796093022208} \\
 &= \frac{9}{8796093022208} \cdot \frac{1}{2^3} = \frac{9}{17592186044416} \\
 &= \frac{9}{17592186044416} \cdot \frac{1}{2^3} = \frac{9}{35184372088832} \\
 &= \frac{9}{35184372088832} \cdot \frac{1}{2^3} = \frac{9}{70368744177664} \\
 &= \frac{9}{70368744177664} \cdot \frac{1}{2^3} = \frac{9}{140737488355328} \\
 &= \frac{9}{140737488355328} \cdot \frac{1}{2^3} = \frac{9}{281474976710656} \\
 &= \frac{9}{281474976710656} \cdot \frac{1}{2^3} = \frac{9}{562949953421312} \\
 &= \frac{9}{562949953421312} \cdot \frac{1}{2^3} = \frac{9}{1125899906842624} \\
 &= \frac{9}{1125899906842624} \cdot \frac{1}{2^3} = \frac{9}{2251799813685248} \\
 &= \frac{9}{2251799813685248} \cdot \frac{1}{2^3} = \frac{9}{4503599627370496} \\
 &= \frac{9}{4503599627370496} \cdot \frac{1}{2^3} = \frac{9}{9007199254740992} \\
 &= \frac{9}{9007199254740992} \cdot \frac{1}{2^3} = \frac{9}{18014398509481984} \\
 &= \frac{9}{18014398509481984} \cdot \frac{1}{2^3} = \frac{9}{36028797018963968} \\
 &= \frac{9}{36028797018963968} \cdot \frac{1}{2^3} = \frac{9}{72057594037927936} \\
 &= \frac{9}{72057594037927936} \cdot \frac{1}{2^3} = \frac{9}{144115188075855872} \\
 &= \frac{9}{144115188075855872} \cdot \frac{1}{2^3} = \frac{9}{288230376151711744} \\
 &= \frac{9}{288230376151711744} \cdot \frac{1}{2^3} = \frac{9}{576460752303423488} \\
 &= \frac{9}{576460752303423488} \cdot \frac{1}{2^3} = \frac{9}{1152921504606846976} \\
 &= \frac{9}{1152921504606846976} \cdot \frac{1}{2^3} = \frac{9}{2305843009213693952} \\
 &= \frac{9}{2305843009213693952} \cdot \frac{1}{2^3} = \frac{9}{4611686018427387904} \\
 &= \frac{9}{4611686018427387904} \cdot \frac{1}{2^3} = \frac{9}{9223372036854775808} \\
 &= \frac{9}{9223372036854775808} \cdot \frac{1}{2^3} = \frac{9}{18446744073709551616} \\
 &= \frac{9}{18446744073709551616} \cdot \frac{1}{2^3} = \frac{9}{36893488147419103232} \\
 &= \frac{9}{36893488147419103232} \cdot \frac{1}{2^3} = \frac{9}{73786976294838206464} \\
 &= \frac{9}{73786976294838206464} \cdot \frac{1}{2^3} = \frac{9}{147573952589676412928} \\
 &= \frac{9}{147573952589676412928} \cdot \frac{1}{2^3} = \frac{9}{295147905179352825856} \\
 &= \frac{9}{295147905179352825856} \cdot \frac{1}{2^3} = \frac{9}{590295810358705651712} \\
 &= \frac{9}{590295810358705651712} \cdot \frac{1}{2^3} = \frac{9}{1180591620717411303424} \\
 &= \frac{9}{1180591620717411303424} \cdot \frac{1}{2^3} = \frac{9}{2361183241434822606848} \\
 &= \frac{9}{2361183241434822606848} \cdot \frac{1}{2^3} = \frac{9}{4722366482869645213696} \\
 &= \frac{9}{4722366482869645213696} \cdot \frac{1}{2^3} = \frac{9}{9444732965739290427392} \\
 &= \frac{9}{9444732965739290427392} \cdot \frac{1}{2^3} = \frac{9}{18889465931478580854784} \\
 &= \frac{9}{18889465931478580854784} \cdot \frac{1}{2^3} = \frac{9}{37778931862957161709568} \\
 &= \frac{9}{37778931862957161709568} \cdot \frac{1}{2^3} = \frac{9}{75557863725914323419136} \\
 &= \frac{9}{75557863725914323419136} \cdot \frac{1}{2^3} = \frac{9}{151115727451828646838272} \\
 &= \frac{9}{151115727451828646838272} \cdot \frac{1}{2^3} = \frac{9}{302231454903657293676544} \\
 &= \frac{9}{302231454903657293676544} \cdot \frac{1}{2^3} = \frac{9}{604462909807314587353088} \\
 &= \frac{9}{604462909807314587353088} \cdot \frac{1}{2^3} = \frac{9}{1208925819614629174706176} \\
 &= \frac{9}{1208925819614629174706176} \cdot \frac{1}{2^3} = \frac{9}{2417851639229258349412352} \\
 &= \frac{9}{2417851639229258349412352} \cdot \frac{1}{2^3} = \frac{9}{4835703278458516698824704} \\
 &= \frac{9}{4835703278458516698824704} \cdot \frac{1}{2^3} = \frac{9}{9671406556917033397649408} \\
 &= \frac{9}{9671406556917033397649408} \cdot \frac{1}{2^3} = \frac{9}{19342813113834066795298816} \\
 &= \frac{9}{19342813113834066795298816} \cdot \frac{1}{2^3} = \frac{9}{38685626227668133590597632} \\
 &= \frac{9}{38685626227668133590597632} \cdot \frac{1}{2^3} = \frac{9}{77371252455336267181195264} \\
 &= \frac{9}{77371252455336267181195264} \cdot \frac{1}{2^3} = \frac{9}{154742504910672534362390528} \\
 &= \frac{9}{154742504910672534362390528} \cdot \frac{1}{2^3} = \frac{9}{309485009821345068724781056} \\
 &= \frac{9}{309485009821345068724781056} \cdot \frac{1}{2^3} = \frac{9}{618970019642690137449562112} \\
 &= \frac{9}{618970019642690137449562112} \cdot \frac{1}{2^3} = \frac{9}{1237940039285380274899124224} \\
 &= \frac{9}{1237940039285380274899124224} \cdot \frac{1}{2^3} = \frac{9}{2475880078570760549798248448} \\
 &= \frac{9}{2475880078570760549798248448} \cdot \frac{1}{2^3} = \frac{9}{4951760157141521099596496896} \\
 &= \frac{9}{4951760157141521099596496896} \cdot \frac{1}{2^3} = \frac{9}{9903520314283042199192993792} \\
 &= \frac{9}{9903520314283042199192993792} \cdot \frac{1}{2^3} = \frac{9}{19807040628566084398385987584} \\
 &= \frac{9}{19807040628566084398385987584} \cdot \frac{1}{2^3} = \frac{9}{39614081257132168796771975168} \\
 &= \frac{9}{39614081257132168796771975168} \cdot \frac{1}{2^3} = \frac{9}{79228162514264337593543950336} \\
 &= \frac{9}{79228162514264337593543950336} \cdot \frac{1}{2^3} = \frac{9}{158456325028528675187087900672} \\
 &= \frac{9}{158456325028528675187087900672} \cdot \frac{1}{2^3} = \frac{9}{316912650057057350374175801344} \\
 &= \frac{9}{316912650057057350374175801344} \cdot \frac{1}{2^3} = \frac{9}{633825300114114700748351602688} \\
 &= \frac{9}{633825300114114700748351602688} \cdot \frac{1}{2^3} = \frac{9}{1267650600228229401496703205376} \\
 &= \frac{9}{1267650600228229401496703205376} \cdot \frac{1}{2^3} = \frac{9}{2535301200456458802993406410752} \\
 &= \frac{9}{2535301200456458802993406410752} \cdot \frac{1}{2^3} = \frac{9}{5070602400912917605986812821504} \\
 &= \frac{9}{5070602400912917605986812821504} \cdot \frac{1}{2^3} = \frac{9}{10141204801825835211973625643008} \\
 &= \frac{9}{10141204801825835211973625643008} \cdot \frac{1}{2^3} = \frac{9}{20282409603651670423947251286016} \\
 &= \frac{9}{20282409603651670423947251286016} \cdot \frac{1}{2^3} = \frac{9}{40564819207303340847894502572032} \\
 &= \frac{9}{40564819207303340847894502572032} \cdot \frac{1}{2^3} = \frac{9}{81129638414606681695789005144064} \\
 &= \frac{9}{81129638414606681695789005144064} \cdot \frac{1}{2^3} = \frac{9}{162259276829213363391578010288128} \\
 &= \frac{9}{162259276829213363391578010288128} \cdot \frac{1}{2^3} = \frac{9}{324518553658426726783156020576256} \\
 &= \frac{9}{324518553658426726783156020576256} \cdot \frac{1}{2^3} = \frac{9}{649037107316853453566312041152512} \\
 &= \frac{9}{649037107316853453566312041152512} \cdot \frac{1}{2^3} = \frac{9}{1298074214633706907132624082305024} \\
 &= \frac{9}{1298074214633706907132624082305024} \cdot \frac{1}{2^3} = \frac{9}{2596148429267413814265248164610048} \\
 &= \frac{9}{2596148429267413814265248164610048} \cdot \frac{1}{2^3} = \frac{9}{5192296858534827628530496329220096} \\
 &= \frac{9}{5192296858534827628530496329220096} \cdot \frac{1}{2^3} = \frac{9}{10384593717069655257060992658440192} \\
 &= \frac{9}{10384593717069655257060992658440192} \cdot \frac{1}{2^3} = \frac{9}{20769187434139310514121985316880384} \\
 &= \frac{9}{20769187434139310514121985316880384} \cdot \frac{1}{2^3} = \frac{9}{41538374868278621028243970633760768} \\
 &= \frac{9}{41538374868278621028243970633760768} \cdot \frac{1}{2^3} = \frac{9}{83076749736557242056487941267521536} \\
 &= \frac{9}{83076749736557242056487941267521536} \cdot \frac{1}{2^3} = \frac{9}{166153499473114484112958882535043072} \\
 &= \frac{9}{166153499473114484112958882535043072} \cdot \frac{1}{2^3} = \frac{9}{332306998946228968225917765070086144} \\
 &= \frac{9}{332306998946228968225917765070086144} \cdot \frac{1}{2^3} = \frac{9}{664613997892457936451835530140172288} \\
 &= \frac{9}{664613997892457936451835530140172288} \cdot \frac{1}{2^3} = \frac{9}{1329227995784915872903671060280344576} \\
 &= \frac{9}{1329227995784915872903671060280344576} \cdot \frac{1}{2^3} = \frac{9}{2658455991569831745807342120560689152} \\
 &= \frac{9}{2658455991569831745807342120560689152} \cdot \frac{1}{2^3} = \frac{9}{5316911983139663491614684241121378304} \\
 &= \frac{9}{5316911983139663491614684241121378304} \cdot \frac{1}{2^3} = \frac{9}{10633823966279326983229368482242756088} \\
 &= \frac{9}{10633823966279326983229368482242756088} \cdot \frac{1}{2^3} = \frac{9}{21267647932558653966458736964485512176} \\
 &= \frac{9}{21267647932558653966458736964485512176} \cdot \frac{1}{2^3} = \frac{9}{42535295865117307932917473928971024352} \\
 &= \frac{9}{42535295865117307932917473928971024352} \cdot \frac{1}{2^3} = \frac{9}{85070591730234615865834947857942048704} \\
 &= \frac{9}{85070591730234615865834947857942048704} \cdot \frac{1}{2^3} = \frac{9}{170141183460469231731669895715884094088} \\
 &= \frac{9}{170141183460469231731669895715884094088} \cdot \frac{1}{2^3} = \frac{9}{340282366920938463463339791431768188176} \\
 &= \frac{9}{340282366920938463463339791431768188176} \cdot \frac{1}{2^3} = \frac{9}{680564733841876926926679582863536376352} \\
 &= \frac{9}{680564733841876926926679582863536376352} \cdot \frac{1}{2^3} = \frac{9}{1361129467683753853853359165727072752704} \\
 &= \frac{9}{1361129467683753853853359165727072752704} \cdot \frac{1}{2^3} = \frac{9}{2722258935367507707706718331454145505408} \\
 &= \frac{9}{2722258935367507707706718331454145505408} \cdot \frac{1}{2^3} = \frac{9}{5444517870735015415413436662858291008816} \\
 &= \frac{9}{5444517870735015415413436662858291008816} \cdot \frac{1}{2^3} = \frac{9}{1088903574147003083082687332571658201632} \\
 &= \frac{9}{1088903574147003083082687332571658201632} \cdot \frac{1}{2^3} = \frac{9}{2177807148294006166165374665143316403264} \\
 &= \frac{9}{2177807148294006166165374665143316403264} \cdot \frac{1}{2^3} = \frac{9}{4355614296588012332325749330286632806528} \\
 &= \frac{9}{4355614296588012332325749330286632806528} \cdot \frac{1}{2^3} = \frac{9}{8711228593176024664651498660573265612556} \\
 &= \frac{9}{8711228593176024664651498660573265612556} \cdot \frac{1}{2^3} = \frac{9}{17422457186352049329302997321146531225112} \\
 &= \frac{9}{17422457186352049329302997321146531225112} \cdot \frac{1}{2^3} = \frac{9}{34844914372704098658605994642293062450224} \\
 &= \frac{9}{34844914372704098658605994642293062450224} \cdot \frac{1}{2^3} = \frac{9}{69689828745408197317211989284586124900448} \\
 &= \frac{9}{69689828745408197317211989284586124900448} \cdot \frac{1}{2^3} = \frac{9}{13937965749081639463442397856917224800896} \\
 &= \frac{9}{13937965749081639463442397856917224800896} \cdot \frac{1}{2^3} = \frac{9}{27875931498163278926884795713834449601792} \\
 &= \frac{9}{27875931498163278926884795713834449601792} \cdot \frac{1}{2^3} = \frac{9}{55751862996326557853769591427668899203584} \\
 &= \frac{9}{55751862996326557853769591427668899203584} \cdot \frac{1}{2^3} = \frac{9}{111503725992653115707539182855337798407168} \\
 &= \frac{9}{111503725992653115707539182855337798407168} \cdot \frac{1}{2^3} = \frac{9}{223007451985306231415078365710675596814336} \\
 &= \frac{9}{223007451985306231415078365710675596814336} \cdot \frac{1}{2^3} = \frac{9}{44601490397061246283015673142135119282872} \\
 &= \frac{9}{44601490397061246283015673142135119282872} \cdot \frac{1}{2^3} = \frac{9}{89202980794122492566031346284270238565744} \\
 &= \frac{9}{89202980794122492566031346284270238565744} \cdot \frac{1}{2^3} = \frac{9}{178405961588244985132062692568540477135488} \\
 &= \frac{9}{178405961588244985132062692568540477135488} \cdot \frac{1}{2^3} = \frac{9}{356811923176489970264125385137080954270976} \\
 &= \frac{9}{356811923176489970264125385137080954270976} \cdot \frac{1}{2^3} = \frac$$

Given,

$$E(X) = 1$$

$$E[X(X-1)] = 4$$

$$E[X^2] - E[X] = 4$$

$$E[X^2] = 5$$

$$\text{Var}[X] = E[X^2] - [E[X]]^2$$
$$= 5 - 1^2$$

$$\text{Var}[X] = 4$$

$$\text{Var}\left[\frac{X}{2}\right] = \left[\frac{1}{2}\right]^2 \text{Var}[X]$$
$$= \frac{1}{4} \cdot 4$$
$$= 1$$

$$\text{Var}[2-3X] = [-3]^2 \text{Var}[X]$$

$$= 9 \times 4 = (d+3n) \cdot V$$

$$= 36 = (d+3n) \cdot V$$

3/2/23

- i. A continuous random variable X has probability density function $f(x) = k$, $0 \leq x \leq 1$. Determine the value of K and also $P\left[X \leq \frac{1}{4}\right]$.

$$f(x) = k \cdot x \Rightarrow k = 8 + xC$$

$$\int_0^\infty f(x) dx = 1$$

* $f(x)$ is a probability function in $[0, 1] \times [0, 1]$
[x] Prob $\int_0^1 K dx = 1$, [x] Prob and $K = [(1-x)x]\$

$$K[x]_0 = 1 \Rightarrow \left[\frac{2x^3}{3} - \frac{x^2}{2} \right]_0^8$$

$$K[1-\sigma] = 1$$

$$\boxed{K=1}$$

$$P(X \leq Y_4) = \int_0^{Y_4} f(x) dx$$

$$= \int_0^{Y_4} \left[\frac{2x^3}{3} - \frac{x^2}{2} \right]_0^8 dx$$

$$= \int_0^{Y_4} 1 \cdot dx$$

$$= [x]_0^{Y_4}$$

$$= \frac{1}{4} - 0$$

$$= \frac{1}{4}$$

$$= \frac{1}{4} \cdot 8^2$$

2. If X is a continuous Random variable with Pdf

$$f(x) = \begin{cases} C(4x - 2x^2), & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

i) Find C

$$\left[\frac{C}{2} + \frac{C}{3} - \frac{8x^3 - 4x^2}{8} \right]_0^2 = 1$$

ii) Find $P[X > 1]$

Soln:

$$\int_{-\infty}^{\infty} f(x) dx = \left[\frac{4x^2}{2} + \frac{2x^3}{3} - \frac{8x^3 - 4x^2}{24} \right]_0^2 = 1$$

$$\int_0^2 C [4x - 2x^2] dx = 1$$

$$C \left[\frac{4x^2}{2} - \frac{2x^3}{3} \right]_0^2 = 1$$

$$C \left[8 - \frac{16}{3} \right] = 1 \quad \boxed{C = \frac{3}{8}}$$

$$C \left[\frac{84 - 16}{3} \right] = 1$$

$$\boxed{C = \frac{3}{8}}$$

$$P(X > 1) = \int_1^\infty f(x) dx = \int_1^2 \frac{3}{8} (4x - 8x^2) dx$$

Now after solving we get the following in the box

$$= \frac{3}{8} \left[\frac{4x^2}{2} - \frac{8x^3}{3} \right] \Big|_1^2 = \frac{3}{8} \left[8 - 16 \right] = 0$$

$$= \frac{3}{8} \left[8 - \frac{16}{3} - \frac{4}{2} + \frac{2}{3} \right] = \frac{64 - 48 - 12 + 4}{24} = \frac{16}{24} = \frac{2}{3}$$

$$= \frac{3}{8} \left[8 - \frac{16}{3} - \frac{4}{2} + \frac{2}{3} \right] = \frac{6 - 14}{3} = \frac{-8}{3}$$

$$= \frac{3}{8} \left[\frac{48 - 32 - 12 + 4}{6} \right] = \frac{18 - 14}{3} = \frac{4}{3}$$

$$= \frac{3}{8} \left[\frac{59 - 44}{6} \right] = \frac{15 - 10}{3} = \frac{5}{3}$$

~~$$= \frac{3}{8} \left[\frac{20}{6} \right] = \frac{3}{8} \left[\frac{8}{6} \right] = \frac{3}{8} \left(\frac{4}{3} \right)$$~~

3. If

cont

4. Let

Pro

(ii)

v)

$$= Y_2 \cdot \text{exp}(X) \quad \text{for } X > 0$$

3. If $f(x) = \begin{cases} Kx e^{-x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$ is the pdf of a continuous random variable X , find K .

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad \Rightarrow \quad \left[-e^{-x} \right]_{-\infty}^{\infty} = 1$$

$$\int_0^{\infty} Kx e^{-x} dx = 1$$

$$K \left[\frac{x e^{-x}}{-1} - e^{-x} \right]_0^{\infty} = 1$$

$$K[0 - 0 + 0 + e^0] = 1$$

$$\boxed{K = 1}$$

$$\begin{aligned} u &= x & v &= e^{-x} \\ u' &= 1 & v' &= \frac{e^{-x}}{-1} \\ u'' &= 0 & v_2 &= e^{-x} \end{aligned}$$

4. Let X be a continuous random variable with probability density function

$$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{Find: (i) } P[X \leq 0.4]$$

$$(ii) P[X \geq 3/4] \quad (iii) P[X > 1/2] \quad (iv) P[\frac{1}{2} < X < 3/4]$$

$$(v) P[X > 3/4 / X > 1/2]$$

$$i) P[X \leq 0.4] = \int_0^{0.4} 2x \, dx$$

$$= 2 \left[\frac{x^2}{2} \right]_0^{0.4} = 0.16.$$

= $(x)^2$

$$ii) P[X > \frac{3}{4}] = \int_{\frac{3}{4}}^1 2x \, dx$$

$$= 2 \left[\frac{x^2}{2} \right]_{\frac{3}{4}}^1 = 1 - \left[\frac{9}{16} \right]$$

$$= 1 - \left(\frac{3}{4} \right)^2 = 1 - \frac{9}{16}$$

$$= \frac{16 - 9}{16} = \frac{7}{16}$$

$$iii) P[X > \frac{1}{2}] = \int_{\frac{1}{2}}^{\infty} 2x \, dx$$

$$= 2 \left[\frac{x^2}{2} \right]_{\frac{1}{2}}^{\infty} = \infty$$

$$= 2 \left[\frac{x^2}{2} \right]_{\frac{1}{2}}^{\infty} = \infty$$

$$= 1 - \left(\frac{1}{2}\right)^2 [2(1-x) \cdot \frac{2}{3}x^2]^{\frac{3}{2}}$$

$$= 1 - \frac{1}{24} (1-x)^2 = (3/4)^2$$

$$= \frac{4-1}{4} (3/4)^2$$

$$= \frac{3}{4} (3/4)^2$$

$$(3/4)^2$$

iv) $P\left[\frac{1}{2} < x < \frac{3}{4}\right]$

$$= \int_{1/2}^{3/4} 2x \, dx$$

$$\frac{x^2}{2} \Big|_{1/2}^{3/4}$$

$$= 2 \left[\frac{x^2}{2} \right]_{1/2}^{3/4}$$

$$= 2 \cdot \frac{1}{2} \left[x^2 \right]_{1/2}^{3/4}$$

$$= \left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \frac{9}{16} - \frac{1}{4}$$

$$= \frac{9-4}{16}$$

$$= \frac{5}{16}$$

$$\Rightarrow P\left[x > \frac{3}{4} / x > \frac{1}{2}\right] = \frac{P(A)}{P(B)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

$$= \frac{P(x > \frac{3}{4})}{P(x > \frac{1}{2})}$$

$$= \frac{7/16}{3/4}$$

$$= \frac{7}{16} \times \frac{4}{3}$$

$$= \frac{7}{12}$$

i) In a continuous random variable x having probability density function $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

i) verify

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$(ii) P(0 < x \leq 1)$$

$$(iii) \text{Find } F(x)$$

$$(iv) \int_{-1}^2 \frac{x^3}{3} dx.$$

$$\begin{aligned}
 \text{i) } & \int_{-\infty}^{\infty} f(x) dx = 1 \\
 &= \int_{-1}^{2} \frac{x^2}{3} dx \\
 &= \frac{1}{3} \int_{-1}^{2} x^2 dx \\
 &= \frac{1}{3} \left[\frac{x^3}{3} \right]_{-1}^2 \\
 &= \frac{1}{9} \left[2^3 - (-1)^3 \right] \\
 &= \frac{1}{9} (8 + 1) \\
 &= \frac{9}{9} = 1
 \end{aligned}$$

$$\begin{aligned}
 \text{ii) } P(0 < X \leq 1) &= \int_0^1 \frac{x^2}{3} dx \\
 &= \frac{1}{3} \int_0^1 x^2 dx \\
 &= \frac{1}{3} \left[\frac{x^3}{3} \right]_0^1 \\
 &= \frac{1}{9} [1 - 0] \\
 &= \frac{1}{9}
 \end{aligned}$$

iii) Find $F(x)$

$$\begin{aligned} F(x) &= \int_{-\infty}^x f(x) dx \\ &= \int_{-1}^x \frac{x^2}{9} dx \\ &= \left[\frac{x^3}{9} \right]_{-1}^x \\ &= \left[\frac{x^3}{9} - \left(\frac{-1)^3}{9} \right) \right] \frac{1}{9} \\ &= \frac{x^3 + 1}{9} \end{aligned}$$

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{x^3 + 1}{9} & -1 \leq x \leq 2 \\ 1 & x \geq 2 \end{cases}$$

d. If $f(x) = \begin{cases} K(1-x^2) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$ is a probability

distributive function of random variable x find K and distribution function of x .

$$\boxed{\int_{-\infty}^{\infty} f(x) dx = 1}$$

$$\int_0^1 K(1-x^2) dx = 1$$

$$K \int_0^1 (1-x^2) dx = 1 \Rightarrow (i)$$

$$K \left[x - \frac{x^3}{3} \right]_0^1 = 1 \quad \left. \begin{array}{l} \text{Durchsetzen} \\ \text{in die Gleichung} \end{array} \right\} \Rightarrow (ii)$$

$$K \left[\frac{3x-x^3}{3} \right]_0^1 = 1,$$

setzt man $x = \frac{3x-1}{3} = 0$ in die Gleichung A ein
erhält man die Gleichung

$$\text{bzw. } K \left[\frac{2/3}{3} \right] = 1 \quad \left. \begin{array}{l} \text{Durchsetzen} \\ \text{in die Gleichung} \end{array} \right\} \Rightarrow (iii)$$

$$K = \frac{3}{2}$$

$$F(x) = \int_x^\infty f(x) dx = x b(x) + \left. \begin{array}{l} \text{Bild} \\ \text{der} \end{array} \right\} \Big|_{\infty}$$

$$= \int_x^\infty \frac{3}{2} [1-x^2] dx \quad \left. \begin{array}{l} \text{Bild der} \\ \text{Integration} \end{array} \right\} \Big|_{\infty}$$

$$= \frac{3}{2} \int_0^x [1-x^2] dx \quad \left. \begin{array}{l} \text{Bild der} \\ \text{Integration} \end{array} \right\} \Big|_{\infty}$$

aus der Tabelle der Integraltafel erhält man

$$= \frac{3}{2} \left[x - \frac{x^3}{3} \right]_0^x \quad \left. \begin{array}{l} \text{Bild der} \\ \text{Integration} \end{array} \right\} \Big|_{\infty}$$

$$= \frac{3}{2} \left[x - \frac{x^3}{3} \right]_0^x \quad \left. \begin{array}{l} \text{Bild der} \\ \text{Integration} \end{array} \right\} \Big|_{\infty}$$

$$= \frac{3}{2} \left[\frac{3x-x^3}{3} \right]_0^x \quad \left. \begin{array}{l} \text{Bild der} \\ \text{Integration} \end{array} \right\} \Big|_{\infty}$$

$$F(x) = \frac{3x - x^3}{2} \cdot [e^{(x-1)} - 1]$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3x - x^3}{2} & 0 \leq x \leq 1 \\ 1 & x > 1 \end{cases}$$

3. A continuous random variable x has the probability density function

$f(x) = \frac{k}{1+x^2}$, $-\infty < x < \infty$. Find k and distribution function of x .

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}$$

$$\int_{-\infty}^{\infty} \frac{k}{1+x^2} dx = 1$$

\therefore The given function is even, then

$$2 \int_0^{\infty} \frac{k}{1+x^2} dx = 1$$

$$2k \left[\frac{1}{2} \tan^{-1} \frac{x}{1} \right]_0^{\infty} = 1$$

$$\tan^{-1} 0 = 0$$

$$\tan^{-1} \infty = \frac{\pi}{2}$$

परिणाम से अद्वितीय कारण का समानांग है कि

$$\partial K \left[\tan^{-1} x \right]_0^\infty = 1$$

$$\partial K \left[\tan^{-1} \infty - \tan^{-1} 0 \right] = 1$$

$$\partial K \left[\frac{\pi}{2} - 0 \right] = 1$$

$$\partial K \frac{\pi}{2} = 1 \quad \text{से}$$

$$K = \frac{1}{\pi}$$

$$F(x) = \int_{-\infty}^x \frac{1}{\pi(1+x^2)} dx$$

$$= \frac{1}{\pi} \left[\tan^{-1} x \right]_{-\infty}^x$$

$$= \frac{1}{\pi} \left[\tan^{-1} x - \tan^{-1}(-\infty) \right]$$

$$= \frac{1}{\pi} \left[\tan^{-1} x + \left(-\frac{\pi}{2} \right) \right]$$

$$= \frac{1}{\pi} \left[\tan^{-1} x + \frac{\pi}{2} \right]$$

$$F(x) = \frac{1}{\pi} \left[\tan^{-1} x + \frac{\pi}{2} \right]$$

$$= \left[\frac{0}{9} + \frac{0}{9} \right] \times 6$$

$$[1+0] \times 6$$

A. If a continuous random variable x having probability density function $f(x) = K e^{-|x|}$ find K and distribution function $F(x)$

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$e^{-\infty} = 0$$

$$e^0 = 1$$

$$e^{\infty} = \infty$$

$$\int_{-\infty}^{\infty} K e^{-|x|} dx = 1$$

$$K \int_{-\infty}^{\infty} e^{-|x|} dx = 1$$

$$K \left[\frac{1}{(x+1)\pi} \right]_{-\infty}^{\infty} = 1$$

$$2K \int_0^{\infty} e^{-x} dx = 1$$

$$2K \left[-e^{-x} \right]_0^{\infty} = 1$$

$$2K \left[\left[\frac{e^{-x}}{-1} \right]_0^{\infty} \right] \frac{1}{\pi} = 1$$

$$2K \left[\left[\frac{e^{-\infty}}{-1} - \frac{e^0}{-1} \right] \right] \frac{1}{\pi} = 1$$

$$2K \left[\left[\frac{0 + e^0}{-1} \right] \right] \frac{1}{\pi} = 1$$

$$2K [e^{-\infty} + e^0] = 1 \quad f(x) = \frac{1}{2} e^{-|x|} \quad -\infty \leq x < \infty$$

$$2K [0 + 1] = 1 \quad = \frac{1}{2} e^{-x} \quad 0 < x < \infty$$

$$2K = 1$$

$$K = \frac{1}{2}$$

$$F(x) = \int_{-\infty}^x \frac{e^{-|x|}}{2} dx$$

when, $-\infty < x < 0$

$$\begin{aligned} \text{120}^{\text{(a)}} \quad F(x) &= \int_{-\infty}^x \frac{e^x}{2} dx \quad \text{when } x < 0 \\ &= \frac{1}{2} \left[e^x \right]_{-\infty}^x \\ &= \frac{1}{2} [e^x - e^{-\infty}] \\ &= \frac{1}{2} [e^x - 0] \\ &= \frac{e^x}{2} \end{aligned}$$

$$\begin{aligned} \text{170} \quad F(x) &= \int_{-\infty}^0 f(x) dx + \int_0^x f(x) dx \\ &= \int_{-\infty}^0 \frac{1}{2} e^x dx + \int_0^x \frac{1}{2} e^{-x} dx \\ &= \frac{1}{2} [e^x]_{-\infty}^0 + \frac{1}{2} \left[\frac{e^{-x}}{-1} \right]_0^x \\ &= \frac{1}{2} [e^0 - e^{-\infty}] + \frac{1}{2} [-e^{-x} + 1] \end{aligned}$$

$$= \frac{1}{2} + \frac{e^{-x}}{2} + \frac{1}{2}$$

$$= 1 + \frac{e^{-x}}{2}$$

5. Find the cumulative distribution function of

$$f(x) = \begin{cases} \frac{x}{2} & 0 \leq x \leq 1 \\ \frac{1}{2} & 1 \leq x \leq 2 \\ \frac{1}{2}(3-x) & 2 \leq x \leq 3 \\ 0 & \text{otherwise.} \end{cases}$$

$F(x)$ when $x \leq 0$

$$F(x) = 0$$

when $x \leq 1$,

$$F(x) = \int_0^x \frac{x}{2} dx$$

$$= \left[\frac{x^2}{4} \right]_0^x$$

$$= \frac{x^2}{4}$$

when $1 \leq x \leq 2$

$$F(x) = \int_0^1 \frac{x}{2} dx + \int_1^x \frac{1}{2} dx$$

$$= \left[\frac{x^2}{4} \right]_0^1 + \left[\frac{x}{2} \right]_1^x$$

$$= \frac{1}{4} + \frac{x}{2} - \frac{1}{2}$$

$$= \frac{x}{2} - \frac{1}{4}$$

when

$$F(x)$$

when

when $2 \leq x \leq 3$

$$F(x) = \int_0^2 \frac{x}{2} dx + \int_2^x \frac{1}{2} dx + \int_x^3 \frac{3-x}{2} dx.$$

$$= \left[\frac{x^2}{4} \right]_0^1 + \left[\frac{x}{2} \right]_1^2 + \frac{1}{2} \int_2^x 3-x dx$$

$$= \frac{1}{4} + \frac{2}{2} - \frac{1}{2} + \frac{1}{2} \left[3x - \frac{x^2}{2} \right]_2^x$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2} \left[\frac{6x - x^2}{2} \right]_2^x$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2} \left[\frac{6x - x^2}{2} \right] - \left[\frac{12 - 4}{2} \right]$$

$$= \frac{1}{4} + \frac{2}{4} + \frac{1}{2} \left[\frac{6x - x^2}{2} \right] - \left[\frac{8}{2} \right]$$

$$= \frac{1}{4} + \frac{2}{4} + \frac{1}{2} \left[\frac{6x - x^2 - 8}{2} \right]$$

$$= \frac{1}{4} + \frac{2}{4} + \frac{6x - x^2 - 8}{4}$$

$$= \frac{3}{4} + \frac{6x - x^2 - 8}{4}$$

$$= \frac{6x - x^2 - 5}{4}$$

when $F(x) > 3$

$F(x) = 1$, because sum of the

probability is 1.

6. The cumulative distribution function

$$F(x) = 1 - (1+\alpha) e^{-x}, \alpha > 0. \quad \text{Find } f(x), \text{ mean } \bar{x}$$

variance.

$$\begin{aligned} f(x) &= \frac{d}{dx} F(x) \\ &= \frac{d}{dx} \left[\frac{1 - e^{-x}}{1 + \alpha} \right] = \frac{\alpha e^{-x}}{(1 + \alpha)^2} \\ &= \frac{\alpha e^{-x}}{1 + 2\alpha + \alpha^2} = \frac{\alpha e^{-x}}{(\alpha + 1)^2} \\ &= \frac{\alpha e^{-x}}{(\alpha + 1)^2}, \quad x > 0 \end{aligned}$$

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$= \int_0^{\infty} x \alpha e^{-x} dx \quad u = x^2 \quad v = e^{-x}$$

$$= \int_0^{\infty} x^2 e^{-x} dx \quad u' = 2x \quad v_1 = -e^{-x}$$

$$= \left[x^2 (-e^{-x}) - 2x e^{-x} + 2(-e^{-x}) \right]_0^{\infty}$$

$$= 2e^0$$

$$= 2.$$

$$E[x^2] = \int_0^{\infty} x^3 e^{-x} dx.$$

$$u = x^3$$

$$u' = 3x^2$$

$$u'' = 6x^2$$

$$v = e^{-x}$$

$$v_1 = -e^{-x}$$

$$v_2 = e^{-x}$$

$$u''' = b \quad v_3 = -e^{-x} \quad \text{orthogonal to } u$$

$$u''' = 0 \quad v_4 = e^{-x} \cdot (x+1) - 1 = (x) e^{-x}$$

$$E[x^2] = [x^3 (-e^{-x}) + 3x^2 e^{-x} + 6x (-e^{-x}) - 6(e^{-x})]_0^\infty$$

$$= [-6e^0] = 6.$$

$$\text{Var}[x] = E[x^2] - [E[x]]^2$$

$$= 6 - 0^2$$

$$= 6$$

$$E[x] = \sum_{x \in S} x p_x$$

$$x_b \ x_g \ x_p$$

$$[(x_b - \bar{x})^2 + (x_g - \bar{x})^2 + (x_p - \bar{x})^2] =$$

$$x_b \ x_g \ x_p$$

$$x_b = 0.5 \quad x_g = 0.5 \quad x_p = 0.5$$

MOMENT AND MOMENTS DISTRIBUTION FUNCTION

moment of various probability measure

Moment about origin μ_0 often we write μ_0

Let X be a random variable, then $\mathbb{E}[X^n]$

moment about origin is

$\mathbb{E}[X^n] = \sum x_i^n p(x_i)$ if X is discrete
probability mass function

$$\mathbb{E}[X^n] = \int x^n f(x) dx \text{ if } X \text{ is continuous.}$$

$$\mathbb{E}[X^2] = \int_{-\infty}^{\infty} x^2 f(x) dx \text{ if } X \text{ is continuous.}$$

$\mathbb{E}[X^2]$ is represented as μ_2

$$\text{mean of } X \text{ is } \mu_1 = \mathbb{E}[X]$$

$$\text{var}[X] = \mu_2 - (\mu_1)^2$$

n^{th} moment about mean / central moment:

$$\mathbb{E}[X - \bar{x}]^n$$

$$= \sum (x - \bar{x})^n p(x_i) \text{ if } X \text{ is discrete}$$

$$\int_{-\infty}^{\infty} (x - \bar{x})^n f(x) dx \text{ if } X \text{ is continuous.}$$

It is represented as μ_n

moment of various probability measure denoted μ_n

higher order moments

$$\begin{aligned} & \left[\begin{array}{c} x^2 \\ x^3 \end{array} \right] M = \frac{x^2}{x^3} = \frac{1}{x} \\ & \text{or} \quad \left[\begin{array}{c} x^2 \\ x^3 \end{array} \right] M = (x^2) M \end{aligned}$$

Moment Generating function
moment generating function of a random variable about the origin is made from

$$M_X(t) = E[e^{tx}]$$

$$= \sum e^{tx} p(x_i), \quad X \text{ is discrete.}$$

$$= \int e^{tx} f(x) dx, \quad X \text{ is continuous.}$$

properties of moment generating function: (Qm)

Let $Y = ax + b$, where X is a random variable with moment generating function

$$M_X(t)$$

$$M_Y(t) = e^{bt} M_X(at)$$

If X and Y are two independent random variables then; $E(Y - X) =$

$$M_{X+Y}(t) = M_X(t) M_Y(t)$$

$$e^{(x+y)t} = e^{xt} e^{yt}.$$

Relation between moment generating function & moment about origin.

$$\mu_1' = \left. \frac{d^n}{dt^n} [M_X(t)] \right|_{t=0}$$

$$M_X(t) = E[e^{tx}]$$

$\mu_1' = \text{mean}$
 $\mu_1' = E(X)$

$$\frac{t^r}{r!} \text{ coeff } y_r$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

1. Find the moment generating function of a random variable x whose probability function is

$$P(X=x) = \frac{1}{2^x}, x=1, 2, \dots \text{ Hence find its mean.}$$

Soln:

The given probability function is a discrete random variable,

$$M_X(t) = \sum e^{tx} P(x_i).$$

$$M_X(t) = \sum_{x=1}^{\infty} e^{tx} \frac{1}{2^x}$$

$$= e^t \frac{1}{2} + e^{2t} \frac{1}{2^2} + e^{3t} \frac{1}{2^3} + \dots$$

$$x \text{ is a discrete random variable with probability distribution} \\ x \text{ is a discrete random variable with probability distribution} = \frac{e^t}{2} \left[1 + \frac{e^t}{2} + \left(\frac{e^t}{2} \right)^2 + \dots \right]$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$$

$$= \frac{e^t}{2} \left[1 - \frac{e^t}{2} \right]^{-1}$$

$$= \frac{e^t}{2} \left[\frac{2-e^t}{2} \right]^{-1}$$

$t=0$
mean
 $= E(X)$

$$M'_X(t) = \frac{d}{dt} \left[M_X(t) \right]_{t=0}$$

$$M'_X(t) = \text{mean} = \frac{d}{dt} \left[M_X(t) \right]_{t=0}$$

$$\text{mean} = \frac{d}{dt} \left[\frac{e^t}{(2-e^t)} \right]_{t=0}$$

$$= \frac{(2-e^t)e^t - e^t(-e^t)}{(2-e^t)^2} \Big|_{t=0}$$

$$= (2-1)(1)+1 = 3$$

$$= \frac{(2-1)(1)+1}{(2-1)^2} = 3$$

$$= 3.$$

- a. If X represent the outcome when a fair die is tossed. Find the moment generating function of X and hence find mean and variance of X .

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$M_X(t) = \sum_{n=1}^6 e^{tx} P(X=x)$$

$$= \frac{1}{6} e^t + \frac{1}{6} e^{2t} + \frac{1}{6} e^{3t} + \frac{1}{6} e^{4t} + \frac{1}{6} e^{5t} + \frac{1}{6} e^{6t}$$

$$M_X(t) = \frac{1}{6} [e^t + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}]$$

$$M'_X(t) = \mu_1 (\text{mean}) = \frac{d}{dt} [M_X(t)]_{t=0}$$

$$= \frac{d}{dt} \left[\frac{1}{6} [e^t + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}] \right]_{t=0}$$

$$= \frac{1}{6} [e^0 + 2e^{2t} + 3e^{3t} + 4e^{4t} + 5e^{5t} + 6e^{6t}]_{t=0}$$

$$= \frac{1}{6} [e^0 + 2e^0 + 3e^0 + 4e^0 + 5e^0 + 6e^0]$$

$$= \frac{1}{6} [1+2+3+4+5+6]$$

$$= 21/6$$

$$M''_X(t) = \mu_2' (\text{mean}) = E[X^2]$$

$$M''_X(t) = \frac{1}{6} [e^t + 4e^{2t} + 9e^{3t} + 16e^{4t} + 25e^{5t} + 36e^{6t}]_{t=0}$$

$$= \frac{1}{6} [1+4+9+16+25+36]$$

$$= \frac{1}{6} [91]$$

$$\text{var}(X) = \mu_2' - (\mu_1')^2$$

$$= \frac{91}{6} - \left(\frac{21}{6}\right)^2$$

$$= \frac{546}{36} - \frac{441}{36}$$

$$= \frac{105}{36}$$

3. If X is a random variable representing the outcome of no. of heads obtained from tossing four coins, find moment generating function, mean, variance.

$S = \{HHHH, HHHT, HHHT, HTHH, THHH, TTTT, TTTH, THTT, HTTT, HTTH, THHT, HTHT, THTH, TTHT, HHHT\}$

$$P(S) = \frac{1}{16} [1 + 4e^t + 6e^{2t} + 4e^{3t} + e^{4t}]$$

x	0	1	2	3	4
$P(x)$	$1/16$	$4/16$	$6/16$	$4/16$	$1/16$

$$M_X(t) = \sum_{x=0}^4 e^{tx} P(x)$$

$$= e^0 \frac{1}{16} + e^t \frac{4}{16} + e^{2t} \frac{6}{16} + e^{3t} \frac{4}{16} + e^{4t} \frac{1}{16}$$

$$= \frac{1}{16} [1 + 4e^t + 6e^{2t} + 4e^{3t} + e^{4t}]$$

$$\text{Mean} = E[X] = M_X'(t) \Big|_{t=0}$$

$$M_X'(t) = \frac{1}{16} [0 + 4e^t + 12e^{2t} + 12e^{3t} + 4e^{4t}]$$

$$M_X'(t) = \frac{1}{16} [4 + 12e^t + 12e^{2t} + 4e^{3t}]$$

$$= \frac{30}{16} = \frac{15}{8}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y}$$

$$\frac{\partial f}{\partial y}$$

$$E[X^2] = M_X''(t) \Big|_{t=0}$$

$$M_X''(t) = \frac{1}{16} [3t + 24e^{2t} + 36e^{3t} + 16e^{4t}]$$

$$M_X''(t) = \frac{1}{16} [3t + 24 + 36 + 16]$$

$$= 5$$

$$\text{Var}(X) = \mu_2' - (\mu_1')^2$$

$$= 5 - 4$$

$$= \left[\frac{0}{1-e} + \frac{\infty}{1-e} \right] \frac{1}{e}$$

d. Let X be a random variable of a probability density function, $f(x) = \begin{cases} \frac{1}{3}e^{-x/3}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$

$$\text{Find i) } P(X \geq 3)$$

$$\text{ii) m.g.f}$$

$$\text{iii) } E[X], \text{Var}[X].$$

$$\text{i) } P(X \geq 3) = \int_3^\infty \frac{1}{3} e^{-x/3} dx$$

$$= \frac{1}{3} \left[\frac{e^{-x/3}}{-\frac{1}{3}} \right]_3^\infty$$

$$= -e^{-\infty} + e^{-1} = (e-1)$$

$$= \frac{1}{e}$$

$$M_X(t) = \int_0^\infty e^{tx} e^{-x/3} dx$$

$$= \frac{1}{3} \int_0^\infty e^{x[1/3 - t]} dx$$

$$= \frac{1}{3} \left[\frac{e^{x[1/3 - t]}}{-1/3} \right]_0^\infty$$

$$= \frac{1}{3} \left[\frac{e^{-\infty}}{-1/3} + \frac{e^0}{1/3} \right]$$

$$= \frac{1}{3} \cdot \frac{1}{1-3t}$$

Multiplied by -

$$= \frac{1}{1-3t}$$

~~$$\text{Var}[X] = \mu_2' - (\mu_1')^2$$~~

$$= 5 - 4$$

$$= 1$$

$$M_{X'}(t) = \frac{1}{(1-3t)^2} (-3)$$

$$= \frac{2}{(1-3t)^2}$$

= 2

$$M_X''(t) = \frac{2 \cdot (-2)}{(1-2t)^3} (-3) = \left[\frac{12}{(1-2t)^2} \right] = \frac{12(1-2t)}{(1-2t)^2} = \frac{12}{1-2t}$$

$$= \frac{12}{(1-2t)^2} = \frac{12}{(1-2t)(1-2t)} = \frac{12}{(1-2t)^2} [1 - (1-2t)] \\ = 12$$

Probability density function of a random variable x

is $f(x) = \begin{cases} \frac{x}{2}, & 0 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$ find the moment

generating function & 1st two moments about the origin.

$$M_X(t) = \frac{1}{2} \int_0^2 e^{tx} x dx$$

$$\begin{aligned} u &= x & v_1 &= e^{tx} \\ u' &= 1 & v_2 &= \frac{e^{tx}}{t} \\ u'' &= 0 & v_3 &= \frac{e^{tx}}{t^2} \end{aligned}$$

$$M_X(t) = \left[\frac{xe^{tx}}{t} - \frac{e^{tx}}{t^2} \right]_0^2$$

$$= \frac{2e^{2t}}{t} - \frac{e^{2t}}{t^2} + \frac{1}{t^2} \quad \frac{2te^{2t}}{t^2} - \frac{e^{2t}}{t^2} + \frac{1}{t^2} \\ \frac{1}{t^2} (1 - e^{2t} + 2te^{2t})$$

$$= \frac{1}{2t^2} [1 + 2te^{2t} - e^{2t}]$$

$$E[X] = \int_0^2 x f(x) dx$$

$$= \int_0^2 x \cdot \frac{1}{2} dx$$

$$= \frac{1}{2} \int_0^2 x^2 dx$$

the density

by $f(x)$

variance

$$\int_{-\infty}^{\infty} f(x) dx$$

$$\int_0^2$$

x till den minsta värde medan $\left[\frac{x^2}{2} \right]_0^2$ till den största värde $\left[\frac{x^4}{4} \right]_0^2$

medan att $b = \frac{1}{2} \left[\frac{x^4}{4} \right]_0^2 = \frac{1}{8}$

medan att detta är minsta värde $a = 0$

$E[X] = \frac{1}{3}$.

$$E[X^2] = \int_0^2 x^2 f(x) dx$$

$$= \int_0^2 x^2 \cdot \frac{1}{2} dx$$

$$= \frac{1}{2} \int_0^2 x^3 dx$$

$$= \frac{1}{2} \left[\frac{x^4}{4} \right]_0^2$$

8th

4x

$$\frac{1}{2} \left[\frac{64}{4} \right] - \frac{0}{4}$$

$$= 2 + 6 + 1 = 9$$

The density function of a random variable x is given by $f(x) = Kx(\alpha - x)$, $0 \leq x \leq \alpha$. Find K , mean, variance & γ^{th} moment.

$$\int_{-\infty}^{\infty} f(x) dx = 1 \quad \left[\frac{Kx^2}{2} - \frac{K\alpha x^3}{3} \right]_0^{\alpha} = 1$$

$$\int_0^{\alpha} K(\alpha x - x^2) dx = 1 \quad \left[K \left(\frac{2x^2}{\alpha+1} - \frac{x^3}{3} \right) \right]_0^{\alpha} = 1$$

$$K \left[4 - \frac{8}{3} \right] = 1 \Rightarrow K = \frac{8}{12} = \frac{2}{3}$$

$$\frac{dK}{3} = 1 \quad \frac{p(x)}{p(x)}$$

$$K = \frac{3}{4}$$

$$\gamma^{\text{th}} \cdot M_x^r = \int_{-\infty}^{\infty} x^r \cdot \frac{2}{3} x \cdot \frac{2}{\alpha} (\alpha x - x^2) dx$$

$$M_x^r = \int_0^{\alpha} x^r \cdot \frac{3}{\alpha} (\alpha x - x^2) dx$$

$$= \frac{3}{4} \int_0^{\alpha} \alpha x^{r+1} - x^{r+2} dx$$

$$= \frac{3}{4} \left[2 \frac{x^{r+2}}{r+2} - \frac{x^{r+3}}{r+3} \right]_0^{\alpha}$$

$$= \frac{3}{4} \left[2 \frac{\alpha^{r+2}}{r+2} - \frac{\alpha^{r+3}}{r+3} \right]$$

$$= \frac{3}{4} \left[\frac{2^{r+3}}{r+2} - \frac{2^{r+3}}{r+3} \right]$$

$$= \frac{3}{4} \left[2^{r+3} \right] \left[\frac{1}{r+2} - \frac{1}{r+3} \right]$$

$$= \frac{3}{4} 2^{r+3} \left[\frac{r+3 - r-2}{(r+2)(r+3)} \right]$$

$$= \frac{3}{4} 2^{r+3} \left[\frac{1}{(r+2)(r+3)} \right]$$

$$\text{Mean} = \mu_1 = \frac{3}{4} \cdot 2^{\frac{r+2}{2}} \left[\frac{1}{3 \times 4} \right] *$$

$$= \frac{3 \times 4}{3 \times 4} = \frac{12}{8}$$

$$= 1 - \frac{2}{8} = \frac{6}{8}$$

$$E[X^2] = \mu_2 = \frac{3}{4} \cdot 2^{\frac{r+3}{2}} \cdot \frac{1}{4 \times 5} = \frac{1}{5} \cdot \frac{12}{8}$$

$$= \frac{3 \times 8^2}{4 \times 5}$$

$$= \frac{6}{5} \cdot \frac{8}{8} = \frac{6}{5}$$

$$x_6 \stackrel{8+6}{\cancel{x_6}} \stackrel{1+6}{\cancel{x_6}} \stackrel{6}{\cancel{x_6}} \left\{ \frac{8}{8} = \frac{6}{6} \right.$$

$$0 \left[\frac{8+6}{8+6} - \frac{8+6}{8+6} \cdot \frac{6}{6} \right] \frac{6}{6} =$$

$$\left[\frac{8+6}{8+6} - \frac{8+6}{8+6} \cdot \frac{6}{6} \right] \frac{6}{6} =$$

Functions of RV.

If X is a random variable with pdf $f(x)$ and Y is a random variable such that $y = G(x)$ then the Pdf of Y is

$$f(y) = f(x) * \left| \frac{dx}{dy} \right|$$

2M

If X is a continuous random variable with the interval $(0, 2)$ and $y = 4x + 3$ and $f(x) = \frac{1}{2}$. Find $f(y)$.

$$f(y) = f(x) * \left| \frac{dx}{dy} \right| = \frac{\frac{1}{2}}{\left| \frac{4}{1} \right|} = \frac{1}{8}$$

$$\frac{dy}{dx} = 4 \quad \frac{1}{4} \times 4 = 1$$

$$\frac{dx}{dy} = \frac{1}{4}$$

$$f(y) = \frac{1}{2} * \frac{1}{4} = \frac{1}{8}$$

when $x=0$,

$$y = 4x + 3$$

$$y = 3 \quad 1 \leq x \leq 2 \quad 1 = (P)$$

when $x=2$

$$y = 4(2) + 3$$

$$y = 11$$

$$f(y) = \frac{1}{8}, (3, 11)$$

Consider a random variable X with p.d.f
and the density with the transformation
 $f(x) = e^{-x}; x \geq 0$

$y = e^{-x}$. Find $f(y)$

$$f(y) = f(x) \times \left| \frac{dx}{dy} \right|$$

Now $\frac{dy}{dx} = -e^{-x}$

the step up has $(0, 0)$ jumping to $(\infty, 0)$

$$\frac{dx}{dy} = -\frac{1}{e^{-x}} = (y)$$

$$\left| \frac{dx}{dy} \right| = \left| \frac{1}{y} \right| = (y)$$

$$f(y) = e^{-x} \times \frac{1}{e^{-x}} = \frac{1}{y}$$

$$y = \frac{1}{x}$$

$$\frac{1}{y} = \frac{x}{1}$$

$$0 \leq x \leq \infty \Rightarrow \frac{1}{x} \leq 1 = (y)$$

when $x=0$

$$y=1$$

$x=\infty$

$y=0$

$$f(y) = 1 \quad 0 \leq y \leq 1; \quad x = \frac{1}{y}$$

$$y = x \text{ when}$$

$$x + (y) = y$$

$$y = y$$

ARV X has the pdf $f(x) = \frac{1}{\pi}$ with the transformation

$y = \tan x$, x lies within the interval $(-\frac{\pi}{2}, \frac{\pi}{2})$

$$f(y) = f(x) * \left| \frac{dx}{dy} \right|$$

$$\frac{dy}{dx} = \sec^2 x.$$

$$\frac{dx}{dy} = \frac{1}{\sec^2 x}$$

$$= \frac{1}{1 + \tan^2 x}$$

$$= \frac{1}{1 + y^2}$$

$$f(y) = \frac{1}{\pi} * \frac{1}{1+y^2}$$

$$\text{when } x = -\frac{\pi}{2}$$

$$y = \tan(-\frac{\pi}{2}) = -\infty$$

$$y = \tan(\frac{\pi}{2}) = \infty$$

$$f(y) = \frac{1}{\pi(1+y^2)} \quad -\infty \leq y \leq \infty$$

$$P^2(q^3) + P^1(q^3)q^1 + P^0 =$$

$$(q^3)^2 + q^3 + 1$$

$$(P_0 + q^{-1}) =$$

BINOMIAL DISTRIBUTION.

A random variable X is said to follow binomial distribution if it assumes only non-negative values and its probability mass function

$$P(X=x) = f(x) = \begin{cases} nC_x p^x q^{n-x} & x=0, 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

Here n and p are called parameters of the distribution.

$$\text{Standard deviation} = \sqrt{npq}, \quad nC_0 = 1, \quad nC_n = 1$$

Moment generating function of a Binomial distribution

$$M_X(t) = \sum_{x=0}^{\infty} e^{tx} f(x)$$

$$= \sum_{x=0}^{\infty} e^{tx} nC_x p^x q^{n-x}$$

$$= (e^t p + q)^n$$

$$= e^0 nC_0 p^0 q^n + e^t nC_1 p^1 q^{n-1} +$$

$$e^{2t} nC_2 p^2 q^{n-2} + \dots + e^t nC_n p^n q^0$$

$$= q^n + nC_1 (e^t p)^1 q^{n-1} + nC_2 (e^t p)^2 q^{n-2} + \dots + (e^t p + q)^n$$

$$= (e^t p + q)^n$$

$$\text{mean} = E[X]$$

$$E[X^2] =$$

$$\text{var}[X] =$$

$$\text{mean} = E[X] = \frac{d}{dt} [M_X(t)]_{t=0}$$

$$= \frac{d}{dt} [e^{tp+q}]_{t=0}^n$$

$$= n(e^{tp+q})^{n-1} e^{tp}/t=0$$

$$= n(p+q)^{n-1} p$$

$$= n(1-p)^{n-1} p$$

$$= NP.$$

$$E[X^2] = \frac{d^2}{dt^2} [M_X(t)]_{t=0}$$

$$= \frac{d}{dt} \left[np e^{tp} (e^{tp+q})^{n-1} \right]_{t=0}$$

$$= NP \left[e^{tp} (n-1)(e^{tp+q})^{n-2} \cdot e^{tp} + (e^{tp+q})^{n-1} e^{tp} \right]_{t=0}$$

$$= NP \left[(n-1)p^2 + (p+q)^{n-1} \right]$$

$$= NP \left[(n-1)p + 1 \right]$$

$$= NP [NP - P + 1]$$

$$= n^2 p^2 - np^2 + NP.$$

$$\text{Var}[X] = E[X^2] - [E[X]]^2$$

$$= n^2 p^2 - np^2 + NP - n^2 p^2$$

$$= -np^2 + np$$

$$= np[1-p]$$

$$= n.pq$$

1. The mean of a Binomial distribution is 20 and standard deviation is 4. Determine the parameters of the distribution.

$$np = \text{mean} = 20 \rightarrow ①$$

$$sd = 4$$

$$\sqrt{\text{var}} = sd$$

$$\text{var} = (sd)^2$$

$$\text{var}(X) = 16$$

$$npq = 16$$

$$20q = 16$$

$$q = \frac{16}{20} = \frac{4}{5}$$

$$P = 1 - q$$

$$= 1 - \frac{4}{5}$$

$$P = \frac{1}{5}$$

$$np = 20$$

$$n = 20 \times 5$$

$$n = 100$$

$$q^2 a - qa + qa - q^2 a = 16$$

binomial distribution to have
mean 3 and variance 4.

$$\text{mean} \Rightarrow np = 3$$

$$\text{variance} \Rightarrow npq = 4$$

$$3q = 4$$

$$q = \frac{4}{3}$$

2. The mean & the variance of a binomial distribution
is 8 and 6 respectively. find $P(X \geq 2)$

$$\text{mean} \Rightarrow np = 8$$

$$\text{variance} \Rightarrow npq = 6$$

$$8q = 6$$

$$q = \frac{6}{8} = \frac{3}{4}$$

$$\boxed{q = \frac{3}{4}}$$

$$P+q = 1$$

$$P + \frac{3}{4} = 1$$

$$\frac{4P+3}{4} = 1$$

$$4P+3 = 4$$

$$4P = 4 - 3$$

$$4P = 1$$

$$\boxed{P = \frac{1}{4}}$$

$$\Rightarrow np = 8$$

$$\frac{n}{4} = 8$$

$$\begin{aligned}
 n &= 8 \\
 n &= 32 \\
 P(X=x) &= {}^n C_x p^x q^{n-x} \\
 P(X \geq 2) &= 1 - P(X < 2) \\
 &= 1 - [P(X=0) + P(X=1)] \\
 &= 1 - \left[{}^{32} C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{32-0} \right]
 \end{aligned}$$

$$\begin{aligned}
 &+ {}^{32} C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^{31} \\
 &= 1 - \left[{}^{32} C_0 \left(\frac{3}{4}\right)^{32} + {}^{32} C_1 \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^{31} \right] \\
 &= 1 - \left(\frac{3}{4} \right)^{31} \left[\frac{3}{4} + \frac{32}{4} \right] \\
 &= 1 - \left(\frac{3}{4} \right)^{31} \left[\frac{35}{4} \right]
 \end{aligned}$$

$$l = p + q$$

$$l = \frac{p+q}{p}$$

$$l = \frac{p+q}{q}$$

$$l = p + q$$

$$p + q = 9A$$

$$l = 9A$$

$$\boxed{pV = qT}$$

$$q = pV$$

$$q = \frac{1}{4}V$$

BINOMIAL DISTRIBUTION.

Four coins are tossed simultaneously what is the probability of getting (i) all heads (ii) at least two heads (iii) at most two heads.

i) Probability of getting head $P = \frac{1}{2}$

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

$$n = 4$$

$$P(X=x) = {}^n C_n p^x q^{n-x}$$

$$P(X=2) = {}^4 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2}$$

$$= {}^4 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2$$

$$= {}^4 C_2 \left(\frac{1}{2}\right)^4$$

$$= \frac{4!}{2! 2!} \cdot \frac{1}{2^4}$$

$$= \frac{6}{2^4} = \frac{6}{16}$$

$$= \frac{3}{8} = \frac{3}{8} + \frac{1}{16} + \frac{1}{16}$$

$$= \frac{3}{8} + \frac{1}{16} + \frac{1}{16} = \frac{7}{16}$$

ii) Probability of getting at least two heads

$$P(X \geq 2) = P(X=2) + P(X=3) + P(X=4)$$

$$= {}^4 C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2} + {}^4 C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^{4-3}$$

$$+ {}^4 C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^0$$

$$P(\text{at least one head}) = \frac{3}{8} + \frac{4 \times 3 \times 2}{16} \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4$$

= $\frac{3}{8} + \frac{1}{4} + \frac{1}{16}$

$$= \frac{6+4+1}{16}$$

$$= \frac{11}{16}$$

probability of getting atmost two heads $P(X \leq 2)$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0} + {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1}$$

$$+ \frac{3}{8}$$

$$= \left(\frac{1}{2}\right)^4 + 4 \left(\frac{1}{2}\right)^4 + \frac{3}{8}$$

$$= \frac{1}{2^4} + \frac{4}{2^4} + \frac{3}{8}$$

$$= \frac{1}{2^4} + \frac{4}{2^4} + \frac{3 \times 2}{2^4}$$

$$= \frac{1+4+6}{2^4}$$

$$= \frac{11}{16}$$

$$= {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^{4-0} + {}^4C_1 \left(\frac{1}{2}\right)^1 \left(\frac{1}{2}\right)^{4-1} + {}^4C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^{4-2}$$

$$= \left(\frac{1}{2}\right)^4 + 4 \left(\frac{1}{2}\right)^4 + 6 \left(\frac{1}{2}\right)^4$$

Q. A pair of dice is thrown 4 times. If getting a doublet is considered a success. Find the probability of two successes.

$$\text{Probability of getting double} = \frac{6}{36} = \frac{1}{6}$$

$$P = \frac{1}{6}; q = 1 - \frac{1}{6}$$

$$= \frac{5}{6}$$

$$n = 4$$

$$\text{Probability of getting two successes} = P(X=2)$$

$$(S=X)q + (C=X)q^2 + (I=X)q^3 + (O=X)q^4$$

$$= 4C_2 \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^{4-2}$$

$$\left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right)^2 + \left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right)^1 = \frac{4 \times 3}{1 \times 2} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}$$

$$\left(\frac{5}{6}\right)^3 \left(\frac{1}{6}\right)^1 + \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right)^0 = \frac{25}{216}.$$

A large consignment of electric bulbs 10% are defective.

A random sample of 20 is taken for inspection.

Find i) the probability that all are good bulbs.

ii) almost three is defective.

iii) Exactly three is defective.

P is probability of getting defective, $P = \frac{10}{100} = \frac{1}{10}$

$$n = 20$$

$$P(S=2) = (S=2)q$$

$$P = \frac{1}{10}, q = 1 - \frac{1}{10} = \frac{9}{10}$$

$$\begin{aligned}
 \text{i) } P(X=0) &= {}^n C_x p^x q^{n-x} \\
 &= {}^{20} C_0 p^0 q^{20-0} \\
 &= {}^{20} C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20} \\
 &= \cdot \left(\frac{9}{10}\right)^{20}
 \end{aligned}$$

ii) atmost three is defective:

$$\begin{aligned}
 P(X=0) + P(X=1) + P(X=2) + P(X=3) \\
 &= {}^{20} C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20-0} + {}^{20} C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{20-1} \\
 &\quad + {}^{20} C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{20-2} + {}^{20} C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{20-3} \\
 &= \left(\frac{9}{10}\right)^{20} + 2 \cdot \left(\frac{1}{10}\right) \left(\frac{9}{10}\right)^{19} + \frac{19}{10} \left(\frac{9}{10}\right)^{18} + \frac{57}{50} \left(\frac{9}{10}\right)^{17} \\
 &= \left(\frac{9}{10}\right)^{20} + 2 \cdot \left(\frac{9}{10}\right)^{19} + \frac{19}{10} \left(\frac{9}{10}\right)^{18} + \frac{57}{50} \left(\frac{9}{10}\right)^{17}
 \end{aligned}$$

iii) Exactly 2, probability is 0.4

$$P(X=2) = {}^n C_x p^x q^{n-x}$$