RANDOM VARIABLE:

A random variable is a function that assigns a real number $X(s)$ to every element in the sample space $s \in S$ corresponding to a random experiment E.

Types of random variables:

* discrete random variables
* continuous random variables.

Discrete random variables:

A random variable x is discrete if it assumes only discrete values.

Example: No. of students inside a class.

Continuous random variables:

If X is a random variable which can take all possible values between certain limits / in an interval, which may be a finite (or) infinite then X is a continuous random variable.

Eg: $f(x) = \{ x^2 - 1 \leq x \leq 5 \}$

PROBABILITY MASS FUNCTION (PMF):

If X is a discrete random variable which can take the values $x_1, x_2, x_3, \ldots$ such that,

$$P(X = x_i) = P_i$$

then P_i is called Probability mass function (or) probability function and it satisfies the condition.

(i) $P_i \geq 0$

(ii) $\sum P_i = 1$

PROBABILITY DENSITY FUNCTION (PDF):

If X is a continuous random variable such that

$$P\left(X - \frac{1}{2}dx \leq X \leq X + \frac{1}{2}dx\right) = f(x)\,dx.$$

then $f(x)$ is said to be probability density function and it also obeys

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

COMMULATIVE DISTRIBUTION FUNCTION [CDF]:

If X is a random variable discrete or continuous then $P(X \leq x)$ is called cummulative distribution function. It is represented by $F(x)$.

if $F(x) = \sum P_i$, $x_i \in x$ Then x is discrete.

if $F(x) = \int_{-\infty}^{\infty} f(x)\,dx$, then x is continuous.

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1. A discrete Random Variable X has the following probability distribution.

x	0	1	2	3	4	5	6	7	8
$P(X=x)$	a	$3a$	$5a$	$7a$	$9a$	$11a$	$13a$	$15a$	$17a$

i) Find the value of a.

ii) Find $P(X<3)$, $P(0<X<3)$, $P(X\geq 3)$

iii) Find the distribution function of x.

i) We know that,

$$\sum P(x_i) = 1$$

$$a+3a+5a+7a+9a+11a+13a+15a+17a = 1$$

$$81a = 1$$

$$a = 1/81$$

ii) $P(X<3) = P(X=0) + P(X=1) + P(X=2)$

$$= a+3a+5a$$

$$= 9a$$

$$= \frac{9}{81}$$

$$= \frac{1}{9}$$

$P(0<X<3) = P(X=1) + P(X=2)$

$$= 3a+5a$$

$$= 8a$$

$$= \frac{8}{81}.$$

$P(X \geq 3) = 1 - P(X < 3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \frac{1}{9}$

$= \frac{8}{9}.$

iii) distribution fn $F(x)$

$F(0) = P(X=0) = a = \frac{1}{81}$

$F(1) = P(X=0) + P(X=1) = a + 3a = \frac{4}{81}$

$F(2) = P(X=0) + P(X=1) + P(X=2) = \frac{9}{81}.$

$F(3) = P(X=0) + P(X=1) + P(X=2) + P(X=3) = \frac{16}{81}.$

$F(4) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) = \frac{25}{81}$

$F(5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) = \frac{36}{81}$

$F(6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) = \frac{49}{81}.$

$F(7) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6) + P(X=7) = \frac{64}{81}.$

$F(8) = \frac{81}{81} = 1$

x	0	1	2	3	4	5	6	7	8
$F(x)$	$\frac{1}{81}$	$\frac{4}{81}$	$\frac{9}{81}$	$\frac{16}{81}$	$\frac{25}{81}$	$\frac{36}{81}$	$\frac{49}{81}$	$\frac{64}{81}$	1

If X is a discrete random variable

then mean $E(X) = \sum x_i P(x_i)$

$E(X^2) = \sum x_i^2 P(x_i)$

$Var(X) = E[x^2] - [E[x]]^2$

2. Determine the constant K given the following probability distribution of discrete random variable X. Also find mean and variance of X.

$X = x$	1	2	3	4	5
$P(X=x_i)$	0.1	0.2	K	$2K$	0.1

to find K:

$$\sum P(x_i) = 1$$

$$0.1 + 0.2 + K + 2K + 0.1 = 1$$

$$3K + 0.4 = 1$$

$$3K = 0.6$$

$$K = 0.2$$

Mean $= \sum x_i P(x_i)$

$= 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.2 + 4 \times 0.4 + 5 \times 0.1$

$= 3.2$

$E(X^2) = \sum x_i^2 P(x_i)$

$= 1^2 \times 0.1 + 2^2 \times 0.2 + 3^2 \times 0.2 + 4^2 \times 0.4 + 5^2 \times 0.1$

$= 11.6$

$$Var(X) = E(X^2) - [E(x)]^2$$
$$= 11.6 - (3.2)^2$$
$$= 1.36.$$

CONDITIONAL PROBABILITY

$$P(A/B) = \frac{P(A \cap B)}{P(B)}.$$

2) A random variable X has the following Probability fn

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

i) Find k

ii) $P[X<6]$, $P[X \geq 6]$

iii) If $P[X \leq c] > \frac{1}{2}$ Find minimum value of c.

iv) Evaluate $P[X<2]$, $P[X>3]$, $P[1<X<5]$

v) $P[1.5 < X < 4.5 / X>2]$

i)
$$0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$
$$9k+10k^2=1$$
$$10k^2+9k-1=0$$
$$\boxed{k=\frac{1}{10}}, \quad k=-1$$

ii) $P[x<6] = P[x=0] + P[x=1] + P[x=2] + P[x=3]$

$+ P[x=4] + P[x=5]$

$= 0 + k + 2k + 2k + 3k + k^2$

$= 8k + k^2$

$= \frac{8}{10} + \frac{1}{100}$

$= \frac{80+1}{100}$

$= \frac{81}{100}$

$P[x \geq 6] = 1 - P[x<6]$

$= 1 - \frac{81}{100}$

$= \frac{100-81}{100}$

$= \frac{19}{100}$

iii) $P[x<2] = P[x=0] + P[x=1]$

$= 0 + k$

$= 1/10$

$P[x>3] = P[x=4] + P[x=5] + P[x=6] + P[x=7]$

$= 3k + k^2 + 2k^2 + 7k^2 + k$

$= 10k^2 + 4k$

$= 10\left(\frac{1}{10}\right)^2 + \frac{4}{10}$

$= \frac{10}{100} + \frac{4}{10}$

$= \frac{10+40}{100} = \frac{50}{100} = \frac{1}{2}$

$$P[1 < X < 5] = P[X=2] + P[X=3] + P[X=4]$$
$$= 2K + 2K + 3K$$
$$= 7K$$
$$= \frac{7}{10}$$

iii) $P[X \leq c] > \frac{1}{2}$

$P(X=0) = 0$

$P(X \leq 1) = 0 + K = \frac{1}{10}$

$P(X \leq 2) = 3K = \frac{3}{10}$

$P(X \leq 3) = 5K = \frac{1}{2}$

$P(X \leq 4) = 8K > \frac{1}{2}$

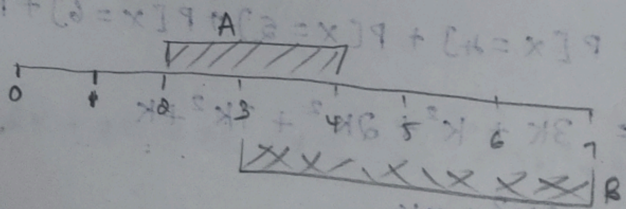
$\therefore \boxed{c = 4}$

v) $P[1.5 < X < 4.5 / X > 2]$

we know that,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

Here.



$$P[1.5 < X < 4.5 / X > 2] = \frac{P[1.5 < X < 4.5] \cap P[X>2]}{P[X>2]}$$

$$= \frac{P(X=3) + P(X=4)}{1 - P(X \leq 2)}$$

4) The pr

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find m

P[X

Soln:

$$= \frac{5K}{1-[P(X=0)+P(X=1)+P(X=2)]}$$

$$= \frac{5K}{1-3K} = \frac{5/10}{1-3/10} = \frac{5/10}{\frac{10-3}{10}} = \frac{5/10}{7/10} = \frac{5}{10} \times \frac{10}{7}$$

$$= 5/7$$

4) The probability function of an infinite discrete distribution is given by; $P[X = x_j] = 1/2^j$, $j = 1, 2, \ldots \infty$

Find mean and $P(X \text{ is even})$, $P(X \geq 5)$

$P[X \text{ divisible by } 3]$.

Soln:

$$\text{Mean} = \sum_{j=1}^{\infty} x_j p_i$$

$$= 1\frac{1}{2} + 2 \cdot \frac{1}{2^2} + 3\frac{1}{2^3} + \cdots$$

$$= \frac{1}{2}\left[1 + 2(1/2) + 3(1/2)^2 + \cdots\right].$$

$$\because (1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \cdots$$

$$\text{Mean} = 1/2\,[1-(1/2)]^{-2}. \Rightarrow \frac{1}{2}\left[\frac{2-1}{2}\right]^{-2} \Rightarrow \frac{1}{2}\left(\frac{1}{2}\right)^{-2}$$

$$= 1/2\,(1/2)^{-2}$$

$$\Rightarrow 1/2\,\frac{1}{(1/2)^2}$$

$$= \frac{2^2}{2}$$

$$= 1/2\left(\frac{1}{1/4}\right) 4^2 (1/2)$$

$$= 2.$$

$$= 4/2$$

$$\boxed{\text{mean} = 2.}$$

$\geq 2]$

$P[X = \text{even}] = P(X=2) + P(X=4) + \dots$

$= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^6} + \dots$

$2^6 = (2^2)^3$

$= 2^2 \cdot 2^2 \cdot 2^2$

$= 2^2 (2^4)$

$= \frac{1}{2^2}\left[1 + \frac{1}{2^2} + \frac{1}{2^4} + \dots\right]$

$= \frac{1}{4}\left[1 - \left(\frac{1}{2^2}\right)\right]^{-1} \Rightarrow \frac{1}{4}\left[1 - \frac{1}{4}\right]^{-1}$

$\cancel{= \frac{1}{4}\left(\frac{1}{2^2}\right)^{-1}}$ $\quad = \frac{1}{4}\left[\frac{4-1}{4}\right]^{-1}$

$\cancel{= \left(\frac{1}{4}\right)(2)}$ $\quad = \frac{1}{4}\left(\frac{3}{4}\right)^{-1}$

$\cancel{= \frac{1}{2}.}$ $\quad = \frac{1}{4}\left(\frac{4}{3}\right)$

$\boxed{P[X = \text{even}] = \frac{1}{3}}$

$P(X \geq 5) = P(X=5) + P(X=6) + P(X=7) + \dots$

$= \frac{1}{2^5} + \frac{1}{2^6} + \frac{1}{2^7} + \dots$

$= \frac{1}{2^5} + \frac{1}{2^5 \cdot 2^1} + \frac{1}{2^5 \cdot 2^2} + \dots$

$= \frac{1}{2^5}\left[1 + \frac{1}{2} + \left(\frac{1}{2}\right)^2 + \dots\right]$

$= \frac{1}{2^5}\left[1 - \frac{1}{2}\right]^{-1} \Rightarrow \left(\frac{2-1}{2}\right)^{-1} \times \frac{1}{2^5}$

$= \frac{1}{2^5}\left(\frac{1}{2}\right)^{-1} \quad = \left(\frac{1}{2}\right)^{-1} \times \frac{1}{2^5}$

$= \frac{1}{2^4} \quad = \frac{1}{(1/2)} \times \frac{1}{2^5} = 2 \times \frac{1}{2^{5}}$

$= \frac{1}{2^4}$

$P[X \text{ divisible by } 3] = P[X=3] + P[X=6] + \dots$

$$= \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^9 + \dots$$

$$= \frac{1}{2^3} + \left(\frac{1}{2^6}\right) + \frac{1}{2^9} + \dots$$

$$= \frac{1}{2^3}\left[1 + \frac{1}{2^3} + \frac{1}{2^6} + \dots\right]$$

$$= \frac{1}{2^3}\left[1 - \frac{1}{2^3}\right]^{-1}$$

$$= \frac{1}{2^3}\left[\frac{7}{8}\right]^{-1}$$

$$= \frac{1}{8} \times \frac{8}{7}$$

$$= \frac{1}{7}.$$

Properties:

* $V(ax+b) = a^2 \, Var(x)$
* $E[ax+b] = a\,E(x) + b$. [Mean]

1. If variance of x is 4. Find $var[3x+8]$.

Soln:

Given, $vari(x) = 4$

$$var(3x+8) = 3^2 \; Var(x)$$

$$= 9 \times 4$$

$$= 36.$$

2. If X is a random variable with $E(X) = 1$ & $E(X(X-1)) = 4$. Find $var[x]$, $var[X/2]$ and $var[2-3x]$.

Given,

$E(X) = 1$

$E[X(X-1)] = 4$

$E[X^2] - E[X] = 4$

$E[X^2] = 5$

$V[X] = E[X^2] - [E[X]]^2$

$= 5 - 1^2$

$V[X] = 4$

$Var\left[\frac{X}{2}\right] = \left[\frac{1}{2}\right]^2 \cdot Var[X]$

$= \frac{1}{4} \cdot 4$

$= 1$

$Var[2-3X] = [-3]^2 \; Var[X]$

$= 9 \times 4$

$= 36.$

3/2/23

1. A continuous random variable X has probability density function $f(x) = k$, $0 \leq x \leq 1$. Determine the value of K and also $P\left[X \leq \frac{1}{4}\right]$.

$f(x) = K.$

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^1 K\,dx = 1.$$

2. If

$f(x$

i) Fin

ii) Fin

Sol

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$$K[x]_0^1 = 1$$

$$K[1-0] = 1$$

$$\boxed{K=1}$$

$$P(X \le 1/4) = \int_0^{1/4} f(x)\,dx$$

$$= \int_0^{1/4} 1.\,dx$$

$$= [x]_0^{1/4}$$

$$= \frac{1}{4} - 0$$

$$= 1/4$$

2. If X is a continuous Random variable with Pdf

$$f(x) = \begin{cases} c\,(4x - 2x^2), & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

i) Find c

ii) Find $P[x > 1]$

Soln:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^2 c\,[4x - 2x^2]\,dx = 1$$

$$c\left[\frac{4x^2}{2} - \frac{2x^3}{3}\right]_0^2 = 1$$

$$c\left[8-\frac{16}{3}\right]=1$$

$$c\left[\frac{24-16}{3}\right]=1$$

$$\boxed{c=\frac{3}{8}}$$

$$P(X>1)=\int_1^{\infty} f(x)\,dx$$

$$=\int_1^2 \frac{3}{8}(4x-2x^2)\,dx$$

$$=\frac{3}{8}\left[\frac{4x^2}{2}-\frac{2x^3}{3}\right]_1^2$$

$$=\frac{3}{8}\left[2\times4-\frac{2\times8}{3}-\frac{4}{2}+\frac{2}{3}\right]$$

$$=\frac{3}{8}\left[8-\frac{16}{3}-\frac{4}{2}+\frac{2}{3}\right]$$

$6-\frac{14}{3}$

$\frac{18-14}{3}$

$\frac{4}{3}$

$$=\frac{3}{8}\left[\frac{48-32-12+4}{6}\right]$$

$$=\frac{3}{8}\left[\frac{52-44}{6}\right]$$

$$=\cancel{\frac{3}{8}\left[\frac{80}{6}\right]}=\frac{3}{8}\left[\frac{8}{6}\right]=\frac{3}{8}\left(\frac{4}{3}\right)$$

3. If f

cont

4. Let

Pro

(ii) P

v) P

$= 1/2.$

3. If $f(x) = \begin{cases} Kxe^{-x} & x > 0 \\ 0 & \text{otherwise} \end{cases}$ is the pdf of a continuous Random variable X. Find K.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{\infty} Kxe^{-x}\,dx = 1$$

$$K\left[\frac{xe^{-x}}{-1} - e^{-x}\right]_0^{\infty} = 1$$

$u = x \quad v = e^{-x}$

$u' = 1 \quad v_1 = \frac{e^{-x}}{-1}$

$u'' = 0 \quad v_2 = e^{-x}$

$$K[0 - 0 + 0 + e^0] = 1$$

$$\boxed{K = 1}$$

4. Let X be a continuous random variable with probability density function

$f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ Find. (i) $P[X \le 0.4]$

(ii) $P[X > 3/4]$ (iii) $P[X \geq 1/2]$ iv) $P[1/2 < X < 3/4]$

v) $P\left[X > 3/4 \,/\, X > 1/2\right]$.

i) $P[X \le 0.4] = \int_0^{0.4} 2x \, dx$

$= 2\left[\frac{x^2}{2}\right]_0^{0.4}$

$= 0.4 \times 0.4$

$= 0.16.$

ii) $P[X > \frac{3}{4}] = \int_{3/4}^{1} 2x \, dx$

$= 2\left[\frac{x^2}{2}\right]_{3/4}^{1}$

$= 2 \cdot \frac{1}{2}\left[x^2\right]_{3/4}^{1}$

$= 1 - \left(\frac{3}{4}\right)^2$

$= 1 - \frac{9}{16}$

$= \frac{16-9}{16}$

$= 7/16$

iii) $P[X > 1/2] = \int_{1/2}^{1} 2x \, dx$

$= 2\left[\frac{x^2}{2}\right]_{1/2}^{1}$

$= 2 \cdot \frac{1}{2}\left[x^2\right]_{1/2}^{1}$

iv) $P[\frac{1}{2} < x$

$= 1 - \left(\frac{1}{2}\right)^2$

$= 1 - \frac{1}{4}$

$= \frac{4-1}{4}$

$= 3/4$

iv) $P\left[\frac{1}{2} < x < \frac{3}{4}\right]$

$$= \int_{1/2}^{3/4} 2x \, dx$$

$$= 2\left[\frac{x^2}{2}\right]_{1/2}^{3/4}$$

$$= \cancel{2} \cdot \frac{1}{\cancel{2}} \left[x^2\right]_{1/2}^{3/4}$$

$$= \left(\frac{3}{4}\right)^2 - \left(\frac{1}{2}\right)^2$$

$$= \frac{9}{16} - \frac{1}{4}$$

$$= \frac{9-4}{16}$$

$$= \frac{5}{16}.$$

v) $P\left[X > \frac{3}{4} / X > 1/2\right]$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(X > 3/4)}{P(X > 1/2)}$$

$$= \frac{7/16}{3/4}$$

$$= \frac{7}{\cancel{16}_4} \times \frac{\cancel{4}}{3}$$

$$= 7/12.$$

1. In a continuous random variable X having probability density function $f(x) = \begin{cases} \frac{x^2}{3}, & -1 < x < 2 \\ 0 & \text{otherwise.} \end{cases}$

i) verify $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(ii) $P(0 < X \leq 1)$

(iii) Find $F(x)$

iv) $\int_{-1}^{2} \frac{x^3}{3}\,dx.$

i) $\int_{-\infty}^{\infty}$

ii) P(

i) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$= \int_{-1}^{2} \frac{x^2}{3}\,dx$

$= \frac{1}{3}\int_{-1}^{2} x^2\,dx$

$= \frac{1}{3}\left[\frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{1}{9}\left[2^3 - (-1)^3\right]$

$= \frac{1}{9}(8+1)$

$= \frac{9}{9}$

$= 1$

probability

ii) $P(0 < X \leq 1) = \int_{0}^{1} \frac{x^2}{3}\,dx$

$= \frac{1}{3}\int_{0}^{1} x^2\,dx$

$= \frac{1}{3}\left[\frac{x^3}{3}\right]_{0}^{1}$

$= \frac{1}{9}[1-0]$

$= 1/9$

iii) Find $F(x)$

$$F(x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-1}^{x} \frac{x^2}{3}\,dx$$

$$= \left[\frac{x^3}{9}\right]_{-1}^{x}$$

$$= \left[\frac{x^3}{9} - \frac{(-1)^3}{9}\right]$$

$$= \frac{x^3+1}{9}$$

$$F(x) = \begin{cases} 0 & x < -1 \\ \frac{x^3+1}{9} & -1 < x < 2 \\ 1 & x \geq 2. \end{cases}$$

2. If $f(x) = \begin{cases} K(1-x^2) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$ is a probability distributive function of random variable X Find k and distribution function of X.

$$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = 1}$$

$$\int_{0}^{1} K(1-x^2)\,dx = 1$$

$$k\int_0^1 (1-x^2)\,dx = 1$$

$$k\left[x-\frac{x^3}{3}\right]_0^1 = 1$$

$$k\left[\frac{3x-x^3}{3}\right]_0^1 = 1$$

$$k\left[\frac{3-1}{3} - 0\right]$$

$$k\left[2/3\right] = 1$$

$$\boxed{k = 3/2}$$

$$F(x) = \int_{\infty}^{x} f(x)\,dx$$

$$= \int_0^x \frac{3}{2}\left[1-x^2\right]dx$$

$$= \frac{3}{2}\int_0^x \left[1-x^2\right]dx$$

$$= \frac{3}{2}\left[x-\frac{x^3}{3}\right]_0^x$$

$$= \frac{3}{2}\left[x-\frac{x^3}{3}\right]$$

$$= \frac{\cancel{3}}{2}\left[\frac{3x-x^3}{\cancel{3}}\right]$$

$$F(x) = \frac{3x - x^3}{2}$$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{3x - x^3}{2} & 0 < x < 1 \\ 1 & x > 1 \end{cases}$$

3. A continuous random variable x has the probability density function

$$f(x) = \frac{k}{1+x^2} \quad -\infty < x < \infty$$. Find k and distribution function of x.

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\boxed{\int \frac{1}{a^2+x^2}\, dx = \frac{1}{a} \tan^{-1} \frac{x}{a}}$$

$\tan^{-1} \infty = \frac{\pi}{2}$

$\tan^{-1} 0 = 0$

$$\int_{-\infty}^{\infty} \frac{k}{1+x^2}\, dx = 1$$

$\because$ The given function is even, then

$$2 \int_{0}^{\infty} \frac{k}{1+x^2}\, dx = 1$$

$$2k \left[\frac{1}{1} \tan^{-1} \frac{x}{1} \right]_0^{\infty} = 1$$

$$2k\left[\tan^{-1}x\right]_0^{\infty} = 1$$

$$2k\left[\tan^{-1}\infty - \tan^{-1}0\right] = 1$$

$$2k\left[\frac{\pi}{2} - 0\right] = 1$$

$$\cancel{2}k\frac{\pi}{\cancel{2}} = 1$$

$$\boxed{k = 1/\pi}$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\pi(1+x^2)}\,dx$$

$$= \frac{1}{\pi}\left[\tan^{-1}x\right]_{-\infty}^{x}$$

$$= \frac{1}{\pi}\left[\tan^{-1}x - \tan^{-1}(-\infty)\right]$$

$$= \frac{1}{\pi}\left[\tan^{-1}x - \left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{\pi}\left[\tan^{-1}x + \frac{\pi}{2}\right]$$

$$F(x) = \frac{1}{\pi}\left[\tan^{-1}x + \frac{\pi}{2}\right]$$

4. A continuous random variable x having probability density function $f(x) = k e^{-|x|}$ find k and distribution function $F(x)$

$e^{-\infty} = 0$

$e^{0} = 1$

$e^{\infty} = \infty$

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{\infty} k e^{-|x|}\, dx = 1$$

$$2k\int_{0}^{\infty} e^{-|x|}\, dx = 1$$

$$2k\int_{0}^{\infty} e^{-x}\, dx = 1$$

$$2k\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = 1$$

$$2k\left[\frac{e^{-\infty}}{-1} - \frac{e^{0}}{-1}\right] = 1$$

$$2k\left[e^{-\infty} + e^{0}\right] = 1$$

$f(x) = \frac{1}{2}e^{+x} \quad -\infty \leq x < 0$

$\quad = \frac{1}{2}e^{-x} \quad 0 < x < \infty$

$$2k\left[0+1\right] = 1$$

$$2k = 1$$

$$\boxed{k = 1/2}$$

When

$x<0$

$x>0$

F

$$F(x) = \int_{-\infty} \frac{e^{-|x|}}{2}\,dx$$

e^{∞} $\quad$ $e^{-x-\infty}$

$-\infty$ $\quad$ 0 $\quad$ ∞

when, $-\infty < x < 0$

$x<0$

$$F(x) = \int_{-\infty}^{x} \frac{e^{x}}{2}\,dx$$

$$= \frac{1}{2}\int_{-\infty}^{x} e^{x}\,dx$$

$$= \frac{1}{2}\left[e^{x}\right]_{-\infty}^{x}$$

$$= \frac{1}{2}\left[e^{x} - e^{-\infty}\right]$$

$$= \frac{1}{2}\left[e^{x} - 0\right]$$

$$= \frac{e^{x}}{2}$$

$x>0$

$$F(x) = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{x} f(x)\,dx$$

$$= \int_{-\infty}^{0} \frac{1}{2}e^{x}\,dx + \int_{0}^{x} \frac{1}{2}e^{-x}\,dx$$

$$= \frac{1}{2}\left[e^{x}\right]_{-\infty}^{0} + \frac{1}{2}\left[\frac{e^{-x}}{-1}\right]_{0}^{x}$$

$$= \frac{1}{2}\left[e^{0} - e^{-\infty}\right] + \frac{1}{2}\left[-e^{-x} + 1\right]$$

$$= \frac{1}{2} - \frac{e^{-x}}{2} + \frac{1}{2}$$

$$= 1 - \frac{e^{-x}}{2}.$$

5. Find the cumulative distributive function of

$$f(x) = \begin{cases} \frac{x}{2} & 0 \le x \le 1 \\ 1/2 & 1 \le x \le 2 \\ 1/2(3-x) & 2 \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

$F(x)$ when $x < 0$

$F(x) = 0$

when $x \le 1$,

$$F(x) = \int_0^x \frac{x}{2}\, dx$$

$$= \left[\frac{x^2}{4}\right]_0^x$$

$$= \frac{x^2}{4}$$

when $1 \le x \le 2$

$$F(x) = \int_0^1 \frac{x}{2}\, dx + \int_1^x \frac{1}{2}\, dx$$

$$= \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^x$$

$$= \frac{1}{4} + \frac{x}{2} - \frac{1}{2}$$

$$= \frac{x}{2} - \frac{1}{4}$$

when $2 \le x \le 3$.

$$F(x) = \int_0^1 \frac{x}{2}\,dx + \int_1^2 \frac{1}{2}\,dx + \int_2^x \frac{3-x}{2}\,dx.$$

$$= \left[\frac{x^2}{4}\right]_0^1 + \left[\frac{x}{2}\right]_1^2 + \frac{1}{2}\int_2^x 3-x\,dx$$

$$= \frac{1}{4} + \frac{2}{2} - \frac{1}{2} + \frac{1}{2}\left[3x - \frac{x^2}{2}\right]_2^x$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[\frac{6x-x^2}{2}\right]_2^x$$

$$= \frac{1}{4} + \frac{1}{2} + \frac{1}{2}\left[\frac{6x-x^2}{2}\right] - \left[\frac{12-4}{2}\right]$$

$$= \frac{1}{4} + \frac{2}{4} + \frac{1}{2}\left[\frac{6x-x^2}{2}\right] - \left[\frac{8}{2}\right]$$

$$= \frac{1}{4} + \frac{2}{4} + \frac{1}{2}\left[\frac{6x-x^2-8}{2}\right]$$

$$= \frac{1}{4} + \frac{2}{4} + \frac{6x-x^2-8}{4}$$

$$= \frac{3}{4} + \frac{6x-x^2-8}{4}$$

$$= \frac{6x-x^2-5}{4}$$

when $F(x) > 3$

$F(x) = 1$, because sum of the probability is 1.

PRATHIKSHA S

6. The cumulative distributive function

$F(x) = 1-(1+x).e^{-x}, x>0.$ Find $f(x)$, mean & variance.

$$f(x) = \frac{d}{dx} F(x)$$

$$= \frac{d}{dx}\left[\underline{1-e^{-x}} - \underset{u\ v}{\underline{xe^{-x}}}\right]$$

$(uv)' = uv_1 - u'v_2 +$

$$= \cancel{e^{-x}} - \cancel{e^{-x}} - x(-e^{-x})$$

$$= xe^{-x} \qquad x>0$$

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$= \int_0^{\infty} x\, x\, e^{-x}\, dx$$

$u = x^2 \quad v = e^{-x}$

$u' = 2x \quad v_1 = -e^{-x}$

$u'' = 2 \quad v_2 = e^{-x}$

$u''' = 0 \quad v_3 = e^{-x}$

$$= \int_0^{\infty} x^2 e^{-x}\, dx$$

$$= \left[x^2(-e^{-x}) - 2xe^{-x} + 2(-e^{-x})\right]_0^{\infty}$$

$$= 2e^0$$

$$= 2.$$

$$E[x^2] = \int_0^{\infty} x^3 e^{-3}\, dx.$$

$u = x^3 \qquad v = e^{-x}$

$u' = 3x^2 \qquad v_1 = -e^{-x}$

$u'' = 6x^3 \qquad v_2 = e^{-x}$

$u''' = 6 \quad v_3 = -e^{-x}$

$u'''' = 0 \quad v_4 = e^{-x}.$

$$E[x^2] = \left[x^3(-e^{-x}) - 3x^2 e^{-x} + 6x(-e^{-x}) - 6(e^{-x})\right]_0^{\infty}$$

$$= [-[-6e^0]]$$

$$= 6.$$

$$Var[x] = E[x^2] - [E[x]]^2$$

$$= 6 - (2)^2$$

$$= 2.$$

MOMENT AND MOMENT GENERATING FUNCTION

Moment about origin :

Let X be a random variable, then r^{th} moment about origin is

$$E[X^r] = \sum x_i^r P(x_i) \text{ if } X \text{ is discrete}$$

Probability mass function

$$E[X^r] = \int_{-\infty}^{\infty} x^r f(x)\,dx \text{ if } X \text{ is continuous.}$$

Probability density function

$E[X^r]$ is represented as μ_r'

i.e. $\mu_1' = \text{mean}$, $\mu_2' = E[X^2]$

$$var[X] = \mu_2' - (\mu_1')^2$$

r^{th} moment about ~~origin~~ mean / central moment :

$$E[X-\bar{x}]^r$$

$$= \sum (x-\bar{x})^r P(x_i) \text{ if } X \text{ is discrete}$$

$$\int_{-\infty}^{\infty} (x-\bar{x})^r f(x)\,dx \text{ if } X \text{ is continuous.}$$

It is represented as μ_r

Moment Generating function

Moment generating function of a random variable about the origin is

$$M_X(t) = E[e^{tx}].$$

$$= \sum e^{tx} p(x_i), \quad x \text{ is discrete.}$$

$$= \int_{-\infty}^{\infty} e^{tx} f(x)\, dx, \quad x \text{ is continuous}$$

properties of moment generating function: (2m)

Let $Y = ax + b$, where X is a random variable with moment generating function $M_X(t)$

$$M_Y(t) = e^{bt} M_X(at)$$

If X and Y are two independent random variable then,

$$M_{X+Y}(t) = M_X(t)\, M_Y(t)$$

$$e^{(X+Y)t} \quad e^{Xt}\, e^{Yt}.$$

Relation between moment generating function & moment about origin.

$$\mu_r' = \frac{d^r}{dt^r}\left[M_X(t)\right]_{t=0}$$

μ_1' at $t=0$, $= mean$

$\mu_2' = E(X^2)$

$$\boxed{M_X(t) = E[e^{tx}]}$$

$$\boxed{\frac{t^r}{r!} \text{ coeff } \mu_r'}$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

1. Find the moment generating function of an random variable x whose probability function is $P(X = x) = \frac{1}{2^x}, x = 1, 2, \dots$ Hence find its mean.

Soln:

The given probability function is a discrete random variable,

$$M_X(t) = \sum e^{tx} P(x_i).$$

$$M_X(t) = \sum_{x=1}^{\infty} e^{tx} \frac{1}{2^x}$$

$$= e^t \frac{1}{2} + e^{2t} \frac{1}{2^2} + e^{3t} \cdot \frac{1}{2^3} + \dots$$

$$= \frac{e^t}{2} \left[1 + \frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \dots \right]$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots$$

$$= \frac{e^t}{2} \left[1 - \frac{e^t}{2} \right]^{-1}$$

$$= \frac{e^t}{2} \left[\frac{2 - e^t}{2} \right]^{-1}$$

$$= \frac{e^t}{2-e^t}$$

$M_x'(t) = \mu_1'$ mean $= \frac{d}{dt}\left[M_X(t)\right]_{t=0}$

$$\frac{u}{v} = \frac{vu' - uv'}{v^2}$$

$$= \frac{d}{dt}\left[\frac{e^t}{2-e^t}\right]_{t=0}$$

$$= \cancel{\frac{d}{dt}\, e^t(}$$

$$= \left[\frac{(2-e^t)\,e^t - e^t\,(-e^t)}{(2-e^t)^2}\right]_{t=0}$$

$$= \frac{(2-1)\,(1)+1}{(2-1)^2}$$

$$= 2.$$

2. If X represent the outcome when a fair die is tossed. Find the moment generating function of X and hence find mean and variance of X.

x	1	2	3	4	5	6
$P(X=x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

$$M_X(t) = \sum_{x=1}^{6} e^{tx}\, P(x_i)$$

$$= \frac{1}{6}e^t + \frac{1}{6}e^{2t} + \frac{1}{6}e^{3t} + \frac{1}{6}e^{4t} + \frac{1}{6}e^{5t} + \frac{1}{6}e^{6t}$$

$$M_X(t) = \frac{1}{6}\left[e^{t} + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}\right]$$

$$M_X'(t) = \mu_1' \text{ (mean)} = \frac{d}{dt}\left[M_X(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[\frac{1}{6}\left[e^{t} + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}\right]\right]_{t=0}$$

$$= \frac{1}{6}\left[e^{t} + 2e^{2t} + 3e^{3t} + 4e^{4t} + 5e^{5t} + 6e^{6t}\right]_{t=0}$$

$$= \frac{1}{6}\left[e^{0} + 2e^{0} + 3e^{0} + 4e^{0} + 5e^{0} + 6e^{0}\right]$$

$$= \frac{1}{6}\left[1+2+3+4+5+6\right]$$

$$= 21/6$$

$$\boxed{M_X''(t) = \mu_2' \cancel{(\text{Variance})} = E[X^2]}$$

$$M_X''(t) = \frac{1}{6}\left[e^{t} + 4e^{2t} + 9e^{3t} + 16e^{4t} + 25e^{5t} + 36e^{6t}\right]_{t=0}$$

$$= \frac{1}{6}\left[1+4+9+16+25+36\right]$$

$$= \frac{1}{6}\left[91\right]$$

$$= 91/6$$

$$\boxed{Var(X) = \mu_2' - (\mu_1')^2}$$

$$= \frac{91}{6} - \left(\frac{21}{6}\right)^2$$

$$= \frac{546}{36} - \frac{441}{36}$$

$$= \frac{105}{36}$$

3. If X is a no. of heads obtaining four toss of outcome of a coin. Find moment generating function, mean...

$S = \{$ HHHH, HHHT, HHTH, HTHH, THHH,
TTTT, TTTH, TTHT, THTT, HTTT,
HTTH, THHT, HTHT, THTH, TTHH, HHTT $\}$

$P(S) = \frac{1}{16}$

x	0	1	2	3	4
$P(x)$	1/16	4/16	6/16	4/16	1/16

$$M_X(t) = \sum_{x=0}^{4} e^{tx} P(x_i)$$

$$= e^0 \frac{1}{16} + e^t \frac{4}{16} + e^{2t} \frac{6}{16} + e^{3t} \frac{4}{16} + e^{4t} \frac{1}{16}$$

$$= \frac{1}{16}\left[e^0 + 4e^t + 6e^{2t} + 4e^{3t} + e^{4t}\right]$$

Mean $= E[X] = M_X'(t)/_{t=0}$

$$M_X'(t) = \frac{1}{16}\left[0 + 4e^t + 12e^{2t} + 12e^{3t} + 4e^{4t}\right]$$

$$M_X'(t) = \frac{1}{16}\left[4 + 12 + 12 + 4\right]$$

$$= \frac{32}{16}$$

$$E[X^2] = M_X''(t)/_{t=0}$$

$$M_X''(t) = \frac{1}{16}\left[4e^{t} + 24e^{2t} + 36e^{3t} + 16e^{4t}\right]$$

$$M_X''(t) = \frac{1}{16}\left[4 + 24 + 36 + 16\right]$$

$$= 5$$

$$Var(X) = \mu_2' - (\mu_1')^2$$

$$= 5 - 4$$

$$= 1$$

4. Let X be a random variable of a probability density function, $f(x) = \begin{cases} \frac{1}{3} e^{-x/3} & , x > 0 \\ 0 & \text{otherwise.} \end{cases}$

Find i) $P(X > 3)$

ii) m.g.f

iii) $E[X]$, $var[X]$.

i) $$P(X>3) = \int_3^{\infty} \frac{1}{3} e^{-x/3}\, dx$$

$$= \frac{1}{3}\left[\frac{e^{-x/3}}{-1/3}\right]_3^{\infty}$$

$$= -e^{-\infty} + e^{-1}$$

$$= 1/e$$

$$M_X(t) = \int_0^\infty \frac{e^{tx} e^{-x/3}}{3} dx$$

$$= \frac{1}{3} \int_0^\infty e^{-x[1/3 - t]} dx$$

$$= \frac{1}{3} \left[\frac{e^{x[1/3 - t]}}{-(1/3 - t)} \right]_0^\infty$$

$$= \frac{1}{3} \left[\frac{e^{-\infty}}{-[1/3 - t]} + \frac{e^0}{1/3 - t} \right].$$

$$= \frac{1}{\cancel{3}} \frac{1}{\frac{1-3t}{\cancel{3}}}$$

multiplied by -

$$= \frac{1}{1-3t}$$

$$\cancel{Var[X] = \mu_2' - (\mu_1')^2}$$

$$\cancel{= 5 - 4}$$

$$\cancel{= 1}$$

$$M_X'(t) = -\frac{1}{(1-3t)^2}(-3)$$

$$= \frac{3}{(1-3t)^2}$$

$$= 2$$

$$M_x''t = 2\frac{(-2)}{(1-3t)^3}(-3) \qquad \left[\frac{u}{v} = \frac{vu'-uv'}{v^2}\right]$$

$$= \frac{18}{(1-3t)^3}$$

$$= 18$$

Probability density function of a random variable x is $f(x) = \begin{cases} \frac{x}{2}, & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$ Find the moment generating function & 1st two moments about the origin.

$$M_x(t) = \frac{1}{2}\int_0^2 e^{tx}\, x\, dx.$$

$u = x \qquad V_1 = e^{tx}$

$u' = 1 \qquad V_2 = \frac{e^{tx}}{t}$

$u'' = 0 \qquad V_3 = \frac{e^{tx}}{t^2}$

$$M_x(t) = \left[\frac{xe^{tx}}{t} - \frac{e^{tx}}{t^2}\right]_0^2$$

$$\frac{2te^{2t}}{t^2} - \frac{e^{2t}}{t^2} + \frac{1}{t^2}$$

$$= \frac{2e^{2t}}{t}\frac{\times t}{\times t} - \frac{e^{2t}}{t^2} - 0 + \frac{1}{t^2} \qquad \frac{1}{t^2}\left(1 - e^{2t} + 2te^{2t}\right)$$

$$= \frac{1}{2t^2}\left[1 + 2te^{2t} - e^{2t}\right].$$

$$E[X] = \int_0^2 x\, f(x)\, dx$$

$$= \int_0^2 x \cdot \frac{x}{2}\, dx$$

$$= \frac{1}{2} \int_0^2 x^2\, dx$$

$$= \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{8}^{\,4}}{3}\right]$$

$$= \frac{4}{3}.$$

$$E[X^2] = \int_0^2 x^2\, f(x)\, dx$$

$$= \int_0^2 x^2 \cdot \frac{x}{2}\, dx$$

$$= \frac{1}{2} \int_0^2 x^3\, dx$$

$$= \frac{1}{2}\left[\frac{x^4}{4}\right]_0^2$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{16}^{\,4}}{\cancel{4}}\right]^2$$

$$= 2.$$

The density

by $f(x)$

variance

$\int_{-\infty}^{\infty} f(x$

$\int_0^2 K$

K

$r^{th}\ \mu_r$

μ_r'

The density function of a random variable x is given by $f(x) = kx(2-x),\ 0 \le x \le 2$. Find k, mean, variance & r^{th} moment.

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_0^2 k(2x - x^2)\,dx = 1$$

$$k\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$k\left[4 - \frac{8}{3}\right] = 1$$

$$\frac{4}{3}k = 1$$

$$k = \frac{3}{4}$$

r^{th} $\mu_r' = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$$\mu_r' = \int_0^2 x^r \frac{3}{4}(2x - x^2)\,dx$$

$$= \frac{3}{4}\int_0^2 2x^{r+1} - x^{r+2}\,dx$$

$$= \frac{3}{4}\left[2\frac{x^{r+2}}{r+2} - \frac{x^{r+3}}{r+3}\right]_0^2$$

$$= \frac{3}{4}\left[2\frac{2^{r+2}}{r+2} - \frac{2^{r+3}}{r+3}\right]$$

$$= \frac{3}{4}\left[\frac{2^{r+3}}{r+2} - \frac{2^{r+3}}{r+3}\right]$$

$$= \frac{3}{4}\left[2^{r+3}\right]\left[\frac{1}{r+2} - \frac{1}{r+3}\right]$$

$$= \frac{3}{4}\, 2^{r+3}\left[\frac{r+3-r-2}{(r+2)(r+3)}\right]$$

$$= \frac{3}{4}\, 2^{r+3}\left[\frac{1}{(r+2)(r+3)}\right]$$

$$\text{Mean} = \mu_1' = \frac{3}{\cancel{4}} \cdot 2^{\cancel{4}\,2} \cdot \frac{1}{3\times 4}$$

$$= \frac{3\times 4}{3\times 4}$$

$$= 1$$

$$E[x^2] = \mu_2' = \frac{3}{\cancel{4}} \cdot 2^{\cancel{5}\,3} \cdot \frac{1}{4\times 5}$$

$$= \frac{3\times \cancel{8}\,2}{\cancel{4}\times 5}$$

$$= \frac{6}{5}$$

Function

$f(x)$

$y = G(x)$

2M

1.

I

with

$f(x) = $

when

when

Functions of RV.

If x is a random variable with pdf $f(x)$ and Y is a random variable such that $y = G(x)$ then the pdf of Y is.

$$f(y) = f(x) * \left|\frac{dx}{dy}\right|$$

2M

1. If X is a continuous random variable with the interval $(0,2)$ and $y = 4x+3$ and $f(x) = \frac{1}{2}$. Find $f(y)$.

$$f(y) = f(x) * \left|\frac{dx}{dy}\right|$$

$$\frac{dy}{dx} = 4$$

$$\frac{dx}{dy} = \frac{1}{4}$$

$$f(y) = \frac{1}{2} * \frac{1}{4}$$

$$= \frac{1}{8}$$

when $x = 0$,

$$y = 4x+3$$

$$y = 3$$

when $x = 2$

$$y = 4(2)+3$$

$$y = 11$$

$$f(y) = \frac{1}{8}, \quad (3,11)$$

Consider a random variable X with pdf $f(x) = e^{-x}; x > 0$ with the transformation $y = e^{-x}$. Find $f(y)$

$$f(y) = f(x) \times \left|\frac{dx}{dy}\right|$$

$$\frac{dy}{dx} = -e^{-x}$$

$$\frac{dx}{dy} = -\frac{1}{e^{-x}}$$

$$\left|\frac{dx}{dy}\right| = \frac{1}{e^{-x}}$$

$$f(y) = e^{-x} \times \frac{1}{e^{-x}}$$

$$= 1$$

$$0 \le x \le \infty$$

when $x = 0$

$y = 1$

$x = \infty$

$y = 0$

$f(y) = 1 \quad 0 \le x \le 1.$

A RV X has the Pdf $x = \frac{1}{\pi}$ with the transformation

$y = \tan x$, x lies with in the interval $(-\pi/2, \frac{\pi}{2})$

$$f(y) = f(x) * \left|\frac{dx}{dy}\right|$$

$$\frac{dy}{dx} = \sec^2 x.$$

$$\frac{dx}{dy} = \frac{1}{\sec^2 x}$$

$$= \frac{1}{1+\tan^2 x}$$

$$= \frac{1}{1+y^2}$$

$$f(y) = \frac{1}{\pi} * \frac{1}{1+y^2}$$

when $x = -\frac{\pi}{2}$

$$y = \tan\left(-\frac{\pi}{2}\right) = -\infty$$

$$y = \tan\left(\frac{\pi}{2}\right) = \infty$$

$$f(y) = \frac{1}{\pi(1+y^2)} \quad -\infty \le x \le \infty$$

BINOMIAL DISTRIBUTION. $p+q=1$

A random variable x is said to follow binomial distribution if it assumes only non-negative values and its probability mass function

$$P(X=x) = f(x) = \begin{cases} n_{C_x} p^x q^{n-x} & x = 0,1,2,\ldots \\ 0 & \text{otherwise} \end{cases}$$

Here n and p are called parameters of the distribution.

Standard deviation $= \sqrt{\text{var}}$ $\qquad {}^nC_0 = 1,\ {}^nC_n = 1$

Moment generating function of a Binomial distribution

$$M_X(t) = \sum_{x=0}^{\infty} e^{tx} f(x)$$

$$= \sum_{x=0}^{\infty} e^{tx}\, n_{C_x} p^x q^{n-x}$$

$$= e^0 n_{C_0} p^0 q^n + e^t n_{C_1} p^1 q^{n-1} + e^{2t} n_{C_2} p^2 q^{n-2} + \ldots + e^{nt} n_{C_n} p^n q^{n-n}$$

$$= q^n + n_{C_1}(e^t p)^1 q^{n-1} + n_{C_2}(e^t p)^2 q^{n-2} + \ldots + (pe^t)^n$$

$$= (e^t p + q)^n$$

mean $= E[X$

$E[X^2] =$

$\text{Var}[X] =$

$$\text{mean} = E[X] = \frac{d}{dt}\left[M_X(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[e^t p + q\right]^n_{t=0}$$

$$= n(e^t p + q)^{n-1} e^t p \Big/_{t=0}$$

$$= n[p+q]^{n-1} p$$

$$= n(1)^{n-1} p$$

$$= np.$$

$$E[X^2] = \frac{d^2}{dt^2}\left[M_X(t)\right]_{t=0}$$

$$= \frac{d}{dt}\left[npe^t (e^t p + q)^{n-1}\right]_{t=0}$$

$$= np\left[e^t (n-1)(e^t p + q)^{n-2} e^t p + (e^t p + q)^{n-1} e^t\right]_{t=0}$$

$$= np\left[(n-1)(p+q)^{n-2} p + (p+q)^{n-1}(1)\right]$$

$$= np\left[(n-1)p + 1\right]$$

$$= np[np - p + 1]$$

$$= n^2p^2 - np^2 + np.$$

$$\text{Var}[X] = E[X^2] - [E[X]]^2$$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$= -np^2 + np$

$= np[1-p]$

$= npq$

2m

1. The mean of a Binomial distribution is [illegible] and standard deviation is 4. Determine the parameters of the distribution.

$np = mean = 20 \rightarrow (1)$

$sd = 4$

$\sqrt{var} = sd$

$var = (sd)^2$

$var(x) = 16$

$npq = 16$

$20q = 16$

$q = \frac{16}{20} = \frac{4}{5}$

$p = 1 - q$

$= 1 - \frac{4}{5}$

$\boxed{p = 1/5}$

$np = 20$

$n = 20 \times 5$

$\boxed{n = 100}$

binomial distribution to have
mean 2 and variance 4.

mean $\Rightarrow np = 3$

variance $\Rightarrow npq = 4$

$3q = 4$

$q = 4/3$

3. The mean & the variance of a binomial distribution is 8 and 6 respectively. Find $P(X) \geq 2$

mean $\Rightarrow np = 8$

variance $\Rightarrow npq = 6$

$8q = 6$

$q = \cancel{6}^{3}/\cancel{8}_{4}$

$\boxed{q = 3/4}$

$p + q = 1$

$p + \frac{3}{4} = 1$

$\frac{4p+3}{4} = 1$

$4p + 3 = 4$

$4p = 4 - 3$

$4p = 1$

$\boxed{p = 1/4}$

$\Rightarrow np = 8$

$n\frac{1}{4} = 8$

$$\frac{n}{4} = 8$$

$$n = 32$$

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$P(X \geq 2) = 1 - P(X < 2)$$

$$= 1 - [P(X=0) + P(X=1)]$$

$$= 1 - \left[{}^{32}C_{0} \left(\frac{1}{4}\right)^{0} \left(\frac{3}{4}\right)^{32-0} + {}^{32}C_{1} \left(\frac{1}{4}\right) \left(\frac{3}{4}\right)^{31} \right]$$

$$= 1 - \left[{}^{32}C_{0} \left(3/4\right)^{32} + {}^{32}C_{0} \left(\frac{1}{4}\right) \left(\frac{3}{4}\right. \right.$$

$$= 1 - \left(\frac{3}{4}\right)^{31} \left[\frac{3}{4} + \frac{32}{4} \right]$$

$$= 1 - \left(\frac{3}{4}\right)^{31} \left[\frac{35}{4} \right]$$

BISON DISTRIBUTION.

Four coins are tossed simultaneously what is the probability of getting (i) 2 heads (ii) atleast two heads (iii) atmost two heads.

i) Probability of getting head $p = 1/2$

$$q = 1 - 1/2 = 1/2$$

$$n = 4$$

$$P(X=x) = {}^{n}C_{x}\, p^{x} q^{n-x}$$

$$P(X=2) = {}^{4}C_{2} \cdot (1/2)^{2} (1/2)^{4-2}$$

$$= {}^{4}C_{2} \, (1/2)^{2} (1/2)^{2}$$

$$= {}^{4}C_{2} \, (1/2)^{4}$$

$$= \frac{\cancel{4}^{2}}{1} \cdot \frac{3}{\cancel{2}} \cdot \frac{1}{2^{4}}$$

$$= \frac{\cancel{6}^{3}}{\cancel{2^4}^{3}}$$

$$= \frac{3}{2^{3}}$$

$$= \frac{3}{8}$$

ii) Probability of getting atleast twoheads

$$P(x \geq 2) = P(X=2) + P(X=3) + P(X=4)$$

$$= {}^{4}C_{2} \, (1/2)^{2} (1/2)^{4-2} + {}^{4}C_{3} \, (1/2)^{3} (1/2)^{4-3}$$

$$+ {}^{4}C_{4} \, (1/2)^{4} (1/2)^{0}$$

$$= \frac{3}{8} + \frac{4 \times \cancel{3} \times \cancel{2}}{1 \times \cancel{2} \times \cancel{3}} \left(\tfrac{1}{2}\right)^4 + 1 \left(\tfrac{1}{2}\right)^4$$

$$= \frac{3}{8} + \frac{1}{4} + \frac{1}{16}$$

$$= \frac{6+4+1}{16}$$

$$= \frac{11}{16}.$$

probability of getting atmost two heads $P(X \le 2)$

$$= P(X=0) + P(X=1) + P(X=2)$$

$$= {}^4C_0 \left(\tfrac{1}{2}\right)^0 \left(\tfrac{1}{2}\right)^{4-0} + {}^4C_1 \left(\tfrac{1}{2}\right)^1 \left(\tfrac{1}{2}\right)^{4-1}$$

$$+ 3/8$$

$$= \left(\tfrac{1}{2}\right)^4 + 4\left(\tfrac{1}{2}\right)^4 + \frac{3}{8}.$$

$$= \frac{1}{2^4} + \frac{4}{2^4} + \frac{3}{8}$$

$$= \frac{1}{2^4} + \frac{4}{2^4} + \frac{3 \times 2}{2^4}$$

$$= \frac{1+4+6}{2^4}$$

$$= \frac{11}{16}.$$

$)^4$

$P(X \leq 2)$

2. A pair of dice is thrown 4 times if getting a doublet is consider a success find the probability of two success.

probability of getting double $= \frac{6}{36} = \frac{1}{6}$

$p = \frac{1}{6}$; $q = 1 - 1/6$

$= 5/6$

$n = 4$

Probability of getting two success $= P(X=2)$

$= {}^4C_2 (1/6)^2 (5/6)^{4-2}$

$= \frac{4 \times 3}{1 \times 2} \times \frac{1}{6} \times \frac{1}{6} \times \frac{5}{6} \times \frac{5}{6}$

$= \frac{25}{216}$.

A large consignment of electric bulb 10% are defective A random sample of 20 is taken for inspection Find i) the probability that all are good bulbs.

ii) almost three is defective

iii) Exactly three is defective.

p is probability of getting defective, $p = \frac{10}{100} = \frac{1}{10}$

$n = 20$

$p = \frac{1}{10}$, $q = 1 - \frac{1}{10} = \frac{9}{10}$.

i) $P(X=0) = {}^{n}C_x \, p^x \, q^{n-x}$

$$= {}^{20}C_0 \, p^0 \, q^{20-0}$$

$$= {}^{20}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20}$$

$$= \left(\frac{9}{10}\right)^{20}$$

${}^{n}C_n = 1$

${}^{n}C_0 = 1$

${}^{n}C_1 = n$

ii) atmost three is defective:

$$P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= {}^{20}C_0 \left(\frac{1}{10}\right)^0 \left(\frac{9}{10}\right)^{20-0} + {}^{20}C_1 \left(\frac{1}{10}\right)^1 \left(\frac{9}{10}\right)^{20-1}$$

$$+ {}^{20}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^{20-2} + {}^{20}C_3 \left(\frac{1}{10}\right)^3 \left(\frac{9}{10}\right)^{20-3}$$

$$\frac{20 \times 19}{1 \times 2} \times \frac{1}{10} \times \frac{1}{10} \times \left(\frac{9}{10}\right)^{18}$$

$$= \left(\frac{9}{10}\right)^{20} + 20 \left(\frac{1}{10}\right) \left(\frac{9}{10}\right)^{19} + \frac{19}{10} \left(\frac{9}{10}\right)^{18} + \frac{57}{50} \left(\frac{9}{10}\right)^{17}$$

$$= \left(\frac{9}{10}\right)^{20} + 2 \left(\frac{9}{10}\right)^{19} + \frac{19}{10} \left(\frac{9}{10}\right)^{18} + \frac{57}{50} \left(\frac{9}{10}\right)^{17}$$

iii) Exactly 2,

$$P(X=2) = {}^{n}C_x \, p^x \, q^{n-x}$$