~~Define~~

Geometric distribution

Suppose that, ~~geometric di~~ independent trails each having probability P, $0<P<1$ of being a success is performed until a success occur

If x equal no of trails require then

PMf

$$p(X=x) = q^{x-1}p. \quad x = 1, 2, 3, \dots$$

Moment generating function of a Geometric distribution.

$$E[e^{tx}] = \sum_{x=1}^{\infty} e^{tx} p(x)$$

$$= \sum_{x=1}^{\infty} e^{tx} q^{x-1} p$$

e^{tx-t} $\frac{e^{tx}}{e^t} \times e^t$

$\because e^{tx} = e^{t(x-1)} e^t$

$= e^{tx} e^{-t} e^{t}$

$\frac{e^{tx} q^{x-1}}{e^t (e^t q)^{x-1}}$

$$= \sum_{x=1}^{\infty} e^t (e^t q)^{x-1} p$$

$e^{tx} . q^x . q^{-1}$

$(e^t . q)^{x-1}$

$$= pe^t \sum_{x=1}^{\infty} (e^t q)^{x-1}$$

$e^{tx} q^x q^{-1}$

$(e^t q)^x q^{-1}$

$$= pe^t [1 + e^t q + (e^t q)^2 + \dots]$$

$e^t (e^t q)^{x-1} p$

$$= pe^t [1 - e^t q]^{-1}$$

$$= \frac{pe^t}{1 - e^t q}$$

$$= \frac{p}{e^{-t}[1 - e^t q]}$$

$$= \frac{p}{e^{-t} - q}$$

to find mean, $M_x'(t) \Rightarrow$

we have $M_x(t) = \dfrac{p}{e^{-t} - q}$ $\quad \dfrac{u}{v} \quad \dfrac{vu' - uv'}{v^2}$

$$M_x'(t) = \frac{(e^{-t} - q) \cdot (0) - p(-e^{-t})}{(e^{-t} - q)^2} \Bigg/_{t=0}$$

$$= \frac{0 + p}{(1-q)^2} \qquad 1 - q = p$$

$$= \frac{p}{p^2}$$

$E[X] = \text{mean} = 1/p$

$$M_x''(t) = \frac{d}{dt}\left[\frac{pe^{-t}}{(e^{-t} - q)^2}\right]$$

$$= \frac{(e^{-t} - q)^2 [-pe^{-t}] - [pe^{-t}]\, 2(e^{-t} - q)(-e^{-t})}{(e^{-t} - q)^4}$$

$$= \frac{(1-q)^2(-p) - p\,2(1-q)(-1)}{(1-q)^4}$$

$$= \frac{-p^3 + 2p^2}{p^4} = \frac{\cancel{p^2}[2-p]}{\cancel{p^4}\,2}$$

$$= \frac{2-p}{p^2} = \frac{2-p}{p^2}$$

$$var[X] = E[X^2] - (E[X])^2$$

$$= \frac{2-p}{p^2} - \frac{1}{p^2}$$

$$= \frac{1-p}{p^2}$$

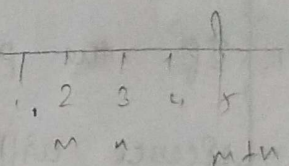
$$var[X] = q/p^2$$

Memoryless property of a geometric distribution 2m

let X be a geometric distribution then for any positive integer m and n $P[X > m+n / X > m] = P[X > n]$ (such that)

$$\because P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{P(X > m+n \cap X > m)}{P(X > m)}$$



$$= \frac{P(X > m+n)}{P(X > m)}$$

for a geometric distribution

$$P(X = r) = pq^{r-1}$$

$$P(X > k) = \sum_{r=k+1}^{\infty} pq^{r-1}$$

$$= pq^k + pq^{k+1} + pq^{k+2} + \cdots$$

$$= pq^k[1 + q + q^2 + \cdots]$$

$$= pq^k[1-q]^{-1}$$

$$= p q^k p^{-1}$$

$$= q^k$$

$$P(X > m+n) = q^{m+n}$$

$$P(X > n) = q^n$$

$$P(X > m) = q^m.$$

$$\frac{P(X > m+n)}{P(X > m)} = \frac{q^{m+n}}{q^m}$$

$$= q^n$$

$$= P(X > n)$$

1. If the probability that an applicant for a drivers licence will pass the road test on any given trial is 0.8 what is the probability that he will pass the test in

i) fourth trial

ii) in fewer than fourth trial

$$P = 0.8, \quad q = 1 - P = 1 - 0.8 = 0.2$$

i) $$P(X = 4) = q^{4-1} P$$

$$= (0.2)^3 (0.8)$$

$$= (0.04)(0.2)(0.8)$$

$$= (0.04)(0.16)$$

$$= 0.0064$$

ii) $P(X < 4) = P(X=1) + P(X=2) + P(X=3)$

$= q^{1-1}p + q^{2-1}p + q^{3-1}p$

$= q^0 (0.8) + (0.2)(0.8) + (0.04)(0.8)$

$= 0.8 + 0.16 + 0.032$

$= 0.992$ $\boxed{0.9984}$

2. Let 1 copy of magazine out of 10 copies bears a special price following geometric distribution. Find its mean & variance.

Mean $= 1/p$ $p = 1/10$ $p + q = 1$

$var[X] = q/p^2$ $q = 1-p$

$q = 1 - \frac{1}{10}$

mean $p = \frac{1}{\frac{1}{10}} = 10$ $q = 9/10$

$var[X] = \frac{9/10}{(1/10)^2} = \frac{9}{10} \times \frac{100}{1} = 90.$

3. If the probability is 0.05 that a certain kind of measuring device will show excessive drift, what is the probability that the sixth of the test will show excessive drift.

$p = 0.05$ $q = 1 - 0.05 = 0.95$

$P(X=6) = q^{6-1}p$

$= q^5 p = (0.95)^5 (0.05)$

$= 0.77378 (0.05)$

$= 0.038689046$

UNIFORM DISTRIBUTION:

A random variable x is said to have continuous uniform distribution over an interval (a, b) if its probability density function is given (constant)

by $f(x) = \begin{cases} \frac{1}{b-a}, & a < x < b \\ 0, & \text{otherwise} \end{cases}$

MOMENT GENERATING FUNCTION OF UNIFORM DISTRIBUTION.

$$M_x(t) = \int_a^b e^{tx} f(x)\, dx$$

$$= \int_a^b \frac{e^{tx}}{b-a}\, dx$$

$$= \frac{1}{b-a} \left[\frac{e^{tx}}{t}\right]_a^b$$

$$= \frac{1}{(b-a)t} (e^{tx})_a^b$$

$$= \frac{1}{(b-a)t} (e^{bt} - e^{at})$$

$$= \frac{1}{(b-a)t} \left[\left[1 + \frac{bt}{1!} + \frac{(bt)^2}{2!} + \frac{(bt)^3}{3!} + \dots\right] - \left[1 + \frac{at}{1!} + \frac{(at)^2}{2!} + \frac{(at)^3}{3!} + \dots\right]\right]$$

$$= \frac{1}{(b-a)t} \left[\frac{t(b-a)}{1!} + \frac{t^2[b^2 - a^2]}{2!} + \frac{t^3[b^3 - a^3]}{3!} + \dots\right]$$

$$= 1 + \frac{t^2(b+a)(b-a)}{(b-a)t} + \frac{t^3(b-a)(a^2+ab+b^2)}{(b-a)t\,2!} + \cdots$$

$$= 1 + \frac{t(b+a)}{2!} + \frac{t^2(a^2+ab+b^2)}{3!} + \cdots$$

$$M_x'(t) = 0 + \frac{(b+a)}{2!} + \frac{2t(a^2+ab+b^2)}{3!} + \cdots$$

$$M_x'(t)/_{t=0} = \frac{b+a}{2}$$

$$= \text{mean} \Rightarrow E[x]$$

$$M_x''(t) = 0 + 0 + \frac{2(a^2+ab+b^2)}{3!} + \cdots$$

$$M_x''(t)/_{t=0} = \frac{a^2+ab+b^2}{3}$$

$$Var[x] = E[x^2] - [E(x)]^2$$

$$= \frac{a^2+ab+b^2}{3} - \frac{(b+a)^2}{4}$$

$$= \frac{a^2+ab+b^2}{3} - \frac{(b^2+a^2+2ab)}{4}$$

$$= \frac{4a^2+4ab+4b^2-3b^2-3a^2-6ab}{12}$$

$$= \frac{a^2+b^2-2ab}{12}$$

$$= \frac{(a-b)^2}{12}$$

If X is uniformly distributed over (0,10). Calculate the probability that $X<3$, $X>6$, $3<X<8$.

$$f(x) = \frac{1}{b-a} = \frac{1}{10-0} = \frac{1}{10}$$

$$P(X<3) = \int_0^3 \frac{1}{10}\,dx$$

$$= \left[\frac{x}{10}\right]_0^3$$

$$= \frac{3}{10}.$$

$$P(X>6) = \int_6^{10} \frac{1}{10}\,dx$$

$$= \left(\frac{x}{10}\right)_6^{10}$$

$$= \frac{10}{10} - \frac{6}{10}$$

$$P(X>6) = \frac{4}{10}.$$

$$P(3<X<8) = \int_3^8 \frac{1}{10}\,dx$$

$$= \left(\frac{x}{10}\right)_3^8$$

$$= \frac{8}{10} - \frac{3}{10}$$

$$= \frac{\cancel{5}\,1}{\cancel{10}\,2}.$$

$$= \frac{1}{2}$$

A random variable X has uniform distribution over $(-3, 3)$ compute probability

i) $P(X < 2)$

ii) $P(|X| < 2)$

iii) $P(|X-2| < 2)$

iv) find k if $P(X > k) = \frac{1}{3}$.

$$f(x) = \frac{1}{b-a}$$

$$= \frac{1}{3-(-3)} = \frac{1}{6}.$$

i) $$P(X<2) = \int_{-3}^{2} \frac{1}{6}\,dx$$

$$= \left[\frac{x}{6}\right]_{-3}^{2}$$

$$= \frac{2}{6} + \frac{3}{6}$$

$$= \frac{5}{6}$$

$$P(X<2) = \frac{5}{6}$$

ii) $$P(|X|<2) \Rightarrow P(-2<X<2) = \int_{-2}^{2} \frac{1}{6}\,dx = \left(\frac{x}{6}\right)_{-2}^{2}$$

$$= \frac{2}{6} + \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$$

iii) $$P(|X-2|<2) \Rightarrow P(-2 < X-2 < 2)$$

$$= P(0 < X < 4) = P(0 < X < 3) = \int_{0}^{3} \frac{1}{6}\,dx$$

$$= 3/6 = 1/2$$

iv) $P(X > K) = 1/3$

$$\int_K^3 1/6 \, dx = 1/3$$

$$\left(\frac{x}{6}\right)_K^3 = 1/3$$

$$\frac{3}{6} - \frac{K}{6} = \frac{1}{3}$$

$$\frac{3-K}{\cancel{6}2} = \frac{1}{\cancel{3}}$$

$$3 - K = 2$$

$$-K = 2 - 3$$

$$-K = -1$$

$$\boxed{K = 1}$$

Buses arrives at a specified bus stop at 15 minutes interval starting at 7am. If a passenger arrives at the bus stop at a random time which is uniformly distributed between 7 and 7.30 am. Find the probability that he waits

i) less than 5 minutes.

ii) atleast 12 minutes for bus.

Let X denotes the time that a passenger arrives between 7 and 7.30 am.

$$f(x) = \frac{1}{b-a} = \frac{1}{30-0} = \frac{1}{30}.$$

7.00 - 7.15 - 7.30

i) 0 - 15 - 30

10-15 25-30

$P(10 < x < 15) + P(25 < x < 30)$

$$= \int_{10}^{15} \frac{1}{30} dx + \int_{25}^{30} \frac{1}{30} dx$$

$$= \frac{15}{30} - \frac{10}{30} + \frac{30}{30} - \frac{25}{30}$$

$$= \frac{45}{30} - \frac{35}{30}$$

$$= \frac{10}{30}$$

$$= 1/3$$

ii) atleast 12 minutes

$P(0 < x < 3) + P(15 < x < 18)$

$$= \frac{1}{30}\left[\int_0^3 dx + \int_{15}^{18} dx\right]$$

$$= \frac{1}{30}(3+3)$$

$$= \frac{6}{30}$$

$$= 1/5.$$

— x —

4. A passenger arrives at a local railway platform at 10 AM knowing that the local train will arrive at sometime uniformly distributed between 10 AM & 10:30 AM

a) what is the probability that he will have to wait longer than 10 minutes.

b) If at 10.15 AM the train has not arrived what is the probability that he'll have to wait atleast 10 additional mins.

$$f(x) = \frac{1}{b-a} = \frac{1}{30-0} = \frac{1}{30}$$

i) waiting longer than 10 minutes

$$P(X>10) = P(10 < X < 30) = \int_{10}^{30} \frac{1}{30}\,dx = \frac{20}{30} = 2/3$$

ii) Probability the passenger waits additional 10 minutes when the train have not arrived at 10:15

$$P(X>15+10) / P(X>15) = P(X>25) / P(X>15)$$

$$= \frac{P(X>25)}{P(X>15)}$$

$$= \frac{1}{30}\int_{25}^{30} dx \Big/ \frac{1}{30}\int_{15}^{30} dx$$

$$= \frac{5}{15}$$

$$= 1/3$$

Exponential distribution:

A continuous r.v X is said to follow an exponential distribution is its probability density function is given by,

$$f(x) = \begin{cases} \alpha e^{-\alpha x} & , x \geq 0, \alpha > 0 \\ 0 & , \text{otherwise.} \end{cases}$$

Moment Generating function:

$$M_X(t) = \int_0^t e^{tx} f(x)\, dx = \int_0^\infty e^{tx} \alpha e^{-\alpha x} dx.$$

$$= \alpha \int_0^\infty e^{-x(\alpha - t)} dx$$

$$= \alpha \left[\frac{e^{-x(\alpha-t)}}{-(\alpha-t)} \right]_0^\infty$$

$$= \alpha \left[e^{-\infty} - \frac{e^0}{-(\alpha-t)} \right]$$

$$= \alpha \left[0 - \frac{1}{-(\alpha-t)} \right]$$

$$= \frac{\alpha}{\alpha - t}$$

$$\text{Mean} = E[X] = M_X'(t) /_{t=0}$$

$$= \frac{-\alpha(-1)}{(\alpha-t)^2} = \frac{\alpha}{(\alpha-t)^2}$$

$$M_X'(t)/_{t=0} = \frac{\alpha}{(\alpha-0)^2} = \frac{\alpha}{\alpha^2} = \frac{1}{\alpha}$$

$$M_X''(t) = \frac{d}{dt}\left(\frac{\alpha}{(\alpha-t)^2}\right) = \alpha\left(\frac{-2(-1)}{(\alpha-t)^3}\right) = \frac{2\alpha}{(\alpha-t)^3}$$

$$M_X''(t)/_{t=0} = \frac{2\alpha}{\alpha^3} = \frac{2}{\alpha^2}, \quad Var(X) = \frac{2}{\alpha^2} - \frac{1}{\alpha^2} = \frac{1}{\alpha^2}$$

The mileage which carowners get a certain kind of radial tyre is a r.v having a exponential distribution which mean 40,000 km, find the probability that one of this tiree will lost

i) atleast 20000 km

ii) atmost 30000 km.

$$\text{Mean} = 40,000$$

$$1/\alpha = 40,000$$

$$\alpha = \frac{1}{40,000}$$

$$f(x) = \alpha e^{-\alpha x}; \quad x \geq 0.$$

$$= \frac{1}{40,000} e^{-\frac{x}{40,000}}$$

atleast 20,000 km

$$P(X \geq 20,000) = \int_{20,000}^{\infty} \frac{1}{40,000} e^{-\frac{x}{40,000}} dx$$

$$= \frac{1}{40000} \left[\frac{e^{-\frac{x}{40000}}}{-\frac{1}{40000}} \right]_{20,000}^{\infty}$$

$$= \left[-e^{-\infty} - \left[-e^{-1/2} \right] \right]$$

$$= e^{-5}$$

$$= 0.606.$$

ii) almost 30,000.

$$P[X \le 30{,}000] = \frac{1}{40{,}000} \int_0^{30{,}000} e^{-\frac{x}{40000}}\, dx$$

$$= \frac{1}{\cancel{40{,}000}} \left[\frac{e^{-\frac{x}{40{,}000}}}{-\cancel{1/40{,}000}} \right]_0^{30{,}000}$$

$$= -e^{-3/4} + e^{0}$$

$$= 0.527.$$

The length of time a person speaks over phone follows exponential distribution with mean 1/6 what is the probability that a person talk for

.2636

i) more than 8 minutes

ii) between 4 and 8 minutes.

$$\text{mean} = \frac{1}{6}.$$

$$\frac{1}{\alpha} = 6$$

$$\Rightarrow \alpha = 1/6$$

$$f(x) = \alpha e^{-\alpha x}$$

$$= \frac{1}{6} e^{-\frac{1}{6}x}$$

$$= \frac{1}{6} e^{-x/6}$$

i) $P(x > 8) = \int_8^\infty \frac{1}{6} e^{-x/6} \, dx$

$$= \frac{1}{6} \int_8^\infty e^{-x/6} \, dx$$

$$= \frac{1}{6} \left[\frac{e^{-x/6}}{-1/6} \right]_8^\infty = \frac{1}{6} \left[\frac{e^{-\infty/6}}{-1/6} - \frac{e^{-8/6}}{-1/6} \right]$$

$$= \frac{1}{6} \left[0 - \frac{e^{-1.33}}{-1/6} \right]$$

$$= \frac{1}{6} \left[- \frac{e^{-1.33}}{-1/6} \right] = e^{-1.33} = 0.264$$

ii) $P(4 < x < 8) = \int_4^8 \frac{1}{6} e^{-x/6} \, dx = \frac{1}{6} \int_4^8 e^{-x/6} \, dx$

$$= \frac{1}{6} \left[\frac{e^{-x/6}}{-1/6} \right]_4^8 = \frac{1}{6} \left[\frac{e^{-8/6}}{-1/6} - \frac{e^{-4/6}}{-1/6} \right]$$

$$= \frac{1}{6} \left[\frac{e^{-8/6} - e^{-4/6}}{-1/6} \right] = -\left[e^{-1.33} - e^{-0.66} \right]$$

$$= -[0.264 - 0.516]$$

$$= 0.253.$$

— x —

~~The time~~ in ~~hours~~ ~~required~~ ~~to repair~~ a machine

The daily consumption of milk in excess is 20,000 gallons is approximately exponetial distribution with mean 3000. The city has a daily stock of 35000 gallons. what is the propability that au two days selected at a random the stock is insufficient for both days.

X - Excess amount of milk consumed in a day.

i.e. Y denotes the daily consumption of milk.

then $X = Y - 20{,}000$

$\frac{1}{\alpha} = 3000$

$\alpha = \frac{1}{3000}$

$$P(Y \geq 35{,}000) = P(X + 20{,}000 > 35{,}000)$$

$$= P(X > 15{,}000)$$

$$= \frac{1}{3000} \int_{15{,}000}^{\infty} e^{\frac{-x}{3000}} dx$$

$$= \frac{1}{3000} \left[\frac{e^{-x/3000}}{-1/3000} \right]_{15{,}000}^{\infty}$$

$$= 0 + e^{-5}$$

$$= e^{-5} = 6.7379 \times 10^{-3}$$

Stock insufficient for two days $= e^{-5} e^{-5}$

$= 0.00004539$.

Memoryless property for an Exponential distribution:

$$P(X > s + t / X > s) = P(X > t) \text{ for } s, t > 0$$

$$P(X > s+t / X > s) = P(X > s + t) \cap P(X > s) / P(X > s)$$

$$= P(X > s + t) / P(X > s)$$

$$P(X > K) = \int_{K}^{\infty} \alpha e^{-\alpha x} dx = \left[\frac{\alpha e^{-\alpha x}}{-\alpha} \right]_{K}^{\infty} = -e^{-\infty} + e^{-K\alpha} = e^{-K\alpha}$$

$$\therefore \frac{P(X > s + t)}{P(X > s)} = \frac{e^{-(s+t)\alpha}}{e^{-s\alpha}} = \frac{e^{-s\alpha} \cdot e^{-t\alpha}}{e^{-s\alpha}} = e^{-t\alpha} = P(X > t)$$

The time in hours required to repair a machine is exp dis with $\lambda = 1/2$.

i) what is the probability that the repair time exceeds 2 hours

(ii) what is the conditional probability that the repair time takes atleast 10hrs given that its duration exceeds 9hrs.

$$\lambda = 1/2$$

i) $P(X>2) = \int_2^\infty \frac{1}{2} e^{-x/2}\, dx = 1/2 \left(\frac{e^{-x/2}}{-1/2} \right)_2^\infty$

$= \cancel{1/2} \times (-\cancel{2}) (e^{-\infty} - e^{-1})$

$= \boxed{0.3678.}$

ii) $P(X \geq 10) / P(X \geq 9) = \dfrac{P(X>10) \cap P(X>9)}{P(X \geq 9)}$

$= \dfrac{P(X>10)}{P(X \geq 9)} = \dfrac{1/2 \int_{10}^\infty e^{-x/2}\, dx}{1/2 \int_9^\infty e^{-x/2}\, dx}$

$= \dfrac{-e^{-\infty} + e^{-10/2}}{-e^{-\infty} + e^{-9/2}} = 0.6065$

Normal distribution.

A continuous random variable x with parameter μ and σ is said to follow normal distribution. The probability ~~distr~~ density function is given by

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} , \quad -\infty < x < \infty$$

$$-\infty < \alpha < \infty$$

$$\alpha > 0$$

The distribution of normal random variable with mean 0 and variance 1 is called standard normal distribution.

Properties :

* The normal curve is a bell shaped curve.
* The curve is symmetric about a vertex axis through the μ.
* The mean, median and mode of a normal distribution go inside.

$$z = \frac{x-\mu}{\sigma}$$

The above mentioned eqn is the normal distribution.

1. In a class of 50 students the average marks of student in a subject is 48. and Standard deviation is 24.

Find

i) no. of students who got above 50.

ii) Between 35 and 50.

μ

X denotes the marks of students which follow normal distribution.

mean $\mu = 48$

Standard deviation, $\sigma = 24$

i) $P(X > 50) = 1 - P(X \leq 50)$

$$z = \frac{x-\mu}{\sigma} = \frac{50-48}{24} = 0.0833$$

$$P(X>50) = 1 - P(X \leq 50)$$

$$= 1 - P(z \leq 0.0833)$$

$$= 1 - 0.53188$$

$$= 0.46812$$

ii) $P(35 \leq X \leq 50)$.

$X = 35$

$$z = \frac{35-48}{24} = \frac{-13}{24} = -0.541$$

$$P(35 \leq X \leq 50) = P(-0.541 \leq z \leq 0.0833)$$

$a < x < b$

$b - a$

$$= P(0.0833) - P(-0.541)$$

$$= 0.53188 - 0.29460$$

$= 0.2372$

The saving bank account of a customer showed an average balance of ₹150 and sd of ₹50 Assume that the account bal is normally distributed then

i) What % of account is over ₹200?

ii) what % of account is between ₹120 & ₹170?

$P(120 \le x \le 170)$

mean $\mu = 150$

SD $\sigma = 50$

i) $P(X > 200) = 1 - P(X \le 200)$

$$Z = \frac{x - \mu}{\sigma} = \frac{200 - 150}{50} = \frac{50}{50} = 1$$

$Z \doteq 0.84134$

$P(X > 200) = 1 - P(X \le 200)$

$= 1 - P(Z \le 0.84134)$

$= 1 - 0.84134$

$= 0.15866. \quad = 15.866\%$

ii) $P(120 \le X \le 170)$

$X = 120$

$$Z = \frac{120 - 150}{50} = \frac{-3}{5} = -0.6 = 0.27425$$ SD

$X = 170$

$$Z = \frac{170 - 150}{50} = \frac{20}{50} = 0.4 = 0.65542$$ SD

$P(120 \le X \le 170) = P(0.27425 \le Z \le 0.65542)$

$= P(0.65542) - P(0.27425)$

$= 0.74215 - 0.60642$

$P(120 < X < 170) = P(-0.6 < z < 0.4)$

$= P(0.4) - P(-0.6)$

$= 0.65542 - 0.27425$

$= 0.38117.$

A ~~team~~ mean weight of 500 students is 151 pounds and σ is 15 pounds. the weight are normally distributed. Find out how many student weight between 120 & 155.

$\text{mean} = \mu = 151$

$\sigma = SD = 15$

$$z = \frac{X - \mu}{\sigma}$$

$P(120 \leq X \leq 155)$

$x = 120, \quad Z = \frac{120 - 151}{15} = \frac{-31}{15} = -2.067$

$x = 155, \quad Z = \frac{155 - 151}{15} = \frac{4}{15} = 0.2667.$

$P(120 \leq X \leq 155) = P(-2.067 \leq z \leq 0.2667)$

$= P(0.2667) - P(-2.067)$

$= 0.60257 - 0.01970.$

$= \cancel{0.40557}.$

$= 0.58287$

$500\, P(120 \leq X \leq 155) = 500(0.58287)$

$= 291.435$

The slump clearance authorities in a city installed 2000 electric lamp in a newly constructed town. If the lamp have an average life of 1000 burning houses. what is the σ of 200hrs? How many lamps might be expected to fail in 700 burning hours?

$P(X \leq 700)$?

$$\text{Mean } \mu = 1000$$

$$\sigma = 200.$$

$$z = \frac{X-\mu}{\sigma} = \frac{700-1000}{200} = \frac{\cancel{300}}{\cancel{200}} = -1.5$$

$$P(X \leq 700) = P(z \leq 1.5)$$

$$= \cancel{0.93319}\ 0.6681$$

$$= \cancel{0.93319}\ 0.6681 \times \underline{2000}. \text{ lamps}.$$

$$= \cancel{1866.38}\ \ 1336.2$$

— x —

UNIT- III.

Two dimensional random variable.

Let S be a sample space and $X = X(S)$ $Y = Y(S)$ be two functions each assigning a real number to each outcome $s \in S$. then (X, Y) is a two dimensional random variable.

* If the possible values of (X, Y) are finite or countably infinite. then (X, Y) is called two dimensional discrete random variable.

* If (X, Y) assumes all the values in a specified region R in XY plane, then (X, Y) is called two dimensional continuous random variable.

Joining probability function of a discrete random variable:

If (X, Y) is a two-dimensional discrete random variable such that,

$$f(x_i, y_j) = P(X = x_i, Y = y_j)$$
$$= P_{ij}$$

It is called the joining probability function (or Join probability mass function of (X, Y) with $P_{ij} \geq 0$ i, j

$$\sum_{i=1}^{n} \sum_{j=1}^{m} P_{ij} = 1$$

Join probability density function: (continuous)

If (X, Y) is a two-dimensional continuous random variable, such that

$$P\left(x - \frac{dx}{2} \le X \le x + \frac{dx}{2}, y - \frac{dy}{2} \le Y \le y + \frac{dy}{2}\right) =$$

$$f(x, y)\, dx\, dy.$$

then $f(x, y)$ is called join probability density function of X, Y and it satisfies,

$$f(x, y) \ge 0 \quad \forall x, y$$

$$\iint_R f(x, y)\, dx\, dy = 1$$

Marginal probability mass function:

The Marginal probability mass function of x is given by

$$f(x) = \sum_{j=1}^{m} P_{ij}$$

$$= P_{i*} \text{ (or) } P_{i.}$$

$$f(y) = \sum_{i=1}^{n} P_{ij}$$

$$= P_{*j}$$

Marginal probability density function:

If (X, Y) is a continuous random variable then the marginal density function

$$f(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$$

$$f(y) = \int_{-\infty}^{\infty} f(x, y)\, dx$$

Conditional probability distribution:

$$P(X/y) = \frac{f(x,y)}{f(y)}$$

$$P(Y/x) = \frac{f(x,y)}{f(x)}$$

(X, Y) two dimensional discrete random variable.

$$P(X/y) = \frac{P_{ij}}{P_{*j}}$$

$$P(Y/x) = \frac{P_{ij}}{P_{i*}}$$

(X, Y) is two dimensional continuous Random Variable

$$P(X/y) = \frac{f(x,y)}{f(y)}$$

$$P(Y/x) = \frac{f(x,y)}{f(x)}$$

Independent Random Variable.

If (X,Y) is discrete random variable

the X and Y are said to be independent.

$$P_{ij} = P_{i*} \times P_{*j}.$$

If (X,Y) is continuous random variable then

X and Y are independent if

$$f(x,y) = f(x)\,f(y)$$

1. The join probability mass function of x and y is

$P(x,y)$		Y: 0	1	2
X	0	0.1	0.04	0.02
	1	0.08	0.2	0.06
	2	0.06	0.14	0.30

compute

i) marginal pmf x and y

ii) $P(X \le 1, Y \le 1)$

iii) check x and y are independent.

i) marginal probability mass function of x

$P(X=0) = P(0,0) + P(0,1) + P(0,2) = 0.1 + 0.04 + 0.02 = 0.16$

$P(X=1) = P(1,0) + P(1,1) + P(1,2) = 0.08 + 0.2 + 0.06 = 0.34$

$P(X=2) = P(2,0) + P(2,1) + P(2,2) = 0.06 + 0.14 + 0.30 = 0.5$

marginal probability density function of y

$P(Y=0) = P(0,0) + P(1,0) + P(2,0) = 0.1 + 0.08 + 0.06 = 0.24$

$P(Y=1) = P(0,1) + P(1,1) + P(2,1) = 0.04 + 0.2 + 0.14 = 0.38$

$P(Y=2) = P(0,2) + P(1,2) + P(2,2) = 0.02 + 0.06 + 0.30 = 0.38$

ii) $P(X \le 1, Y \le 1) = P(0,0) + P(0,1) + P(1,0) + P(1,1)$

$= 0.1 + 0.04 + 0.08 + 0.2$

$= 0.42$

iii) X and Y are independent,

$P_{ij} = P_{i*} \times P_{*j} \quad \forall i, j$

$P(X=0, Y=0) = P(X=0)\, P(Y=0)$

$0.1 \ne 0.16 \times 0.24$ It is not independent

2. From the following table for bivariate distribution of (X, Y) find

i) $P(X \le 1)$ (ii) $P(Y \le 3)$ (iii) $P(X \le 1, Y \le 3)$

iv) $P(X \le 1 / Y \le 3)$ v) $P(Y \le 3 / X \le 1)$

vi) $(X+Y \le 4)$ vii) marginal distribution of X

viii) Marginal distribution of Y ix) The conditional distribution of X given $Y = 2$. x) Examine X and Y are independent or not xi) Find $E(Y-2X)$

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

i) $P(X \le 1) = P(X=0) + P(X=1)$

$= P(0,1)\,P(0,2) \ldots P(0,6) + P(1,1)\,P(1,2), \ldots P(1,6)$

$\frac{8}{32} + \frac{4}{8} + \frac{8}{16}$ $\frac{8}{32} + \frac{5}{8} = \frac{1}{4} + \frac{5}{8} = \frac{7}{8}$

$= 7/8$

ii) $P(Y \le 3) = P(Y=1) + P(Y=2) + P(Y=3)$

$= P(0,1) + P(1,1) + P(2,1) + P(0,2) + P(1,2) + P(2,2) + P(0,3) + P(1,3) + P(2,3)$

$= \frac{23}{64}$

iii) $P(X \le 1, Y \le 3) = P(0,1) + P(0,2) + P(0,3) + P(1,1) + P(1,2) + P(1,3)$

$= \frac{1}{32} + \frac{2}{16} + \frac{1}{8}$

$= \frac{1+4+4}{32} = 9/32$

iv) $P(X\leq 1 / Y\leq 3) = \dfrac{P(X\leq 1, Y\leq 3)}{P(Y\leq 3)}$

$= \dfrac{9/\cancel{32}}{23/\cancel{64}\,2}$

$= \dfrac{18}{23}$.

vi) $P(X+Y\leq 4) = P(0,1)+P(0,2)+P(0,3)+P(0,4)+P(1,1)+$

$P(1,2)+P(1,3)+P(2,1)+P(2,2)$

$= 13/32$.

v) $P(Y\leq 3 / X\leq 1) = \dfrac{P(X\leq 1, Y\leq 3)}{P(X\leq 1)}$

$= \dfrac{9/\cancel{32}\,4}{7/\cancel{8}}$

$= \dfrac{9}{\cancel{32}\,4} \times \dfrac{\cancel{8}}{7}$

$= \dfrac{9}{28}$

vii) marginal distribution of x

$P(X=0) = \frac{1}{32} + \frac{2}{32} + \frac{2}{32} + \frac{3}{32} = 8/32$

$P(X=1) = \frac{1}{16} + \frac{1}{16} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{2}{16} + \frac{4\times 2}{8\times 2} = \frac{10}{16}$

$P(X=2) = \frac{1}{32} + \frac{1}{32} + \frac{1}{64} + \frac{1}{64} + \frac{2}{64} = \frac{4}{64} + \frac{2\times 2}{32\times 2} = 8/64 = 4/32$

$= 2/16 = 1/8$

viii) Marginal distribution of Y

$P(Y=1) = \frac{1}{16} + \frac{1}{32} = \frac{3}{32}$

$P(Y=2) = 3/32$

$P(Y=3) = \frac{1}{32} + \frac{1}{8} + \frac{1}{64} = \frac{2}{64} + \frac{1}{64} + \frac{8}{64} = \frac{11}{64}$

$P(Y=4) = \frac{2}{32} + \frac{1}{8} + \frac{1}{64} = \frac{4}{64} + \frac{8}{64} + \frac{1}{64} = \frac{13}{64}$

$P(Y=5) = \frac{2}{32} + \frac{1}{8} = \frac{2}{32} + \frac{4}{32} = \frac{6}{32}$

$P(Y=6) = \frac{3}{32} + \frac{1}{8} + \frac{2}{64} = \frac{6}{64} + \frac{8}{64} + \frac{2}{64} = \frac{16}{64}$.

ix) $P(X/Y=2)$

$P(X=0/Y=2) = \frac{P(0,2)}{P(Y=2)} = 0.$

$P(X=1/Y=2) = \frac{P(1,2)}{P(Y=2)} = \frac{1/16}{3/32} = 2/3$

$P(X=2/Y=2) = \frac{P(2,2)}{P(Y=2)} = \frac{1/32}{3/32} = 1/3.$

x) $\because$ X & Y are independent.

$P_{ij} = P_{i*} \times P_{*j}$

$P(0,1) = 0 \neq 8/32 \times 3/32$

$\therefore$ X & Y are not independent.

xi) $E[Y-2X] = E[Y] - 2E[X]$

$E[X] = \sum x_i P(x_i)$

$= x_0 \times P(X=0) + x_1 P(X=1) + x_2 P(X=2)$

$= 0\ 8/32 + 1\ 10/16 + 2\ 4/32$

The joint probability mass for if (X,Y) is given by $P(x,y) = k(2x+3y)$, $x = 0,1,2$; $y = 1,2,3$. Find all the marginal distribution and conditional distribution. Also find the probability distribution of $X+Y$ and $P(X+Y>3)$

x \ y	1	2	3
0	3k	6k	9k
1	5k	8k	11k
2	7k	10k	13k

$\sum \sum P_{ij} = 1$

$3k + 6k + 9k + 5k + 8k + 11k + 7k + 10k + 13k = 1$

$72k = 1$

$k = 1/72$

X \ Y	1	2	3
0	3/72	6/72	9/72
1	5/72	8/72	11/72
2	7/72	10/72	13/72

Marginal distribution of X:

$$P(X=0) = \frac{6}{72} + \frac{3}{72} + \frac{9}{72} = \frac{18}{72}$$

$$P(X=1) = \frac{5}{72} + \frac{8}{72} + \frac{11}{72} = \frac{24}{72}$$

$$P(X=2) = \frac{10}{72} + \frac{7}{72} + \frac{13}{72} = \frac{30}{72}$$

Marginal distribution of Y:

$$P(Y=1) = \frac{3}{72} + \frac{5}{72} + \frac{7}{72} = \frac{15}{72}$$

$$P(Y=2) = \frac{6}{72} + \frac{8}{72} + \frac{10}{72} = \frac{24}{72}$$

$$P(Y=3) = \frac{9}{72} + \frac{11}{72} + \frac{13}{72} = \frac{33}{72}$$

Conditional distribution:

$$P(X=0/Y=1) = \frac{P(X=0, Y=1)}{P(Y=1)} = \frac{3/72}{15/72} = \frac{3}{72} \times \frac{72}{15} = \frac{3}{15}$$

$$P(X=1/Y=1) = \frac{P(X=1, Y=1)}{P(Y=1)} = \frac{5/72}{15/72} = 5/15$$

$$P(X=2/Y=1) = \frac{P(X=2, Y=1)}{P(Y=1)} = \frac{7/72}{15/72} = 7/15$$

$$P(X=0/Y=2) = \frac{P(X=0, Y=2)}{P(Y=2)} = \frac{6/72}{24/72} = 6/24 = 1/4$$

$$P(X=1/Y=2) = \frac{P(X=1, Y=2)}{P(Y=2)} = \frac{8/72}{24/72} = 8/24 = \frac{1}{2}$$

$$P(X=2/Y=2) = \frac{P(X=2, Y=2)}{P(Y=2)} = \frac{10/72}{24/72} = \frac{10}{24} = \frac{5}{12}$$

$$P(X=0/Y=3) = \frac{P(X=0, Y=3)}{P(Y=3)} = \frac{9/72}{33/72} = 9/33$$

$$P(X=1/Y=3) = \frac{P(X=1, Y=3)}{P(Y=3)} = \frac{11/72}{33/72} = 11/33$$

$$P(X=2/Y=3) = \frac{P(X=2, Y=3)}{P(Y=3)} = \frac{13/72}{33/72} = 13/33$$

$$P(Y=1/X=0) = \frac{P(Y=1, X=0)}{P(X=0)} = \frac{3/72}{18/72} = 3/18$$

$$P(Y=2/X=0) = \frac{P(Y=2, X=0)}{P(X=0)} = \frac{6/72}{18/72} = 6/18$$

$$P(Y=3/X=0) = \frac{P(Y=3, X=0)}{P(X=0)} = \frac{9/72}{18/72} = 9/18$$

$$P(Y=1/X=1) = \frac{P(Y=1, X=1)}{P(X=1)} = \frac{5/72}{24/72} = 5/24$$

$$P(Y=2/X=1) = \frac{P(Y=2, X=1)}{P(X=1)} = \frac{8/72}{24/72} = 8/24$$

$$P(Y=3/X=1) = \frac{P(Y=3, X=1)}{P(X=1)} = \frac{11/72}{24/72} = 11/24$$

$$P(Y=1/X=2) = \frac{P(Y=1, X=2)}{P(X=2)} = \frac{7/72}{30/72} = 7/30$$

$$P(Y=2/X=2) = \frac{P(X=2, Y=2)}{P(X=2)} = \frac{10/72}{30/72} = 10/30$$

$$P(Y=3/X=2) = \frac{P(X=2, Y=3)}{P(X=2)} = \frac{13/72}{30/72} = 13/30$$

probability distribution:

$X+Y$		
1	$P(0,1)$	$= 3/72$
2	$P(0,2)+P(1,1)$	$= 11/72$
3	$P(0,3)+P(1,2)+P(2,1)$	$= 24/72$
4	$P(1,3)+P(2,2)$	$= 21/72$
5	$P(2,3)$	$= 13/72$

$P(X+Y > 3)$

$= P(X+Y=4) + P(X+Y=5)$

$= P(1,3) + P(2,2) + P(2,3)$

$= \frac{21}{72} + \frac{13}{72}$

$= \frac{44}{72}.$

Suppose the probability density function of a continuous distribution is given by

$$f(x,y) = \begin{cases} \frac{6}{5}(x+y^2), & 0 \leq x \leq 1,\ 0 \leq y \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

obtain the marginal distribution of X and Y

also find $P(1/4 \leq y \leq 3/4)$

marginal distribution:

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

$$= \int_0^1 \frac{6}{5}(x+y^2)\, dy$$

$$= \frac{6}{5}\left[xy + \frac{y^3}{3}\right]_0^1$$

$$= \frac{6}{5}\left[x + 1/3\right]$$

marginal distribution of y:

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$$

$$= \int_0^1 \frac{6}{5}(x+y^2)\, dx$$

$$= \frac{6}{5}\left[\frac{x^2}{2} + y^2 x\right]_0^1$$

$$= \frac{6}{5}\left[1/2 + y^2\right]$$

$$P\left(\frac{1}{4} \le y \le 3/4\right) = \int_{1/4}^{3/4} f(y)\, dy$$

$$= \int_{1/4}^{3/4} \frac{6}{5}\left[y^2 + \frac{1}{2}\right] dy$$

$$= \frac{6}{5}\left[\frac{y^3}{3} + \frac{y}{2}\right]_{1/4}^{3/4}$$

$$= \frac{6}{5}\left[\frac{2y^3 + 3y}{6}\right]_{1/4}^{3/4}$$

$$= \frac{1}{5}\left[2(3/4)^3 + 3(3/4) - 2(1/4)^3 + 3(1/4)\right]$$

$$= \frac{1}{5}\left[2\left(\frac{27}{64}\right) + 3\left(\frac{3}{4}\right) - 2\left(\frac{1}{64}\right) + \frac{3}{4}\right]$$

$$= \frac{1}{5}\left[\frac{54}{64} + \frac{9}{4} - \frac{2}{64} + \frac{3}{4}\right]$$

$$= \frac{1}{5}\left[\frac{54}{64} + \frac{144}{64} - \frac{2}{64} + \frac{48}{64}\right]$$

$$= \frac{1}{5}\left[\frac{246}{64} - \frac{2}{64}\right] = \frac{1}{5}\left[\frac{244}{64}\right]$$

$$= \frac{244}{320} = 0.7625$$

The pdf of a continuous random variable is

$$f(x,y) = \begin{cases} \frac{x^3 y^3}{16}, & 0 \le x \le 2, \quad 0 \le y \le 2 \\ 0, & \text{otherwise.} \end{cases}$$

check whether X and Y are independent.

$$f(x,y) = f(x)\, f(y)$$

marginal distribution:

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$$

$$= \int_0^2 \frac{x^3 y^3}{16}\, dy$$

$$= \frac{1}{16} \left[\frac{x^3 y^4}{4} \right]_0^2$$

$$= \frac{1}{16} \frac{x^3 2^4}{4}$$

$$= \frac{x^3}{4}$$

$$f(y) = \int_0^2 \frac{x^3 y^3}{16}\, dx$$

$$= \frac{1}{16} y^3 \left[\frac{x^4}{4} \right]_0^2$$

$$= y^3/4$$

$$f(x)\, f(y) = \frac{x^3}{4} \frac{y^3}{4} = \frac{x^3 y^3}{16} = f(x,y)$$

$\therefore$ X & Y are independent.

The join pdf of rv is $f(x,y)=\begin{cases} k(6-x-y), & 0<x<2, \\ 0 & \end{cases}$

$2<y<4$

otherwise

Find i) the value of k

ii) $P(X<1, Y<3)$;

iii) $P(X+Y<3)$

iv) $P(X<1 / Y<3)$

i) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$$

$$\int_{2}^{4}\int_{0}^{2} k(6-x-y)\, dx\, dy = 1$$

$$k\int_{2}^{4}\left[6x - \frac{x^2}{2} - yx\right]_0^2 dy = 1$$

$$k\int_{2}^{4} 6(2) - \frac{(2)^2}{2} - 2y\; dy = 1$$

$$k\int_{2}^{4} (12 - 2 - 2y)\, dy = 1$$

$$k\int_{2}^{4} (10 - 2y)\, dy = 1$$

$$k\left[10y - \frac{\cancel{2}y^2}{\cancel{2}}\right]_2^4 = 1$$

$40-16-20+4$

$44-36 = 8$

$$k\left[40-16-20+4\right]=1$$

$$\boxed{k=1/8}$$

$$P(X<1, Y<3) = \int_2^3 \int_0^1 \frac{1}{8}\left[6-x-y\right] dx\, dy.$$

$$= \frac{1}{8}\int_2^3 \left[6x-\frac{x^2}{2}-xy\right]_0^1 dy$$

$$= \frac{1}{8}\int_2^3 \left[6-\frac{1}{2}-y\right] dy$$

$$= \frac{1}{8}\int_2^3 \left[\frac{12-1-2y}{2}\right] dy$$

$$= \frac{1}{8}\cdot\frac{1}{2}\int_2^3 \left[11-2y\right] dy$$

$$= \frac{1}{8}\cdot\frac{1}{2}\left[11y-\frac{2y^2}{2}\right]_2^3$$

$$= \frac{1}{16}\left[\left[11(3)-\frac{2(3)^2}{2}\right]-\left[11(2)-\frac{2(2)^2}{2}\right]\right]$$

$$= \frac{1}{16}\left[\left[33-\frac{18}{2}\right]-\left[22-\frac{8}{2}\right]\right]$$

$$= \frac{1}{16}\left[\frac{66-18-44+8}{2}\right]$$

$$= \frac{1}{\cancel{16}_4}\left[\frac{\cancel{12}^3}{2}\right] = 3/8 = 0.375$$

3/4

$$P(X<1 / Y<3) = \frac{P(X<1, Y<3)}{P(Y<3)}$$

$$P(Y<3) = \int_2^3 f(y)\,dy$$

$$f(y) = \int_0^2 f(x,y)\,dx$$

$$= \int_0^2 \frac{1}{8}(6-x-y)\,dx$$

$$= \frac{1}{8}\left[6x - \frac{x^2}{2} - yx\right]_0^2$$

$$= \frac{1}{8}[12 - 2 - 2y]$$

$$= \frac{10-2y}{8}$$

$$= \frac{2(5-y)}{2(4)}$$

$$f(y) = \frac{5-y}{4}$$

$$P(Y<3) = \int_2^3 \frac{5-y}{4}\,dy$$

$$= \frac{1}{4}\int_2^3 5-y\,dy = \frac{1}{4}\left[5y - \frac{y^2}{2}\right]_2^3$$

$$= \frac{1}{4}\left[15 - \frac{9}{2} - 10 + \frac{4}{2}\right]$$

$$= \frac{1}{4}\left[5 - 5/2\right]$$

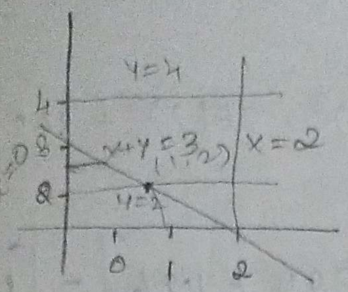
$$= 1/4\left[5/2\right]$$

$$= 5/8$$

$$P(X<1 / Y<3) = \frac{P(X<1, Y<3)}{P(Y<3)}$$

$$= \frac{3}{8} \times \frac{8}{5} = \frac{3}{5}$$

$P(X+Y<3)$



$$P(X+Y<3) = \int_2^3 \int_0^{3-y} \tfrac{1}{8}(6-x-y)\,dx\,dy$$

$$= \frac{1}{8}\int_2^3 \left[6x - \frac{x^2}{2} - xy\right]_0^{3-y} dy$$

$$= \frac{1}{8}\int_2^3 \left[6(3-y) - \frac{(3-y)^2}{2} - [(3-y)y]\right] dy$$

$$= \frac{1}{8}\int_2^3 18-6y - \frac{(9-6y+y^2)}{2} - 3y+y^2\,dy$$

$$= \frac{1}{8}\int_2^3 \frac{18\times2 - 12y - 9 + 6y - y^2 - 6y + 2y^2}{2}\,dy$$

$$= \frac{1}{16}\int_2^3 36 - 12y - 9 + \cancel{6y} - y^2 - \cancel{6y} + 2y^2\,dy$$

$$= \frac{1}{16}\int_2^3 y^2 - 12y + 27\,dy$$

$$= \frac{1}{16}\left[\frac{y^3}{3} - \frac{12y^2}{2} + 27y\right]_2^3$$

$$= \frac{1}{16}\left[\frac{2y^3 - 36y^2 + 27\times6y}{6}\right]_2^3$$

$$= \frac{1}{96}\left[2y^3 - 36y^2 + 162y\right]_2^3$$

$$= \frac{1}{96}\left\{\left[2(3)^3 - 36(3)^2 + 162(3)\right] - \left[2(2)^3 - 36(2)^2 + 162(2)\right]\right\}$$

$$= \frac{1}{96}\left\{\left[54 - 324 + 486\right] - \left[16 - 144 + 324\right]\right\}$$

$$= \frac{1}{96}\left[54 - 324 + 486 - 16 + 144 - 324\right]$$

$$= \frac{1}{96}\left[684 - 664\right]$$

$$= \frac{20}{96}$$

$$= \frac{10}{48}$$

$$= \frac{5}{24}$$

Find the joint pdf of X & Y is $f(x,y) = \begin{cases} 8xy & 0<x<1 \\ & 0<y<x \\ 0 & \text{Otherwise} \end{cases}$

Find $P(Y < 1/8 \,/\, X < 1/2)$ & $f(y/x)$

marginal distribution of X

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$= \int_{0}^{x} 8xy\,dy$$

$$= 8x\left[\frac{y^2}{2}\right]_0^{x}$$

$$= \frac{\cancel{8}^{4}x^3}{\cancel{2}}$$

$$= 4x^3$$

$$P(Y < 1/8 \,/\, X < 1/2) = \frac{P(X < 1/2,\ Y < 1/8)}{P(X < 1/2)}$$

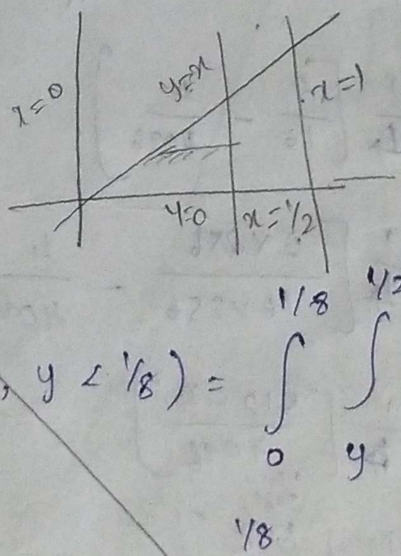
$$P(X < 1/2) = \int_0^{1/2} 4x^3\,dx$$

$$= \left[\frac{\cancel{4}x^4}{\cancel{4}}\right]_0^{1/2}$$

$$= (1/2)^4$$

$$= \frac{1}{16}$$

4 8 16
$2\times2\times2\times2$



$$P(X < 1/2,\ y < 1/8) = \int_0^{1/8}\int_y^{1/2} 8xy\,dx\,dy$$

$$= 8\int_0^{1/8}\left[\frac{x^2y}{2}\right]_y^{1/2} dy$$

$$= 8\int_0^{1/8} \frac{(1/2)^2 y}{2} - \frac{y^2\cdot y}{2}\,dy$$

$$= 8\int_0^{1/8} \frac{\frac{1}{4}\cdot y}{2} - \frac{y^3}{2}\,dy$$

$$P(x < 1/2, y < 1/8) = \int_0^{1/8} \int_y^{1/2} 8xy \, dx \, dy$$

$$= 8\int_0^{1/8} \left[\frac{x^2}{2} y\right]_y^{1/2} dy$$

$$= \frac{8}{2}\int_0^{1/8} (x^2 y)_y^{1/2} \, dy$$

$$= \frac{8}{2}\int_0^{1/8} \left(\frac{1}{2}\right)^2 y - y^2 \cdot y \, dy$$

$$= 4\int_0^{1/8} \frac{y}{4} - y^3 \, dy$$

$$= 4\int_0^{1/8} \frac{y - 4y^3}{4} \, dy = \frac{4}{4}\int_0^{1/8} y - 4y^3 dy$$

$$= \left[\frac{y^2}{2} - \frac{4y^4}{4}\right]_0^{1/8} = \left[\frac{2y^2}{4} - \frac{4y^4}{4}\right]_0^{1/8}$$

$$f\left(\frac{y}{x}\right) = \frac{f(x,y)}{f(x)} = \frac{8xy}{4x^3} = \frac{2y}{x^2}$$

$$= \frac{1}{4}\left[2y^2 - 4y^4\right]_0^{1/8}$$

$$= \frac{1}{4}\left[2\left(\frac{1}{8}\right)^2 - 4\left(\frac{1}{8}\right)^4\right]$$

$$= \frac{1}{4}\left[2\left(\frac{1}{64}\right) - 4\left(\frac{1}{4096}\right)\right]$$

$$= \frac{1}{4}\left(\frac{2}{64} - \frac{4}{4096}\right) = \frac{1}{4}\left(\frac{2\times 64}{64\times 64} - \frac{4}{4096}\right)$$

$$= \frac{1}{4}\left(\frac{128-4}{4096}\right) = \frac{1}{4}\left(\frac{124}{4096}\right)$$

$$= \frac{31}{4096}$$ ✓

If X and Y are random variable, covariance of

$Cov(X,Y) = E[XY] - E[X] \cdot E[Y]$

Properties:

1) (covariance) $\cdot\ cov(aX, bY) = ab\ cov(X,Y)$

2) $cov(X+a ; Y+b) = cov(X,Y)$

3) $cov(aX+b, cY+d) = ac\ cov(X,Y)$

4) $2\ cov(X,Y) = Var(X_1) + Var(X_2) - Var(X_1 - X_2)$

NOTE:

If X and Y are independent, then covariance of

~~E[XY]. E[~~ $cov(X,Y) = 0$.

Karl Pearson's co-efficient of correlation:

Let X and Y be given random variable

$$r(X,Y) = r_{XY} = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$$

calculate the ~~covariant~~ correlation coefficient for the following heights of fathers X and sons Y.

X	65	66	67	67	68	69	70	72
Y	67	68	65	68	72	72	69	71

x	y	xy	x^2	y^2
65	67	4355	4225	4489
66	68	4488	4356	4624
67	65	4355	4489	4225
67	68	4556	4489	4624
68	72	4896	4624	5184
69	72	4968	4761	5184
70	69	4830	4900	4761
72	71	5112	5184	5041

karl pearson's coefficient of correlation :

$$V(x,y) = V_{xy} = \frac{\operatorname{cov}(x,y)}{\sigma_x \sigma_y}$$

$$\operatorname{Cov}(x,y) = E(x,y) - E(x)\cdot E(y)$$

$$E[x] = \frac{1}{n}\sum x = \frac{544}{8} = 68$$

$$E[y] = \frac{1}{n}\sum y = \frac{552}{8} = 69$$

$$E[xy] = \frac{1}{n}\sum xy = \frac{37560}{8} = 4695$$

$$\operatorname{cov}(x,y) = 4695 - (68 \times 69) = 3$$

$$\sigma_x = \sqrt{\tfrac{1}{n}\sum x^2 - (\bar{x})^2} = \sqrt{\frac{37028}{8} - (68)^2} = 2.121$$

$$\sigma_y = \sqrt{\frac{1}{n}\sum y^2 - (\bar{y})^2} = \sqrt{\frac{38732}{8} - (69)^2} = 2.345$$

$$V_{xy} = \frac{3}{4.973} = 0.603$$

The joint pmf of X and Y is given below.

$y \backslash x$	-1	$+1$
0	1/8	3/8
1	2/8	2/8

Find correlation coefficient of X and Y.

$$\text{cov}(X, Y) = E(X, Y) - E(X) \cdot E(Y)$$

$$E[X] = \sum x_i P(x_i) = (-1)\frac{3}{8} + 1\left(\frac{5}{8}\right) = 2/8$$

$$E[Y] = \sum y_i P(y_i) = \frac{0 \cdot 4}{8} + 1 \cdot \frac{4}{8} = 1/2$$

$$E[X, Y] = \sum \sum x_i y_i \, P(x_i, y_i)$$

$$= (0)(-1) \cdot \frac{1}{8} + 0(1)\frac{3}{8} + (1)(-1)\frac{2}{8} + (1)(1)\frac{2}{8}$$

$$= 0$$

$$E[X^2] = \sum x^2 P(x_i)$$

$$= (-1)^2 \frac{3}{8} + 1^2 \frac{5}{8} = 1$$

$$E[Y^2] = 0^2\left(\frac{4}{8}\right) + 1^2\left(\frac{4}{8}\right) = 1/2$$

$$\text{Var}[X] = E[X^2] - [E[X]]^2 = 1 - (1/4)^2 = 15/16$$

$$\text{Var}[Y] = E[Y^2] - [E[Y]]^2 = 1/2 - (1/2)^2 = 1/2$$

$$\sigma_x = \frac{\sqrt{5}}{4}, \quad \sigma_y = \frac{1}{\sqrt{2}}$$

$$\text{cov}(X, Y) = E(X, Y) - E(X)E(Y) = -1/2$$

Let [illegible] probability function

$f(x,y) = \frac{x+y}{21}$, $x = 1, 2, 3$ $y = 1, 2$ Find correlation of X and Y.

$y \backslash x$	1	2	3
1	2/21	3/21	4/21
2	3/21	4/21	5/21

$E[X] = \sum x_i P(x_i) = 1 \cdot 5/21 + 2 \cdot 7/21 + 3 \cdot 9/21 = \frac{46}{21}$

$E[Y] = \sum y_i P(y_i) = 1 \cdot 9/21 + 2 \cdot \frac{12}{21} = \frac{33}{21}$

$E[X^2] = \sum x_i^2 P(x_i) = 1^2 \frac{5}{21} + 2^2 \frac{7}{21} + 3^2 \frac{9}{21} = \frac{14}{21}.$

$E[Y^2] = \sum y_i^2 P(y_i) = 1^2 \cdot 9/21 + 2^2 \frac{12}{21} = \frac{57}{21}.$

$E[XY] = \sum x_i y_i P(x_i y_i) = 1 \cdot 1 \cdot \frac{2}{21} + 1 \cdot 2 \cdot \frac{3}{21} + 1 \cdot 3 \cdot \frac{4}{21} + 2 \times 1 \times 3/21 + 2 \cdot 2 \cdot 4/21 + 2 \cdot 3 \cdot 3/21 = \frac{72}{21}.$

$Cov(x,y) = E[XY] - E[X] \cdot E[Y] = -6/441.$

$Var[X] = E[X^2] - [E(X)]^2 = 278/441$

$Var[Y] = E[Y^2] - [E[Y]]^2 = \frac{108}{441}.$

$\sigma_x = \sqrt{var(x)} = \sqrt{278/441} = \frac{16.67}{21} = 0.7938$

$\sigma_y = \sqrt{var(y)} = \sqrt{108/441} = \frac{10.39}{21} = 0.4948$

$r_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y} = -0.0346$ ans

$= \frac{-6/441}{1.2886} = \frac{-0.0136}{1.2886}$

Suppose that the 2D RV (X, Y) has the joint pdf

$$f(x,y) = \begin{cases} x+y, & 0 < x < 1,\ 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

Find the correlation between X and Y check whether X and Y are independent.

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_0^1 (x+y)\,dy = \left[xy + \frac{y^2}{2}\right]_0^1$$

$$= \frac{x+1}{2}.$$

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^1 x+y\,dx = \left[\frac{x^2}{2} + yx\right]_0^1 = y + \frac{1}{2}$$

$$E[X] = \int_{-\infty}^{\infty} x\,f(x)\,dx = \int_0^1 x\left[x + \frac{1}{2}\right]dx$$

$$= \int_0^1 x^2 + \frac{x}{2}\,dx = \left[\frac{x^3}{3} + \frac{x^2}{4}\right]_0^1 = 7/12$$

$$E[Y] = \int_{-\infty}^{\infty} y\,f(y)\,dy = \int_0^1 y\left(y + \frac{1}{2}\right)dy = 7/12$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2\,f(x)\,dx = \int_0^1 x^2\left(x + \frac{1}{2}\right)dx = \int_0^1 x^3 + \frac{x}{2}\,dx$$

$$= \left[\frac{x^1}{4} + \frac{x^3}{6}\right]_0^1 = \frac{1}{4} + \frac{1}{6} = \frac{10}{24} = \frac{5}{12}$$

$$E[Y^2] = \int_0^1 y^2\,f(y)\,dy = \frac{5}{12}$$

$$Var[X] = E[X^2] - [E[X]]^2 = \frac{5}{12} - \frac{49}{144} = \frac{11}{144}$$

$$Var[Y] = E[Y^2] - [E[Y]]^2 = 11/144$$

$$E[XY] = \iint xy \, f(x,y) \, dx \, dy$$

$$= \int_0^1 \int_0^1 xy \, (x+y) \, dx \, dy$$

$$= \int_0^1 \int_0^1 x^2y + y^2x \, dx \, dy$$

$$= \int_0^1 \left[\frac{x^3}{3} y + y^2 \frac{x^2}{2} \right]_0^1 dy$$

$$= \int_0^1 \frac{y}{3} + \frac{y^2}{2} \, dy$$

$$= \left[\frac{y^2}{6} + \frac{y^3}{6} \right]_0^1 = \frac{2}{6} = \frac{1}{3}.$$

$$\mathrm{cov}(X,Y) = E[XY] - E[X]\,E[Y]$$

$$= 1/3 - 7/12 \cdot 7/12 = -1/144$$

$$r_{XY} = \frac{-1/144}{\sqrt{\frac{11}{144}}\sqrt{\frac{11}{144}}} = -\frac{1}{11}.$$

$$f(x,y) = 2 - x - y \quad 0 < x < 1, \; 0 < y < 1$$