

Unit - 5 Design of Experiment

completely Randomized Design

One way Classification : (columnwise)

One way classification observation are classified according to one factor this is exhibited column-wise.

$$\begin{array}{l} H_0 \\ H_1 \end{array}$$

Step 1 : Find N number of observations

2 : Find T. total of observation

3 : Corrected factor : $\frac{T^2}{N}$ sum of square of w/1

4 : $TSS = \sum x_1^2 + x_2^2 + \dots + \frac{T^2}{N}$

5 : Column Sum Square : $SSC = \frac{(\sum x_1)^2}{N_1} + \frac{(\sum x_2)^2}{N_2} + \dots - \frac{T^2}{N}$

6 : Sum of Square error : $SSE = TSS - SSC$

Form ANOVA Table :

Source of variation	Sum of squares	Degrees of freedom	Mean square	Variation ratio
Between column	SSC	C-1	$MSC = \frac{SSC}{C-1}$	$F = \frac{MSC}{MSE}$
Error	SSE	N-C	$MSE = \frac{SSE}{N-C}$	or
Total	TSS	N-1		$F = \frac{MSE}{MSC}$

- The following are the no. of mistakes made in 5 successive day of 4 technicians working in a lab. test at the level of significance $\alpha = 0.01$ whether the difference among the sample means can be attributed to chance.

Tech I	II	III	IV	Total	D.F.
6	14	10	9	47	-1
14	9	12	12	47	-1
8	10	7	8	33	-3
10	10	10	8	40	3
11	11	11	11	44	7
				-1 9 5 0	13

Subtracting each value by 10

H_0 : There is no difference in technicians

H₀: There is no difference in technicians.

t-test

x_1^2	x_2^2	x_3^2	x_4^2	
16	16	0	1	
16	1	4	4	
0	4	9	4	
4	0	25	0	
1	16	1	1	

37 37 37 37 39 10 10 10 10 10

$$TSS = \sum x_1^2 + \sum x_2^2 + \sum x_3^2 + \sum x_4^2 - \frac{T^2}{N}$$

$$= 37 + 37 + 39 + 10 - 8.45 = 114.55$$

$$\frac{ST}{\sqrt{N}} = \frac{(\sum x_1)^2}{N_1} + \frac{(\sum x_2)^2}{N_2} + \frac{(\sum x_3)^2}{N_3} + \frac{(\sum x_4)^2}{N_4} - \frac{T^2}{N}$$

$$= 14 \left(\frac{(-1)^2}{5} + \frac{(9)^2}{5} + \frac{(5)^2}{5} \right) + 0 - 8.45 = 12.95$$

$$SSE = TSS - SSC = 101.6$$

ANOVA Table:

	Square of variation	Sum of Square	D.F.	Mean Square	Ratio
Between columns	SSC = 12.95	C-1	MSC = $\frac{SSC}{C-1}$		
Error	SSE = 101.6	N-C	MSE = $\frac{SSE}{N-C}$	$\frac{MSC}{MSE}$	

321	322	323	324	325	326	327	328	329	330
321	322	323	324	325	326	327	328	329	330
321	322	323	324	325	326	327	328	329	330
321	322	323	324	325	326	327	328	329	330
321	322	323	324	325	326	327	328	329	330

Table value = $F_C(16, 3)$

= 26.83

local critical value at 5% significant level $\Rightarrow 1.473 < 26.83$

so H_0 is accepted.

As complete randomised design experiment with 10 plots and 3 treatments gave the following results:

plot no. 1 2 3 4 5 6 7 8 9 10

Treatment A-B C A C C A B A A-B

yield 5 4 3 7 5 9 3 4 1 8 7

analyse the result for treatment effect.

as per student's t-test

mean square error = 1.473

A	B	C	total	x_1^2	x_2^2	x_3^2	
5	4	3	12	25	16	9	
7	4	5	16	49	16	25	
3	7	1	11	9	49	1	
1			1	1	1	1	
total	16	15	9	40	84	35	

H_0 : There is no significant difference in treatment

H_1 : There is significant difference in treatment

$$N = 10, CT = 40, \text{Corrected factor } = \frac{CT^2}{N} = \frac{40^2}{10} = 160$$

$$TSS = \sum x_1^2 + \sum x_2^2 + \sum x_3^2 + \sum x_4^2 - \frac{CT^2}{N}$$

$$\text{Ans. } = 40 + 84 + 81 + 35 - 160$$

units. = 40 + 84 + 81 + 35 - 160

$$SSC = \frac{(\sum x_1)^2}{N_1} + \frac{(\sum x_2)^2}{N_2} + \frac{(\sum x_3)^2}{N_3} + \frac{(\sum x_4)^2}{N_4} - \frac{CT^2}{N}$$

$$= \frac{(16)^2}{4} + \frac{(15)^2}{3} + \frac{(9)^2}{3} - 160 = 6$$

$$SSE = TSS - SSC = 40 - 6 \\ = 34$$

ANOVA Table:

Source of variation sum of squares d.f. mean square variation ratio.

Between columns $SSC = 6, C-1 = 1, MSC = \frac{SSC}{C-1} = \frac{6}{2} = 3, MME = \frac{MSE}{C-1} = \frac{3}{1} = 3$

Error $SSE = 34, N-C = 10-3 = 7, MSE = \frac{SSE}{N-C} = \frac{34}{7} = 4.857, F = \frac{MSE}{MSE} = \frac{4.857}{3} = 1.62$

$F = \frac{4.857}{1.62} = 3$

Randomized Block Design
Two way Classification : (here check row & col also)

ANOVA Table :

Source of variation	Sum of Squares	d.f	mean sum square	Variance ratio
Between Column	SSC	C-1	$MSC = \frac{SSC}{C-1}$	$F_C = \frac{MSC}{MSE}$
Between row	SSR	R-1	$MSR = \frac{SSR}{R-1}$	$F_R = \frac{MSR}{MSE}$
Error	SSE	(R-1)(C-1)	$MSE = \frac{SSE}{(R-1)(C-1)}$	

1. An experiment was design to study the performance of 4 different detergents in 3 different engines. The results are shown below

	Engine 1	Engine 2	Engine 3
Det 1	45	43	51
Det 2	47	46	52
Det 3	48	50	55
Det 4	42	37	49

Perform ANOVA and test at 1% level whether there are difference in detergent or in the engine.

Subtracting each value by 40

	E1	E2	E3	tot	x_1^2	x_2^2	x_3^2	Yerr
D1	-5	3	11	19	25	9	121	
D2	7	6	12	25	49	36	144	
D3	8	10	15	33	64	100	225	
D4	2	-3	9	14	4	9	81	
Tot	22	16	47	85	142	154	571	

$$N = 12, T = 85, \text{ corrected factor } \frac{T^2}{N} = \frac{85^2}{12} = 602.08$$

H₀: There is no significant difference in detergent or in engine.

H₁: There is significant difference in detergent or in engine

$$TSS = \sum X_1^2 + \sum X_2^2 + \sum X_3^2 - \frac{T^2}{N}$$

$$= 14.2 + 15.4 + 57.1 - 602.08 = 264.92$$

$$SSC = \frac{(\sum X_1)^2}{N_1} + \frac{(\sum X_2)^2}{N_2} + \frac{(\sum X_3)^2}{N_3} - \frac{T^2}{N}$$

$$= \frac{(22)^2}{4} + \frac{(16)^2}{4} + \frac{(47)^2}{4} - 602.08 = 135.17$$

$$SSR = \frac{(\sum Y_1)^2}{N_1} + \frac{(\sum Y_2)^2}{N_2} + \frac{(\sum Y_3)^2}{N_3} + \frac{(\sum Y_4)^2}{N_4} - \frac{T^2}{N}$$

$$= \frac{(19)^2}{3} + \frac{(25)^2}{3} + \frac{33^2}{3} + \frac{8^2}{3} - 602.8$$

$$= 110.91$$

$$SSE = TSS - SSC - SSR$$

$$= 264.92 - 135.17 - 110.91 = 0$$

$$= 18.84$$

$$d.f. = 8(3) - 8(1) - 8(3) + 1 = 6$$

ANOVA :

$$\text{Sum of Squares} / \text{d.f.} = \text{Mean Square} / \text{d.f.} = \text{Ratio.}$$

$$\text{between column} = \frac{SSC}{c-1} = \frac{135.17}{3-1} = 67.58 \quad F_c = \frac{MSC}{MSE}$$

$$= \frac{67.58}{67.58} = 1 \quad F_c = 67.58 / 3.14$$

$$\text{between row} = \frac{SSR}{r-1} = \frac{110.91}{4-1} = 36.97 \quad F_R = \frac{MSR}{MSE}$$

$$\text{Error} = \frac{SSE}{(R-1)(C-1)} = \frac{18.84}{3 \times 2} = 3.14 \quad MSE = \frac{SSE}{(R-1)(C-1)} = \frac{18.84}{3 \times 2} = 3.14$$

$$= 3.14 \text{ uncorrected} = 11.77$$

Significant level $F_{0.05}(2,6) = 4.23$

$$\text{Table value} = F_c(2,6) = 10.92 < 21.52$$

$$\text{Table value} = F_R(3,6) = 9.78 < 11.77$$

H_0 is rejected in both cases / rejected along columnwise and along row-wise.

Three varieties of A,B,C of crop tested in Randomized block design with replication analyse the experiment.

A	G	C	S	A	8	B	9	
B	8	A	4	B	6	C	9	
C	7	B	6	C	10	A	7	
	1	2	3	4	Tot	x_1^2	x_2^2	x_3^2
								x_4^2
A	6	4	8	6	24	36	16	64
B	8	6	6	9	29	64	36	36
C	7	5	10	9	31	49	25	100
tot	21	15	24	24	84	149	77	200
						Excess	Deficit	

H₀: There is no significant difference

H₁: There is significant difference

$$N = 120, H - T = 84, \text{ corrected factor} = \frac{T^2}{N} = 588$$

$$TSS = 149 + 77 + 200 + 198 - 588 = 36$$

$$SSC = \frac{(21)^2}{3} + \frac{(15)^2}{3} + \frac{(24)^2}{3} + \frac{(24)^2}{3} - 588 = 18$$

$$SSR = \frac{(24)^2}{4} + \frac{(29)^2}{4} + \frac{31^2}{4} - 588 = 6.5$$

$$SSE = 36 - 18 - 6.5 = 11.5$$

ANOVA :

src of variation sum of d.f. mean sum ratio.

$$\text{between col} \quad SSC = 18 \quad MSc = \frac{18}{3} = 6 \quad F_{C-1} = 7$$

$$\begin{array}{lll} \text{between row} & SSR = R-1 & MSR = \frac{6.5}{2} \\ & 6.5 & = 3-1=2 \\ & & = 3.25 \end{array}$$

$$\begin{array}{lll} \text{Error} & SSE = (R-1)(C-1) & MSE = \frac{11.5}{6} \\ & 11.5 & = (3 \times 2 - 6) \\ & & = 1.91 \end{array}$$

partial factor 1, 2nd, 3rd, 4th, 5th, 6th, 7th, 8th, 9th, 10th, 11th, 12th, 13th, 14th, 15th, 16th, 17th, 18th, 19th, 20th, 21st, 22nd, 23rd, 24th, 25th, 26th, 27th, 28th, 29th, 30th, 31st, 32nd, 33rd, 34th, 35th, 36th, 37th, 38th, 39th, 40th, 41st, 42nd, 43rd, 44th, 45th, 46th, 47th, 48th, 49th, 50th, 51st, 52nd, 53rd, 54th, 55th, 56th, 57th, 58th, 59th, 60th, 61st, 62nd, 63rd, 64th, 65th, 66th, 67th, 68th, 69th, 70th, 71st, 72nd, 73rd, 74th, 75th, 76th, 77th, 78th, 79th, 80th, 81st, 82nd, 83rd, 84th, 85th, 86th, 87th, 88th, 89th, 90th, 91st, 92nd, 93rd, 94th, 95th, 96th, 97th, 98th, 99th, 100th, 101st, 102nd, 103rd, 104th, 105th, 106th, 107th, 108th, 109th, 110th, 111st, 112nd, 113rd, 114th, 115th, 116th, 117th, 118th, 119th, 120th, 121st, 122nd, 123rd, 124th, 125th, 126th, 127th, 128th, 129th, 130th, 131st, 132nd, 133rd, 134th, 135th, 136th, 137th, 138th, 139th, 140th, 141st, 142nd, 143rd, 144th, 145th, 146th, 147th, 148th, 149th, 150th, 151st, 152nd, 153rd, 154th, 155th, 156th, 157th, 158th, 159th, 160th, 161st, 162nd, 163rd, 164th, 165th, 166th, 167th, 168th, 169th, 170th, 171st, 172nd, 173rd, 174th, 175th, 176th, 177th, 178th, 179th, 180th, 181st, 182nd, 183rd, 184th, 185th, 186th, 187th, 188th, 189th, 190th, 191st, 192nd, 193rd, 194th, 195th, 196th, 197th, 198th, 199th, 200th, 201st, 202nd, 203rd, 204th, 205th, 206th, 207th, 208th, 209th, 210th, 211st, 212nd, 213rd, 214th, 215th, 216th, 217th, 218th, 219th, 220th, 221st, 222nd, 223rd, 224th, 225th, 226th, 227th, 228th, 229th, 230th, 231st, 232nd, 233rd, 234th, 235th, 236th, 237th, 238th, 239th, 240th, 241st, 242nd, 243rd, 244th, 245th, 246th, 247th, 248th, 249th, 250th, 251st, 252nd, 253rd, 254th, 255th, 256th, 257th, 258th, 259th, 260th, 261st, 262nd, 263rd, 264th, 265th, 266th, 267th, 268th, 269th, 270th, 271st, 272nd, 273rd, 274th, 275th, 276th, 277th, 278th, 279th, 280th, 281st, 282nd, 283rd, 284th, 285th, 286th, 287th, 288th, 289th, 290th, 291st, 292nd, 293rd, 294th, 295th, 296th, 297th, 298th, 299th, 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586th, 587th, 588th, 589th, 590th, 591st, 592nd, 593rd, 594th, 595th, 596th, 597th, 598th, 599th, 600th, 601st, 602nd, 603rd, 604th, 605th, 606th, 607th, 608th, 609th, 610th, 611st, 612nd, 613rd, 614th, 615th, 616th, 617th, 618th, 619th, 620th, 621st, 622nd, 623rd, 624th, 625th, 626th, 627th, 628th, 629th, 630th, 631st, 632nd, 633rd, 634th, 635th, 636th, 637th, 638th, 639th, 640th, 641st, 642nd, 643rd, 644th, 645th, 646th, 647th, 648th, 649th, 650th, 651st, 652nd, 653rd, 654th, 655th, 656th, 657th, 658th, 659th, 660th, 661st, 662nd, 663rd, 664th, 665th, 666th, 667th, 668th, 669th, 670th, 671st, 672nd, 673rd, 674th, 675th, 676th, 677th, 678th, 679th, 680th, 681st, 682nd, 683rd, 684th, 685th, 686th, 687th, 688th, 689th, 690th, 691st, 692nd, 693rd, 694th, 695th, 696th, 697th, 698th, 699th, 700th, 701st, 702nd, 703rd, 704th, 705th, 706th, 707th, 708th, 709th, 710th, 711st, 712nd, 713rd, 714th, 715th, 716th, 717th, 718th, 719th, 720th, 721st, 722nd, 723rd, 724th, 725th, 726th, 727th, 728th, 729th, 730th, 731st, 732nd, 733rd, 734th, 735th, 736th, 737th, 738th, 739th, 740th, 741st, 742nd, 743rd, 744th, 745th, 746th, 747th, 748th, 749th, 750th, 751st, 752nd, 753rd, 754th, 755th, 756th, 757th, 758th, 759th, 760th, 761st, 762nd, 763rd, 764th, 765th, 766th, 767th, 768th, 769th, 770th, 771st, 772nd, 773rd, 774th, 775th, 776th, 777th, 778th, 779th, 780th, 781st, 782nd, 783rd, 784th, 785th, 786th, 787th, 788th, 789th, 790th, 791st, 792nd, 793rd, 794th, 795th, 796th, 797th, 798th, 799th, 800th, 801st, 802nd, 803rd, 804th, 805th, 806th, 807th, 808th, 809th, 810th, 811st, 812nd, 813rd, 814th, 815th, 816th, 817th, 818th, 819th, 820th, 821st, 822nd, 823rd, 824th, 825th, 826th, 827th, 828th, 829th, 830th, 831st, 832nd, 833rd, 834th, 835th, 836th, 837th, 838th, 839th, 840th, 841st, 842nd, 843rd, 844th, 845th, 846th, 847th, 848th, 849th, 850th, 851st, 852nd, 853rd, 854th, 855th, 856th, 857th, 858th, 859th, 860th, 861st, 862nd, 863rd, 864th, 865th, 866th, 867th, 868th, 869th, 870th, 871st, 872nd, 873rd, 874th, 875th, 876th, 877th, 878th, 879th, 880th, 881st, 882nd, 883rd, 884th, 885th, 886th, 887th, 888th, 889th, 890th, 891st, 892nd, 893rd, 894th, 895th, 896th, 897th, 898th, 899th, 900th, 901st, 902nd, 903rd, 904th, 905th, 906th, 907th, 908th, 909th, 910th, 911st, 912nd, 913rd, 914th, 915th, 916th, 917th, 918th, 919th, 920th, 921st, 922nd, 923rd, 924th, 925th, 926th, 927th, 928th, 929th, 930th, 931st, 932nd, 933rd, 934th, 935th, 936th, 937th, 938th, 939th, 940th, 941st, 942nd, 943rd, 944th, 945th, 946th, 947th, 948th, 949th, 950th, 951st, 952nd, 953rd, 954th, 955th, 956th, 957th, 958th, 959th, 960th, 961st, 962nd, 963rd, 964th, 965th, 966th, 967th, 968th, 969th, 970th, 971st, 972nd, 973rd, 974th, 975th, 976th, 977th, 978th, 979th, 980th, 981st, 982nd, 983rd, 984th, 985th, 986th, 987th, 988th, 989th, 990th, 991st, 992nd, 993rd, 994th, 995th, 996th, 997th, 998th, 999th, 1000th, 1001st, 1002nd, 1003rd, 1004th, 1005th, 1006th, 1007th, 1008th, 1009th, 1010th, 1011st, 1012nd, 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Three way classification:

Source of variation	Sum of squares	d.f	mean sum of squares	F ratio.
between columns	SSC	K-1	$MSC = \frac{SSC}{K-1}$	$F_C = \frac{MSE}{MSE}$
between rows	SSR	K-1	$MSR = \frac{SSR}{K-1}$	$F_R = \frac{MSR}{SSE}$
between treatment	SST	K-1	$MST = \frac{SST}{K-1}$	$F_T = \frac{MST}{SSE}$
Error	SSE	(K-1)(K-2)	$MSE = \frac{SSE}{(K-1)(K-2)}$	

The following Latin square of a design when 4 varieties of seeds are being tested set up the analysis of variance - tables and state your conclusion.

A	105	B	95	C	125	D	115
B	115	D	125	A	105	B	105
D	115	C	95	B	105	A	115
B	95	A	135	D	115	C	115

$$X_1, X_2, X_3, X_4, S_{\text{tot}} = X_1^2 + X_2^2 + X_3^2 + X_4^2$$

$$Y_1 = 5 = 1, -5 = -1, 25 = 5, 15 = 3, 8 = 1, 1 = 1, 25 = 5, 9 = 1$$

$$Y_2 = 15 = 3, 25 = 5, 5 = 1, 5 = 1, 10 = 1, 9 = 25 = 1, 1 = 1$$

$$Y_3 = 15 = 3, -5 = -1, 5 = 1, 15 = 3, 6 = 1, 9 = 1, 1 = 1, 9 = 1$$

$$Y_4 = -5 = -1, 35 = 7, -5 = -1, 15 = 3, 8 = 1, 1 = 1, 49 = 1, 1 = 1$$

$$\text{tot} = 6, 10, 6, 10, 32, 24, 76, 28, 28$$

H_0 : There's no significant difference.

H_1 : There is significant difference.

$$N = 16, T = 32, \text{ corrected factor } \frac{T^2}{N} = \frac{32^2}{16} = 64$$

$$TSS = 20 + 76 + 28 + 28 - 64 = 88$$

$$SSC = \frac{(6)^2}{4} + \frac{(10)^2}{4} + \frac{6^2}{4} + \frac{10^2}{4} - 64 = 4$$

$$SSR = \frac{(8)^2}{4} + \frac{(10)^2}{4} + \frac{6^2}{4} + \frac{8^2}{4} - 64 = 2$$

One way classification:

	A	B	C	D	Tot
Row	1	-1	+1	-1	0
Col	1	5	3	-5	10
Row Col	1	3	5	-3	10
Tot	8	10	6	8	32

$$A = 1 \quad B = -1 \quad C = 5 \quad D = 3$$

$$SST = \frac{(z_1)^2}{N_1} + \frac{(\sum z_2)^2}{N_2} + \frac{(\sum z_3)^2}{N_3} + \frac{(\sum z_4)^2}{N_4}$$

$$SST = \frac{(12)^2}{4} + 0 + \frac{(10)^2}{4} + \frac{(10)^2}{4} = 60$$

$$SST = 60$$

$$SSE = TSS - SSC - SSR - SST_{error}$$

ANOVA:

Source of variation	Sum of squares	d.f	MSS	ratio.
B/w rows	$SSR = 20$	$df_A = 3$	$\frac{20}{3} = 6.67$	$F_R = \frac{6.67}{0.67} = 9.92$
B/w cols	$SSC = 4$	$\frac{4}{3} = 1.33$	$F_C = \frac{10}{1.33} = 7.51$	
B/w treatment	$SST = 22$	$df_T = 3$	$\frac{22}{3} = 7.33$	$F_T = \frac{10}{7.33} = 1.36$

$$\text{Error} \quad SSE = 60 \quad (k-1)(k-2) \quad \frac{60}{6} = 10 = 2.5 \quad F_E = 2.5 / 7.33 = 0.34$$

$$\text{Table value} = F_R(6,3), F_C(6,3), F_T(6,3)$$

$$= 8.94$$

$$14.92 > 8.94, (7.51, 1.36) < 8.94$$

Hypothesis accepted along column wise and treatment rejected along row wise.

* There is no significant difference in column and treatment.

There is significant difference in row.

C 25 B 23 A 20 D 20
 C 19 D 19 C 21 B 18

Subtract by 20

B 19 A 14 D 17 C 20
 C 20 B 21 A 15

D 17 C 20 B 21 A 15

C 5 B 3 A 0 D 8

A -1 D -1 C 1 B -2

B -1 A -6 D -3 C 0

D -3 C 0 B 1 A -5

	x_1	x_2	x_3	x_4	tot	x_1^2	x_2^2	x_3^2	x_4^2
y_1	5	3	0	0	8	25	9	0	0
y_2	-1	-1	1	-2	-3	1	1	1	4
y_3	-1	-6	-3	0	-10	1	36	9	0
y_4	-3	0	1	-5	-7	9	0	1	25
tot	0	-4	-1	-7	-12	36	46	11	29

$$N = 16, T = -12 \quad \text{corrected factor} = \frac{T^2}{N} = \frac{144}{16} = 9$$

$$\text{TSS} = 36 + 46 + 11 + 29 - 9 = 113$$

$$\text{SSC} = 0 + \frac{(-4)^2}{4} + \frac{(-1)^2}{4} + \frac{(-7)^2}{4} = 16.5 - 9 = 7.5$$

$$\text{SSR} = \frac{8^2}{4} + \frac{(-3)^2}{4} + \frac{(-10)^2}{4} + \frac{(-7)^2}{4} - 9 = 46.5$$

A	0	-1	-6	-5	tot -12
B	3	-2	-1	1	1
C	5	1	0	0	6
D	0	-1	-3	-3	-7

$$\text{SST} = \frac{(-12)^2}{4} + \frac{1}{4} + \frac{6^2}{4} + \frac{(-7)^2}{4} - 9 = 48.5$$

$$\text{SSE} = 113 - 7.5 - 46.5 - 48.5 = 10.5$$

ANOVA:

src of variation	sum of square	d.f	MSS	ratio.
B/W rows	$\text{SSR} = 46.5$	$4-1=3$	MSR = 15.5	$F_c = \frac{2.5}{1.15}$
B/W cols	$\text{SSC} = 7.5$	3	MSL = 2.5	$= 1.42$

$$\begin{array}{l}
 \text{B/w treatment} \quad \text{SST} = 48.5 \quad 3 \quad \text{MST} = 16.16 \quad F_R = \frac{15.5}{1.75} = 8.8 \\
 \text{Error} \quad \text{SSE} = 10.5 \quad 6 \quad \text{MSE} = 1.75 \quad F_T = \frac{16.16}{1.75} = 9.23 \\
 \end{array}$$

$$\begin{array}{l}
 \text{Table value} = (6,3) = 8.94 \\
 9.23 > 8.94, (8.857, 1.42) < 8.94
 \end{array}$$

H_0 is accepted along column wise and row-wise rejected along treatment.

	A	B	C	D	E	F	G
O-A	P-A	P-B	P-C	P-D	P-E	P-F	P-G
P-A	R-A	R-B	R-C	R-D	R-E	R-F	R-G
P-B	R-B	R-C	R-D	R-E	R-F	R-G	R-H
P-C	R-C	R-D	R-E	R-F	R-G	R-H	R-I
P-D	R-D	R-E	R-F	R-G	R-H	R-I	R-J
P-E	R-E	R-F	R-G	R-H	R-I	R-J	R-K
P-F	R-F	R-G	R-H	R-I	R-J	R-K	R-L
P-G	R-G	R-H	R-I	R-J	R-K	R-L	R-M

$F = \frac{\text{MST}}{\text{MSE}}$ is called F-ratio or F-statistic.

$$E(Y) = P_A P_B + M_A S_B + S_A P_B + S_B P_A$$

$$\text{C.R.} = P_A - Q_A + \frac{(P_A - Q_A)^2}{P_A} + \frac{P_A^2}{P_A} + Q_A - \frac{Q_A^2}{P_A} = 0.328$$

$$\text{E.dft} = P_A \frac{(P_A - 1)(Q_A)}{P_A} + \frac{(Q_A - 1)(P_A)}{P_A} + \frac{P_A - 1}{P_A} = 20.8$$

	A	B	C	D	E	F	G
O-A	P-A	P-B	P-C	P-D	P-E	P-F	P-G
P-A	R-A	R-B	R-C	R-D	R-E	R-F	R-G
P-B	R-B	R-C	R-D	R-E	R-F	R-G	R-H
P-C	R-C	R-D	R-E	R-F	R-G	R-H	R-I
P-D	R-D	R-E	R-F	R-G	R-H	R-I	R-J
P-E	R-E	R-F	R-G	R-H	R-I	R-J	R-K
P-F	R-F	R-G	R-H	R-I	R-J	R-K	R-L
P-G	R-G	R-H	R-I	R-J	R-K	R-L	R-M

$$E.Y = P_A P_B + \frac{P_A}{P_A} + \frac{P_B}{P_B} + \frac{(P_A - 1)(Q_A)}{P_A} + \frac{(Q_A - 1)(P_A)}{P_A} = 7.28$$

$$\text{S.C.E.} = \text{Sum of all } O_{ij} - C.R. - E.Y = 32.2$$

Other

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