Canonical cover (or) Minimal cover

A canonical cover F_c for F is a set of dependencies such that F logically implies all dependencies in F_c, and F_c logically implies all dependencies in F. **Furthermore, F_c must have the following properties:**

1. No functional dependency in F_c contains an extraneous attribute.
2. Each left side of a functional dependency in F_c is unique. That is, there are no two dependencies $\alpha_1 \rightarrow \beta_1$ and $\alpha_2 \rightarrow \beta_2$ in F_c such that $\alpha_1 = \alpha_2$.

Extraneous attributes

An attribute of an FD is said to be extraneous if we can remove it without changing the closure of the set of FD.

Steps for finding Minimal cover

STEP1: For a given set of FD, decompose each FD using decomposition rule (Armstrong Axiom) if the right side of any FD has more than one attribute.

STEP2: Now make a new set of FD having all decomposed FD.

STEP3: Find closure of the left side of each of the given FD **by including that FD and excluding that FD,** if closure in both cases are same then that FD is redundant and we remove that FD from the given set, otherwise if both the closures are different then we do not exclude that FD.

STEP4: Repeat step 3 till all the FDs in FD set are complete.

STEP5: After STEP 4, find resultant FD.

STEP 6: Next selecting those FD's (from step5) which are having more than one attribute on its left.
[Ex]: FD of the form, $AD \rightarrow C$ has two attributes at its left, let's check their importance, i.e. whether they both are important or only one.
STEP 6 a: Find Closure AD+

STEP 6 b: Find Closure A+

STEP 6 c: Find Closure D+

Compare Closure of STEP (6a, 6b, 6c) if the closure of AD+, A+, D+ are not equivalent, hence in FD $AD \rightarrow C$, both A and D are important attributes and cannot be removed, otherwise, remove the redundant attribute.

Problem1: Given a relational Schema R(A, B, C, D) and set of Function Dependency FD = { $B \rightarrow A$, $AD \rightarrow BC$, $C \rightarrow ABD$ }. Find the canonical cover?

Solution: Given FD = {$B \rightarrow A$, $AD \rightarrow BC$, $C \rightarrow ABD$ }, now decompose the FD using decomposition rule.

Step1:

1. $B \rightarrow A$
2. $AD \rightarrow B$ (using decomposition rule on $AD \rightarrow BC$)
3. $AD \rightarrow C$ (using decomposition rule on $AD \rightarrow BC$)
4. $C \rightarrow A$ (using decomposition rule on $C \rightarrow ABD$)
5. $C \rightarrow B$ (using decomposition rule on $C \rightarrow ABD$)
6. $C \rightarrow D$ (using decomposition rule on $C \rightarrow ABD$)

Step 2: New set of FD = { $B \rightarrow A$, $AD \rightarrow B$, $AD \rightarrow C$, $C \rightarrow A$, $C \rightarrow B$, $C \rightarrow D$ }

Step 3: The next step is to find closure of the left side of each of the given FD by including that FD and excluding that FD, if closure in both cases are same then that FD is redundant and remove that FD from the given set, otherwise if both the closures are different then , do not exclude that FD.

Calculating closure of all FD {$B \rightarrow A$, $AD \rightarrow B$, $AD \rightarrow C$, $C \rightarrow A$, $C \rightarrow B$, $C \rightarrow D$ }

1a. Closure B+ = BA using FD = { $B \rightarrow A$, $AD \rightarrow B$, $AD \rightarrow C$, $C \rightarrow A$, $C \rightarrow B$, $C \rightarrow D$ }

1b.Closure B+ = B using FD = {$AD \rightarrow B$, $AD \rightarrow C$, $C \rightarrow A$, $C \rightarrow B$, $C \rightarrow D$ }

From 1a and 1b, found that both the Closure(by including B $\rightarrow$ A and excluding B $\rightarrow$ A) are not equivalent, hence FD B $\rightarrow$ A is important and cannot be removed from the set of FD.

2 a. Closure AD+ = ADBC using FD ={ B $\rightarrow$A, AD $\rightarrow$ B, AD $\rightarrow$ C, C $\rightarrow$ A, C $\rightarrow$ B, C $\rightarrow$ D }

2 b. Closure AD+ = ADCB using FD = { B $\rightarrow$ A, AD $\rightarrow$ C, C $\rightarrow$ A, C $\rightarrow$ B, C $\rightarrow$ D }

From 2a and 2b,found that both the Closure (by including AD $\rightarrow$ B and excluding AD $\rightarrow$ B) are equivalent, hence FD AD $\rightarrow$ B is not important and can be removed from the set of FD.

Hence resultant FD = { B $\rightarrow$ A, AD $\rightarrow$ C, C $\rightarrow$ A, C $\rightarrow$ B, C $\rightarrow$ D }

3 a. Closure AD+ = ADCB using FD = { B $\rightarrow$A, AD $\rightarrow$ C, C $\rightarrow$ A, C $\rightarrow$ B, C $\rightarrow$ D }

3 b. Closure AD+ = AD using FD = { B $\rightarrow$ A, C $\rightarrow$ A, C $\rightarrow$ B, C $\rightarrow$ D }

From 3 a and 3 b, found that both the Closure (by including AD $\rightarrow$ C and excluding AD $\rightarrow$ C) are not equivalent, hence FD AD $\rightarrow$ C is important and cannot be removed from the set of FD.

Hence resultant FD = { $B \rightarrow A$, $AD \rightarrow C$, $C \rightarrow A$, $C \rightarrow B$, $C \rightarrow D$ }

4 a. Closure $C+$ = CABD using FD = { $B \rightarrow A$, $AD \rightarrow C$, $C \rightarrow A$, $C \rightarrow B$, $C \rightarrow D$ }

4 b. Closure $C+$ = CBDA using FD = { $B \rightarrow A$, $AD \rightarrow C$, $C \rightarrow B$, $C \rightarrow D$ }

From 4a and 4b,found that both the Closure (by including $C \rightarrow A$ and excluding $C \rightarrow A$) are equivalent, hence FD $C \rightarrow A$ is not important and can be removed from the set of FD.

Hence resultant FD = {$B \rightarrow A$, $AD \rightarrow C$, $C \rightarrow B$, $C \rightarrow D$}
5a.Closure $C+$= CBDA using FD={$B \rightarrow A$, $AD \rightarrow C$, $C \rightarrow B$, $C \rightarrow D$}
5b. Closure $C+$ = CD using FD={$B \rightarrow A$,$AD \rightarrow C$, $C \rightarrow D$}

From 5a and 5b, found that both the Closure (by including $C \rightarrow B$ and excluding $C \rightarrow B$) are not equivalent, hence FD $C \rightarrow B$ is important and cannot be removed from the set of FD.

Hence resultant FD = { $B \rightarrow A$, $AD \rightarrow C$, $C \rightarrow B$, $C \rightarrow D$ }
6 a. Closure $C+$ = CDBA using FD = { $B \rightarrow A$, $AD \rightarrow C$, $C \rightarrow B$, $C \rightarrow D$ }
6 b. Closure $C+$ = CBA using FD = { $B \rightarrow A$, $AD \rightarrow C$, $C \rightarrow B$ }

From 6a and 6b, found that both the Closure(by including C $\rightarrow$ D and excluding C $\rightarrow$ D) are not equivalent, hence FD C $\rightarrow$ D is important and cannot be removed from the set of FD.

Hence resultant FD = { B → A, AD → C, C → B, C → D }

Step 5: Now FD= { B → A, AD → C, C → B, C → D }

Step6: Check the left side of FD AD → C has two attributes, let's check their importance, i.e. whether they both are important or only one.
Closure AD+ = ADCB using FD = { B →A, AD → C, C → B, C → D }
Closure A+ = A using FD= {B →A, AD → C, C → B, C → D}
Closure D+ = D using FD= {B →A, AD → C, C → B, C → D}
Since the closure of AD+, A+, D+ that found are not all equivalent, hence in FD AD → C, both A and D are important attributes and cannot be removed.

Hence resultant FD = { B → A, AD → C, C → B, C → D }

FD = { B → A, AD → C, C → BD } is Canonical Cover of FD = { B → A, AD → BC, C → ABD }.

Problem2: Given a relational Schema R(W, X, Y, Z) and set of FDs are = { W → X, Y → X, Z → WXY, WY → Z }. Find the canonical cover?

Solution: Given FD = { W → X, Y → X, Z → WXY, WY → Z }, now decompose the FD using decomposition rule(Armstrong Axiom).

1. W → X
2. Y → X
3. Z → W (using decomposition rule on Z → WXY)
4. Z → X (using decomposition rule on Z → WXY)
5. Z → Y (using decomposition rule on Z → WXY)
6. WY → Z

Now set of FD = { W → X, Y → X, WY → Z, Z → W, Z → X, Z → Y }

The next step is to find closure of the left side of each of the given FD by including that FD and excluding that FD, if closure in both cases are same then that FD is redundant and remove that FD from the given set, otherwise if both the closures are different then do not exclude that FD.

Calculating closure of all FD { W → X, Y → X, Z → W, Z → X, Z → Y, WY → Z }

1 a. Closure W+ = WX using FD = { W → X, Y → X, Z → W, Z → X, Z → Y, WY → Z }

1 b. Closure W+ = W using FD = { Y → X, Z → W, Z → X, Z → Y, WY → Z }

From 1 a and 1 b, found that both the Closure (by including W → X and excluding W → X) are not equivalent, hence FD W → X is important and cannot be removed from the set of FD.

Hence resultant FD = { W → X, Y → X, Z → W, Z → X, Z → Y, WY → Z }

2 a. Closure Y+ = YX using FD = { W → X, Y → X, Z → W, Z → X, Z → Y, WY → Z }

2 b. Closure Y+ = Y using FD = { W → X, Z → W, Z → X, Z → Y, WY → Z }

From 2 a and 2 b found that both the Closure (by including Y → X and excluding Y → X) are not equivalent, hence FD Y → X is important and cannot be removed from the set of FD.

Hence resultant FD = {W → X, Y → X, Z → W, Z → X, Z → Y, WY → Z}

3 a. Closure Z+ = ZWXY using FD = { W → X, Y → X, Z → W, Z → X, Z → Y, WY → Z }

3 b. Closure Z+ = ZXY using FD = { W → X, Y → X, Z → X, Z → Y, WY → Z }

From 3 a and 3 b, found that both the Closure (by including Z → W and excluding Z → W) are not equivalent, hence FD Z → W is important and cannot be removed from the set of FD.

Hence resultant FD = { W → X, Y → X, Z → W, Z → X, Z → Y, WY → Z }

4 a. Closure Z+ = ZXWY using FD = { W → X, Y → X, Z → W, Z → X, Z → Y, WY → Z }

4 b. Closure Z+ = ZWYX using FD = { W → X, Y → X, Z → W, Z → Y, WY → Z }

From 4 a and 4 b, found that both the Closure (by including Z → X and excluding Z → X) are equivalent, hence FD Z → X is not important and can be removed from the set of FD.

Hence resultant FD = { W → X, Y → X, Z → W, Z → Y, WY → Z }

5 a. Closure Z+ = ZYWX using FD = { W → X, Y → X, Z → W, Z → Y, WY → Z }

5 b. Closure Z+ = ZWX using FD = { W → X, Y → X, Z → W, WY → Z }

From 5 a and 5 b, found that both the Closure (by including $Z \rightarrow Y$ and excluding $Z \rightarrow Y$) are not equivalent, hence FD $Z \rightarrow X$ is important and cannot be removed from the set of FD.

Hence resultant FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow W$, $Z \rightarrow Y$, $WY \rightarrow Z$ }

6 a. Closure WY+ = WYZX using FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow W$, $Z \rightarrow Y$, $WY \rightarrow Z$ }

6 b. Closure WY+ = WYX using FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow W$, $Z \rightarrow Y$ }

From 6 a and 6 b, found that both the Closure (by including $WY \rightarrow Z$ and excluding $WY \rightarrow Z$) are not equivalent, hence FD $WY \rightarrow Z$ is important and cannot be removed from the set of FD.

Hence resultant FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow W$, $Z \rightarrow Y$, $WY \rightarrow Z$ }

Since FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow W$, $Z \rightarrow Y$, $WY \rightarrow Z$ } is resultant FD.

Next check the FD $WY \rightarrow Z$ has two attributes at its left, let's check their importance, i.e. whether they both are important or only one.

Closure $WY^+ = WYZX$ using FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow W$, $Z \rightarrow Y$, $WY \rightarrow Z$ }

Closure $W^+ = WX$ using FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow W$, $Z \rightarrow Y$, $WY \rightarrow Z$ }

Closure $Y^+ = YX$ using FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow W$, $Z \rightarrow Y$, $WY \rightarrow Z$ }

Since the closure of WY^+, W^+, Y^+ , found that they are not equivalent, hence in FD $WY \rightarrow Z$, both W and Y are important attributes and cannot be removed.

Hence resultant FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow W$, $Z \rightarrow Y$, $WY \rightarrow Z$ }.

FD = { $W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow WY$, $WY \rightarrow Z$ } is Canonical Cover of FD={$W \rightarrow X$, $Y \rightarrow X$, $Z \rightarrow WXY$, $WY \rightarrow Z$ }.

Problem3: Given a relational Schema R(V, W, X, Y, Z) and set of Function Dependency FD = { $V \rightarrow W$, $VW \rightarrow X$, $Y \rightarrow VXZ$ }. Find the canonical cover?

Solution: Given FD={ $V \rightarrow W$, $VW \rightarrow X$, $Y \rightarrow VXZ$ }. Now decompose the FD using decomposition rule.

1. $V \rightarrow W$
2. $VW \rightarrow X$

3. $Y \rightarrow V$ (using decomposition rule on $Y \rightarrow VXZ$)
4. $Y \rightarrow X$ (using decomposition rule on $Y \rightarrow VXZ$)
5. $Y \rightarrow Z$ (using decomposition rule on $Y \rightarrow VXZ$)

Now set of FD = { $V \rightarrow W$, $VW \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$ }.

The next step is to find closure of the left side of each of the given FD by including that FD and excluding that FD, if closure in both cases is same then that FD is redundant and remove that FD from the given set, otherwise if both the closures are different then do not exclude that FD.

Calculating closure of all FD { $V \rightarrow W$, $VW \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$ }.

1 a. Closure V+ = VWX using FD = {$V \rightarrow W$, $VW \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$}

1 b. Closure V+ = V using FD = {$VW \rightarrow X$, $Y \rightarrow V$, $Y \rightarrow X$, $Y \rightarrow Z$ }

From 1 a and 1 b, found that both the Closure(by including $V \rightarrow W$ and excluding $V \rightarrow W$) are not equivalent, hence FD $V \rightarrow W$ is important and cannot be removed from the set of FD.

Hence resultant FD = {V → W, VW → X, Y → V, Y → X, Y → Z }.

2 a. Closure VW+ = VWX using FD = { V → W, VW → X, Y → V, Y → X, Y → Z }

2 b. Closure VW+ = VW using FD = { V → W, Y → V, Y → X, Y → Z }

From 2 a and 2 b, found that both the Closure(by including VW → X and excluding VW → X) are not equivalent, hence FD VW → X is important and cannot be removed from the set of FD.

Hence resultant FD = { V → W, VW → X, Y → V, Y → X, Y → Z }.

3 a. Closure Y+ = YVXZW using FD = { V → W, VW → X, Y → V, Y → X, Y → Z }

3 b. Closure Y+ = YXZ using FD = { V → W, VW → X, Y → X, Y → Z }

From 3 a and 3 b, found that both the Closure(by including Y → V and excluding Y → V) are not equivalent, hence FD Y → V is important and cannot be removed from the set of FD.

Hence resultant FD = { V → W, VW → X, Y → V, Y → X, Y → Z }.

4 a. Closure Y+ = YXVZW using FD = { V → W, VW → X, Y → V, Y → X, Y → Z }

4 b. Closure Y+ = YVZWX using FD = { V → W, VW → X, Y → V, Y → Z }

From 4 a and 4 b, found that both the Closure(by including Y → X and excluding Y → X) are equivalent, hence FD Y → X is not important and can be removed from the set of FD.

Hence resultant FD = { V → W, VW → X, Y → V, Y → Z }.

5 a. Closure Y+ = YZVWX using FD = { V → W, VW → X, Y → V, Y → Z }

5 b. Closure Y+ = YVWX using FD = { V → W, VW → X, Y → V }

From 5 a and 5 b, found that both the Closure(by including Y → Z and excluding Y → Z) are not equivalent, hence FD Y → Z is important and cannot be removed from the set of FD.

Hence resultant FD={V→W,VW→ X,Y → V,Y→ Z }.

Since FD = { V → W, VW → X, Y → V, Y → Z } is resultant FD.

Now, check the left side of FD, VW → X has two attributes at its left, let's check their importance, i.e. whether they both are important or only one.

Closure VW+ = VWX using FD = { V → W, VW → X, Y → V, Y → Z }

Closure V+ = VWX using FD = { V → W, VW → X, Y → V, Y → Z }

Closure W+ = W using FD = { V → W, VW → X, Y → V, Y → Z }

Since the closure of VW+, V+, W+ found that all the Closures of VW and V are equivalent, hence in FD VW → X, W is not at all an important attribute and can be removed.

Hence resultant FD={V → W,V→ X, Y → V, Y → Z }
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FD = { V → WX, Y → VZ } is Canonical Cover of FD = { V → W, VW → X, Y → VXZ }.