Bayes Risk Lower Bounds

- Intuitively, if $I(\mathscr{P}, w)$ is small, then Bayes risk should be close to Null risk

$$\text{Bayes Risk} \geq 1 + \frac{I(\mathscr{P}, w) + \log(1 + R_0)}{\log(1 - R_0)}$$

Mapping to Our Problem

- $\Theta = \{(u, v) : u, v \in \mathbb{R}^n\}$

- w is the Gaussian distribution over Θ

- $\mathscr{P}_{(u,v)}$ is the distribution of $(\alpha/\sqrt{n}) \cdot Q^{(1)}(u \otimes v) + Q^{(1)} \cdot \text{vec}(G)$

  - By rotational invariance : $Q^{(1)} \cdot \text{vec}(G) \sim N(0, I_t)$

  - $\mathscr{P}_{(u,v)} = N(\mu = (\alpha/\sqrt{n}) \cdot Q^{(1)}(u \otimes v), \Sigma = I_t)$