

Bayes Risk Lower Bounds

- Is $r^{(1)}$ enough to pick good $Q^{(2)}$?
- Bayes risk lower bounds:
 - Framework used by Simchowitz, El Aloui, Recht '18 to obtain matrix-vector lower bounds for a related problem
- Θ be a parameter space and $\{\mathcal{P}_\theta : \theta \in \Theta\}$ be a set of distributions
- Suppose $\theta \sim w$ and $x \sim \mathcal{P}_\theta$ and x is given to us
 - Bayes risk lower bounds show how much we can say about θ

Intuition about Bayes Risk

- All \mathcal{P}_θ are the same \implies Can't say anything about θ from x
 - Need to capture how different \mathcal{P}_θ are
- One way to capture:

$$I(\mathcal{P}, w) = \inf_Q E_{\theta \sim w} [d_{\text{KL}}(\mathcal{P}_\theta \parallel Q)]$$

- If all \mathcal{P}_θ are “close” to some Q , then $I(\mathcal{P}, w)$ is “small”