Linear Measurements vs Matrix-Vector Products

- If A is $n \times n$, matrix-vector products can be simulated with n linear measurements
- *n* linear measurements are **at least** as powerful as one matrix-vector product
- Trace can be computed exactly with one linear measurement
 - Requires $\Omega(n)$ matrix-vector products for exact trace
 - Sun, Woodruff, Yang, Zhang '21 (Triangle detection)
 - Requires $\Omega(1/\varepsilon^2)$ for approximating unto $1 \pm \varepsilon$
 - Wimmer, Wu, Zhang '14

Upper Bounds

- With n^2 linear measurements, can read entire matrix
 - 1 round and n^2 linear measurements suffice
- For constant r, $O(\log n)$ iterations of power method suffice
 - $O(\log n)$ rounds and O(n) linear measurements per round suffice
- What is the measurements-vs-rounds tradeoff?