Feature map for Polynomials

• A finite dimensional φ such that $\langle \varphi(q), \varphi(k) \rangle = \langle q, k \rangle^p$?

 $\varphi: x \mapsto x^{\otimes p}$

• If $x \in \mathbb{R}^h$, then $x^{\otimes p} \in \mathbb{R}^{h^p}$

•
$$(x^{\otimes p})_{(i_1,i_2,\ldots,i_p)} = x_{i_1} \cdot x_{i_2} \cdot \cdots \cdot x_{i_p}$$

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Linear Attention using Polynomials