- Initialize $x \leftarrow 0 \in \mathbb{R}^d$

- On update (i, Δ) :

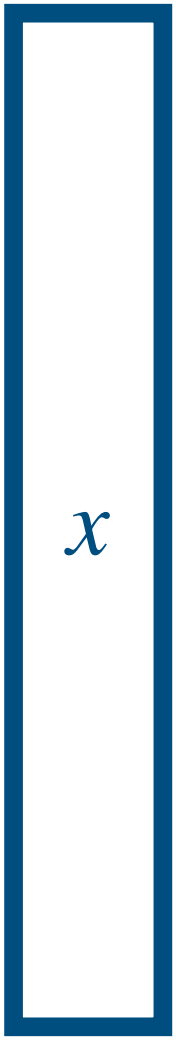
- Set $x[i] \leftarrow x[i] + \Delta$

- Answer queries about x at the end of the stream using **small space**

- What was $\|x\|_\infty$ or $x[i]$ or $\|x\|_2$ ...

- **Update time**: Time to process a new update

Turnstile Streaming



Updates: (i_1, Δ_1) (i_2, Δ_2) $\ldots$ (i_m, Δ_m)

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Updates: (i_1, Δ_1) (i_2, Δ_2) … (i_m, Δ_m)

Our Results

- $F_p(x) = \sum_{i=1}^{d} |x_i|^p$

- **Theorem** [K, Pagh, Thorup, Woodruff FOCS ’23]: For $p > 2$, there is an algorithm using $\tilde{O}(d^{1-2/p})$ space and an update time of $O(1)$
  - Space is optimal unto polylog factors!
  - Time in the WordRAM model with $O(\log d)$ bits word size
  - Improves on poly(d) update time of Andoni ‘17