

# Bayes Risk Lower Bounds

- Decision rule  $\mathfrak{d} : x \mapsto a$  and loss function  $L : \theta \times a \mapsto \{0, 1\}$ 
  - Decision rule sees  $x$  and outputs some action  $a$
  - Loss function evaluates how good  $a$  is on  $\theta$
- Risk =  $E_{\theta \sim w}[E_{x \sim \mathcal{P}_\theta} L(\theta, \mathfrak{d}(x))]$
- Bayes risk is the minimum possible risk
- Null-risk is  $\inf_a E_{\theta \sim w}[L(\theta, a)]$

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- Intuitively, if  $I(\mathcal{P}, w)$  is small, then Bayes risk should be close to Null risk

$$\text{Bayes Risk} \geq 1 + \frac{I(\mathcal{P}, w) + \log(1 + R_0)}{\log(1 - R_0)}$$