

Bayes Risk Lower Bounds

- Alg: $x \mapsto a$ and loss function $L: \theta \times a \mapsto \{0, 1\}$

- Decision rule sees x and outputs some action a

- Loss function evaluates how good a is on θ

- $\text{Risk} = E_{\theta \sim \mathcal{V}} [E_{x \sim \mathcal{P}_{\theta}} L(\theta, \text{Alg}(x))]$

• **Bayes risk is the minimum possible risk**

- Null-risk is $\inf_a E_{\theta \sim \nu} [L(\theta, a)]$

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- Alg : $x \mapsto a$ and loss function $L : \theta \times a \mapsto \{0, 1\}$
 - Decision rule sees x and outputs some action a
 - Loss function evaluates how good a is on θ
- Risk = $E_{\theta \sim w}[E_{x \sim \mathcal{P}_\theta} L(\theta, \text{Alg}(x))]$
- Bayes risk is the minimum possible risk
- Null-risk is $\inf_a E_{\theta \sim w}[L(\theta, a)]$

Bayes Risk Lower Bounds

- Intuitively, if $I(\mathcal{P}, w)$ is small, then Bayes risk should be close to Null risk

$$\text{Bayes Risk} \geq 1 + \frac{I(\mathcal{P}, w) + \log(1 + R_0)}{\log(1 - R_0)}$$