# A New Parameter and Our Result

- $c_{f,S} \leq c_f[S] \leq S \cdot c_{f,S}$

- For $f(y) = y^p$, $c_f[s] = s^{p-1} = c_{f,s}$

- **Theorem:** Given a super-additive function $f$ that is "approximately invertible", there is a **two round** protocol using $O(c_f[s] \cdot \text{polylog}(n)/\varepsilon^2)$ bits of communication to approximate $\sum_i f(x_i)$ up to $1 \pm \varepsilon$

- $\Omega(c_f[s]/\varepsilon^2)$ **lower bound** for a restricted class of functions

- Suggests $c_f[s]$ captures the complexity better

19

$$f(y_1 + \cdots + y_s) \le \frac{c_f[s]}{s}\left(\sqrt{f(y_1)} + \cdots + \sqrt{f(y_s)}\right)^2$$

$$\boxed{f(y_1 + \cdots + y_s) \leq c_{f,s}(f(y_1) + \cdots + f(y_s)) \text{ for all } y_i}$$

# A New Parameter and Our Result

$$f(y_1 + \cdots + y_s) \le \frac{c_f[s]}{s}\left(\sqrt{f(y_1)} + \cdots + \sqrt{f(y_s)}\right)^2$$

- $c_{f,s} \le c_f[s] \le s \cdot c_{f,s}$
- For $f(y) = y^p$, $c_f[s] = s^{p-1} = c_{f,s}$
- **Theorem:** Given a super-additive function $f$ that is "approximately invertible", there is a **two round** protocol using $O(c_f[s] \cdot \text{polylog}(n)/\varepsilon^2)$ bits of communication to approximate $\sum_i f(x_i)$ up to $1 \pm \varepsilon$
- $\Omega(c_f[s]/\varepsilon^2)$ **lower bound** for a restricted class of functions
  - Suggests $c_f[s]$ captures the complexity better

# Key Observations