Proof Ideas

- Suffices to produce a distribution of instances for which there is no deterministic algorithm

- Sample $u, v \in \mathbb{R}^n$ and $G \in \mathbb{R}^{n \times n}$ all with independent Gaussian coordinates

$$A = \frac{\alpha}{\sqrt{n}} u v^T + G$$

- $\|G\|_2 \leq 2\sqrt{n}$

- If α large, say > 10, algorithm must approximate u and v to output LRA

What can a deterministic algorithm do?

- Starts with deterministic matrices $S_1^{(1)}, \ldots S_t^{(1)}$ and obtains

$$\langle \text{vec}(S_i^{(1)}), (\alpha/\sqrt{n}) \cdot u \otimes v \rangle + \langle \text{vec}(S_i^{(1)}), \text{vec}(G) \rangle$$

- Based on responses, pick $S_1^{(2)}, \ldots, S_t^{(2)}$ and so on
- Assume $\text{vec}(S_i^{(j)})$ are orthonormal w.l.o.g
- Are first round responses enough to pick good measurements in second round?