Bayes Risk Lower Bounds

- Decision rule $\mathfrak{d} : x \mapsto a$ and loss function $L : \theta \times a \mapsto \{0, 1\}$

- Decision rule sees x and outputs some action a

- Loss function evaluates how good a is on θ

- Risk = $E_{\theta \sim w}[E_{x \sim \mathscr{P}_\theta} L(\theta, \mathfrak{d}(x))]$

- Bayes risk is the minimum possible risk

- Null-risk is $\inf_{a} E_{\theta \sim w}[L(\theta, a)]$

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Bayes Risk Lower Bounds

- Intuitively, if $I(\mathscr{P}, w)$ is small, then Bayes risk should be close to Null risk

$$\text{Bayes Risk} \geq 1 + \frac{I(\mathscr{P}, w) + \log(1 + R_0)}{\log(1 - R_0)}$$