Intuition about Bayes Risk

- All $\mathscr{P}_\theta$ are the same $\Longrightarrow$ Can't say anything about θ from x
  - Need to capture how different $\mathscr{P}_\theta$ are
- One way to capture:

$$I(\mathscr{P}, w) = \inf_Q E_{\theta \sim w}[d_{\mathsf{KL}}(\mathscr{P}_\theta \,\|\, Q)]$$

- If all $\mathscr{P}_\theta$ are "close" to some Q, then $I(\mathscr{P}, w)$ is "small"

Bayes Risk Lower Bounds

- Decision rule $\mathfrak{d} : x \mapsto a$ and loss function $L : \theta \times a \mapsto \{0, 1\}$
    - Decision rule sees x and outputs some action a
    - Loss function evaluates how good a is on θ
- Risk = $E_{\theta \sim w}[E_{x \sim \mathscr{P}_\theta} L(\theta, \mathfrak{d}(x))]$
- Bayes risk is the minimum possible risk
- Null-risk is $\inf_a E_{\theta \sim w}[L(\theta, a)]$