After projecting:- First consider only 1 direction.

$\tilde{\mu}_i = v^T \mu_i$

& $\tilde{y}_i = v^T \tilde{x}_i$ where $\tilde{x}_i = x_i - \mu_i$

then scatter in-between (covariance matrix). $(\tilde{S}_B)$ $= \sum_{i=0}^{c} (\tilde{\mu}_i - \tilde{\mu})(\tilde{\mu}_i - \tilde{\mu})^T N_i$

within class Scatter matrix

$$\tilde{S}_w = \sum_{i=1}^{c} \sum_{x_k \in X_i} (\tilde{x}_k - \tilde{\mu}_i)(\tilde{x}_k - \tilde{\mu}_i)^T$$

~~Scatter~~

$$e = \frac{\sum_{i=0}^{c} v^T(\mu_i - \mu)(\mu_i - \mu)^T v \; N_i}{\sum_{i=1}^{c} \sum_{x_k \in X_i} v^T (x_k - \mu_i)(x_k - \mu_i) v}$$

$e = \frac{e_1}{e_2}$

~~$\frac{de_1}{dv} = v^T(\mu_i - \mu)(\mu_i - \mu)^T v$~~

$$e = \frac{v^T S_B v}{v^T S_W v}$$

$$\frac{de}{dv} = 0 \Rightarrow \frac{\left(\frac{d}{dv} v^T S_B v\right) v^T S_W v - \left(\frac{d}{dv} v^t S_W v\right) v^t S_B v}{(v^t S_W v)^2} = 0$$

$\Rightarrow (v^T S_W v)(S_B v) - (S_W v)(v^T S_B v)$

$$S_B v = \underbrace{\left(\frac{v^T S_B v}{v^T S_W v}\right)}_{\lambda} (S_W v)$$

$S_B v = \lambda S_W v$ → Maximise $\lambda$.